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# TIME PREFERENCE AND INTERNATIONAL LENDING AND BORROWING IN AN OVERLAPPING GENERATIONS MODEL

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ABSTRACT

Two economies, each represented by a Diamond-type overlapping generations model, are joined together. Capital formation and welfare are compared under autarky and openness. Most of the analysis concentrates on the case where the two countries only differ in their pure rates of time preference. The following propositions are established:

1. If the rate of population growth is positive, the country with the higher value of consumption while young has a current account deficit in the steady state.
2. The country with the higher pure rate of time preference has a steady-state current account deficit.
3. If the model is stable, the common open economy steady-state capital-labor ratio lies between the two autarky capital-labor ratios.
4. Outside the steady state, the country with the higher pure rate of time preference will not necessarily be the one to run a current account deficit.
5. The welfare of the old generation alive when the two autarkic countries join in open economy is unaffected by this international economic integration. The welfare of the young generation in the country with the high pure rate of time preference is lower under openness than under autarky, while in the country with the low pure rate of time preference it is higher.
6. The stationary utility level of the country with the high pure rate of time preference is always higher under openness than under autarky. The effect of openness on the stationary utility level of the low pure rate of time preference country is ambiguous. If it is higher, openness can be said to constitute a potentially pareto-superior regime for the low pure rate of time preference country. Because of the short-run welfare loss it incurs after international economic integration, the open economy is not pareto-superior to the closed economy for the high pure rate of time preference country.

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## Time Preference and International Lending and Borrowing in an Overlapping Generations Model

### 1. Introduction

In its most abstract form the pure theory of international trade demonstrates that the reason for exchange across national boundaries is the same as for any kind of exchange: gains can be realized by the contracting parties that would otherwise have been unobtainable. To isolate the underlying determinants of these potential gains, the trading economies are evaluated in a hypothetical **autarky** or pre-trade situation. If the relative prices of tradeable goods differ across countries in the hypothetical **autarky** situation, there is prima facie evidence of potential gains from international commodity arbitrage and, to the extent that technology and internal factor mobility permit, from reallocation of resources and specialization in production.

Differences in relative prices between countries in the absence of trade can be attributed to six factors: differences in technology, differences in factor endowments, differences in tastes, differences in scale, differences in market structuring (monopoly, unionization of labor markets, etc.) and differences in taxes, subsidies and other man-made wedges between buyers' and sellers' prices.

This paper ignores all determinants of trade other than taste differences. The particular taste parameter whose effect on the pattern of trade will be analyzed is the pure rate of time preference. To isolate the contribution of this parameter, the formal model will have but one homogeneous physical output, produced in both countries with an identical technology. The model is non-trivial because two commodities with identical physical characteristics available at different points in time are distinct economic goods. In such a one-commodity world, international trade is always mirrored by international lending or borrowing: there can be international trade in commodities if and only if there is international lending and borrowing. Thus the model incorporates costless international mobility of output (consumption and real capital) and of financial capital. Labor is assumed to be immobile between countries. Section 2 reproduces some results on the one-country overlapping generations model first developed by Diamond [1965]. This model describes the **autarky** situation of the two countries. Section 3 analyzes the open, **integrated world economy.**

In a recent paper Findlay [1978] has approached the problem of international trade, time preference and interest equalization from an "Austrian" viewpoint. This paper can be taken as a neoclassical complement to Findlay's analysis.

## 2. The Autarchy Equilibrium

The world economy consists of two countries, the "home" country and the "foreign" country, identical in every respect except in taste. Each country is represented by competitive output and factor markets, two overlapping generations, and an identical, well-behaved constant returns to scale production function f.

### Notation

$c_t^1$ consumption while young by a member of generation t.

$c_t^2$ consumption while old by a member of generation t.

$L_t$ size of generation t.

$K_t$ capital stock in period t.

$k_t = K_t/L_t$ .

$w_t$ real wage in period t.

$r_t$ interest rate on savings carried from period t-1 into period t.

$n$ one-period proportional rate of growth of population. $L_{t+1} = (1+n) L_t$; $n \geq 0$.

$\rho$ pure rate of time preference; $\rho \geq 0$.

$A_t$ wealth in period t.

$a_t = A_t/L_t$ .

$B_t$ trade balance surplus of the home country in period t.

$b_t = B_t/L_t$ .

$G_t$ current account surplus of the home country in period t.

$g_t = G_t/L_t$ .

Momentary equilibrium

The momentary or single-period equilibrium of the closed economy is summarized in equations (1) - (7).

(1) $$\max_{c_t^1, c_t^2} u(c_t^1, c_t^2)$$

(2) $$w_t - c_t^1 \leq c_t^2(1 + r_{t+1})^{-1}$$

(3) $$c_t^1, c_t^2 \geq 0$$

(4) $$w_t = f(k_t) - k_t f'(k_t)$$

(5) $$r_t = f'(k_t)$$

(6) $$w_t - c_t^1 = k_{t+1} (1 + n)$$

(7) $$k_t \geq 0.$$

$u$ is twice differentiable, strictly quasi-concave and increasing in $c^1$ and $c^2$. $u_1(0, c^2) = u_2(c^1, 0) = +\infty$; $u_1(\infty, c^2) = u_2(c^1, \infty) = 0$. $f$ is twice differentiable; $f(0) = 0$; $f' > 0$; $f'' < 0$; $f'(0) = +\infty$ and $f'(\infty) = 0$.

Individuals within a given country are identical, within and across generations. People live for two periods, work in the first period of their lives and retire in the second. Labour supply is inelastic. The individual's optimization problem is given in equations (1), (2) and (3). The utility of lifetime consumption is maximized subject to the lifetime budget constraint (2). The conditions on u ensure that the budget constraint will hold with equality and that an interior solution will be obtained for $c_t^1$ and $c_t^2$. Equations (4) and (5) state that the labour market and capital rental market are competitive. The capital stock at the beginning of period t+1 equals the value of the saving in period t. This economy-wide capital market equilibrium condition is given in equation (6). Real capital is the only store of value. All saving in period t is performed by the young members of generation t. The old dissave previously accumulated wealth. The conditions on f ensure that $k_t$ will be positive.

When we consider the consequences of different pure rates of time preference the utility function $u(c_t^1, c_t^2)$ will be specialized to the following additively separable form:

$$(8) \qquad u(c_t^1, c_t^2) = v(c_t^1) + (1 + \rho)^{-1} v(c_t^2)$$

Here $\rho$ is the (constant) pure rate of time preference. More generally, the pure rate of time preference is the marginal rate of substitution between consumption when young and consumption when old, when equal amounts are consumed in both periods, minus one. It can be a function of the level of consumption:

$$\rho(c) = \frac{u_1(c, c)}{u_2(c, c)} - 1.$$

The general solution of (1) - (7) is straightforward and can be found e.g., in Diamond [1965]. The interior first-order conditions for the individual are:

(9) $$\frac{u_1(c_t^1, c_t^2)}{u_2(c_t^1, c_t^2)} = 1 + r_{t+1}$$

For the utility function of equation (8) this becomes:

(9') $$\frac{v'(c_t^1)(1+\rho)}{v'(c_t^2)} = 1 + r_{t+1}$$

The consumer's intertemporal equilibrium is graphed in Figure 1. Equations (9) and (2) [the latter holding with equality] can be solved for $c_t^1$ and $c_t^2$ as functions of $w_t$ and $r_{t+1}$.

(10a) $$c_t^1 \equiv c^1(w_t, r_{t+1})$$

(10b) $$c_t^2 = c^2(w_t, r_{t+1})$$ 1/

Similarly, (9') and (2) can be solved to yield:

(10a') $$c_t^1 = c^1(w_t, r_{t+1}, \rho)$$

(10b') $$c_t^2 = c^2(w_t, r_{t+1}, \rho)$$

If consumption in both periods is a normal good, $1 > c_w^1, c_w^2 > 0$. We shall assume this to be the case.

Steady-state equilibrium

The steady state or long-run equilibrium is that sequence of momentary equilibria in which the lifetime per capita consumption profile remains constant from generation to generation. It can be characterized by the following set of equations.

(11) $$\frac{u_1(c^1,c^2)}{u_2(c^1,c^2)} = 1 + r$$

or

(11') $$\frac{v'(c^1)(1+\rho)}{v'(c^2)} = 1 + r$$

(12) $$w - c^1 = \frac{c^2}{1+r}$$

(13) $$w = f(k) - kf'(k)$$

(14) $$r = f'(k)$$

(15) $$w - c^1 = k(1 + n)$$

By substituting (13) and (14) into (12) and (15) we obtain two equations that represent the stationary decentralized (or competitive) consumption possibility locus.

(16a) $$f(k) - kf'(k) = c^1 + \frac{c^2}{1+f'(k)}$$

(16b) $$f(k) - kf'(k) = c^1 + k(1 + n)$$

These two equations can be solved for $c^2$ as a function of $c^1$.

(17) $$c^2 = \psi(c^1);\ \psi' = -(1+f')\ [1 + \frac{k(n-f')}{1+f'} f''(1+n+kf'')^{-1}]$$

For the Cobb-Douglas production function $f(k) = k^{\alpha}$; $0 < \alpha < 1$, the stationary decentralized consumption possibility locus has the shape of OF in Figure 2.[3/] Its slope is $\psi' = \frac{(1+n)(1+\alpha^2 k^{\alpha-1})}{(1-\alpha)\alpha k^{\alpha-1}-(1+n)}$. At the origin (with k=0), the slope of OF is $\frac{\alpha}{1-\alpha}(1+n) > 0$. The capital-labor ratio, k, increases monotonically as one moves from 0 to F. As k approaches infinity (which would be for $c^1 < 0$) the slope of the consumption possibility locus becomes -1. When $c^1 = 0$ at F, the slope of OF is $-\left(1+\alpha^2(1+n)(1-\alpha)^{-1}\right)(1-\alpha)^{-1} < 0$. At the golden rule capital-labor ratio, defined by $f'(k) = n$, the slope of OF is $-(1 + f'(k)) = 1+n$.[4/] I shall assume in what follows that OF is as in Figure 2: strictly concave towards the $c^2$ axis for non-negative values of $c^1$ and $c^2$. This is certainly the case for the Cobb-Douglas production function but need not be true for alternative, otherwise well-behaved neoclassical production functions. It is e.g., quite possible for OF, while remaining strictly concave towards the origin, to be upward-sloping at F. This would reflect extreme overaccumulation of capital: an increase in k at high values of k depresses the marginal product of capital to such an extent that a reduction in both $c^1$ and $c^2$ is required to generate the additional saving required to maintain the higher value of k.

A stationary competitive equilibrium is described by (16a) and (16b) and a tangency of a private budget constraint with slope $1 + f'(k) = 1+r$ to an indifference curve:

(18) $$u_1(c^1,c^2)/u_2(c^1,c^2) = 1 + f'(k)$$

or

(18') $$\frac{v'(c^1)(1+\rho)}{v'(c^2)} = 1 + f'(k)$$

At capital-labor ratios above the golden rule level the budget constraint cuts the OF curve from below if the production function is Cobb-Douglas. In what follows I shall assume that this condition holds.

In Figure 2 I have drawn such an equilibrium at $E_3$, corresponding t a capital-labor ratio in excess of the golden rule capital-labor ratio stationary competitive equilibrium is inefficient in the sense that a lower capital-labor ratio that does not lie below the golden rule ca labor ratio can sustain higher stationary paths of per capita consumption if only physical resource constraints are taken as binding. The physical resource constraint faced by an omniscient and omnipotent social planner is $c^2_{t-1} L_{t-1} + c^1_t L_t \leq f(k_t) L_t + K_t - K_{t+1}$ or $c^2_{t-1}/1+n + c^1_t \leq f(k_t) + k_t - k_{t+1} (1+n)$. The stationary resource constraint is

(19) $$\frac{c^2}{1+n} + c^1 \leq f(k) - nk.$$

A social planner aiming to maximize the stationary value of per capita resources available for consumption by the young and the old chooses k such that $f'(k) = n$. The social planner's stationary consumption possibility locus is $S_1 - S_2$ in Figure 2. The decentralized competitive economy will settle somewhere on OF if the economy has a unique, stable equilibrium.[5/] The potential inefficiency due to overaccumulation reflects the dual role of capital in the model. Capital serves as an input in the production function and as the unique store of value through which private agents can transfer resources from their productive early years into retirement. It would indeed be surprising if this double task could be discharged efficiently by a single medium.

The potential inefficiency can be interpreted as an externality. The perceived private market rate of transformation between consumption while young and consumption while old differs from the actual social rate of transformation which is a function of the aggregate private saving decision. The slope of the short run and long run perceived private transformation locus is, from the private budget constraint (2).

(20) $$\left.\frac{\partial c_t^2}{\partial c_t^1}\right|_{SR,\ LR,\ priv.} = -(1 + r_{t+1}) = -(1 + f'(k_{t+1}))$$

Combining the private budget constraint (2) with the competitive factor pricing conditions (4) and (5) and the economy-wide capital market equilibrium assumption (6), we obtain the short-run social transformation locus.

$$c_t^2 = [f(k_t)-k_t f'(k_t)][1+f'(\frac{f(k_t)-k_t f'(k_t)-c_t^1}{1+n})] - c_t^1[1+f'(\frac{f(k_t)-k_t f'(k_t)-c_t^1}{1+n})]$$

The slope of the short-run social transformation locus is:

$$(21)\qquad \left.\frac{\partial c_t^2}{\partial c_t^1}\right|_{SR,\ Soc.} = -[1 + f'(k_{t+1}) + k_{t+1}\ f''(k_{t+1})]$$

The slope of the long-run social transformation locus is given by $\psi'$ in (17) which is reproduced below.

$$(22)\qquad \left.\frac{\partial c_t^2}{\partial c_t^1}\right|_{LR,\ Soc.} = -[(1 + f') + k(n - f')\ f''(1 + n + kf'')^{-1}]$$

The short-run transformation locus differs from the long-run locus in that with the former $k_t$ is treated as predetermined and only the current period effects on consumption and capital formation are considered. Only if there is a corner solution with zero saving by members of generation t, i.e., only if $w_t - c_t^1 = 0$ and therefore $k_{t+1} = c_t^2 = 0$, will the short-run perceived private transformation locus have the same slope as the actual social transformation locus. Our assumptions on the utility function rule out such a corner solution. The social terms of trade between $c^1$ and $c^2$ are therefore less favorable than the perceived private terms of trade. An increase in $c_t^1$ will, given the inherited capital-labor ratio $k_t$, lower $k_{t+1}$. This will raise the marginal product of capital and thus the interest rate. It could raise it sufficiently to permit a larger value of $c_t^2$ to be associated with a larger value of $c_t^1$. This would be the case if $\frac{d}{dk}[kf'(k)] < -1$. It would require a very low elasticity of substitution between capital and labor for the short-run social transformation locus to be upward sloping. With a Cobb-Douglas production function e.g., the short-run social transformation locus is downward sloping with

$$\left.\frac{\partial c_t^2}{\partial c_t^1}\right|_{SR,\ Soc.} = -\left[1 + \alpha^2 \frac{[(1-\alpha)k_t^{\alpha}-c_t^1]^{\alpha-1}}{1+n}\right].$$

A possible configuration of short-run and long-run transformation loci is represented in Figure 3. An equilibrium such as $E_1$ represents a steady-state capital-labor ratio below the golden rule. This cannot be characterized as inefficient because of (temporary) sacrifices in consumption that would be required to reach a higher capital-labor ratio.

Stability

Substituting (10a) into (6) and using (4) and (5) we obtain a first-order difference equation in $k_t$ that describes the evolution of the model from arbitrary initial conditions:

(23) $$f(k_t) - k_t f'(k_t) - c^1(f(k_t) - k_t f'(k_t),\ f'(k_{t+1})) = k_{t+1}(1+n)$$

The model will be locally stable when $\left|\frac{dk_{t+1}}{dk_t}\right| < 1$ i.e., when

(24) $$\left|(c_w^1 - 1)\, kf''[1 + n + c_r^1 f'']^{-1}\right| < 1.$$

The interpretation of (24) is that an increase in k will lead to excess supply in the capital market. It does not stimulate saving to such an extent that excess demand results. Note that the assumption that goods are normal ($0 < c_w^1 < 1$) and the assumption that an increase in the interest rate reduces current consumption ($c_r^1 < 0$) are insufficient to ensure stability.

We can now establish five propositions about the autarky equilibria of two economies that are identical in all respects except for their pure rates ot time preference. The utility function is given in (8).

Proposition 1: Given $w_t$ and $r_{t+1}$ a consumer with a higher pure rate of time preference will choose a higher value of consumption when young, $c_t^1$.

Proof: We want to show that the consumption function $c_t^1 = c^1(w_t, r_{t+1}, \rho)$ has the property $c_\rho^1 = \frac{\partial c^1}{\partial \rho} > 0$. Differentiating $\frac{v'(c^1)}{v'\left((w-c^1)(1+r)\right)} = 1 + r$ we obtain

$$\frac{\partial c^1}{\partial \rho} = \frac{v'(c^1)}{1+\rho} \left[ v''(c^1) + [(1+\rho)\frac{v'(c^1)}{v'(c^2)}]^2 \frac{v''(c^2)}{1+\rho} \right]^{-1}.$$

The utility function is increasing in $c^1$ and $c^2$, so $v' > 0$. The slope of an indifference curve $\left.\frac{\partial c^2}{\partial c^1}\right|_{u=\bar{u}} = \frac{-v'(c^1)}{v'(c^2)}(1+\rho)$. Strict convexity of an indifference curve is equivalent to: $v''(c^1) + [(1+\rho)\frac{v'(c^1)}{v'(c^2)}]^2 \frac{v''(c^2)}{1+\rho} < 0$. Thus $\frac{\partial c^1}{\partial \rho} > 0$.

Proposition 2: Starting from a given $k_t$ (and therefore a given $w_t$), a higher pure rate of time preference is associated with a lower value of $k_{t+1}$ (and a higher value of $c_t^1$) if a higher interest rate does not reduce saving.

Proof: For the case of the additively separable utility function, (23) can be rewritten as: $f(k_t) - k_t f'(k_t) - c^1(f(k_t) - k_t f'(k_t), f'(k_{t+1}), \rho) = k_{t+1}(1+n)$. Thus

$$\frac{\partial k_{t+1}}{\partial \rho} = \frac{-c_\rho^1}{1+n+c_r^1 f''(k_{t+1})}.$$

A sufficient, but not necessary, condition for $\frac{\partial k_{t+1}}{\partial \rho}$ to be negative is $c_r^1 < 0$.

Proposition 3: Given $\rho$, a lower value of $k_t$ will be associated with a lower value of $k_{t+1}$, if consumption in both periods is a normal good and a higher interest rate does not reduce saving.

Proof: From (23) we obtain $\frac{\partial k_{t+1}}{\partial k_t} = \frac{-(1-c_w^1)k_t f''(k_t)}{1+n+c_r^1 f''(k_{t+1})}$. This will be positive if $0 < c_w^1 < 1$ and $c_r^1 \leq 0$.

Propositions 2 and 3 immediately imply the following:

Proposition 4: Consider two economies, identical in all respects except the pure rate of time preference which is higher for the home country than for the foreign country. Starting from any common initial capital-labor ratio at $t = t_0$, the capital-labor ratio of the high time preference country will be below that of the low time preference country for all $t > t_0$.

Proposition 4 tells us something about the behavior of the two economies were they to start from the same initial condition. It might still seem possible that if the high pure rate of time preference country were to start off with an initial capital-labor ratio, $k_{t_0}$, sufficiently above the initial capital-labor ratio of the low pure rate of time preference country, $\bar{k}_{t_0}$, $k_t$ could remain above $\bar{k}_t$ indefinitely. To show that if $k_{t_0} > \bar{k}_{t_0}$ the two paths of the capital-labor ratio $\{k_t\}$ and $\{\bar{k}_t\}$ will cross, it suffices to show that the steady-state capital-labor ratio of the high time preference country, k, is less than the steady-state capital-labor ratio of the low time preference country, $\bar{k}$. This is shown in the proof of Proposition 5. After the two paths cross for the first time, they never cross again because of Propositions 2 and 3.

Proposition 5: Under autarky, the country with the higher pure rate of time preference will have the lower steady-state capital-labor ratio if the model is locally stable, consumption in both periods is a normal good and an increase in the interest rate lowers consumption in period 1.

Proof: Consider again the consumption function $c^1_t = c^1(w_t, r_{t+1}, \rho)$. From Proposition 1 we know that $c^1_\rho > 0$. We find the effect of a higher value of $\rho$ on steady-state k from $f(k) - kf'(k) - c^1 = k(1+n)$. Differentiating this yields: $\frac{\partial k}{\partial \rho} = c^1_\rho[(c^1_w - 1)kf'' - (c^1_r f'' + 1 + n)]^{-1}$. With $c^1_\rho > 0$, $\frac{\partial k}{\partial \rho} < 0$ requires $(c^1_w - 1)kf'' - (c^1_r f'' + 1 + n) < 0$. This is ensured by stability (equation (24)) if $0 < c_w < 1$ and $c^1_r \leq 0$.

## 3. The Two-Country Equilibrium

The two countries that will be linked together in an international commodity market and an international capital market are identical in all respects, except for the pure rate of time preference. The home country is assumed to have a higher rate of pure time preference than the foreign country. For the home country we adopt the notation developed for the autarchic economy. All variables associated with the foreign country are distinguished by a bar above the relevant variable. Comparing the two countries we have:

(25a) $v = \bar{v}$

(25b) $f = \bar{f}$

(25c) $n = \bar{n}$

(25d) $L_t = \bar{L}_t$

(25e) $\rho > \bar{\rho}$

Output can be moved costlessly between the two countries.

In this "pure absorption" model[6/] international trade and international lending and borrowing (international capital mobility) are part and parcel of the same transaction. In a one commodity model the only way to pay for an extra unit of output today is with a promise of future output. Each trade balance transaction has to involve credit. There can be no current quid-pro-quo. We make the convenient assumption of perfect international mobility of financial capital: ownership claims to domestic and foreign real capital are perfect substitutes in private portfolios in both countries. This means that interest rates will be equalized in the world economy:

(26) $r_t = \bar{r}_t$ for all t.

There is a single world capital market. With international capital mobility it is essential to distinguish between the capital stock used in production in the home country, $K_t$, and the value of claims on real capital, domestic or foreign, owned by domestic residents, $A_t$. In the national income accounts the corresponding distinction is between domestic income or product and national income or product. Location and ownership of physical capital no longer coincide. From interest equalization and identical, linear homogeneous production functions it follows that capital-labor ratios and wage rates are equal in the two countries.

(27a) $k_t = \bar{k}_t$

(27b) $w_t = \bar{w}_t$

The world economy can be summarized as in equations (28) - (34).[7/]

(28) $$\max_{c_t^1, c_t^2} \; v(c_t^1) + (1+\rho)^{-1} \, v(c_t^2)$$

$$(29) \qquad w_t - c_t^1 = \frac{c_t^2}{1+r_{t+1}}$$

$$(30) \qquad \max_{\bar{c}_t^1, \bar{c}_t^2} \quad v(\bar{c}_t^1) + (1+\bar{\rho})^{-1} v(\bar{c}_t^2)$$

$$(31) \qquad w_t - \bar{c}_t^1 = \frac{\bar{c}_t^2}{1+r_{t+1}}$$

$$(32) \qquad w_t = f(k_t) - k_t f'(k_t)$$

$$(33) \qquad r_{t+1} = f'(k_{t+1})$$

$$(34) \qquad w_t - \frac{1}{2}(c_t^1 + \bar{c}_t^1) = k_{t+1}(1+n)$$

The utility maximization problems of the representative household in the home country and in the foreign country are given in equations (28) - (29) and (30) - (31) respectively. Openness is reflected in the equalization of interest rates and in the world capital market equilibrium condition (34). This equation states that world saving in period t, $(w_t - c_t^1) L_t + (\bar{w}_t - \bar{c}_t^1) \bar{L}_t$, equals the world capital stock in period t+1, $K_{t+1} + \bar{K}_{t+1} = A_{t+1} + \bar{A}_{t+1}$.

The individual optimization programs yield the first-order conditions

$$(35a) \qquad \frac{v'(c_t^1)(1+\rho)}{v'(c_t^2)} = 1 + r_{t+1}$$

$$(35b) \qquad \frac{v'(\bar{c}_t^1)(1+\rho)}{v'(\bar{c}_t^2)} = 1 + r_{t+1}$$

Given the predetermined value of $k_t$ $(=\bar{k}_t)$, the seven equations (35a), (35b) (29), (31) - (34) determine the momentary equilibrium values of $c_t^1$, $c_t^2$, $\bar{c}_t^1$, $\bar{c}_t^2$, $w_t$, $r_{t+1}$ and $k_{t+1}$.

The stability conditions for this world-economy model are very similar to those for the closed economy given in (24). As in the closed economy, we can solve (35a) and (29) for $c_t^1$ as a function of $w_t$, $r_{t+1}$ and $\rho$, and (35b) and (31) for $\bar{c}_t^1$ as a function of $w_t$, $r_{t+1}$ and $\bar{\rho}$.

(36a) $$c_t^1 = c^1(w_t, r_{t+1}, \rho)$$

(36b) $$\bar{c}_t^1 = \bar{c}^1(w_t, r_{t+1}, \bar{\rho})$$

Substituting (32) and (33) into (36a), (36b) and (34) we obtain the following first-order difference equation in $k_t$.

(37) $$f(k_t)-k_tf'(k_t) - \frac{1}{2}\left[c^1\left(f(k_t)-k_tf'(k_t),f'(k_{t+1}),\rho\right)+\bar{c}^1\left(f(k_t)-k_tf'(k_t),f'(k_{t+1}),\bar{\rho}\right)\right] = k_{t+1}(1+n)$$

This model will be locally stable if

(38) $$\left| (c_w^1 + \bar{c}_w^1 - 2)kf''[2(1+n) + (c_r^1 + \bar{c}_r^1)f'']^{-1} \right| < 1.$$ 8/

The interpretation of this stability condition is the same as in the closed economy case. In the current model a higher value of k should be associated with excess supply in the world capital market. Again, the assumptions of normal goods and a non-negative effect on saving of an increase in the interest rate do not suffice to establish stability. Note that when tastes are identical the model reduces to the closed economy. All international trade and international borrowing and lending is due to taste differences.

The balance of payments accounts

In a two-country world it suffices to analyze the balance of payments accounts of one of the countries. Without loss of generality I focus on the home country.

(39a) $$B_t \equiv -\bar{B}_t$$

$$(39b) \qquad G_t \equiv -\bar{G}_t$$

As there is no official settlements balance, the current account surplus (deficit) of a country is identically equal to its capital account deficit (surplus).

The balance of trade surplus is the excess of domestic product over domestic absorption. Domestic absorption is the sum of consumption and domestic capital formation.

$$B_t = L_t \, f(k_t) - c_t^1 \, L_t - c_t^2 \, L_{t-1} - (K_{t+1} - K_t)$$

or

$$(40) \qquad b_t = f(k_t) - c_t^1 - \frac{c_{t-1}^2}{1+n} - \left((1+n)k_{t+1} - k_t\right)$$

The current account surplus is the excess of national product over domestic absorption. National product equals domestic product plus net foreign investment income. If $A_t$ is the wealth of the home country at the beginning of period t, net claims on the rest of the world are $A_t - K_t$ and net foreign investment income $r_t(A_t - K_t)$. The country's wealth at the beginning of period t consists of the accumulated saving of the members of generation t-1.

$$A_t = (w_{t-1} - c_{t-1}^1) \, L_{t-1}$$

or

$$(41) \qquad a_t = \frac{w_{t-1} - c_{t-1}^1}{1+n}$$

National wealth can, and with different pure rates of time preference will differ from the value of the domestic capital stock because of the scope for international borrowing and lending. There is a presumption that a nation consisting of people with high rates of time preference will tend to be a net foreign borrower. The truth of this presumption will be investigated in what follows.

The current account surplus of the home country is given by:

$$G_t = L_t\, f(k_t) + r_t(A_t - K_t) - c_t^1\, L_t - c_{t-1}^2\, L_{t-1} - (K_{t+1} - K_t)$$

or

$$(41) \qquad g_t = f(k_t) + r_t(a_t - k_t) - c_t^1 - \frac{c_{t-1}^2}{1+n} - \left((1+n)k_{t+1} - k_t\right)$$

With constant returns to scale in production and marginal productivity factor pricing, $f(k_t) = w_t + r_t k_t$. (41) can therefore be rewritten as:

$$(41') \qquad g_t = w_t + r_t a_t - c_t^1 - \frac{c_{t-1}^2}{1+n} - \left((1+n)k_{t+1} - k_t\right)$$

Equivalently, the current account surplus can be viewed as the net foreign investment by the home country. Net foreign investment is the excess of domestic wealth accumulation (saving) over domestic capital formation, i.e.,

$$G_t = A_{t+1} - A_t - (K_{t+1} - K_t)$$

or

$$(42) \qquad g_t = (1+n)\, a_{t+1} - a_t - \left((1+n)k_{t+1} - k_t\right)$$

Steady-state equilibrium

The steady state equilibrium of the two-country model is given in equations (43) - (50).

$$(43) \qquad \frac{(1+\rho)v'(c^1)}{v'(c^2)} = 1 + r$$

$$(44) \qquad (1 + \bar{\rho})\, \frac{v'(\bar{c}^1)}{v'(\bar{c}^2)} = 1 + r \qquad \underline{9/}$$

$$(45) \qquad w - c^1 = \frac{c^2}{1+r}$$

(46) $w - \bar{c}^1 = \frac{\bar{c}^2}{1+r}$

(47) $w - \frac{1}{2}(c^1 + \bar{c}^1) = k(1+n)$

(48) $b = f(k) - c^1 - \frac{c^2}{1+n} - nk$

or

(48') $b = (n-r)(a-k)$

(49) $g = f(k) + r(a-k) - c^1 - \frac{c^2}{1+n} - nk$

or

(49') $g = n(a-k)$

(50) $a = \frac{w-c^1}{1+n}$

Neither the steady-state trade balance nor the steady-state current account balance need be in equilibrium. As regards the current account balance this is the result of a non-zero rate of population growth and different tastes. If e.g., the home country is a net lender to the rest of the world ($a > k$), steady-state equilibrium requires a current account surplus sufficient to maintain the real per capita value of net claims on the rest of the world, i.e., a surplus of $n(a-k)$ (equation (49')). A steady-state current account surplus will be associated with a steady-state trade balance surplus if the interest rate is less than the rate of population growth, with a steady-state trade balance deficit otherwise (48').

Because the real wage and the interest rate are equalized throughout the world economy, the present discounted value of lifetime resources and lifetime consumption is equal for residents of both countries ((45) and (46)). If and only if the two countries have identical lifetime consumption paths [$c^1 = \bar{c}^1$ and $c^2 = \bar{c}^2$] will there be no net foreign lending or borrowing.

Each economic agent "lives within his means" in the sense that the present discounted value of lifetime resources (period 1 labor income) equals the present discounted value of lifetime consumption. Still except when tastes are identical or population is stationary, one country will be steadily accumulating claims on the other.

We next establish the following three propositions about the steady-state equilibrium of the world economy.

Proposition 6: The country whose residents have a higher value of consumption when young, will run a steady-state current account deficit.

Proof: From (50) we know that $a = \frac{w-c^1}{1+n}$ and $\bar{a} = \frac{w-\bar{c}^1}{1+n}$. Thus $c^1 > \bar{c}^1$ implies $a < \bar{a}$. Rewrite (47) as $a - k + \bar{a} - k = 0$. Therefore $a - k = -(\bar{a} - k) < 0$. The current account surplus is $n(a - k) < 0$. This proposition unlike the following two, does not depend on the additively separable specification of the utility function.

Proposition 7: The country with the higher pure rate of time preference has a steady-state current account deficit.

Proof: From Proposition 6 we know that it suffices to show that $\rho > \bar{\rho}$ implies $c^1 > \bar{c}^1$. This was shown in Proposition 1.

Proposition 8: The common steady-state open economy capital-labor ratio lies between the two autarky capital-labor ratios if the model is stable under autarky.

Proof: Let $k^*$ be the common open economy capital-labor ratio, $k_H$ the autarky domestic capital-labor ratio and $k_F$ the autarky foreign capital-labor ratio. We again assume that $\rho > \bar{\rho}$. For brevity we write steady-state consumption in period 1 as $c^1(k, \rho)$ and $\bar{c}^1(k, \bar{\rho})$, with $c^1_\rho, \bar{c}^1_{\bar{\rho}} > 0$. From the fact that consumption in period 1 is an increasing function of $\rho$, it follows that $\bar{c}^1(k, \bar{\rho}) < \frac{1}{2}[c^1(k, \rho) + \bar{c}^1(k, \bar{\rho})] < c^1(k, \rho)$. To establish the truth of Proposition 8 we must rank the k's determined by the following three capital market equilibrium conditions.

$$f(k_H) - k_H f'(k_H) - c^1(k_H, \rho) = k_H(1+n)$$

$$f(k^*) - k^* f'(k^*) - \frac{1}{2}[(c^1(k^*, \rho) + \bar{c}^1(k^*, \bar{\rho}))] = k^*(1+n)$$

$$f(k_F) - k_F f'(k_F) - \bar{c}^1(k_F, \rho) = k_F(1+n)$$

Assume that $k^*$ is below the lower of the two **autarky** capital-labor ratios, $k_H$. In the **autarky** case, at a value of k below $k_H$ such as $k^*$, there would be excess demand (i.e., excess saving) in the capital market--if the model is stable. A fortiori there would be excess demand if, in the home country capital market equilibrium condition, $c^1(k^*, \rho)$ is replaced by the smaller world-average level of consumption $\frac{1}{2}(c^1(k^*, \rho) + \bar{c}^1(k^*, \bar{\rho}))$. Thus capital market equilibrium requires $k^* > k_H$. By exactly analogous reasoning, it is established that $k^* < k_F$.

Diagrammatically, the stationary equilibrium of the two-country model can be represented as in Figure 4. OF is the stationary decentralized consumption possibility locus for the hypothetical average inhabitant of this two-country world economy, i.e., someone with a lifetime consumption pattern $\frac{1}{2}(c^1 + \bar{c}^1)$, $\frac{1}{2}(c^2 + \bar{c}^2)$. It incorporates the world capital market equilibrium condition (47) and the hypothetical average world citizen's budget constraint (the sum of the two private budget constraints (45) and (46). I.e., OF represents $\frac{1}{2}(c^2 + \bar{c}^2)$ as a function of $\frac{1}{2}(c^1 + \bar{c}^1)$, solved from:

$$f(k) - kf'(k) - \frac{1}{2}(c^1 + \bar{c}^1) = k(1+n) \quad (51)$$

$$f(k) - kf'(k) - \frac{1}{2}(c^1 + \bar{c}^1) = \frac{\frac{1}{2}(c^2 + \bar{c}^2)}{1+f'(k)} \quad (52)$$

OF also represents the stationary autarky consumption possibility locus for each country individually.

While (51) and (52) must be satisfied in a full stationary equilibrium of the world economy, they are not a complete characterization of the stationary decentralized consumption possibility locus. Residents of each country must satisfy their budget constraints individually and not merely on average, across both countries as in (52). In Figure 4, the indifference curve of each country is tangent to the common world capital market line. The home country, after integration into the world economy, consumes at $E_H^T$. Its high level of $c^1$ is a reflection of its high pure rate of time preference. $E_F^T$ is the open economy consumption equilibrium of the foreign country with the low pure rate of time preference. Average world consumption (and world production in both countries) is at $E_W^T$. Consistency requires that the distance $E_F^T - E_W^T = E_W^T - E_H^T$. The home country borrows from the foreign country. From (50) and (47) we obtain that $a-k = \frac{1}{2}\left(\frac{\bar{c}^1 - c^1}{1+n}\right) < 0$. The home country's debt to the foreign country grows at the natural rate of growth. As we have drawn it, $E_W^T$ is at a capital-labor ratio below the golden rule, i.e. $r > n$. The home country therefore runs a trade balance surplus. If the equilibrium had been beyond $E^*$, the home country would still run a current account deficit but with $n > r$ it would now also run a trade balance deficit. The autarky equilibrium for the home country is at $E_H^{NT}$, at a capital-labor ratio below the common post-trade capital-labor ratio. The autarky equilibrium for the foreign country is at $E_F^{NT}$, at a capital-labor ratio above the common open economy capital-labor ratio.

Non-steady-state behavior

The behavior of the two-country model outside the steady state is not much more complicated than its steady-state behavior. There is only one state variable--the capital-labor ratio. Provided the model is stable, convergence to the steady-state equilibrium will therefore be monotonic. The current account and trade account surpluses in period t can also be expressed as a function of $k_{t-1}$. Consider the current account surplus of the home country:

$$(53) \quad g_t = f(k_t) + r_t\left(\frac{w_{t-1} - c^1_{t-1}}{1+n} - k_t\right) - c^1_t - \frac{c^2_{t-1}}{1+n} - \left((1+n)k_{t+1}-k_t\right).$$

$c^1_t$ can be expressed as a function of $w_t$ and $r_{t+1}$ (and $\rho$). $c^1_{t-1}$ and $c^2_{t-1}$ can be expressed as functions of $w_{t-1}$ and $r_t$. Using the competitive factor pricing conditions we can write the current account surplus as

$$(54) \quad g_t = \mu(k_{t+1}, k_t, k_{t-1})$$

From (37) we know that $k_t$ can be expressed as a function of $k_{t-1}$. Equation (54) can therefore be rewritten as:

$$(54') \quad g_t = \gamma(k_{t-1})$$

Will the country with the higher pure rate of time preference run a current account deficit throughout the non-steady state adjustment process, the way it does in the steady state? Corresponding to (53) is the current account surplus equation for the foreign country:

$$(55) \quad \bar{g}_t = f(k_t) + r_t\left(\frac{w_{t-1}-\bar{c}^1_{t-1}}{1+n} - k_t\right) - \bar{c}^1_t - \frac{c^2_{t-1}}{1+n} - \left((1+n)k_{t+1}-k_t\right).$$

In this two-country world $g_t + \bar{g}_t \equiv 0$. Comparing (53) and (55) we see that with $\rho > \bar{\rho}$, $\bar{c}^1_t < c^1_t$, $\bar{c}^1_{t-1} < c^1_{t-1}$ and $\bar{c}^2_{t-1} > c^2_{t-1}$. The excess of period 1 consumption in the home country over period 1 consumption in the foreign country makes for a current account deficit in the home country. The excess of period 2 consumption in the foreign country over period 2 consumption in the home country has the opposite effect. The home country will run a current account deficit if and only if

$$(56) \quad \frac{r_t(c^1_{t-1}-\bar{c}^1_{t-1})}{1+n} + c^1_t - \bar{c}^1_t > \frac{\bar{c}^2_{t-1}-c^2_{t-1}}{1+n}$$

Using the private sector budget constraints (29) and (31) this can be rewritten as

$$(56') \qquad c_t^1 - \bar{c}_t^{-1} > \frac{c_{t-1}^1 - \bar{c}_{t-1}^{-1}}{1+n}$$

With n > 0 this condition is certainly satisfied in the steady state with $c_t^1 = c_{t-1}^1$ and $\bar{c}_t^{-1} = \bar{c}_{t-1}^{-1}$, because $c^1 > \bar{c}^{-1}$. Outside the steady state it will be satisfied if the increase in total consumption by the younger generation between period t-1 and period t in the home country exceeds that in the foreign country. This can be seen by rewriting (56') as

$$(56'') \qquad L_t c_t^1 - L_{t-1} c_{t-1}^1 > L_t \bar{c}_t^{-1} - L_{t-1} \bar{c}_{t-1}^{-1}$$

The current account deficit equals the excess of domestic capital formation over national saving. This holds for both countries. With perfect international capital mobility $k_t$ is the same in both countries in each period. Domestic capital formation will therefore be the same in the two countries. The home country will therefore run a current account deficit i.f.f. domestic saving falls short of foreign saving, i.e., i.f.f. $L_t(w_t - c_t^1) - L_{t-1}(w_{t-1} - c_{t-1}^1) < L_t(w_t - \bar{c}_t) - L_{t-1}(w_{t-1} - \bar{c}_{t-1}^{-1})$. This is equivalent to (56"). While with $\rho > \bar{\rho}$, each generation in the home country saves less than the corresponding generation in the foreign country, national saving in the home country need not be less in any given period than national saving in the foreign country. National saving is the sum of saving by the young and dissaving by the old. In the home country the young save less but as a consequence the old dissave less. Consider the simple example in which the young in the home country save a constant fraction $\eta$ of their labor income and the young in the foreign country save a constant fraction $\bar{\eta}$; $0 < \eta, \bar{\eta} < 1$; $\eta < \bar{\eta}$. The home country will then run a current account deficit provided the total amount of income going to labor is not declining. This can be seen as follows. With the constant proportional saving assumption total home country saving $S_H$ minus total foreign saving $S_F$ can be written as $S_H - S_F = L_t(1-\eta)w_t - L_{t-1}(1-\eta)w_{t-1} - L_t(1-\bar{\eta})w_t + L_{t-1}(1-\eta)w_{t-1}$. Thus $S_H - S_F < 0$ i.f.f. $(\bar{\eta}-\eta)(w_t - \frac{w_{t-1}}{1+n}) < 0$. With $\bar{\eta} > \eta$ this requires $L_t\ w_t > L_{t-1}\ w_{t-1}$. If total labor income declines, the dissaving of the old will, at constant savings rates out of labor income, not be matched by the saving of the young. The country with the higher savings rate out of labor income will therefore, summing over the young and the old, be dissaving faster than the country with the lower savings rate. Identifying the higher savings rate with a lower pure rate of time preference, the country with the lower pure rate of time preference will be running a current account deficit. In the steady state the (in general endogenous) constant savings rate in the home country is lower than in the foreign country and labor income grows at the natural rate of growth n. Outside the steady state labor income may well decline during the adjustment process, especially if the initial capital-labor ratio is above the steady-state capital-labor ratio. The adjustment process then involves a monotonic decline in k which would hurt labor's share, especially if the elasticity of substitution is low. Note that it is not necessary for the approach to the steady-state current account deficit to be monotonic. The home country could alternate between periods of deficits and surpluses although ultimately it will run a deficit.

Scale differences

The model can be generalized in a straightforward manner to include scale differences. For a steady state to exist, the rates of population growth in the two countries must be the same, but the scale of the two countries, as measured by the size of their populations, $L_t$ and $\bar{L}_t$ need not be the same. A restatement of the model with allowance for unequal populations involves minimal changes. All of equations (28) - (33) are retained. The world capital market equilibrium condition (34) becomes: $(w_t-c_t^1)L_t + (w_t-\bar{c}_t^1)\bar{L}_t = K_{t+1} + \bar{K}_{t+1}$. With a common rate of growth of population, n, the ratio of $L_t$ to $\bar{L}_t$ will be constant, say at $L/\bar{L}$. The world capital market equilibrium condition can now be written as[10/]

$$(57) \qquad f(k_t) - k_t f'(k_t) - [c_t^1 \cdot (\frac{L}{L+\bar{L}}) + \bar{c}_t^1 \cdot (\frac{\bar{L}}{L+\bar{L}})] = k_{t+1}(1+n)$$

The lifetime budget constraint of the representative or average world consumer can be obtained from the budget constraints of the consumers in the two countries, (29) and (31). This yields:

$$(58) \qquad f(k_t)-k_t f'(k_t) - [c_t^1(\frac{L}{L+\bar{L}}) + \bar{c}_t^1(\frac{\bar{L}}{L+\bar{L}})] = \frac{c_t^2(\frac{L}{L+\bar{L}}) + \bar{c}_t^2(\frac{\bar{L}}{L+\bar{L}})}{1+f'(k_{t+1})}$$

Figure 5 illustrates the stationary equilibrium of the world economy with $\frac{L}{L+\bar{L}} = \frac{1}{4}$. The equal population size case considered earlier and illustrated in Figure 4 is the special case of the more general model with $\frac{L}{L+\bar{L}} = \frac{\bar{L}}{L+\bar{L}} = .5$. Because the population of the home country is only a third of that of the foreign country, $E_F^T-E_W^T$ equals one-third of $E_W^T-E_H^T$ in Figure 5.

Gains from participation in the world economy

The positive analysis of the determinants of international trade and lending can be complemented in a natural way by the analysis of the gains from participation in the international economy.[11/] When evaluating the gains or losses from participation in the world economy, welfare in the world economy will be compared with the welfare that would have been achieved under continued autarky. Both the short-run effects on the welfare of the generations alive at the moment that the two countries are integrated and the long-run effects will be considered.

Short-run effects

The two countries evolve under autarky up to and including period t-1. In period t the two national capital markets are integrated into a world capital market. It does not matter whether this change of regime is anticipated or unanticipated. In period t there are two generations alive: the old, born in t-1, and the young, born in t. Financial integration does not in any way affect the opportunity set of the old generation. Its members consume the savings made in t-1 plus accumulated interest. The interest rate faced by the generation born in t-1 is the relevant autarky rate, $r_t^H$ for the home country and $r_t^F$ for the foreign country. It is predetermined when the change to the open economy takes place. The welfare of the old generations is therefore unaffected by the integration into the world economy.

The wage rate faced by the generation born in period t is also predetermined in t and independent of the change of regime in that period. Young residents of the home country face a wage rate $w_t^H = f(k_t^H)-k_t^H f'(k_t^H)$ while in the foreign country the wage rate is: $w_t^F = f(k_t^F)-k_t^F f'(k_t^F)$. Thus the welfare of the younger generation of the home country under openness, relative to what it would have been under continued autarky, will improve if the common world interest rate $r_{t+1}$ is greater than the autarky interest rate, $r_{t+1}^H$. Equivalently, let $\{k_\tau^H\}$ be the sequence of capital-labor ratios for the home country under autarky, $\{k_\tau^F\}$ the sequence of capital-labor ratios for the foreign country under autarky and $\{k_\tau;\ \tau = t+1,\ t+2,\ \ldots\}$ the sequence of common, open economy capital-labor ratios. Then the young generation of the home country alive when the autarkic economies are opened up in $\tau=t$ will be better off than it would have been under continued autarky if $k_{t+1} < k^H_{t+1}$. By the same argument the young generation of the foreign country alive during the transition to openness will be better off than they would have been under autarky if $k_{t+1} < k^F_{t+1}$ or $r_{t+1} > r^F_{t+1}$, where $\{r^F_\tau\}$ is the sequence of interest rates under autarky for the foreign country. Diagramatically the welfare reducing effect of a lower interest rate, given the wage rate, can be represented by a downward pivot of the consumer's budget constraint in Figure 1 from $w_t - w_t(1 + r_{t+1})$ to $w_t - w_t(1 + r'_{t+1})$. A tangency of an indifference curve to the new budget line necessarily corresponds to a lower level of welfare. Proposition 9 below states that the young generation of the home country (with the high pure rate of time preference) will suffer a welfare loss as a result of integration into the world economy while the young generation of the foreign country enjoys a welfare gain. The argument assumes that at the time of integration, t, the capital-labor ratio of the home country is not above the capital-labor ratio of the foreign country. This will always be the case if either the two countries started off from the same initial condition (Proposition 4) or, from arbitrary initial conditions, the system had been evolving under autarky for a sufficiently long period of time (Propositions 2, 3 and 5).

Proposition 9: If $k^H_t \leq k^F_t$ and if an increase in the interest rate increases saving then $k^H_{t+1} < k_{t+1} < k^F_{t+1}$ or $r^F_{t+1} < r_{t+1} < r^H_{t+1}$.

Proof: The proof is by contradiction. Assume that $r_{t+1} > r^H_{t+1}$. From Propositions 2 and 3 we know that $r^H_{t+1} > r^F_{t+1}$. Therefore if $c^1_r < 0$ and $\bar{c}^1_r<0$ saving by generation t under openness in both the home country and the foreign country is higher than it would have been under autarky. Thus the per capita open economy saving by generation t in the home country exceeds $(1+n)k^H_{t+1}$ and the per capita open economy saving by generation t in the foreign country exceeds $(1+n)k^F_{t+1}$. With the same size populations in both countries, average per capita saving by generation t in the world economy therefore certainly exceeds $(1+n)k^H_{t+1}$. ($r^H_{t+1} > r^F_{t+1}$ implies $k^H_{t+1} < k^F_{t+1}$.) Average per capita saving by generation t in the world economy equals the average per capita capital stock in the world economy in period t+1. Therefore $k_{t+1} > k^H_{t+1}$. This contradicts the assumption that $r_{t+1} > r^H_{t+1}$. Exactly analogous reasoning yields the result that $k_{t+1} < k^F_{t+1}$.

The impact effect of participation in the world economy on welfare is therefore unambiguous. The opportunity sets of the retired citizens of both countries alive at the moment of the transition are unaffected. Their welfare is the same as under continued autarky. The utility of the young in the high time preference country is lowered, while the utility of the young in the low time preference country is raised. The economic intuition is clear. Under autarky the low time preference country will have a higher real wage and a lower interest rate. During the period that the two countries are integrated into a world economy, the young citizens of the low time preference country inherit a high real wage from the autarky phase. The interest rate at which they can transfer resources into retirement is higher than under autarky, however. There has been an outflow of capital from the low time preference, low interest country to the high time preference, high interest country, and the common world rate of interest lies between the two interest rates that would have prevailed under continued autarky. The budget constraint of the low time preference country's young generation pivots up, that of the high time preference country's young generation pivots down.

The budget constraint of the generation born in period t+1 in the high time preference country will have an abscissa, $w_{t+1}$, which is higher than it would have been under continued autarky, $w^H_{t+1}$. The interest rate faced by this generation, $r_{t+2}$, is lower than it would have been under continued autarky. The opposite holds for the low time preference country. This pattern is repeated in every following period. Thus the effect on welfare is ambiguous, as is evident from the budget constraints drawn in Figures 6.a and 6.b.

Long-run effects

It is not possible, except in special cases, to make unambiguous welfare comparisons by comparing steady-state utility levels--the ranking of the stationary utility levels may not be the same as the ranking of the utility levels achieved during the transition from one steady state to another. Nevertheless, the stationary utility levels achieved under autarky and openness are of interest. They are analyzed next. There are several cases to consider, illustrated in Figure 7a-g. In Figure 7a the autarky capital-labor ratios of both countries are in the inefficient region, above the golden rule capital-labor ratio. The autarky equilibrium of the home country is at $E^{NT}_H$, that of the foreign country at $E^{NT}_F$ (Proposition 5). The golden rule capital-labor ratio is at E*. The open economy equilibrium of the home country is at $E^T_H$, that of the foreign country at $E^T_F$. World equilibrium is at $E^T_W$. The open economy capital-labor ratio is between the two closed economy capital-labor ratios (Proposition 8). Overaccumulation (relative to the golden rule) is reduced in the foreign country, increased in the home country. Because the slope of the autarky budget constraint of the home country is steeper than the slope of the open economy budget constraint, and because of the strict convexity of the indifference curves, the stationary level of utility for the home country (the high pure rate of time preference country) is always strictly greater under openness than under autarky. This has to be balanced against the short-run losses incurred when the transition occurs. In Figure 7a, these short-term losses are reflected in the need of the home country to raise the steady-state capital-labor ratio from $E_H^{NT}$ to $E_W^T$. In Figure 7a the stationary utility level of the foreign country is less under openness than under autarky. Since the foreign country experiences a short-run welfare gain when the two countries are integrated into the world economy, one cannot argue that openness makes the foreign country unambiguously worse off. Figure 7b shows that it is possible for the stationary utility level of the foreign country to be higher under openness than under autarky. Since the foreign country decumulates capital from $E_F^{NT}$ to $E_W^T$, it can be said that in Figure 7b the foreign country experiences a potential pareto-improvement as a result of openness.

In Figure 7c both countries have autarky equilibria at capital-labor ratios below the golden rule value. The stationary utility level of the home country under openness (at $E_H^T$) is always higher than under autarky (at $E_H^{NT}$). However, the home country needs to accumulate capital from $E_H^{NT}$ to $E_W^T$ and will incur utility losses during the adjustment to the new steady state. The stationary utility level of the foreign country under openness, at $E_F^T$ is below that under autarky at $E_F^{NT}$. It decumulates from $E_F^{NT}$ to $E_W^T$ and enjoys a short-run welfare gain during the transition. Figure 7d shows that when both countries have autarky equilibria below the golden rule, the stationary utility level of the foreign country may be higher under openness than under autarky.

The argument does not change qualitatively when the autarky equilibria of the two countries "straddle" the golden rule, with the foreign country's autarky capital-labor ratio above, and the home country's autarky capital-labor ratio below the golden rule capital-labor ratio. Figures 7e, f and g illustrate three possible outcomes. [It does not matter on which side of the golden rule the open economy world equilibrium falls.] In all cases, the stationary utility level of the home country is greater under openness. This then has to be weighted against the short-run losses incurred during the transition. The foreign country gains in the short run and may have a lower (Figure 7e) or a higher (Figures 7f and 7g) stationary level of utility. Figure 7g illustrates the very special case in which the world equilibrium is at the golden rule.

An important clue to these welfare comparisons is provided by the fact that international trade and financial mobility do not expand the consumption possibility set of the "average" world consumer under openness beyond that available to the individual countries under autarky. OF in Figures 4 and 7 is the stationary, decentralized consumption possibility locus for each of the countries under autarky and for the integrated world economy. The same applies in the short run: there are no potential gains in production from specialization. The only difference that openness makes is that a world-wide capital market is substituted for two national capital markets. The equilibrium in this world-wide capital market will be an average of the two autarky equilibria in the national capital markets.

## 4. Conclusion

Diamond's overlapping generations model is extended to a two-country world. A number of results are obtained. The country whose residents consume more in the first period of their lives (at a given wage rate and interest rate) has a steady-state current account deficit if the rate of population growth is positive. The country with the higher pure rate of time preference has a steady-state current account deficit. The common stationary open economy capital-labor ratio lies between the two autarky capital-labor ratios, with the lower autarky capital-labor ratio associated with the country whose residents have the higher pure rate of time preference. Outside the steady state, the country with the higher pure rate of time preference will not necessarily be the one to run a current account deficit. The welfare of the old generation alive when the two autarkic countries join in open economy is unaffected by this international economic integration. The welfare of the young generation in the country with the high pure rate of time preference is lower under openness than under autarky, while in the country with the low pure rate of time preference it is higher. The stationary utility level of the country with the high pure rate of time preference is always higher under openness than under autarky. The effect of openness on the stationary utility level of the low pure rate of time preference country is ambiguous. If it is higher, openness can be said to constitute a potentially pareto-superior regime for the low pure rate of time preference country. Because of the short-run welfare loss it incurs after international economic integration, the open economy is not pareto-superior to the closed economy for the high pure rate of time preference country.

The open economy version of the overlapping generations model can be extended in a number of directions. Within the context of the one-commodity model, budgetary and financial policy can be studied; this includes public sector lending and borrowing, taxation of wage and property income, social security, etc. The two-commodity structure of the Heckscher-Ohlin model can be grafted onto the overlapping generations framework. This allows potential gains from trade through specialization in production. If one of the goods is a capital good, the distinction between international mobility of financial capital--international lending and borrowing--and international trade in capital goods becomes important. One could have either or both. The two-commodity structure permits the analysis of tariffs, quotas and other trade policies, familiar from the static Heckscher-Ohlin model, in addition to the financial and budgetary policies that can be studied in the one-commodity overlapping generations model. Finally, uncertainty can be introduced into the model--most easily into the two-commodity version. Without futures markets for commodities, households have to make labor supply and saving decisions while young, without knowing the future spot prices of commodities with certainty. The scope for further developments appears to be considerable.

# FOOTNOTES

I would like to thank Jeff Carmichael, Jon Eaton and Smith Freeman for useful comments.

1/ For a related approach, which does not, however, incorporate the overlapping generations model see Stiglitz [1970].

2/ $$\frac{\partial c_t^1}{\partial w_t} = \frac{-[u_{12}-(1+r_{t+1})u_{22}]}{[u_{11}-(1+r_{t+1})u_{21}](1+r_{t+1})^{-1}-u_{21}+(1+r_{t+1})u_{22}}$$

$$\frac{\partial c_t^1}{\partial r_{t+1}} = \frac{u_2-c_t^2(1+r_{t+1})^{-1}[u_{21}-(1+r_{t+1})u_{22}]}{u_{11}-(1+r_{t+1})[2u_{21}-(1+r_{t+1})u_{22}]}$$

$$\frac{\partial c_t^2}{\partial w_t} = (1+r_{t+1})\left(1 - \frac{\partial c_t^1}{\partial w_t}\right)$$

$$\frac{\partial c_t^2}{\partial r_{t+1}} = -(1+r_{t+1})\frac{\partial c_t^1}{\partial r_{t+1}} + \frac{c_t^2}{1+r_{t+1}}$$

3/ The historical origins of Figure 2 are not completely clear. It was a well-established part of the "Yale blackboard tradition" when I came to Yale as a graduate student in 1971. James Tobin suggests that David Cass may have been the first one to utilize it. It has since been used as an expository device by Ihori [1978], Buiter [1979] and Carmichael [1979].

4/ In what follows I shall assume that the golden rule capital-labor ratio is less than the capital-labor ratio at F. This need not be the case. From (16b) it is clear that with a Cobb-Douglas production function, $c^1 = 0$ implies either $k = 0$ (at the origin) or $(1 - \alpha)k^{\alpha-1} = (1+n)$. [This requires that k be less than unity at F.] Thus at F, $k^{\alpha-1} = \frac{1+n}{1-\alpha}$. At the golden rule, $k^{\alpha-1} = \frac{n}{\alpha}$. It is likely, but not inevitable, that k at F exceeds the golden rule capital-labor ratio.

5/ The assumptions we have made so far suffice to ensure the existence of a long-run equilibrium (Carmichael [1979]). A (probably overly strong) sufficient condition for the uniqueness of the steady-state equilibrium is that preferences be homothetic (Carmichael [1979]).

6/ I owe this description to Bill Branson.

7/ For brevity I omit the non-negativity constraints. The conditions on u in (1) continue to apply, so the private budget constraint holds with equality and there are interior solutions for $c_t^1$ and $c_t^2$.

8/ Using the more general utility functions $u(c_t^1, c_t^2)$ for the home country and $\bar{u}(\bar{c}_t^1, \bar{c}_t^2)$ for the foreign country we can solve for $c_t^1$ and $\bar{c}_t^1$ as functions of $w_t$ and $r_{t+1}$.

(36a') $c_t^1 = c^1(w_t, r_{t+1})$ (36b') $\bar{c}_t^1 = \bar{c}^1(w_t, r_{t+1})$

The stability analysis carries through unchanged for this more general case.

9/ With the general utility functions of footnote 7, equations (43) and (44) are replaced by

(43') $\frac{u_1(c^1, c^2)}{u_2(c^1, c^2)} = 1 + r$ and (44') $\frac{\bar{u}_1(\bar{c}^1, \bar{c}^2)}{\bar{u}_2(\bar{c}^1, \bar{c}^2)} = 1 + r.$

10/ Note that $\bar{k}_t = \bar{K}_t/\bar{L}_t$. With international capital mobility, $\bar{k}_t = k_t$ for all t.

11/ For simplicity we again assume both countries to have the same size.

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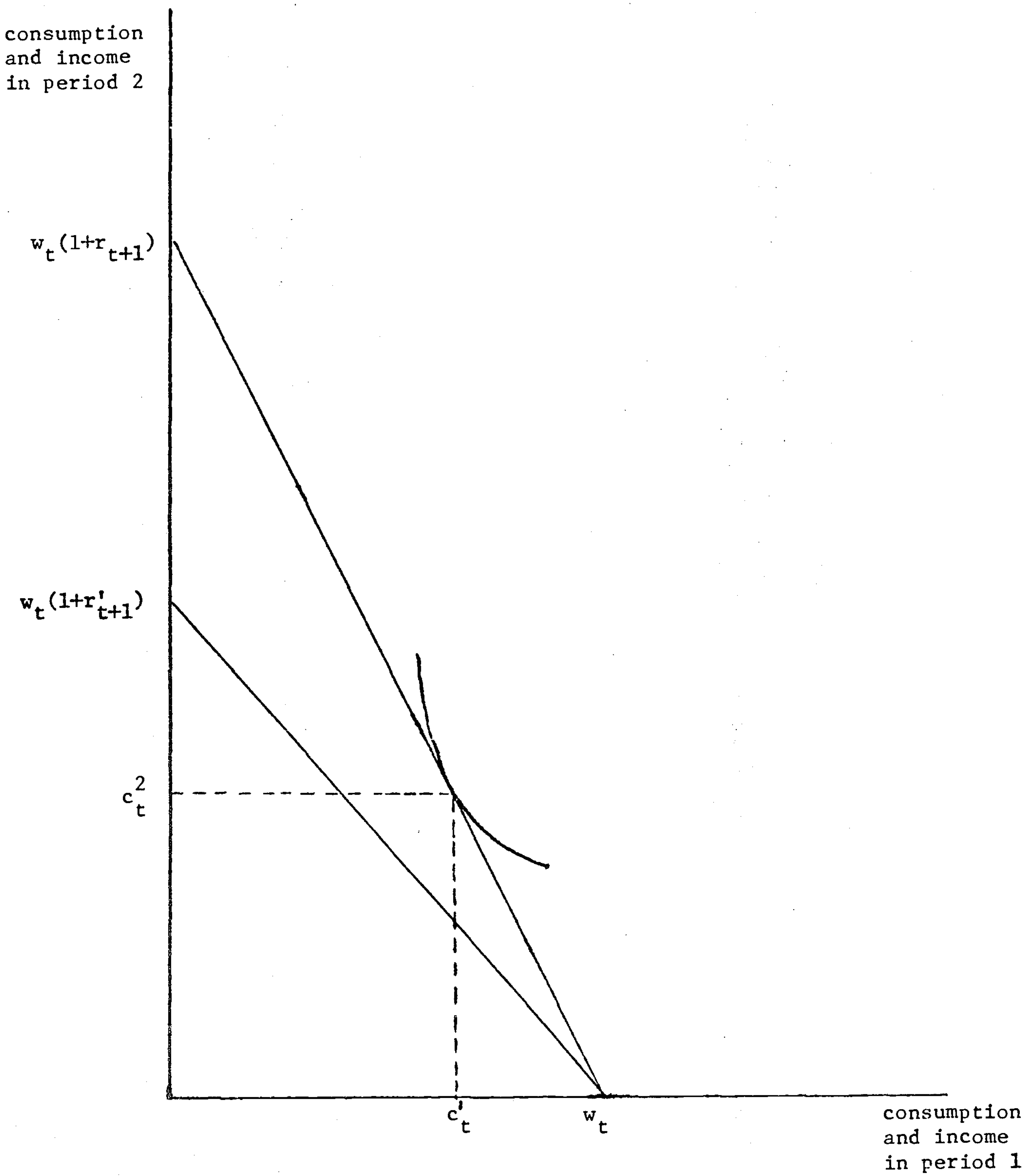
FIGURE 1

The Consumer's Intertemporal Equilibrium



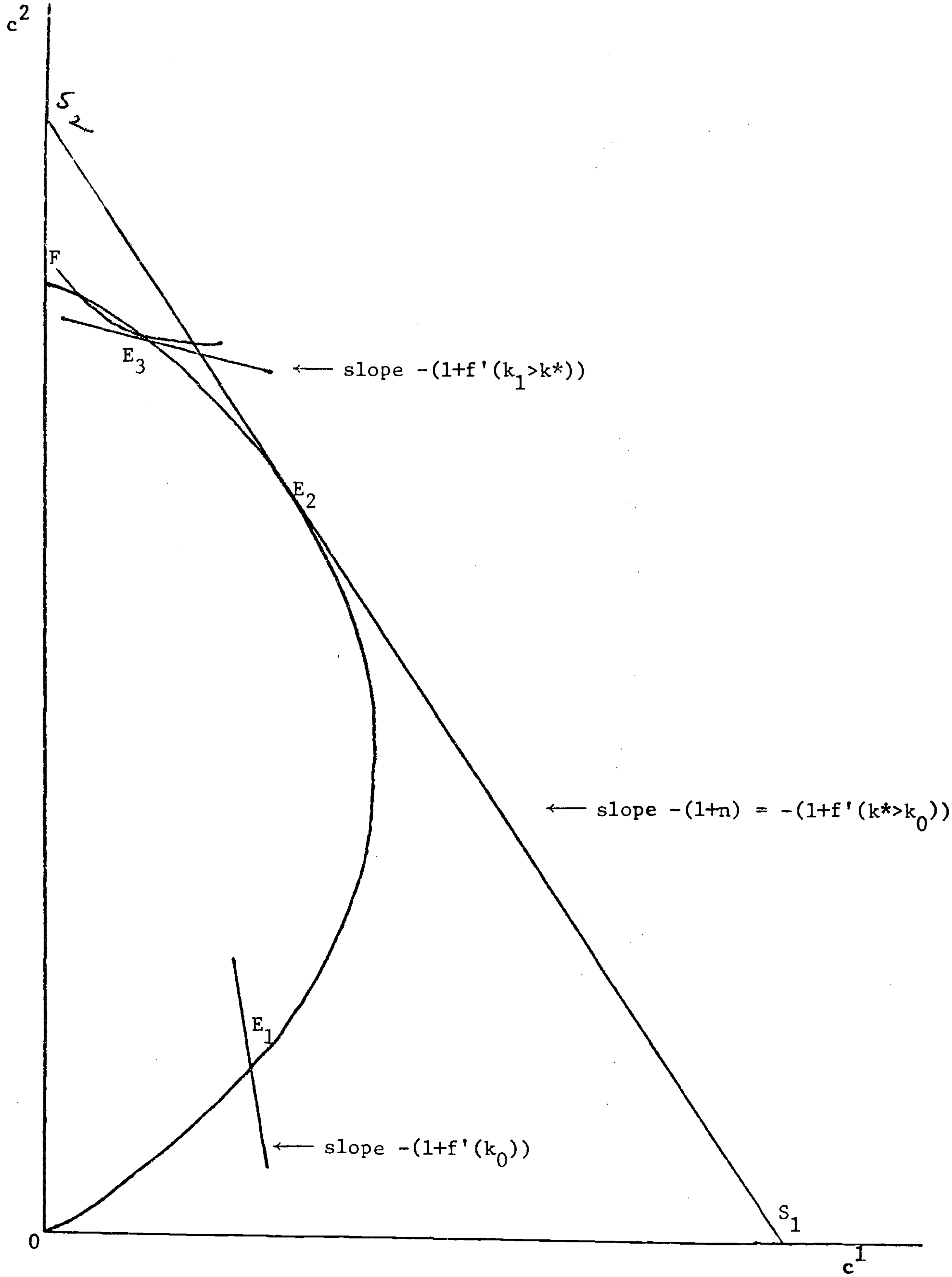



FIGURE 2

The Stationary Competitive Consumption Possibility Locus

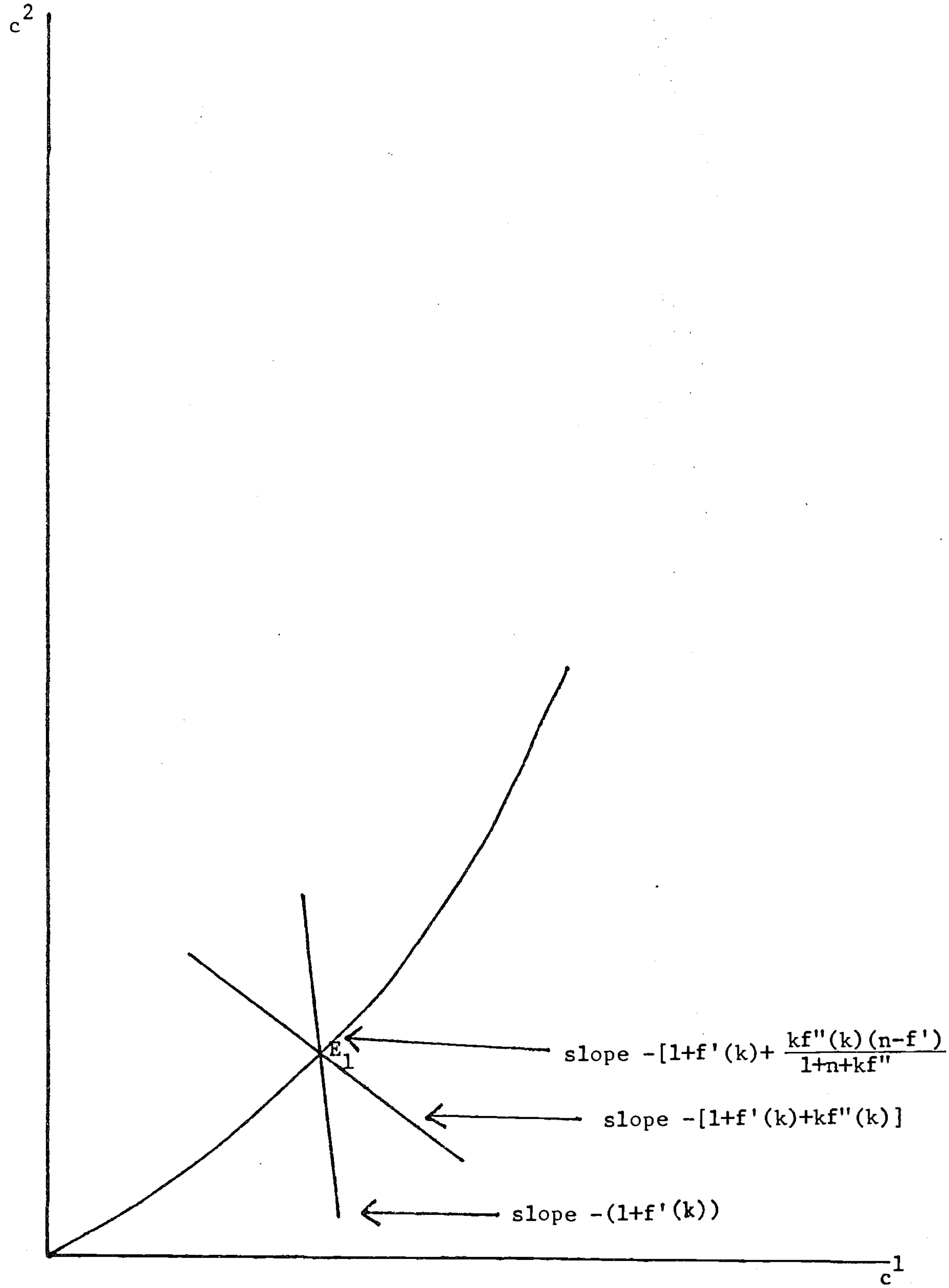



FIGURE 3

Short-run and Long-run Private and Social Transformation Loci

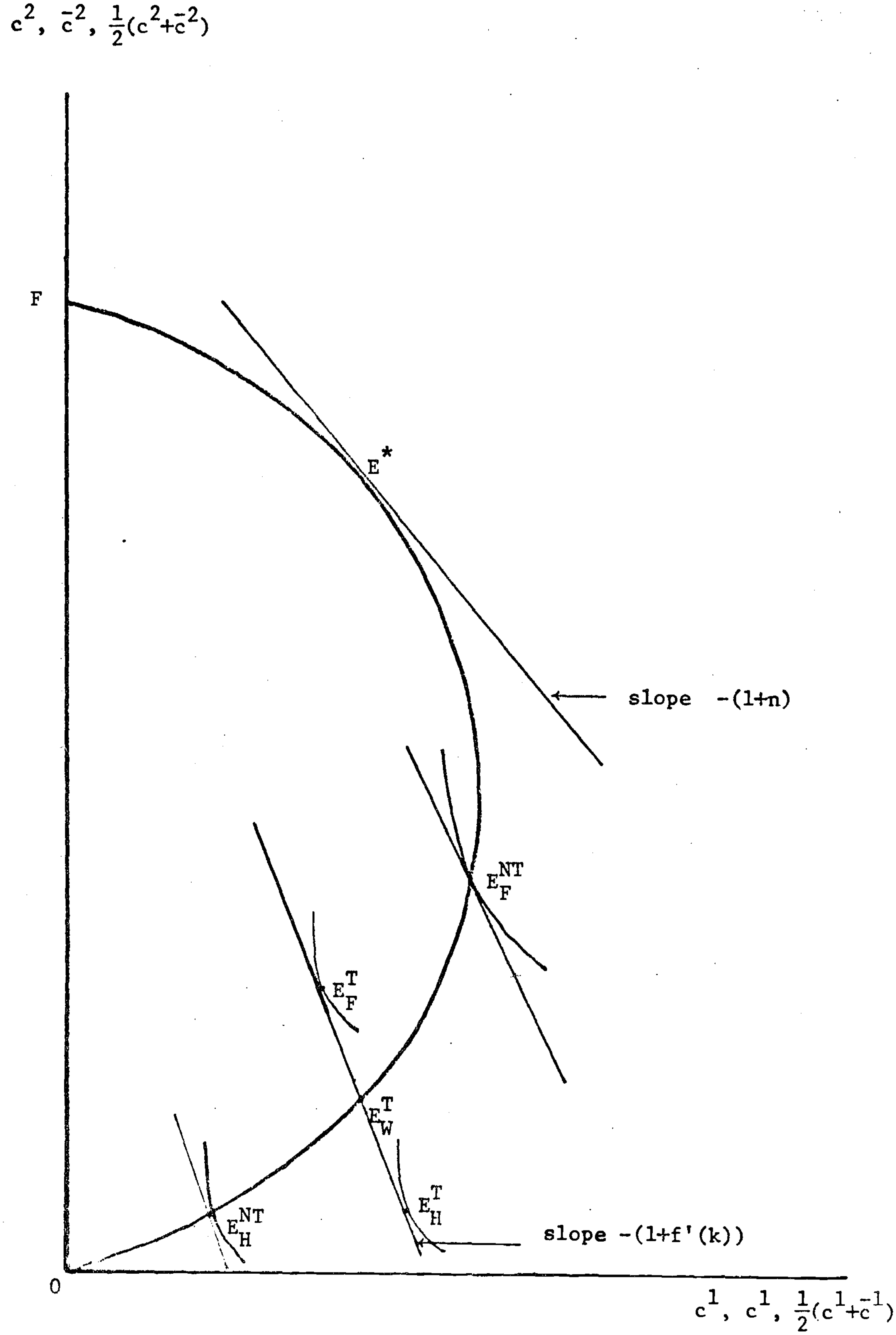



FIGURE 4

Autarky and Open Economy Equilibria for Both Countries

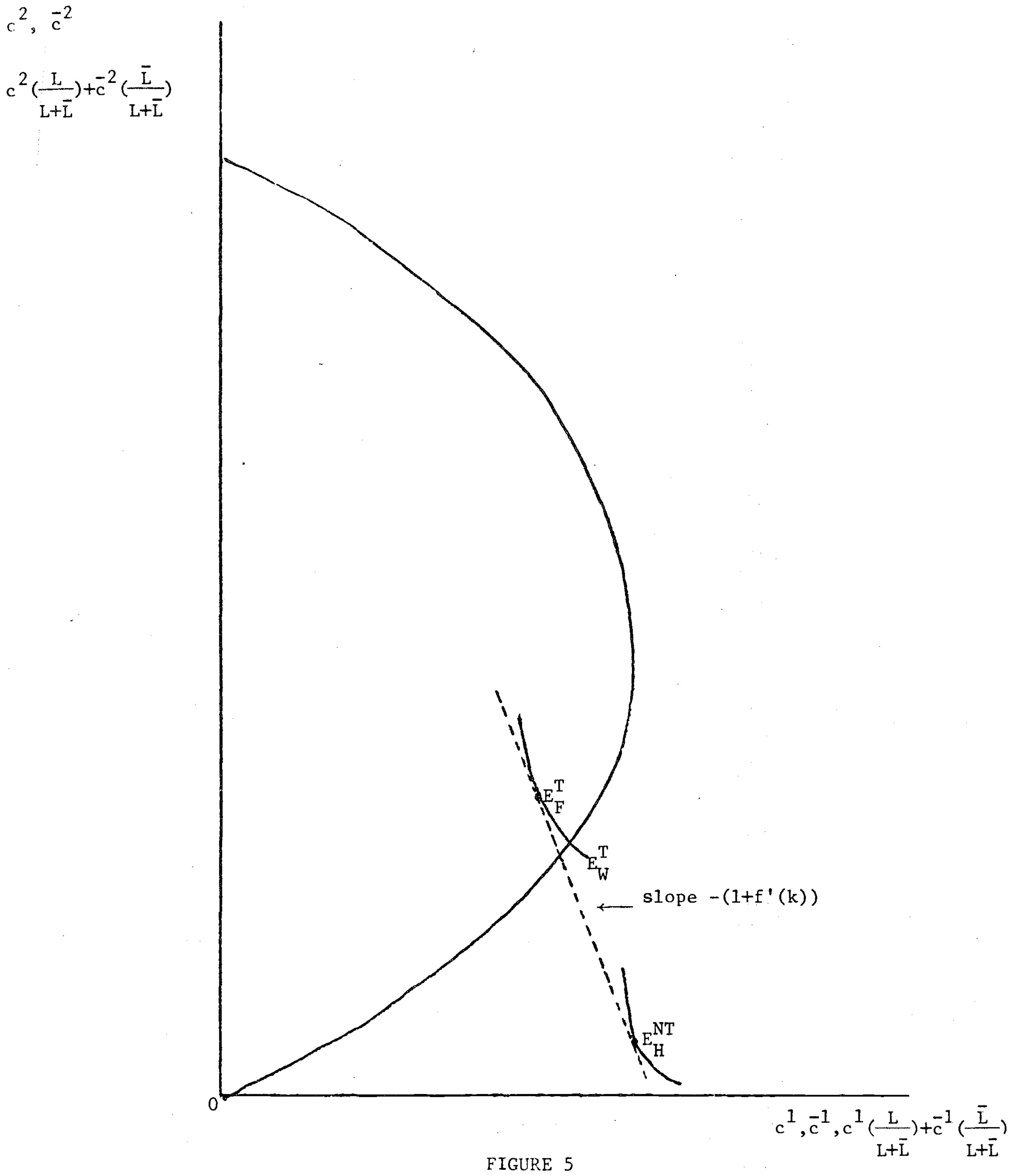
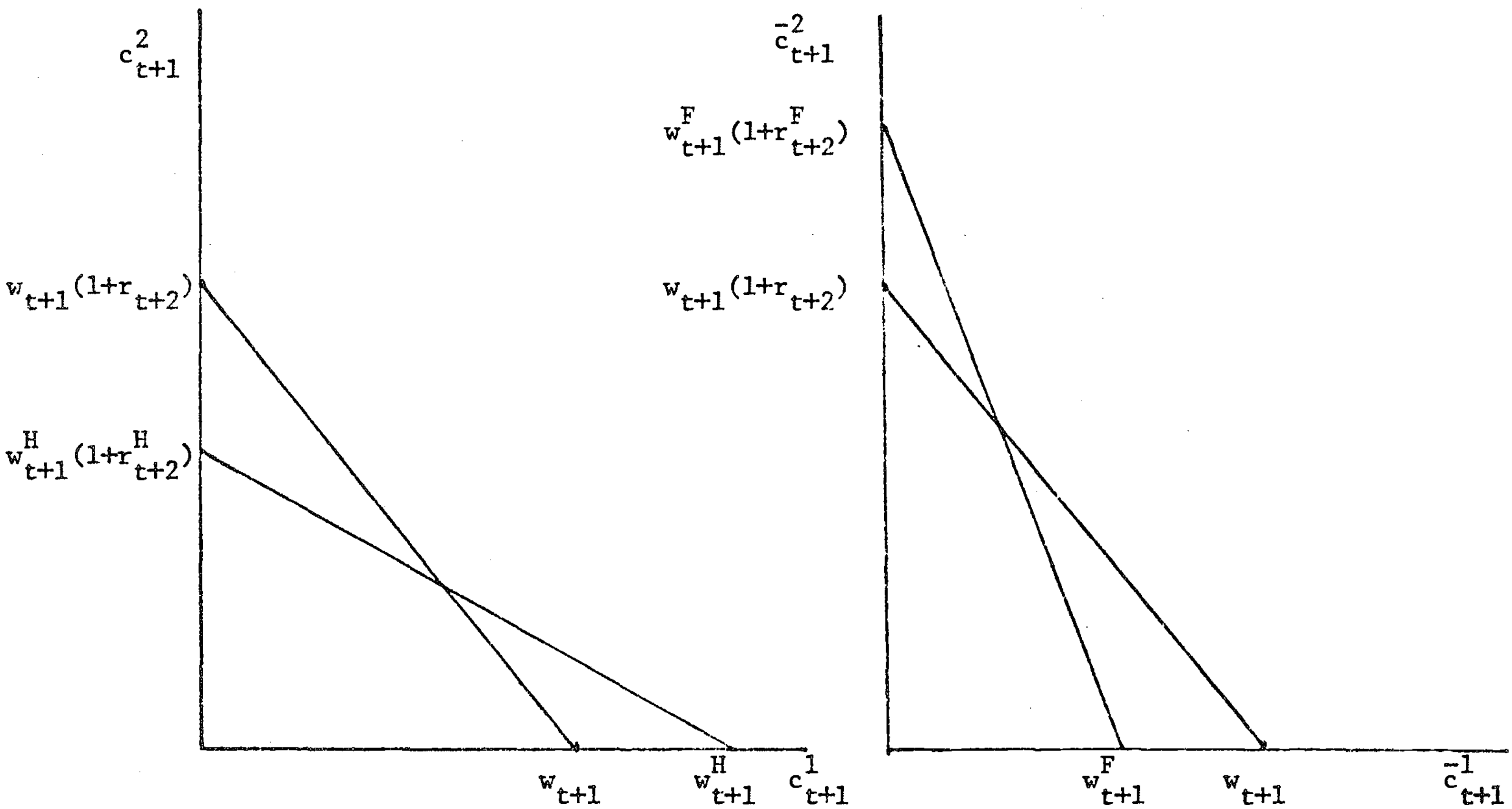



FIGURE 5

Open Economy Equilibrium When the Home Country is Smaller than the Foreign Country

FIGURE 6




(a)

Autarky and open economy budget constraints of members of generation t+1 in the high time preference country.

(b)

Autarky and open economy budget constraints of members of generation t+1 in the low time preference country.

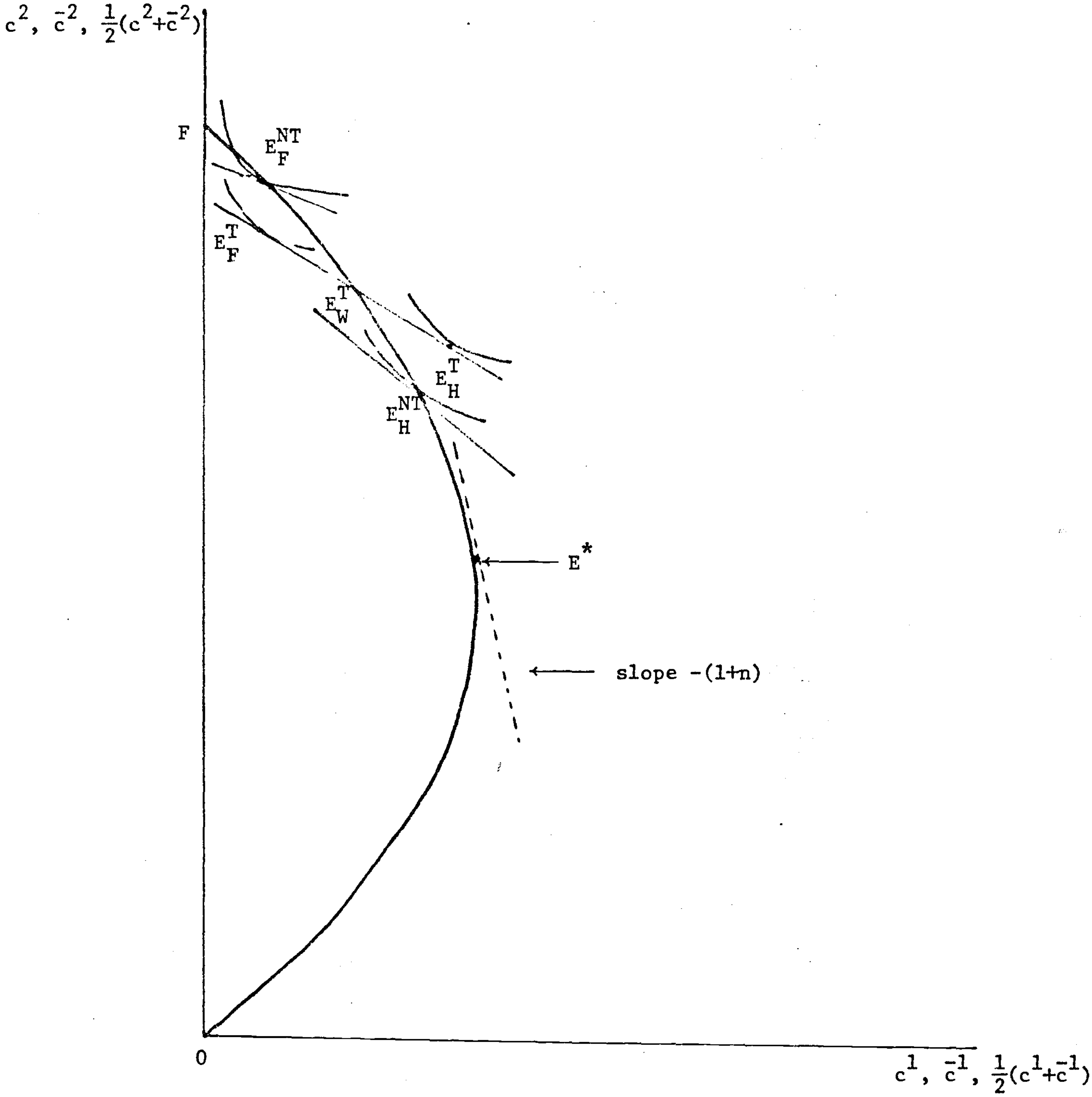



FIGURE 7a

Comparison of stationary utility levels when both countries' autarky equilibria are in the inefficient region.

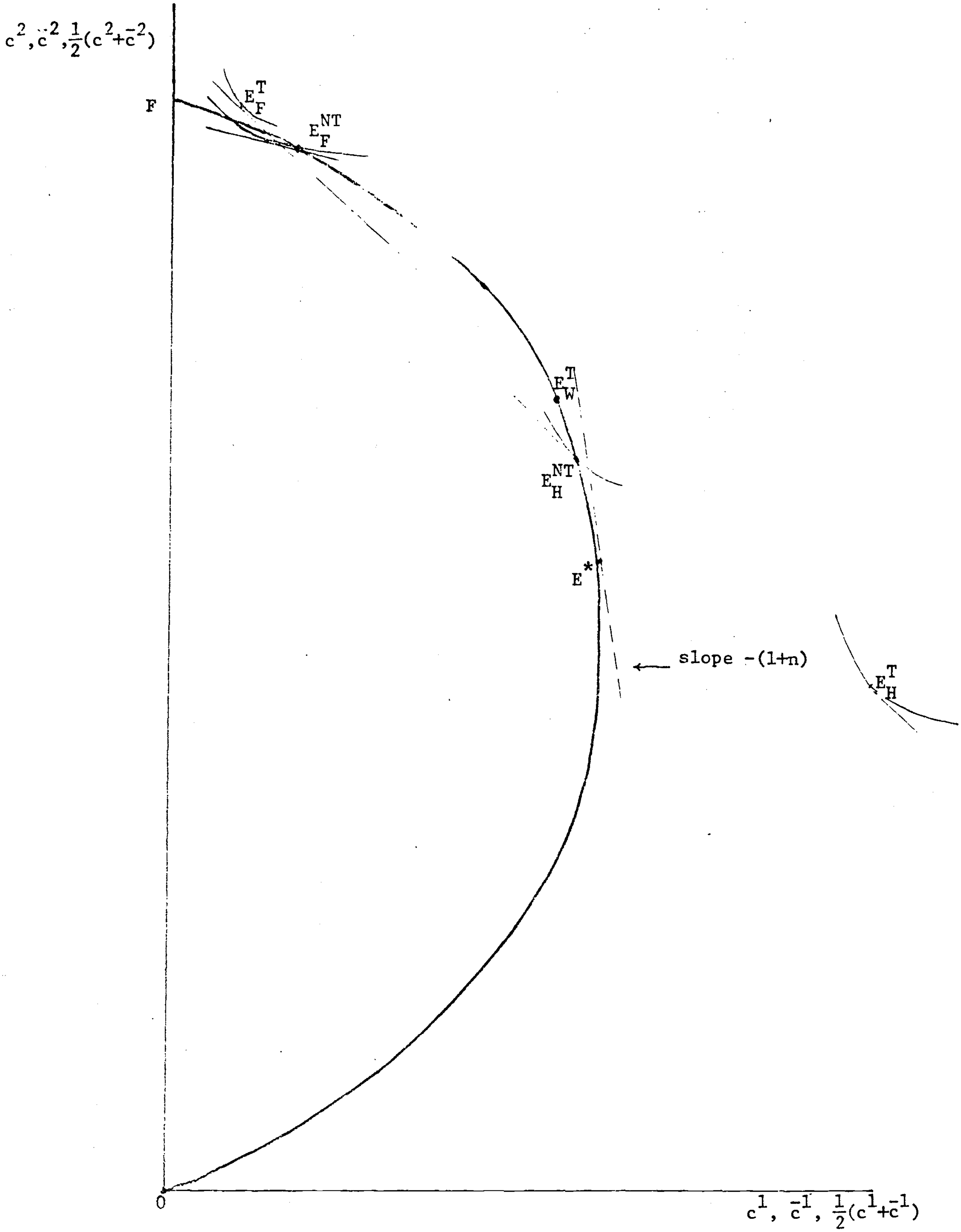



FIGURE 7b

Comparison of stationary utility levels when both countries' autarky equilibria are in the inefficient region.

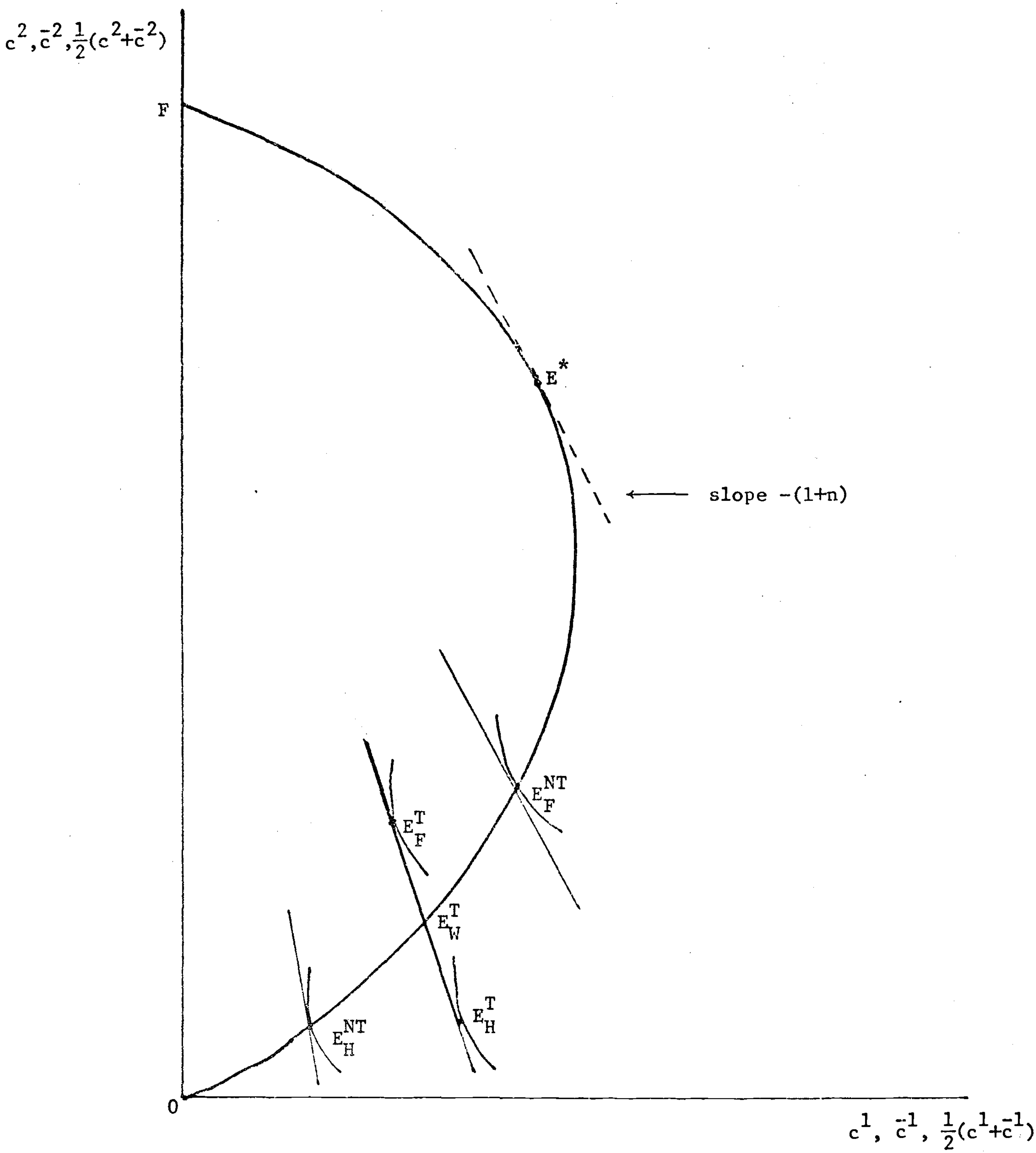



FIGURE 7c

Comparison of stationary utility levels when both countries' autarky equilibria are below the golden rule.

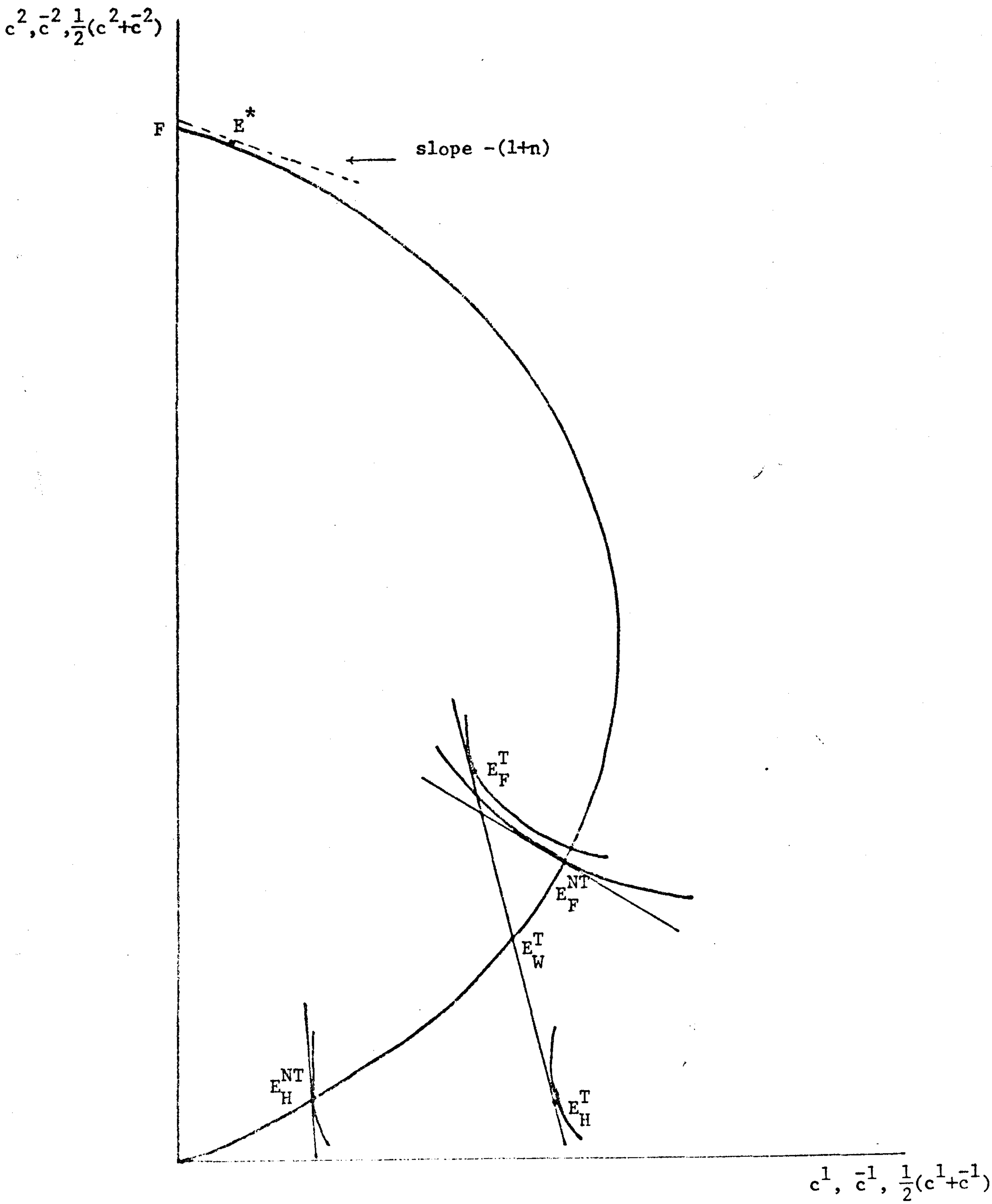



FIGURE 7d

Comparison of stationary utility levels when both countries' autarky equilibria are below the golden rule.

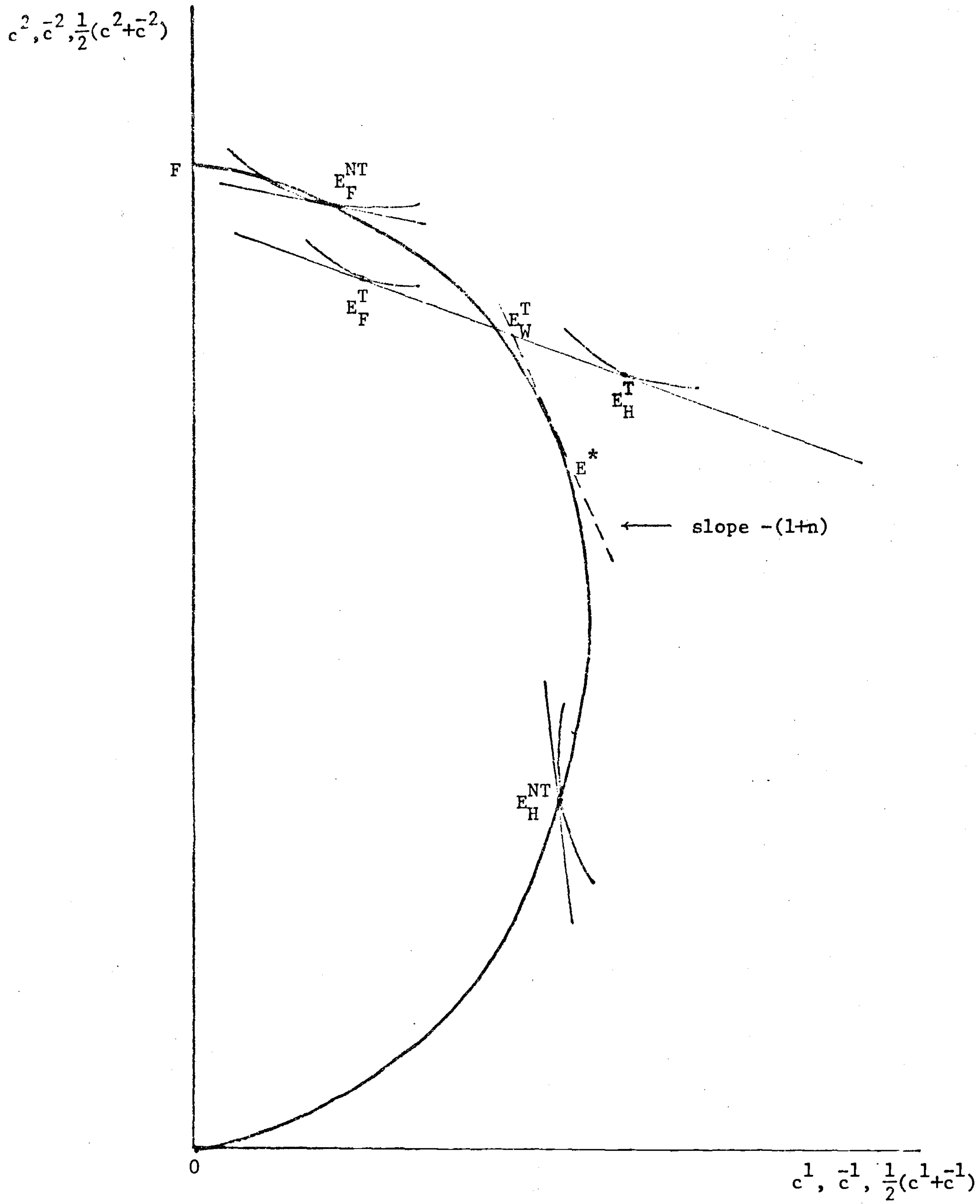



FIGURE 7e

Comparison of stationary utility levels when the autarky equilibria straddle the golden rule.

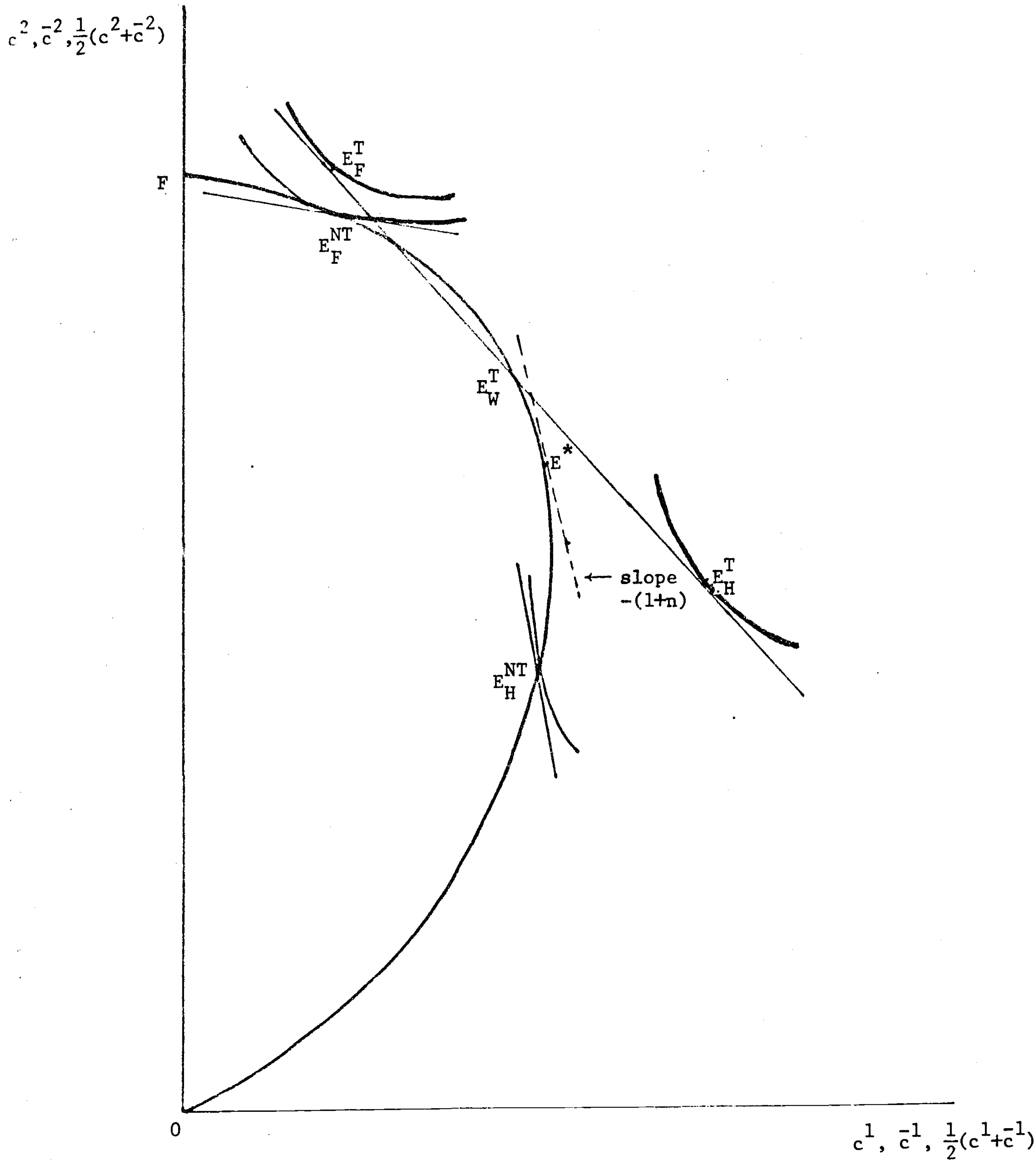



FIGURE 7f

Comparison of stationary utility levels when the autarky equilibria straddle the golden rule.

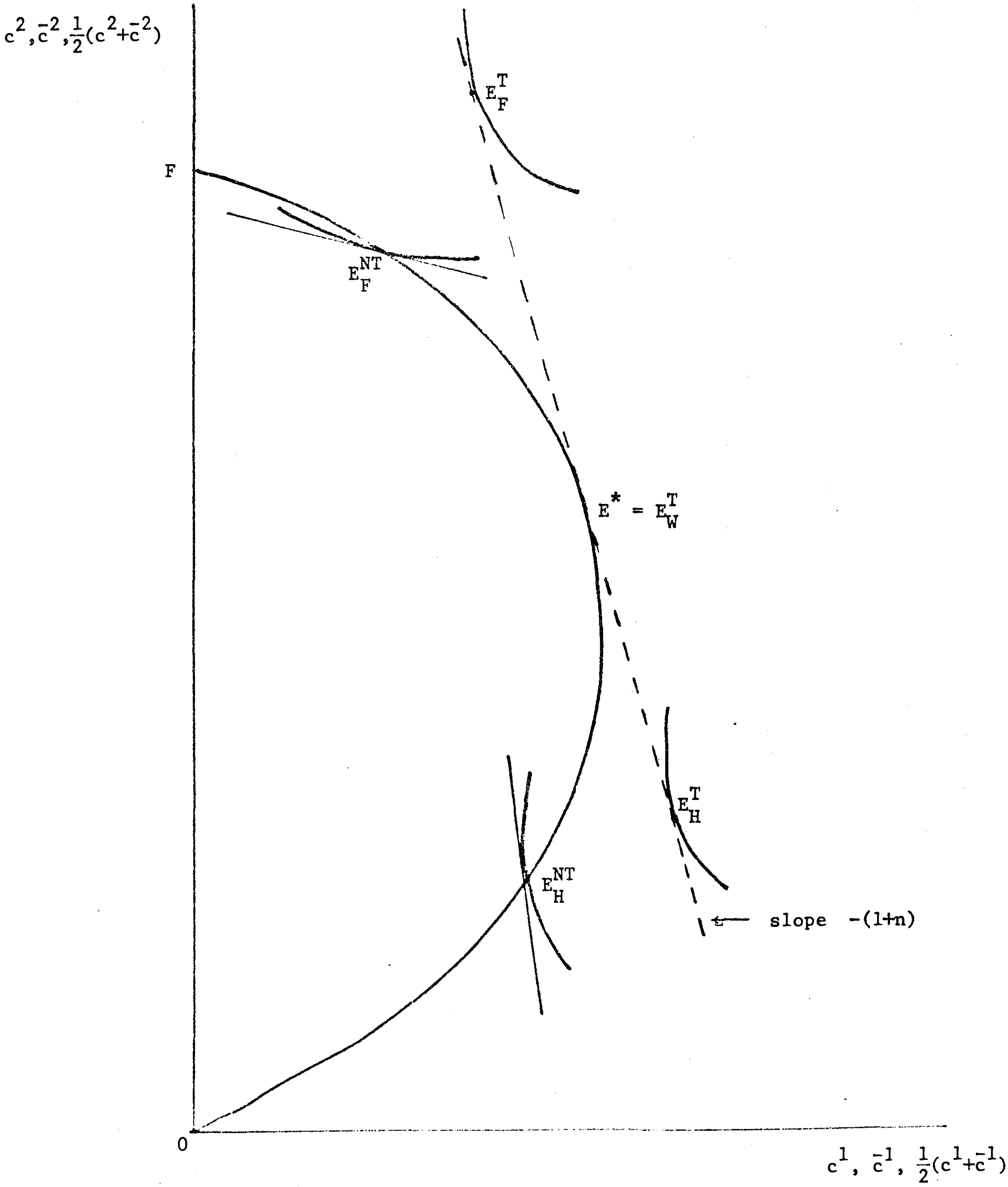



FIGURE 7g

Comparison of stationary utility levels when the world economy equilibrium is at the golden rule.