The Young Person's Guide to

Neutrality, Price Level Indeterminacy, Interest Rate Pegs, and Fiscal Theories of the Price Level[1]

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ABSTRACT

The paper establishes the following:

First, money is *neutral* even if there is a is a non-zero stock of non-monetary nominal public debt, because the government adjust real taxes to satisfy its intertemporal budget constraint.

Second, Woodford's fiscal theory of the price level, according to which for certain fiscal rules the (initial) price level is independent of the nominal money stock, is invalid because it represents a 'solution' to an ill-posed general equilibrium problem. It combines an overdetermined fiscal-financial programme with an unwarranted weakening of the government's intertemporal budget constraint, requiring it to hold only in equilibrium, and only for arbitrarily restricted configurations of public spending, taxes and initial debt stocks.

Third, there is price level determinacy under an exogenous nominal interest rate rule if the transactions technology has cash-in-advance features. The price level is *hysteretic* in this case.

Finally, it is not possible to draw inferences about the historical process of technological improvements in the transactions technology leading to a cashless economy, by studying the limiting behaviour, as a transactions efficiency index takes on successively higher values, of a sequence of histories, each one of which is indexed, for all time, by a *given* level of efficiency.

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I. Introduction

Two of the oldest questions in monetary economics concern the neutrality of money and the determinacy of the price level when the monetary authority pegs the nominal rate of interest (see e.g. Hume [1752], Fisher [1896, 1907, 1911, 1930], Wicksell [1989, 1907], Keynes [1923],Hayek [1931], Patinkin [1965], Patinkin and Steiger [1989], Tobin [1976], Lucas [1972], Brock [1974], Gale [1982], Sargent and Wallace [1982], Canzoneri, Henderson and Rogoff [1983], Grandmont [1983] and McCallum [1986, 1997a]). In a number of important and influential recent papers Woodford [1994, 1995, 1996, 1997], has revisited these venerable issues using modern dynamic optimising models. His investigations extended to two further issues that turn out to be intimately related to the earlier duo: fiscal theories of the price level (see also McCallum [1997b], Luttmer [1997] and Canzoneri, Cumby and Diba [1998]) and the determinacy of the price level when the efficiency of the transactions mechanism is enhanced to the point that money becomes redundant.

This paper demonstrates that all four issues can be analysed and resolved in a simple dynamic optimising model using only the two most basic tools of the trade. The first is verifying the model's equilibrium conditions for zero-degree homogeneity of the real endogenous variables and for first-degree homogeneity of the endogenous nominal prices in various exogenous nominal asset stocks[1]. The second is equation counting. Once the equilibrium conditions are written down in a transparent manner, all the results follow by direct inspection of these equilibrium conditions.

The government's intertemporal budget constraint plays a central role in resolving all four issues. This constraint requires that the initial value of the government's interest-bearing liabilities is equal to the present discounted value of its current and future primary (non-interest) budget surpluses plus the present discounted value of its current and future seigniorage (net issuance of non-interest-bearing 'base' money). Proper consideration of the government's intertemporal budget constraint leads to the conclusion Woodford's fiscal theory of the price level, according to which for certain fiscal rules the equilibrium price level sequence is independent of the nominal money stock sequence, is invalid, because it represents a 'solution' to an ill-posed general equilibrium problem. It also implies that there is no price level indeterminacy when the government pegs the nominal interest rate, if the transactions technology has 'cash-in-advance' features, although the equilibrium price level will be hysteretic, or history-dependent, in this case.

The vehicle for analysing these issues in this paper is a simple endowment economy [2] in which money plays a transactions role. The government's real exhaustive spending programme is exogenously given. So is either the sequence of nominal money stocks or the sequence of nominal interest rates. Initial stocks of its monetary and non-monetary liabilities are predetermined. Current and future taxes are constrained by the requirement that the government's intertemporal budget constraint or solvency constraint must always be satisfied. The medium of exchange or transactions medium is assumed to be the non-interest-bearing financial liability of the government, *fiat* money. No attempt is made to derive this identification of the transactions medium and a specific government financial liability from deep transactions-microeconomic first principles. The interesting questions all arise in the context of non-negative nominal interest rate equilibria, in which money is (weakly) dominated as a store of value and is held only to the extent that it facilitates exchange.

In addition to emphasising the importance of the public and private sector balance sheets, the paper also questions the relevance for monetary policy of arguments based on the study of the limiting behaviour of economies, when some parameter indexing the efficiency of the transactions mechanism increases without bound. Woodford [1997] has considered under what conditions the equilibrium price sequence of such economies converges to an equilibrium price sequence of the cashless economy. I argue that for policy purposes it is more relevant to consider improvements in transactions efficiency occurring in real (calendar) time in a single economy, rather than a comparison of a sequence of economies, each of which is endowed with a fixed transactions efficiency.

II. The Model

To create the most transparent benchmark, I choose a model without private sector nominal rigidities of any kind. Clearly, in a model with nominal price stickiness, the pre-determined initial price level would automatically ensure price level determinacy[3]. The model also exhibits first-order debt neutrality. If the government and private sectors satisfy their solvency constraints, then for any given sequence of government spending and nominal money stocks, the substitution of lump-sum taxes financing for bond financing leaves all equilibrium real and nominal variables unchanged.

II.1 The Private Sector

A representative infinite-lived competitive consumer has preferences over consumption paths represented by [4]

$$u_t = \sum_{j=0}^{\infty} \frac{1}{1-\epsilon} c_{t+j}^{1-\epsilon} \left(\frac{1}{1+\delta} \right)^j \qquad \delta,\ \epsilon > 0,\ c \geq 0\ , \qquad \textbf{(1)}$$

where c_t is private consumption in period t. There are three stores of value available to the household: *(i)* non-interest-bearing money, which is a liability of the government, and serves as the numeraire; *(ii)* one-period nominal pure discount debt, with a single payment on maturity of one unit of money; and *(iii)* one-period real or index-linked debt, with a payment on maturity of one unit of output. There is a single homogeneous non-storable commodity which can be used for private consumption and public consumption. The representative consumer receives an exogenous endowment $y_t > 0$ of the commodity each period t. Households pay a lump-sum tax whose real value in period t is τ_t.

Money is a non-interest-bearing liability of the government and is costless to produce. It does not yield any direct utility nor is it a productive input in the production process. It does provide indirect utility, however, through a real resource-saving role in the transactions process. Real resources expended in transforming disposable income net of household saving, e_t (henceforth *consumer expenditure*), into consumer goods ready for actual consumption by households, c_t (henceforth *consumption*), are a decreasing function of the *beginning-of-period* real stock of money balances, M_t/P_t, where M_t is the nominal stock of money at the beginning of period t and P_t the period t price level (see Feenstra [1986] and McCallum [1986]). The use of the beginning-of period money stock in the shopping function gives it a cash-in-advance flavour. For simplicity a constant returns to scale Cobb-Douglas specification is adopted for the shopping function[5].

$$e_t = \left(1 + \eta\left(\frac{c_t}{M_t/P_t}\right)^{\beta}\right)c_t \qquad \eta \geq 0,\ \beta > 0 \tag{2}$$

As η decreases, the efficiency of the monetary transactions process improves. In the limit as $\eta \to 0$, the transactions technology improves to the point that money becomes redundant as a transactions medium.

Most of the substantive conclusions of the paper would be unaffected if I had adopted a money-in-the-direct-utility-function approach or a cash-in-advance approach instead. However, the model's price level (in)determinacy properties under a nominal interest rate peg *are* affected if an alternative ('cash-in-arrears') specification of the shopping function is adopted which allows the *end-of-period* money stock in period t to facilitate transactions during period t. Equation (2) is then replaced by

$$e_t = \left(1 + \eta\left(\frac{c_t}{M_{t+1}/P_t}\right)^{\beta}\right)c_t \qquad \eta \geq 0,\ \beta > 0 \tag{3}$$

Unless stated otherwise, the shopping function specification of equation (2) will be used.

The household budget identity constraint is

$$M_{t+1} - M_t + \left(\frac{1}{1+i_{t+1}}\right)B_{t+1} - B_t + P_t[\left(\frac{1}{1+r_{t+1}}\right)b_{t+1} - b_t] \equiv P_t(y_t - \tau_t - e_t)$$
$$e_t,\ M_t \geq 0 \tag{4}$$

where B_t is the number of one-period nominal bonds, b_t the number of one-period index-linked bonds held at the beginning of period t, i_{t+1} is the one-period nominal interest rate between periods t and $t+1$ and r_{t+1} the one-period real rate of interest.

With efficient financial markets and in the absence of uncertainty, expected and actual pecuniary rates of return on the two non-monetary assets are equalized. Thus

$$\frac{P_{t+1}}{P_t}(1 + r_{t+1}) = 1 + i_{t+1} \tag{5}$$

The nominal market value of total private financial wealth at the beginning of period *t+1* is

$$A_{t+1} \equiv M_{t+1} + B_{t+1} + P_{t+1} b_{t+1} \qquad \textbf{(6)}$$

The household budget identity can be rewritten as

$$A_{t+1} \equiv (1 + i_{t+1})[A_t + P_t(y_t - e_t - \tau_t)] - i_{t+1} M_{t+1} \qquad \textbf{(7)}$$

The household solvency constraint is [6]:

$$lim_{\ell \to \infty} \prod_{j=1}^{\ell} \left(\frac{1}{1 + i_{t+j}} \right) A_{t+\ell} \equiv 0$$

or **(8)**

$$A_t \equiv \sum_{\ell=0}^{\infty} \prod_{j=1}^{\ell} \left(\frac{1}{1+i_{t+j}} \right) \left[P_{t+\ell}(e_{t+\ell} + \tau_{t+\ell} - y_{t+\ell}) + \frac{i_{t+\ell+1}}{1+i_{t+\ell+1}} M_{t+\ell+1} \right]$$

It is important to note, anticipating the later discussion of the so-called fiscal theories of the price level, that the household solvency constraint (8) has to hold for all sequences of prices, interest rates and wages, and for all initial values of financial wealth, A_t and not just for equilibrium sequences of these variable; it is an identity, not merely an equilibrium condition.

The solution to the household optimisation problem includes the following first-order conditions (M^d denotes the nominal demand for money):

$$\left(\frac{c_{t+1}}{c_t}\right)^{\epsilon} = \left(\frac{1 + r_{t+1}}{1 + \delta}\right)\left(\frac{1 + \eta^{\frac{1}{1+\beta}}(1 + \beta)\left(\frac{i_t}{\beta}\right)^{\frac{\beta}{1+\beta}}}{1 + \eta^{\frac{1}{1+\beta}}(1 + \beta)\left(\frac{i_{t+1}}{\beta}\right)^{\frac{\beta}{1+\beta}}}\right) \tag{9}$$

$$\frac{M_t^d}{P_t} = c_t\left(\frac{\eta\beta}{i_t}\right)^{\frac{1}{1+\beta}} \tag{10}$$

Equation (9) is the familiar consumption Euler equation, modified to reflect money's use as an intermediate input in the household production function. Equation (10) shows that optimal real money balances are proportional to private consumption and vary inversely with the short nominal rate of interest.

If the shopping function with the end-of-period money stock given in (3) had been adopted instead, the Euler equation for consumption (9) and the money demand function (10) would have to be replaced with (11) and (12) respectively.

$$\left(\frac{c_{t+1}}{c_t}\right)^{\epsilon} = \left(\frac{1 + r_{t+1}}{1 + \delta}\right)\left(\frac{1 + \eta^{\frac{1}{1+\beta}}(1 + \beta)\left(\frac{i_{t+1}}{\beta(1+i_{t+1})}\right)^{\frac{\beta}{1+\beta}}}{1 + \eta^{\frac{1}{1+\beta}}(1 + \beta)\left(\frac{i_{t+2}}{\beta(1+i_{t+2})}\right)^{\frac{\beta}{1+\beta}}}\right) \tag{11}$$

$$\frac{M_{t+1}^d}{P_t} = c_t\left(\frac{\eta\beta(1+i_{t+1})}{i_{t+1}}\right)^{\frac{1}{1+\beta}} \tag{12}$$

Equations (3), (11) and (12) only play a role in the discussions of the fiscal theories of the price level in Section IV and of price level determinacy under a nominal interest rate peg in Section VI.1.

II.2 The government sector.

I view the government as a consolidated general government and central bank. Its budget identity is.

$$M_{t+1} - M_t + \frac{1}{1+i_{t+1}}B_{t+1} - B_t + P_t(\frac{1}{1+r_{t+1}}b_{t+1} - b_t) \equiv P_t(g_t - \tau_t) \qquad \textbf{(13)}$$

where real exhaustive public spending, public consumption spending, $g_t \geq 0$, is exogenous.

The nominal market value of the government's financial liabilities, including the stock of base money, at the beginning of period *t+1* is.

$$D_{t+1} \equiv M_{t+1} + B_{t+1} + P_{t+1}b_{t+1} \qquad \textbf{(14)}$$

The government's budget identity can be rewritten as

$$D_{t+1} \equiv (1 + i_{t+1})[D_t + P_t(g_t - \tau_t)] - i_{t+1}M_{t+1} \qquad \textbf{(15)}$$

The government's fiscal-financial-monetary programme satisfies the no-Ponzi finance condition or solvency constraint

$$lim_{j\to\infty}\prod_{\ell=1}^{j}\left(\frac{1}{1 + i_{t+\ell}}\right)D_{t+j} \equiv 0$$

$$or \tag{16}$$

$$D_t \equiv \sum_{\ell=0}^{\infty}\prod_{j=1}^{\ell}\left(\frac{1}{1+i_{t+j}}\right)\left[P_{t+\ell}(\tau_{t+\ell} - g_{t+\ell}) + \frac{i_{t+\ell+1}}{1+i_{t+\ell+1}}M_{t+\ell+1}\right]$$

Note again that the intertemporal budget constraint or solvency constraint of the government is required to hold for all initial debt stocks, D_t, and for all sequences of government spending, nominal money stocks, nominal interest rates and price levels, and not merely for equilibrium sequences. It is an identity, not just an equilibrium condition. This means that the sequence of lump-sum taxes is residually determined to satisfy the solvency constraint (both when the nominal money stock sequence is exogenous and when the nominal interest rate sequence is exogenous). Abandoning this fundamental requirement for a well-posed general equilibrium produces Woodford's fiscal theory of the price level.

When the government follows a *monetary rule*, it specifies a sequence of positive nominal money stocks, taking the initial values of the outstanding stocks of money, M_1, and its two debt instruments, B_1 and b_1 as given. The choice of financing between taxes and issuance of its three debt instruments is arbitrary, as long as the solvency constraint (19) is satisfied.

Taxes are the residual financing instrument, in the sense that, with exogenous public spending and monetary financing (or alternatively with government spending and the sequence of nominal interest rates given), the government chooses (a rule for) its sequence of current and future taxes which ensures that the present discounted value of current and future taxes satisfies its intertemporal budget constraint.[7] With lump-sum taxes and the representative agent set-up, the timing of taxes is irrelevant. Only their present discounted value matters, and this will therefore be an endogenous variable in the model. One example of a tax rule that satisfies the government's solvency constraint is the rule that keeps the nominal value of the government debt constant each period, that is, for $t \geq 1$

$$D_{t+1} = D_t = \bar{D}$$

or

$$\tau_t = g_t + \frac{i_{t+1}}{1+i_{t+1}}\left(\frac{\bar{D} - M_{t+1}}{P_t}\right) \tag{17}$$

In what follows, whenever taxes are taken to be residually determined to satisfy the government solvency constraint, this balanced budget rule can be assumed to hold, without loss of generality.

For simplicity the monetary rule is a fixed, non-contingent, or 'open-loop' rule, although this paper's results go through for a wide range of contingent rules, feedback rules or closed-loop rules.

When the government follows a *nominal interest rate rule*, it specifies a sequence for the current and future nominal interest rate. For simplicity, the nominal interest rule is a fixed, non-contingent or 'open-loop' rule, unless otherwise specified. The nominal interest rates set by the monetary authority are non-negative in each period. In any given period, the government still takes as given (or pre-determined) the stocks of all monetary and non-monetary government liabilities issued the previous period and carried into the current period by the private sector. Again, the present discounted value of current and future taxes is endogenously determined by the requirement that the government satisfy its intertemporal budget constraint for all price, interest rate and money stock sequences and for all initial stocks of public debt.

II.3 The aggregate resource constraint

The economy's aggregate resource constraint is

$$y_t = e_t + g_t \qquad \textbf{(18)}$$

II.4 Equilibrium.

Let $t = 1$ be the initial date. The economy has no terminal date, so the equilibrium conditions (19) to (23) hold for $t = 1,2,...$. Under both a monetary rule and a nominal interest rate rule, equilibrium is characterized by

$$\left(\frac{c_{t+1}}{c_t}\right)^{\epsilon} = \left(\frac{1 + r_{t+1}}{1 + \delta}\right)\left(\frac{1 + \eta^{\frac{1}{1+\beta}}(1 + \beta)\left(\frac{i_t}{\beta}\right)^{\frac{\beta}{1+\beta}}}{1 + \eta^{\frac{1}{1+\beta}}(1 + \beta)\left(\frac{i_{t+1}}{\beta}\right)^{\frac{\beta}{1+\beta}}}\right) \qquad \textbf{(19)}$$

$$\frac{M_t}{P_t} \geq c_t\left(\frac{\eta\beta}{i_t}\right)^{\frac{1}{1+\beta}} \qquad \textbf{(20)}$$

(where (20) holds with strict equality when $\eta, > 0$).

$$e_t = c_t\left(1 + \eta^{\frac{1}{1+\beta}}\left(\frac{i_t}{\beta}\right)^{\frac{\beta}{1+\beta}}\right) \qquad \textbf{(21)}$$

$$1 + r_{t+1} = (1 + i_{t+1})\frac{P_t}{P_{t+1}} \qquad \textbf{(22)}$$

$$y_t = e_t + g_t \qquad \textbf{(23)}$$

$$\frac{M_1 + B_1}{P_1} + b_1 = \sum_{t=1}^{\infty}\prod_{j=2}^{t}\left(\frac{1}{1 + r_j}\right)[\tau_t - g_t + \left(\frac{i_{t+1}}{1+i_{t+1}}\right)\frac{M_{t+1}}{P_t}] \qquad \textbf{(24)}$$

III. Monetary Rules: the Conditional and Unconditional Neutrality of Money and Outside Nominal Assets

In this section $\eta > 0$ and money is useful as a transactions medium. When the government follows a monetary rule, the equilibrium conditions (19) to (24) are complemented by the following initial conditions for the government's financial liabilities:

$$M_1 = \bar{M}_1 > 0 \ ; \quad B_1 = \bar{B}_1 \ ; \quad b_1 = \bar{b}_1 \qquad \textbf{(25)}$$

Given $\{g_t , M_{t+1}\}$; $t = 1, 2, ...$, the values of $\{c_t , e_t , i_{t+1} , r_{t+1}, P_t \}$; $t = 1, 2, ...$, and

$\sum_{t=1}^{\infty} \prod_{j=2}^{t} \tau_t/(1 + r_j)$ are determined in equilibrium.

Without loss of generality, I can restrict the permissible sequences of the nominal money stock to those supporting equilibria with non-negative short nominal interest rates. Negative nominal interest rates would be inconsistent with equilibrium, as this would create arbitrage opportunities for households who would borrow at the negative nominal rate from the government and invest these loans in zero nominal interest rate money.

The equilibrium conditions determine only the present discounted value of the government's current and future taxes. The timing of the tax payments, and any associated variations in the sequences of the three debt instruments after period *1*, are of no significance for either real or nominal equilibrium values. This is not surprising, in this representative agent model with its first-order debt neutrality.

I now introduce the concepts of conditional and unconditional neutrality.

Definition 1: Unconditional Neutrality.

A set of assets is (jointly) unconditionally neutral if the same proportional change in the initial and all future values of each stock leaves real variables <u>including real taxes and real government debt</u> unchanged and raises all nominal prices by the same proportion.

Definition 2: Conditional Neutrality.

A set of assets is (jointly) conditionally neutral if the same proportional changes in the initial and all future values of each stock leaves real variables <u>other than real taxes and real government debt</u> unchanged and raises all nominal prices by the same proportion, but *requires a change in real taxes for the government to satisfy its intertemporal budget constraint because the change in the price levels alters the real value of the government's debt..*

The following propositions follow immediately.

Proposition 1.

The 'outside'[8] nominal assets (money and all nominally denominated public debt instruments) are jointly unconditionally neutral.

Proof: Proposition 1 states that M and B are jointly unconditionally neutral. The proof is trivial. Multiply M_t , $t = 1, 2,...$, B_1 ,, P_t , $t = 1, 2,...$ by the same constant $1+ \mu > 0$. All real equilibrium values $\{c_t , e_t , i_{t+1} , r_{t+1} \}$, $t = 1, 2,$ and $\sum_{t=1}^{\infty} \prod_{j=2}^{t} \tau_t /(1 + r_j)$ are unchanged[9]□.

Proposition 2.

Money is conditionally neutral but not unconditionally neutral unless the initial value of the non-monetary nominal public debt instruments equals zero.

Proof: The proof of Proposition 2 is also straightforward. The public debt stocks, including the nominally denominated public debt stocks, do not affect the determination of the equilibrium values of $\{c_t , e_t , i_{t+1} , r_{t+1} \}$, $t = 1, 2, ...$. From (20), holding with equality, the price level in each period changes by the same proportion as the money stock. Consider the government's intertemporal budget constraint for period 1, (24). P_t changes by the same proportion as M_t and nominal and real interest rates are unchanged. Thus, to continue to satisfy the government's intertemporal budget constraint (24), the present discounted value of current and future taxes will have to change according to (26). Any change the sequence of current and future taxes satisfying (26) will ensure (conditional) neutrality of money. Note that (17) satisfies (26).

$$-\left(\frac{B_1}{P_1}\right)\frac{dP_1}{P_1} = \sum_{t=1}^{\infty}\prod_{j=2}^{t}\left(\frac{1}{1 + r_j}\right)d\tau_t \tag{26}$$

□

The intuition is clear. Consider the case where current and future nominal money stocks alone increase by a proportion, with all other outside nominal assets stocks and payment streams constant. Current and future prices increase by that same proportion. If the initial value of the nominal public debt instruments outstanding is positive, the increase in the price level reduces their real value. The present discounted value of future taxes has to fall by the same amount if the government is to continue to satisfy its intertemporal budget constraint. If the initial value of the nominal debt instruments is negative, the present discounted value of taxes must fall.

If instead of a representative agent model of household behaviour we had adopted an overlapping generations model, any change in the present discounted value of current and future taxes would alter the real equilibrium of the economy, to the extent that it redistributes wealth between generations. Such intergenerational distributional effects can be neutralized if the government has unrestricted age-specific lump-sum taxes and transfers at its disposal (see e.g. Buiter and Kletzer [1997]). In a two-period OLG model, versions of Propositions 1 and 2 hold if the government can tax the young and transfer the proceeds to the old.

An implication of Propositions 1 or 2 is that if the only government debt is index-linked debt, money will be unconditionally neutral.

Finally, he neutrality propositions of this section go through unchanged if the 'cash-in-arrears' specification of the transactions technology given in equation (3) is substituted for the 'cash-in-advance' specification of equation (2).

IV. A Fiscal Theory of the Price Level?

Woodford's fiscal theory of the price level (Woodford [1995]) states that, for certain fiscal rules, the equilibrium price level sequence is independent of the sequence of nominal money stocks. This section demonstrates that this theory is the result of two complementary errors, resulting in an ill-posed general equilibrium model.

The first error is the specification of an overdetermined fiscal-financial programme: for given initial stocks of the government debt instruments: both the sequence of real public spending and the sequence of real taxes net of the real value of new monetary issues are given exogenously. In general, the real public debt sequence then becomes non-stationary, and the government solvency constraint need not be satisfied.

The second error is an unwarranted change in the assumptions about when the government solvency constraint applies. Woodford no longer requires that the government solvency constraint hold for all sequences of price levels and interest rates. Instead he requires only that the solvency constraint hold *in equilibrium*, that is, for equilibrium sequences of prices and other endogenous variables. In addition, he imposes an arbitrary restriction on the permissible configurations of the exogenous public spending and revenue sequences and the predetermined initial stock of non-monetary nominal debt. This relaxations violate the normal rules for constructing a well-posed general equilibrium model.

Household and government decision rules, whether derived from optimising behaviour, as is the case for households in this model, or imposed in an ad-hoc manner, as is the case for the government in this example, are constrained by intertemporal budget constraints that must hold for all price sequences (and other sequences of endogenous variables) and for all initial non-monetary debt stocks. These decision rules, derived for arbitrary price sequences, then determine, jointly with the market-clearing conditions, initial conditions and other system-wide constraints, the equilibrium sequences of prices and other endogenous variables. The budget constraints must be satisfied, however, both for equilibrium and out-of-equilibrium sequences of the endogenous variables in order for these budget constraints to co-determine these equilibrium sequences.

It is instructive to reproduce Woodford's argument in some detail. I will develop the fiscal approach when the government follows a monetary rule (as in Woodford [1995]). As shown below, the fiscal approach can never be valid when the transactions technology has cash-in-advance features (as in equation (2)), but is internally consistent, albeit ill-posed, when the transactions technology has cash-in-arrears features (as in equation (3). Exactly the same strictures apply when the government follows an exogenous nominal interest rate rule.

Let $\tilde{D}$ denote the nominal value of the non-monetary debt of the government, that is

$$\tilde{D}_t \equiv D_t - M_t \qquad \textbf{(27)}$$

The government budget identity can be rewritten as

$$\tilde{D}_{t+1} \equiv (1+i_{t+1})[\tilde{D}_t + P_t(g_t - \tau_t) - (M_{t+1} - M_t)] \qquad \textbf{(28)}$$

As in the previous sub-section, the government spending sequence is exogenous, the initial stocks of the government's non-monetary debt instruments are given, and the sequence of nominal money stocks is exogenous[10]. A key change in assumptions is that the sequence of real taxes now consists of an exogenous component, $\bar{\tau}_t$, minus a component that exactly offsets the seigniorage revenue of the government, that is

$$\tau_t = \bar{\tau}_t - \frac{M_{t+1} - M_t}{P_t} \tag{29}$$

Given the tax rule (36), the government's budget identity becomes

$$\tilde{D}_{t+1} \equiv (1+i_{t+1})[\tilde{D}_t + P_t(g_t - \bar{\tau}_t)] \tag{30}$$

With positive nominal interest rates, the process in (30) will in general be non-stationary. This does not necessarily mean that the government's solvency constraint will be violated: the growth rate of the non-monetary debt could, even in the long run, be less than the rate of interest even if the sequence $\{g_t - \bar{\tau}_t\}$ is exogenous, as long as the present discounted value of the sequence of government primary surpluses net of seigniorage revenues $\{P_t(\bar{\tau}_t - g_t)\}$ is at least equal to the value of the initial stock of non-monetary public debt. The intertemporal budget constraint of the government can be rewritten as in (31). Following Woodford, it is only required to hold *in equilibrium*.

$$\frac{\tilde{D}_t}{P_t} = \frac{B_t}{P_t} + b_t = \sum_{\ell=0}^{\infty} \prod_{j=1}^{\ell} \left(\frac{1}{1 + r_{t+j}} \right) [\bar{\tau}_{t+\ell} - g_{t+\ell}] \tag{31}$$

The equilibrium is characterised by the initial values of all nominal and real asset stocks, equations (19), (20) (holding with equality), (22) and

$$y_t = c_t\left(1 + \eta^{\frac{1}{1+\beta}}\left(\frac{i_t}{\beta}\right)^{\frac{\beta}{1+\beta}}\right) + g_t \qquad \textbf{(32)}$$

$$B_{t+1} + P_{t+1}b_{t+1} \equiv (1 + i_{t+1})[B_t + P_t b_t + P_t(g_t - \bar{\tau}_t)] \qquad \textbf{(33)}$$

$$\frac{B_t}{P_t} = \sum_{\ell=0}^{\infty}\prod_{j=1}^{\ell}\left(\frac{1}{1 + r_{t+j}}\right)[\bar{\tau}_{t+\ell} - g_{t+\ell}] - b_t \qquad \textbf{(34)}$$

There is a unique value of the initial price level, P_1 that permits the government solvency constraint (34) to be satisfied (Woodford [1995]). With the transactions technology used in this paper (both for the cash-in-advance and cash-in-arrears versions), money and consumption are strongly non-separable, as is evident from the appearance of the nominal interest rate in the resource constraint (32). This means that even in the simplest case, where the endowment sequence $\{y_t\}$, the real public spending sequence $\{g_t\}$, the sequence of real taxes minus real seigniorage $\{\bar{\tau}_t\}$, the sequence of index-linked debt $\{b_t\}$ and the nominal money stock sequence $\{M_t\}$ are constant, there does not in general exist a stationary solution for the model's real (or nominal) equilibrium variables. From (32), constant consumption requires a constant nominal interest rate. From (32) and (20), holding with equality, a constant nominal interest rate requires a constant stock of real money balances. With a constant nominal money stock, this requires a constant price level. The solvency constraint reduces in this case to

$$\frac{B_t}{P_t} = \left(\frac{1+\rho}{\rho}\right)(\bar{\tau} - g) - b \qquad \textbf{(35)}$$

so the nominal debt stock would have to be constant, which would only be true for a very restricted set of initial conditions and parameter values.

With exogenously given sequences of government spending and taxes, the solvency constraint will in general be violated, unless there is an initial non-zero stock of non-monetary nominal debt, whose real value can be equated to the present value of future primary surpluses minus the value of the initial stock of index-linked public debt, through an appropriate assignment of the initial price level.

Note that for this approach to work, the following arbitrary restriction on the fiscal-financial programme must be satisfied for all $t \geq 1$,

$$sgn\{B_t\} = sgn\{\sum_{j=0}^{\infty}\prod_{\ell=1}^{j}\left(\frac{1}{1+r_\ell}\right)(\bar{\tau}_{t+j} - g_{t+j}) - b_t\} \qquad \textbf{(36)}$$

or, in the case of a stationary environment,

$$sgn\{B_t\} = sgn\{\left(\frac{1+r}{r}\right)(\bar{\tau} - g) - b\} \qquad \textbf{(37)}$$

The necessity of condition (36) (or, in the case of a stationary environment (37) is obvious from (34) and non-negativity of the price level. For instance, if there is a positive stock of non-monetary nominal debt outstanding, the present value of current and future primary surpluses (net of seigniorage) minus the value of the outstanding stock of index-linked debt must be positive, for it to be possible to determine the initial price level from an equation such as ((34) or (35). The condition given in (36) (or, in a stationary environment

(37)) is an arbitrary restriction on the fiscal-financial programme of the government. Governments could e.g. have index-linked debt in excess of the present value of their future primary surpluses (the right-hand side of (36) is negative, and have a positive stock of nominal non-monetary debt as well. Or the government could be 'super-solvent', with a negative stock of nominal non-monetary debt and a positive excess of the present discounted value of their primary surpluses net of seigniorage over the value of their outstanding stock of index-linked debt. It also is immediately apparent, that the fiscal theory of the price level dissolves if the government only has index-linked debt[11]: with $B_t = 0$, condition (36) or (37) are almost surely violated.

The fiscal theory of the price level also fails if government spending and tax sequences are set in nominal terms and there is no index-linked public debt. Let $\{ T_t \}$ be the sequence of nominal taxes, where $T_t \equiv \bar{T}_t - (M_{t+1} - M_t)$ and $\bar{T}_t$ is exogenous. Let $\{ \bar{G}_t \}$ be the exogenous sequence of nominal public spending. We assume that $b_t = 0$. The government solvency constraint becomes

$$B_t = \sum_{\ell=0}^{\infty} \prod_{j=1}^{\ell} \left(\frac{1}{1+i_{t+j}} \right) (\bar{T}_{t+\ell} - \bar{G}_{t+\ell}) \qquad \textbf{(38)}$$

and there is nothing in the government solvency constraint to pin down the price level. The theory therefore requires there to be nominal non-monetary debt and index-linked tax or spending sequences or index-linked debt. If public spending and taxes were set in nominal terms, the price level could again be determinate if there were an non-zero outstanding stock of index-linked bonds. In that case, condition (39), analogous to (36) would have to be satisfied

$$sgn\{b_t\} = sgn\{\sum_{j=0}^{\infty} \prod_{\ell=1}^{j} \left(\frac{1}{1 + r_\ell} \right) \left(\frac{\bar{T}_{t+j} - \bar{G}_{t+j}}{P_{t+j}} \right) - \frac{B_t}{P_t}\} \qquad \textbf{(39)}$$

With equation (34), and with condition (36) satisfied, we are as close as the model can get to a fiscal theory of the price level. With the transactions technology (2), which depends on the initial stock of nominal money balances, Woodford's theory faces the further problem that the monetary equilibrium condition for the initial period, depends on the predetermined nominal interest i_1 , which is inherited from period *0*.

$$\frac{M_1}{P_1} = c_1 \left(\frac{\eta\beta}{i_1} \right)^{\frac{1}{1+\beta}} \qquad \textbf{(40)}$$

It is therefore not true in this model that the price level sequence (or even just the initial price level) is determined without any reference to the nominal money stock sequence. If the price level were independent of M_1 a larger value of M_1 would, with i_1 predetermined, imply a higher value of period *1* consumption, c_1. From the resource constraint (32) we get a violation of one of the equilibrium conditions, if the original value of c_1 was consistent with equilibrium.[12]

If instead we adopt the cash-in-arrears technology (3), the resulting money demand function and resource constraint, given by equations (41) and (42), replacing (20) and (32) respectively, are more promising from the point of view of the fiscal theory of the price level

$$\frac{M_{t+1}}{P_t} = c_t \left(\frac{\eta\beta(1+i_{t+1})}{i_{t+1}} \right)^{\frac{1}{1+\beta}} \qquad \textbf{(41)}$$

$$y_t = c_t\left(1 + \eta^{\frac{1}{1+\beta}}\left(\frac{i_{t+1}}{\beta(1+i_{t+1})}\right)^{\frac{\beta}{1+\beta}}\right) + g_t \tag{42}$$

What we get, however, is at most a fiscal theory of the *initial* price level. If the real interest rate sequence depends on the nominal money stock sequence, it is clear from (34) that we don't have a fiscal theory of the initial price level, P_1. Assume the real interest rate sequence is independent of the nominal money stock sequence. In that case the initial price level, P_1, is independent of the nominal money stock sequence. From (41) and (42) it follows that i_2 and c_1 are functions of M_2. From the budget identity (33) it follows that B_2 depends on the nominal money stock sequence and thus *that* P_2 and all subsequent price levels depend on the money stock. By influencing the evolution of the nominal non-monetary debt (through its effect on the nominal interest rate), the money stock co-determines the equilibrium price sequence[13]. In fact, it is easily checked that the real interest rate sequence is not independent of the nominal money stock sequence[14]. The price level sequence, including the initial price level, therefore depends on the nominal money stock sequence.

The reason for this is that the transactions technology of this model incorporates a strong form of non-separability of money and consumption. This is reflected in the Euler equation for consumption , which shows that, out of steady state, consumption growth depends both on the real and on the nominal interest rates. In order to get an equilibrium in which the initial price level is independent of the money stock, either a strict cash-in-advance or an (end-of-period) money-in-the utility function approach with money and consumption entering separably. The following example, taken from McCallum [1997b] illustrates this.

Households optimise the objective functional given in (43) subject to the earlier household budget identity and solvency constraint. The transactions technology is dropped.

Real public spending, g, real taxes plus real seigniorage, $\bar{\tau}$, and the stock of index-linked debt, b, are constant.

$$u_t = \sum_{j=0}^{\infty} \left(\frac{1}{1-\epsilon} c_{t+j}^{1-\epsilon} + \phi \frac{1}{1-\epsilon} \left(\frac{M_{t+1}}{P_t}\right)^{1-\epsilon} \right) \left(\frac{1}{1+\delta} \right)^j \qquad \epsilon,\ \phi\ ,\ \delta > 0 \tag{43}$$

The optimal household programme is characterised by

$$\left(\frac{c_{t+1}}{c_t} \right)^{\epsilon} = \frac{1+r_{t+1}}{1+\delta} \tag{44}$$

and

$$\left(\frac{M_{t+1}}{P_t c_t} \right)^{\epsilon} = \phi \left(\frac{1+i_{t+1}}{i_{t+1}} \right) \tag{45}$$

Since $y = c + g$, the only equilibrium that does not violate the government solvency constraint is given by (46)

$$\left(\frac{M_{t+1}}{P_t (y - g)} \right)^{\epsilon} = \phi \left(\frac{1+i_{t+1}}{i_{t+1}} \right) \tag{46}$$

$$1 + \delta = 1 + r_{t+1} = (1 + i_{t+1}) \frac{P_t}{P_{t+1}} \tag{47}$$

$$\frac{B_t}{P_t} = \left(\frac{1+\delta}{\delta}\right)(\bar{\tau} - g) - b \qquad \textbf{(48)}$$

Here we have the pure fiscal theory of the *initial* price level[15]. The initial price level is uniquely determined by equation (48) for $t = 1$, and is proportional to the initial stock of nominal non-monetary debt (a strict quantity theory of nominal bonds). Given this initial price level, all future price levels are determined from (49) and

$$\left(\frac{M_{t+1}\left[\left(\frac{1+\delta}{\delta}\right)(\bar{\tau} - g) - b\right]}{B_t}\right)^{\epsilon} = \phi\left(\frac{(1+\delta)B_{t+1}/B_t}{(1+\delta)(B_{t+1}/B_t) - 1}\right)(y - g) \qquad \textbf{(49)}$$

Future price levels (for $t \geq 2$) are not independent of the sequence of nominal money stocks, because the nominal money stock sequence affects the nominal interest rate and thus the evolution of the future stock of nominal non-monetary debt. Given B_t (and thus P_t) a larger value of M_{t+1} implies a lower nominal interest rate and therefore, with a constant real interest rate, a lower future nominal debt stock, B_{t+1} and a lower future price level.

In summary, the flaws in the fiscal theory of the price level are that the government's fiscal-financial programme is overdetermined and that the government solvency constraint is required to hold only for equilibrium price level sequences rather than for all price sequences, and only for arbitrarily restricted configurations of public spending, revenues and initial debt stocks. The discussion also makes clear that even if one is willing to live with these flaws, the fiscal theory of the price level is only a purely fiscal theory of the *initial* price level. Even then further restrictions have to be imposed on the way money is introduced into the model.

A Household Debt Theory of the Price Level?

Woodford's approach can be applied to the private sector solvency constraint as well. Assume that government follows a fiscal-financial rule consistent with government solvency, say the balanced budget rule, given in (17), which keeps constant the nominal value of the public debt. The real government spending sequence and the nominal money stock sequence are exogenous as before. Now overdetermine the private sector's consumption and portfolio allocation programme by fixing exogenously the initial value of private consumption, $c_1 = \bar{c}_1 > 0$. The remaining conditions governing private sector behaviour remain in effect[16]. With initial consumption fixed at an arbitrary initial value, the private sector solvency constraint will not in general be satisfied. If we now also weaken the private sector solvency constraint by requiring it to hold only in equilibrium, there may exist a unique sequence of prices which allows the private sector to satisfy its solvency constraint. In a stationary environment that would be the price sequence that keeps the real value of private financial wealth constant. This price level sequence equates the real value of the private sector's nominal stocks of government liabilities with the present discounted value of the future primary private sector surpluses and the outstanding stock of private sector holdings of real public debt. That is, for all $t \geq 1$,

$$\frac{B_t}{P_t} = \sum_{\ell=0}^{\infty} \prod_{j=1}^{\ell} \left(\frac{1}{1+r_{t+j}} \right) \left[e_{t+\ell} + \tau_{t+\ell} - y_{t+\ell} + \left(\frac{i_{t+\ell+1}}{1+i_{t+\ell+1}} \right) \frac{M_{t+\ell+1}}{P_{t+\ell}} \right] - b_t \qquad \textbf{(50)}$$

Condition (51), analogous to (39) will have to be satisfied for this to be possible, that is, for all $t \geq 1$,

$$sgn\{B_t\} = sgn\{\sum_{\ell=0}^{\infty}\prod_{j=1}^{\ell}\left(\frac{1}{1+r_{t+j}}\right)\left[e_{t+\ell}+\tau_{t+\ell}-y_{t+\ell}+\left(\frac{i_{t+\ell+1}}{1+i_{t+\ell+1}}\right)\frac{M_{t+\ell+1}}{P_{t+\ell}}\right]-b_t\} \qquad \textbf{(51)}$$

For this ‘solution’ too, it remains true that two wrongs don’t make a right.

If it is correct that Woodford’s fiscal theory of the price level derives from an ill-posed general equilibrium construct, is there any point in testing its empirical implications, such as the relationship between the government’s non-monetary nominal debt and the general price level? Presumably, as long as there are no logical inconsistencies in the model, empirical testing would be interesting, because my rejection of Woodford’s approach is, ultimately, a rejection of some of his assumptions. These assumptions happen to be so fundamental to the construction of any general equilibrium model (and indeed to any model of rational choice subject to budget constraints), that I would not consider the fiscal approach to be a high-priority candidate for empirical verification or refutation. This view does not seem to be universally shared, however (see e.g. Canzoneri, Cumby and Diba [1998]).

It will be clear that Woodford’s fiscal theory of the price level is quite distinct from Sargent and Wallace’s [1981] fiscal theory of inflation. In their “Unpleasant Monetarist Arithmetic” paper, government policy follows two regimes. For a finite period of time, the growth rate of the nominal money stock is fixed exogenously. The primary deficit as a fractions of GDP is exogenous and constant throughout. During this interval, government borrowing is determined residually. Following the fixed interval, the ratio of non-monetary public debt to GDP is stabilized at the value achieved at the end of the interval. The central point is that, if the (exogenous) monetary growth rate is reduced temporarily without any change in the (exogenous) primary government deficit-GDP ratio, non-monetary public debt will accumulate at a faster rate for as long as the lower growth rate of nominal money is in effect. When following the fixed interval of lower monetary growth, the government's non-monetary debt-GDP ratio is stabilised, monetary growth is determined residually. If the real interest rate exceeds the growth rate of real GDP, the higher debt-GDP ratio reached at the end of the fixed interval of lower monetary growth, implies a higher subsequent growth rate of nominal money and a higher rate of inflation. When velocity is a function of the expected rate of inflation, it is even possible that the inflation rate rises even during the interval of lower monetary growth. While in the Sargent-Wallace model inflation is a monetary phenomenon, ultimately, for unchanged fiscal fundamentals (real taxes and spending) money is a fiscal phenomenon. In Sargent and Wallace, the government satisfies its solvency constraint for all price sequences[17] and the fiscal-financial programme is not overdetermined. Theirs is a valid, well-posed, theory.

V. Neutrality and price level determinacy in a world without money.

We now return to the model of section II. Consider the limiting case of the economy where the efficiency of the payment mechanism has improved to the point that money has become redundant. This occurs when $\eta = 0$. In the limit, as $\eta \to 0$, real money demand goes to zero and e tends to c. The real equilibrium variables of the model approach their equilibrium value at $\eta = 0$. Discussion of what happens to nominal variables in the limit is postponed.

The equilibrium when $\eta = 0$, is characterized by:

$$\left(\frac{c_{t+1}}{c_t} \right)^{\epsilon} = \frac{1 + r_{t+1}}{1 + \delta} \quad \textbf{(52)}$$

$$y_t = c_t + g_t \quad \textbf{(53)}$$

$$\frac{B_t}{P_t} + b_t = \sum_{j=0}^{\infty} \left(\prod_{\ell=1}^{j} \left(\frac{1}{1 + r_\ell} \right) (\tau_{t+j} - g_{t+j}) \right) \quad \textbf{(54)}$$

$$1 + r_{t+1} = (1 + i_{t+1}) \frac{P_t}{P_{t+1}} \quad \textbf{(55)}$$

Given $\{g_t\}$, $t = 1, 2, ...$, the equilibrium conditions (52), (53) and (54) determine the equilibrium values of the real endogenous variables: $\{c_t, r_{t+1}\}$; $t = 1, 2, ...$.

It is immediately apparent from (54) that, *if the real present discounted value of future taxes,* $\sum_{t=1}^{\infty} \prod_{j=2}^{t} \tau_t/(1 + r_j)$ *, is exogenous,* P_t is strictly proportional to B_t, the nominal value of the initial stock of nominally denominated non-monetary public debt. This strict quantity theory of nominal non-monetary public debt is just a version for a demonetised economy of the fiscal theory of the price level considered in Section IV. As in Section IV, it is clear that, for given initial stocks of government liabilities, a given real spending programme (and a given sequence of monetary financing), the present discounted value of future taxes should be treated as an endogenous variable, which assumes the value required to satisfy the government's solvency constraint.

It is also apparent from equation (54) and the requirement that $P \geq 0$, that this quantity theory of nominal public debt could only hold if condition (36) is satisfied, the same arbitrary restriction that had to be satisfied for the fiscal theory of the price level to apply in an economy with money. Note that this implies, again as in Section IV, that for the demonetised economy to have a nominal anchor, exogenous nominal and real payment streams (or (non-monetary) nominal and real assets and liabilities), both have to enter in the government's intertemporal budget constraint.

In the demonetised economy, money has become solely a numeraire. While securities may be denominated in terms of money, interest payments and repayment of principal are not made in money, which no longer has any physical existence. We know from general equilibrium theory that the numeraire need not be a good or bundle of goods in the commodity space. Indeed, the numeraire could be something entirely fictitious, such as phlogiston. Only relative prices matter and are determinate. The model of this paper follows the literature in identifying the numeraire with the medium of exchange. In reality, bounded rationality arguments favour such an identification, but the two functions, numeraire and medium of exchange, are logically and functionally distinct[18]. When $\eta = 0$, there no longer is a medium of exchange and money will not be held in any positive nominal interest rate equilibrium. No nominal outside asset appears in any of the equilibrium conditions other than the government's (or the household's) intertemporal budget constraint. If something called money is designated as numeraire, the price level in terms of money will be determinate only if there are outside claims denominated in the numeraire that are willingly held in equilibrium and if the real taxes are exogenous, that is, if the government's fiscal-financial programme is overdetermined. Unsurprisingly, I reject this over-determination and treat real taxes as residually determined to satisfy the government solvency constraint for all price sequences and for all initial debt stocks. I therefore conclude that both the price level and the real present value of taxes are indeterminate in the demonetised economy when the government follows a monetary rule.

With real taxes endogenously determined to satisfy the government solvency constraint, the limit as $\eta \to 0$, the price level, P_t, is not the price level when $\eta = 0$. By the monetary equilibrium condition, holding with equality as long as $\eta > 0$, we see that, for a given sequence of the nominal money stock, the price level increases without bound as $\eta \to 0$, that is, $\lim_{\eta \to 0} P_t = \infty$. When $\eta = 0$, the price level is indeterminate.

VI. Nominal interest rate rules

VI.1 Price level determinacy under a fixed nominal interest rule in an economy with money

We return to the case where money has a transactions function, $\eta > 0$, but now suppose the government pegs the nominal interest rate each period at a positive value:

$$i_t = \bar{i}_t > 0 \qquad t = 1, 2, \ldots \tag{56}$$

In each period, t, the government and households take as given (or predetermined) the financial assets carried into that period. We first consider the cash-in-advance transactions technology (2).

Equilibrium

The equilibrium of the economy under an exogenous (or open-loop) nominal interest rate rule is given by equations (19) to (23), for $t = 1, 2, ...$, the government's intertemporal budget constraint for period 1, (24) and the initial conditions for the three financial asset stocks..

Given $\{g_t , i_{t+1} \}, t = 1, 2,$ the equilibrium conditions then determine $\{c_t , e_t , M_{t+1} , r_{t+1} ,$ $, P_t \}; t = 1, 2, ...$, and $\sum_{t=1}^{\infty} \prod_{j=2}^{t} \tau_t/(1 + r_j)$.

With the cash-in-advance transactions technology, the price level is determinate even under an exogenous nominal interest rate policy. The reason is that the initial stocks of public debt (monetary and non-monetary) are predetermined and because, *with the transactions technology (2) and the money demand function (10) implied by it*, the initial, predetermined money stock, M_1, determines the initial price level. P_1 , given the exogenous nominal interest rate i_1 and the level of consumption c_1, which is independent of the sequence of nominal prices , given the sequence of nominal interest. With the initial price level pinned down by the pre-determined initial nominal money stock, the entire equilibrium price sequence is determinate.

Key to this result is the assumption that transactions during period t are facilitated by the real value of the nominal money stock in existence at the beginning of that period (equation (2)). Since the initial stock of money is inherited from the previous period, the price level is determinate but *hysteretic*. This means that the steady state price level sequence[19] depends on the steady state nominal money stock sequence, but this steady state nominal money stock sequence cannot be determined from the steady state equilibrium conditions of the model alone. It requires knowledge of the steady-state nominal money stock sequence which is a function of the initial, pre-determined, nominal money stock, M_1. I summarize this as Proposition 3.

Proposition 3.

Suppose the transactions technology requires the use of predetermined initial money balances. Then the price level is determinate but hysteretic under an exogenous nominal interest rate peg.

The following two propositions follow by inspection.

Proposition 4.

Suppose the transactions technology requires the use of predetermined initial money balances and that the nominal interest rate is pegged. Money is conditionally neutral, in the sense that a given proportional increase in the initial nominal money stock is associated with an equal proportional increase in all prices and in all future nominal money stocks[20].

Proposition 5.

Suppose the transactions technology requires the use of predetermined initial money balances and that the nominal interest rate is pegged. The set of all monetary and non-monetary nominal government debt instruments, is jointly unconditionally neutral, in the *sense that a given proportional increase in the initial nominal money stock and the initial stocks of nominal public debt is associated with an equal proportional increase in all prices and in all future nominal money stocks. No change in the present discounted value of taxes is required to satisfy the government's intertemporal budget constraint.*

How plausible is the transactions assumption embodied in equation (2)? It certainly is in the spirit of cash-in-advance models that require households to choose their transactions balances before the consumption markets open. It is nevertheless instructive to consider the alternative possibility, embodied in equation (3), that the end-of-period money stock (a choice variable during the period in question) enters as an argument in the shopping function.

The equilibrium conditions for this 'cash-in-arrears' shopping function are the same except for the replacement of equations (19), (20) and (21), by, respectively, equations (11), (12) and (3). It is immediately apparent that the price level now is indeterminate under an exogenous nominal interest rate peg. The initial nominal money stock, M_1 , and the other initial financial asset stocks, enter the equilibrium conditions only through the household and public sector solvency constraints. Assume the initial value of the aggregate nominal government liabilities, monetary and non-monetary, is positive. A higher initial price level, P_1 , would imply a lower real initial value of government's financial liabilities and of the private sector's initial financial wealth. Let the nominal money stock in period *2* and in all later periods be higher by the same proportion as P_1 A reduction in the present value of the sequence of real taxes equal to the reduction in the initial real value of the government's financial liabilities would restore the original real equilibrium[21] and increase all nominal wages and prices by the same proportion as P_1. End-of period real money balances, M_{t+1}/P_t , $t = 1, 2, ...$, would be invariant, but nominal money stocks and nominal prices would be indeterminate.

This suggests the following proposition.

Proposition 6.

Suppose the transactions technology permits the use of end-of-period money balances. Then the price level is indeterminate under an exogenous nominal interest rate peg.

McCallum [1986] argues that the price level is determinate even when the nominal interest rate is pegged. He derives his result for the limiting case of a general monetary rule which links the nominal money stock to the nominal interest rate. In our model, such a rule could be expressed as follows[22]:

$$M_t = \epsilon_0 + \epsilon_1(i_t - \bar{i}) \qquad \epsilon_1 > 0$$

It is clear that in his approach, the nominal money stock is not a predetermined variable, so our Proposition 6 should apply. McCallum considers the limiting behaviour of such an economy as $\epsilon_1 \to \infty$. As $\epsilon_1 \to \infty$, the monetary rule converges to the nominal interest rate peg, $i_t = \bar{i}$. The determinate (indeed unique) minimal state solution for the price level for finite values of ϵ_1 has a well-behaved limit as $\epsilon_1 \to \infty$. All that this establishes, however, is that the limiting equilibrium price level sequence as $\epsilon_1 \to \infty$ is *an* equilibrium price level sequence for the case where $\epsilon_1 = 0$. Since the price level when $\epsilon_1 = 0$ is indeed indeterminate, it is neither surprising, nor a source of comfort for policy makers, that *one* of the continuum of possible price levels consistent with an interest rate pegging policy is indeed the limiting price level supported by a sequence of values of the monetary policy rule parameter that approximates the interest rate peg.

The sequence of monetary policy rule parameter values involved in this limiting process is a sequence of complete histories of 'parallel economies', each of which operates, for all time, under a constant value of this parameter. It does therefore not describe the behaviour over time of a single economy whose policy parameter converges gradually to that consistent with an interest rate peg.

VI.2 Price level determinacy under an interest pegging policy in an economy without money

Consider again the case where the transactions technology requires the use of the initial money stock (equation (2))[23]. Now $\eta = 0$ and money demand is zero. The equilibrium of this economy can be characterized by equations (52) to (55), for $t = 1, 2, \ldots$, and the initial conditions for the real and nominal asset stocks.

Given $\{g_t , i_{t+1} \}, t = 1, 2, \ldots,$ the equilibrium conditions then determine the values of $\{c_t , r_{t+1} , P_t /P_{t+1} \}, t = 1, 2, \ldots$, and $\sum_{t=1}^{\infty} \prod_{j=2}^{t} \tau_t/(1 + r_j)$.

Both the general price level and the present value of current and future taxes are indeterminate if real taxes are residually determined to satisfy the government solvency constraint. If the real present discounted value of future taxes is kept constant, we have a strict quantity theory of nominal non-monetary public debt, provided condition (36) is satisfied.

Restricting ourselves to the case where real taxes adjust to satisfy the government solvency constraint[24], price level indeterminacy can be avoided only by providing some other nominal anchor. A plausible candidate might seem dropping the assumption that the interest rate rule is open-loop and to assume instead that the nominal interest rate is a function of current, past or anticipated future price levels. An example of such a rule, given in Woodford [1997], is

$$i_t = f_t(P_t) \tag{57}$$

The function f_t is strictly increasing, continuous and strictly positive for all positive price levels. A simple example is the linear interest rate rule

$$i_t = \alpha P_t \qquad \alpha > 0 \tag{58}$$

It is obvious that, when money has a transactions role ($\eta > 0$) an interest rate rule such as (57) or (58) is sufficient to give price level determinacy (see e.g. Buiter [1995, 1997] and Buiter, Corsetti and Pesenti [1997]). However, when $\eta = 0$, and money has no transactions role, price level indeterminacy prevails even with the nominal interest rate given by (57) or (58). From equations (55) and (58) it follows that

$$1 + r_t = \frac{P_t}{P_{t+1}} + \alpha P_t \tag{59}$$

Consumption and the real interest rate are determined by equations (52) and (53) . They are independent of the nominal interest rate sequence [25]. From equations (59) and (54) it follows that the price level and the present discounted value of taxes are still indeterminate. There is no boundary condition to pin down the initial value P_1. The same holds for the more general interest rate function given in (57). Neither the price level nor the rate of inflation (or the nominal interest rate) are determinate.

Woodford's analysis is not inconsistent with this conclusion. He does not assert that the price level is determinate in the demonetised, or cashless, economy. His proposition is that the limit, as $\eta \to 0$, of the equilibrium price sequence for the economy with money, is well-behaved and that this limit is *an* equilibrium price sequence of the cashless economy (Woodford [1997]). While the model of this paper supports that conclusion, I question its policy relevance.

The first reason is that, since *any* initial price level is consistent with equilibrium in the cashless economy, the fact that equilibrium for the monetary economy converges to one of a continuum of possible price level equilibria for the demonetised economy, offers scant comfort to those contemplating the eventual disappearance of money in countries with advanced financial systems.

The second and more fundamental reason is that the limiting process considered by Woodford is not economically interesting. Woodford's proposition is that the limit, as $\eta \to 0$, of the equilibrium for the economy with money, is an equilibrium of the cashless economy. Will this come as a relief to the central bankers of the cashless future? Once the economy is demonetised ($\eta = 0$), price level indeterminacy is present. If we think of the improvement in the transactions technology as taking place in real time (η falls period-by-period and reaches 0 after a finite number of periods), the indeterminacy *after* η has reached zero will cause indeterminacy problems even in the periods *before* η reaches zero.

This can be demonstrated as follows. Instead of assuming η to be constant, assume the following: $\eta_t \geq 0$; $\eta_1 > 0$; η_t is a non-increasing function of t, and $\eta_{t+N} = 0$ for some finite $N > 1$. The price level will be indeterminate in this economy from period $t+N$ on. Since $P_{t+N-1} = P_{t+N}(1 + r_{t+N})/(1 + i_{t+N})$, the price level is also indeterminate in all earlier periods.

Woodford's process of considering the limit as $\eta \to 0$ is not a process taking place in calendar time. It involves sequences of 'parallel histories', each successive one indexed by a lower, but constant, value of η. This mathematical convergence concept is clearly inappropriate as a guide to the behaviour of an economy faced, in calendar time, with a declining sequence of η.'s and reaching a cashless state at some finite future date. The need to distinguish between the two amounts to the theorist's version of the familiar econometric adage: do not draw time series inferences from cross-sectional regressions.

VII. Conclusions

The paper makes seven points.

First, when money has a transactions role, money is *conditionally neutral* if there is a non-zero stock of nominally denominated non-monetary government financial liabilities. Neutrality is obtained because the government adjusts the path of real taxes to satisfy its intertemporal budget constraint.

Second, money and all non-monetary nominal government liabilities are jointly *unconditionally neutral*. Neutrality is obtained without any change in real taxes.

Third, Woodford's fiscal theory of the price level is invalid because it represents a 'solution' to an ill-posed general equilibrium problem. It combines an overdetermined fiscal-financial programme with an unwarranted weakening of the government's intertemporal budget constraint, requiring it to hold only in equilibrium and for arbitrarily restricted configurations of public spending, revenues and initial debt stocks..

Fourth, as long as the economy is not demonetised, there is price level determinacy under an fixed nominal interest rate rule if the transactions technology has cash-in-advance features.

Fifth, there is price level indeterminacy under a fixed nominal interest rate rule if the transactions technology has 'cash-in-arrears' features.

Sixth, in a demonetised or cashless economy there is price level indeterminacy even if the interest rate rule makes the nominal interest rate a function of the price level.

Finally, a historical process of technological improvements in the transactions technology, evolving towards a cashless economy, is completely different from the mathematical construct of the limiting behaviour (as the transactions efficiency index takes on successively higher values), of a sequence of histories, each one of which is indexed, for all time, by a *given* level of transactions efficiency. The first concerns the behaviour over time of a given economy undergoing technological improvements in its transactions technology and converging to (or even reaching) a cashless state. The second concerns the comparison of 'parallel universes', each with its own constant level of transactions efficiency, and each with a complete history, but completely disconnected one from the other. Central bankers, financial regulators and other policy makers are faced with technological change occurring in real time, that is, with history and the anticipation of future events. Even those among them who enjoy science fiction do not contemplate moving to a parallel universe, tempting though this may seem at times.

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ENDNOTES

1. or exogenous nominal payments streams.

2. An earlier version of the paper used a Ramsey exogenous growth model with capital and labour as the two inputs.

3. If the nominal price level were sticky, or predetermined in the short run, but became flexible in the long run (say through the expiration of the long-term non-contingent nominal contracts that give rise to price or money wage stickiness in many Keynesian models), the long-run (steady-state) price level, while determinate, would be *hysteretic*. Its steady-state value could not be determined from the steady state equilibrium conditions alone, but would depend on the initial conditions and possibly on the transition path to the steady state as well.

4. The conclusions would be unaffected if the household instantaneous utility function were generalised to a twice continuously differentiable function $u(c)$, with $u' > 0$, $u'' < 0$ which satisfies the Inada conditions.

5. This particular specification is found in Sims [1994]. The conclusions would be unaffected if the household shopping function were generalised to $e_t = c_t[1 + s(\eta, \frac{c_t}{M_t/P_t})]$, with s continuously differentiable, $\eta \geq 0$, $s(\eta, cP/M) > 0$ for $\eta > 0$ and $c > 0$, $s_1 < 0$, $s_2 > 0$ for $\eta > 0$, $s(0, cP/M) = s(\eta, 0) = 0$, and $\lim_{cP/M \to \infty} s_2(\,.\, , cP/M) = \infty$.

6. We adopt the notational convention that $\prod_{j=n}^{n-1} \left(\frac{1}{1 + i_j} \right) = 1$.

7. From now on I refer to this tax rule as the assumption that the present discounted value of the sequence of current and future taxes is endogenously determined in such a way as to satisfy the government's intertemporal budget constraint at each point in time.

8. An 'outside' claim is an asset of the private sector that is not also a liability of the private sector, that is, an outside claim is a claim that is in non-zero net supply to the private sector as a whole.

9. Note that $\sum_{t=1}^{\infty}\left(\prod_{j=2}^{t}\frac{1}{1+i_j}\right)P_t\tau_t = P_1\sum_{t=1}^{\infty}\left(\prod_{j=2}^{t}\frac{1}{1+r_j}\right)\tau_t$.

10. We still restrict the analysis to sequences of exogenous variables and initial conditions that support a positive equilibrium nominal interest rate sequence.

11. or only index-linked debt and foreign-currency-denominated debt.

12. The resource constraint (38) depends, in period *1*, on the predetermined nominal interest rate i_1.

13.If the government followed an exogenous nominal interest rate rule instead of an exogenous nominal money stock rule, the initial price level and all future price levels would be determined 'independently' of the sequence of nominal money stocks. That, of course, not a very helpful result, since the nominal money stock sequence is endogenous in this case. The flaws in the fiscal theory of the price level are independent of whether the government follows a monetary rule or an interest rate rule.

14. Consider doubling all current and future nominal money stocks. If the real interest sequence and P_1 were to be unaffected, it must be true (from (41), and (42), that i_2 falls. From the government's budget identity, for a given P_1 , a lower nominal interest rate i_2 implies a lower value of B_2 and, if the real interest rate sequence is unaffected, a lower value of P_2 . Except for a very restricted set of parameter configurations and initial conditions, the fall in the nominal interest rate and the fall in the future price level will alter the real interest rate in period *2*.

15. The condition $sgn\{B_t\} = sgn\{\left(\frac{1+\delta}{\delta}\right)(\bar{\tau} - g) - b\}$ must of course be satisfied.

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16.An even more dramatic example would be to fix exogenously the entire private consumption sequence $\{c_t\}$, but such sequences would of course not, in general, satisfy the necessary conditions for optimality.

17. When monetary growth becomes endogenous, in phase two, conditions must be satisfied to ensure that enough real seigniorage can be extracted to finance the deficit through monetary issuance.

18. The British Guinea is an example of a unit of account that was not a means of payment. In

medieval Iceland, dried fish were used as the unit of account, but (fortunately for the Icelanders)not as the medium of exchange. I am indebted to Anne Sibert for the dried fish.

19. Stationary sequences of the real variables supported by constant values of *g* and *I*.

20. However, a change in the present discounted value of taxes is required to satisfy the government's intertemporal budget constraint if there is a non-zero stock of non-monetary nominal public debt.

21. Except of course for the present value of the sequence of real taxes and the initial real stock of government financial liabilities.

22. McCallum's monetary rule also includes a time trend and a lagged value of the nominal money stock, and involves the logarithm of the money stock rather than its level, but this does not affect the argument.

23. The results of this subsection go through even if we use the end-of-period specification of equation (3).

24. Woodford [1997] does not suffer from the overdeterminacy problems of Woodford [1995]. The budgetary rule in Woodford [1997] is, except possibly in the initial period, a balanced budget rule like the one given in equation (20), so the government solvency constraint is always satisfied.

25. It is indeed independent of any nominal interest rate rule.