Measuring Fiscal Sustainability*

WILLEM H. BUITER
PROFESSOR OF INTERNATIONAL MACROECONOMICS
UNIVERSITY OF CAMBRIDGE
AND CONSULTANT, EUROPEAN I AND II DEPARTMENTS,
INTERNATIONAL MONETARY FUND

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CONTENTS

ABSTRACT. This note surveys some empirically implementable indices of the sustainability of a country's fiscal-financial-monetary program

1. INTRODUCTION.

The traditional financial performance criteria (government budget deficit limits, ceilings on domestic credit (expansion) by the banking system, ceilings on credit extended to the non-financial public sector, floors on international reserves) that are to be observed by countries under IMF programs are inherently myopic. Recent attempts to adjust, correct and extend these measures can all be interpreted as moves to take a longer-term view of fiscal, financial and monetary policy conditionality. This change of approach is motivated by a recognition of two facts. First, even if the private sector were myopic, longer-run fiscal-financial and monetary dynamics will inevitably work their way into future short-run economic performance. Today's long-run becomes tomorrows short run. Second, and probably even more importantly, private agents, far from being myopic, are forward-looking in wage and price behavior and in the positions they assume and the prices they are willing to pay in domestic and international financial market. This means that even short-run performance depends on current and past anticipations of medium-and long-term behaviour of policy instruments and variables exogenous to the economy in question. The long-run casts its shadow forward into the present through anticipating actions of rational, calculating private agents.

If a longer-run perspective is to be taken (even if we are only or mainly interested in short-term performance), the right place, indeed the only place, to start is the comprehensive balance sheet or present value budget constraint of the consolidated public sector. This will permit us to think systematically about three sets of issues central to the Fund's concerns. First, the issue of government solvency or sovereign default. Second, the issue of financial crowding out of private saving and investment by goverment borrowing and third, the monetary (and hence the inflation) implications of alternative fiscal-financial-monetry programs.

2. SOME BASIC ACCOUNTING.

We start in equation 1 from the basic single-period budget identity (sources and uses of funds) of the combined public sector or CPS, that is, consolidated (or combined) non-financial public sector (NFPS) and public sector financial institutions (PFI). The latter include the central bank and all other public sector financial agencies such as development banks, import-export banks and publicly owned commercial banks.

$$\begin{aligned} & C_t - T_t - E_t N_t^* - F_t + A_t - PRIV_t + i_t B_{t-1}^d + i_t^* E_t (B_{t-1}^* - R_{t-1}^*) \\ \equiv \; & B_t^d - B_{t-1}^d + E_t (B_t^* - B_{t-1}^*) + H_t - H_{t-1} - E_t (R_t^* - R_{t-1}^*) \end{aligned} \tag{1}$$

C_t is the nominal value of government consumption spending in period t.

T_t is the nominal value of taxes net of transfers and subsidies in period t. It includes, with a negative sign, social security health and retirment benefits and public sector pension benefits.

E_t is the nominal spot exchange rate (the domestic currency price of foreign exchange in period t.

N_t^* is the foreign currency value of foreign aid.

F_t is the nominal value of the gross cash flow from the public sector capital stock in period t.

A_t is the nominal value of gross domestic capital formation in the public sector in period t.

$PRIV_t$ is the nominal value of privatization proceeds in period t.

i_t is the nominal interest rate on domestic currency denominated public debt in period t.

B^d_{t-1} is the nominal face value of the net stock of domestic currency-denominated interest-bearing liabilities of the consolidated NFPS and PFI, including arrears, outstanding at the beginning of period t. This includes the net non-monetary financial liabilities of the central bank vis-a-vis the private sector and the foreign sector.

i^*_t is the nominal interest rate on foreign currency denominated public debt in period t.

B^*_{t-1} is the foreign currency face value of the net stock of foreign currency-denominated interest-bearing liabilities of the consolidated NFPS and PFI, excluding official foreign exchange reserves, but including arrears, outstanding at the beginning of period t. This includes the net non-monetary financial liabilities of the central bank that are denominated in foreign exchange.

R^*_{t-1} is the foreign currency value of the stock of official international reserves (denominated in foreign currency) at the beginning of period t.

H_{t-1} is the nominal stock of non-interest bearing base money or high-powered money outstanding at the beginning of period t.

We also define the following:

$$H_t \equiv CU_t + RR_t \tag{2}$$

$$P_t(K_t - K_{t-1}) \equiv A_t - DEP_t - \frac{P_t}{P^k_t} PRIV_t \tag{3}$$

$$DEP_t \equiv P_t \delta_t K_{t-1} \tag{4}$$

$$F_t \equiv P_t \rho_t K_{t-1} \tag{5}$$

CU_t is the nominal stock of domestic currency in the hands of the public at the end of period t.

RR_t is the nominal value of bank reserves held with the central bank at the end of period t.

P_t is the domestic GDP deflator in period t.

K_t is the public sector capital stock at the end of period t valued at current reproduction cost, that is, measured in physical units, which are assumed to be real GDP units. The nominal reproduction cost of public sector capital is therefore assumed to be the GDP deflator, although a capital production cost index distinct from the GDP deflator could be added without complications.

DEP_t is the nominal value of public sector capital consumption or depreciation in period t.

P_t^k is the domestic currency value of the price obtained for a unit of public sector capital privatized in period t.

δ_t is the proportional rate of physical depreciation of the public sector capital stock in period t.

ρ_t is the gross real cash (or financial) rate of return on public sector capital in period t.

The current or consumption account primary surplus (that is, the non-interest, non-investment, non-privatization) surplus of the consolidated public sector and central bank, S_t^c , is defined in equation 6

$$S_t^c \equiv T_t + E_t N_t^* - C_t \tag{6}$$

The conventionally defined primary (non-interest) surplus of the consolidated public sector and central bank, S_t , is defined in equation 7. Unlike S^c it includes capital formation and privatization.

$$\begin{aligned} S_t &\equiv S_t^c + F_t + PRIV_t - A_t \\ &\equiv S_t^c + F_t - [P_t(K_t - K_{t-1}) + DEP_t] + \left(\frac{P_t^k - P_t}{P_t^k} \right) PRIV_t \end{aligned} \tag{7}$$

Public sector gross dissaving by the consolidated public sector and central bank or the consumption account deficit of the consolidated public sector and central bank, D_t^c , is defined in equation 8.

$$D_t^c \equiv -S_t^c - (F_t - DEP_t) + i_t B_{t-1}^d + i_t^* E_t (B_{t-1}^* - R_{t-1}^*) \tag{8}$$

The conventionally defined financial deficit or borrowing requirement of the CPS, D_t , is defined in equation 9

$$\begin{aligned} D_t &\equiv D_t^c + A_t - DEP_t - PRIV_t \\ &\equiv D_t^c + P_t(K_t - K_{t-1}) - \left(\frac{P_t^k - P_t}{P_t^k} \right) PRIV_t \\ &\equiv -S_t + i_t B_{t-1}^d + i_t^* E_t (B_{t-1}^* - R_{t-1}^*) \end{aligned} \tag{9}$$

From equations 1 and 3 to 5 we obtain equation 10

$$C_t - T_t - E_t N_t^* + [\frac{P_t - P_t^k}{P_t^k}]PRIV_t + i_t B_{t-1} + i_t^* E_t(B_{t-1}^* - R_{t-1}^*) - (F_t - DEP_t)$$
$$\equiv \quad -P_t(K_t - K_{t-1}) + B_t^d - B_{t-1}^d + E_t[B_t^* - R_t^* - (B_{t-1}^* - R_{t-1}^*)] + H_t - H_{t-1} \quad (10)$$

We also define the following.

$$\sigma_t \equiv (H_t - H_{t-1})/(P_t Y_t) \quad (11)$$

σ_t is *seigniorage* as a fraction of GDP, that is the change in the nominal stock of base money divided by nominal GDP.

$$B_t \equiv B_t^d + E_t(B_t^* - R_t^*) \quad (12)$$

B_t is the nominal face value (measured in domestic currency) of the total net stock of non-monetary *financial* debt of the CPS at the end of period t.

$$\bar{B}_t \equiv B_t - P_t K_t \quad (13)$$

$\bar{B}_t$ is the nominal face value of the total net stock of non-monetary tangible liabilities of the government at the end of period t. It subtracts the public sector capital stock valued at current reproduction cost from the net stock of non-monetary financial liabilities .

In what follows, lower-case characters stand for the corresponding upper-case character as a fraction of GDP. Letting Y_t denote real GDP in period t, we therefore have:

$c_t \equiv C_t/(P_t Y_t)$; the public sector consumption-GDP ratio in period t.

$\tau_t \equiv T_t/(P_t Y_t)$; taxes net of transfers as a fraction of GDP in period t.

$n_t^* \equiv E_t N_t^*/(P_t Y_t)$; foreign aid as a fraction of GDP in period t.

$dep_t \equiv DEP_t/(P_t Y_t)$; public sector capital depreciation as a fraction of GDP.

$f_t \equiv F_t/(P_t Y_t)$; public sector capital income as a fraction of GDP.

$priv_t \equiv PRIV_t/(P_t Y_t)$; privatization receipts as a fraction of GDP in period t.

$b_t^d \equiv B_t^d/(P_t Y_t)$; the ratio of the net stock of interest-bearing debt denominated in domestic currency at the end of period t to period t *GDP*.

$b_t^* \equiv E_t B_t^*/(P_t Y_t)$; the ratio of the net stock of interest-bearing debt denominated in foreign currency at the end of period t to period t *GDP*.

$\rho_t^* \equiv E_t R_t^*/(P_t Y_t)$; the ratio of the stock of foreign exchange reserves at the end of period to to period t *GDP*.

$s_t^c \equiv S_t^c/(P_t Y_t)$; the current or consumption account primary surplus as a fraction of GDP.

$s_t \equiv S_t/(P_t Y_t)$; the primary surplus as a fraction of GDP.

$d_t^c \equiv D_t^c/(P_tY_t)$; CPS gross dissaving or the CPS current account deficit, as a fraction of GDP.

$d_t \equiv D_t/(P_tY_t)$; the CPS financial deficit or borrowing requirement as a fraction of GDP.

$b_t \equiv B_t/(P_tY_t)$; the nominal value of the total net stock of non-monetary *financial* liabilities of the CPS at the end of period t as a fraction of period t GDP.

$\bar{b}_t \equiv \bar{B}_t/(P_tY_t)$; the nominal value of the total net stock of non-monetary liabilities of the CPS at the end of period t, .as a fraction of period t GDP.

We also define the following:

$\pi_t \equiv (P_t/P_{t-1}) - 1$; the rate of inflation in period t.

$g_t \equiv (Y_t/Y_{t-1}) - 1$; the rate of growth of real GDP in period t.

$\epsilon_t \equiv (E_t/E_{t-1}) - 1$; the rate of depreciation of the nominal exchange rate in period t.

$r_t \equiv [(1+i_t)/(1+\pi_t)] - 1$; the period t domestic real interest rate.

We now can rewrite the identity in equation 10 as equation 14 or 15 below.

$$b_t \equiv \frac{1}{(1+\pi_t)(1+g_t)} b_{t-1} + \frac{\epsilon_t}{(1+\pi_t)(1+g_t)} (b_{t-1}^* - \rho_{t-1}^*) + d_t - \sigma_t \tag{14}$$

$$b_t \equiv \left(\frac{1+r_t}{1+g_t}\right) b_{t-1} + \left(\frac{(1+\epsilon_t)(1+i_t^*) - (1+i_t)}{(1+\pi_t)(1+g_t)}\right) (b_{t-1}^* - \rho_{t-1}^*) - s_t - \sigma_t \tag{15}$$

The term $\left(\frac{(1+\epsilon_t)(1+i_t^*)-(1+i_t)}{(1+\pi_t)(1+g_t)}\right)(b_{t-1}^* - \rho_{t-1}^*)$ corrects for possible deviations from uncovered international interest parity. This is necessary because in the term $\left(\frac{1+r_t}{1+g_t}\right) b_{t-1}$, all debt has the domestic real interest rate imputed to it.

Identities 14 and 15 follow the unhelpful but common practice of lumping together current transactions and capital transactions in the primary deficit, $-s_t$, and in the financial deficit, d_t. It would be conceptually cleaner to use the alternative representations of equations 14 and 15 given in equations 16 and 17 below. Note that in equation 16, (minus) net public sector capital income $-(f_t - dep_t)$ is grouped together with interest payments $[i_tB_{t-1}^p + i_t^*E_t(B_{t-1}^* - R_{t-1}^*)]/(P_tY_t)$, while privatization and (the negative of) net public sector capital formation are treated as financing items on a par with explicit borrowing.

$$\begin{aligned} \bar{b}_t \quad \equiv \quad & \frac{1}{(1+\pi_t)(1+g_t)} \bar{b}_{t-1} + d_t^c \\ & + \left(\frac{P_t - P_t^k}{P_t^k}\right) priv_t - \frac{\pi_t}{(1+\pi_t)(1+g_t)} k_{t-1} + \frac{\epsilon_t}{(1+\pi_t)(1+g_t)} (b_{t-1}^* - \rho_{t-1}^*) \\ & -\sigma_t \end{aligned} \tag{16}$$

Where

$$d_t^c \equiv -s_t^c + \frac{i_t}{(1+\pi_t)(1+g_t)}b_{t-1} + \frac{i_t^*(1+\epsilon_t)}{(1+\pi_t)(1+g_t)}(b_{t-1}^* - \rho_{t-1}^*) - \left(\frac{\rho_t - \delta_t}{1+g_t}\right)k_{t-1}$$

and

$$-s_t^c \equiv c_t - \tau_t - n_t^*$$

$$\begin{aligned}\bar{b}_t \equiv & \left(\frac{1+r_t}{1+g_t}\right)\bar{b}_{t-1} - s_t^c \\ & + \left(\frac{P_t - P_t^k}{P_t^k}\right)priv_t + \left(\frac{r_t - (\rho_t - \delta_t)}{1+g_t}\right)k_{t-1} + \left(\frac{(1+\epsilon_t)(1+i_t^*) - (1+i_t)}{(1+\pi_t)(1+g_t)}\right)(b_{t-1}^* - \rho_{t-1}^*) \\ & -\sigma_t \end{aligned} \quad (17)$$

Note from equation 17 that three conceptually different valuations of public sector capital are relevant for the evolution of the public sector's stock of non-monetary debt. The first is the current reproduction cost of a unit of public sector capital, assumed to be equal to P_t. This enters the definition of net non-monetary liabilities as a fraction of GDP, $\bar{b}_t \equiv [B_t^d + E_t(B_t^* - R_t^*) - P_tK_t]/(P_tY_t)$. The second is the price obtained by the government for a unit of privatized public sector capital, P_t^k. Equations16 and 17 emphasize the obvious point that privatization only reduces the rate of increase in the government's net non-monetary liabilities if the privatization price exceeds the current reproduction cost of government capital [1] . The third is the "continuation value" of a unit of public sector capital in the public sector, V_t . If the public sector capital stock is expected to remain in the public sector forever, the continuation value in the public sector of a unit of public sector capital is given by equation 18 (if the discount rates i_{t+j} are treated as non-stochastic).

$$V_t = \lim_{N\to\infty} E_t \sum_{k=0}^{N-1} \prod_{j=0}^{k} \left(\frac{1}{1+i_{t+j}}\right) P_{t+j}(\rho_{t+j} - \delta_{t+j}) \quad (18)$$

[2]

[1] Presumably, with a rational private sector, P_t^k cannot exceed the present discounted value of the quasi-rents the capital is expected to earn after privatization.

[2] If the objective of the government's privatization policy were to maximize its net worth, V_t would, under risk-neutrality, be determined from the following recursion relation: (α) $V_t = \frac{P_t(\rho_t - \delta_t)}{1+i_t} + Max\ E_t[\frac{V_{t+1}\ ,\ P_{t+1}^k}{1+i_t}]$ If the public sector capital stock is expected to remain in the public sector forever, equation (α) implies equation (β) (if the discount rates i_{t+j} are treated as non-stochastic). (β)

Thus the value to the government of a unit of public sector capital that is expected always to remain in the public sector is the present discounted value of its expected future quasi-rents in the public sector. The continuation value of the public sector capital stock in the public sector does not show up directly (as V_t) in the budget constraints, but the stream of quasi-rents whose present discounted value it is, does appear.

3. FISCAL SUSTAINABILITY INDICATOR #1: THE PUBLIC DEBT-GDP RATIO.

While in the never-never land of pure theory, unbounded debt-GDP ratios are not inconsistent with government solvency and sustainable policy, *de facto* debt-GDP ratios will of course have to remain bounded. Especially at the beginning of a stabilization program, a government trying to establish or regain credibility, may use its a (sequence of) declining public debt-GDP ratio(s) as a signal of its ability to maintain long-run solvency. Equations 14 or 15 describe what will happen to the debt-GDP ratio during the next period.

Equation 14 requires as data inputs the growth rate of nominal GDP $((1+\pi_t)(1+g_t)-1)$, the initial total public debt-GDP ratio (b_{t-1}), the proportional rate of depreciation of the nominal exchange rate (ϵ_t), the initial net foreign debt-GDP ratio $(b^*_{t-1}-\rho^*_{t-1})$, the financial surplus of the public sector (d_t) and seigniorage as a proportion of GDP (σ_t).

Equation 15 requires as data inputs the growth rate of real GDP (g_t), the domestic real interest rate (r_t), the initial total public debt-GDP ratio (b_{t-1}), the proportional rate of depreciation of the nominal exchange rate (ϵ_t), the initial net foreign debt-GDP ratio $(b^*_{t-1}-\rho^*_{t-1})$, the growth rate of nominal GDP $((1+\pi_t)(1+g_t)-1)$, the foreign nominal interest rate (i^*_t), the domestic nominal interest rate (i_t), the primary surplus of the public sector (s_t) and seigniorage as a proportion of GDP (σ_t).

Pitfalls: the financial deficit-GDP ratio and the primary deficit-GDP ratios can be distorted (made to look smaller in the short-run at the expense of being made larger in the long run) by the following tricks:

(1) Speeding up privatization receipts, if the privatized assets would have yielded positive future net cash flow to the government had they remained in the public sector (that is, if they had a positive continuation value in the public sector).

$V_t = \lim_{N\to\infty} E_t \sum_{k=0}^{N-1} \prod_{j=0}^{k} \left(\frac{1}{1+i_{t+j}}\right) P_{t+j}(\rho_{t+j}-\delta_{t+j}) + \lim_{N\to\infty} E_t \prod_{j=0}^{N-1} \left(\frac{1}{1+i_{t+j}}\right) V_{t+1+j}$ If specu- tcol

lative bubbles are ruled out, $\lim_{N\to\infty} E_t \prod_{j=0}^{N-1} \left(\frac{1}{1+i_{t+j}}\right) V_{t+1+j} = 0$ and equation (β) expresses the tcol
value to the government of a unit of public sector capital that is expected alway to remain in the public sector as the present discounted value of its expected future quasi-rents in the public sector.

(2) Cutting back government capital formation, if the present discounted value of the future net cash flow to the government from these investment projects would have exceeded the cost of the projects.

To avoid these problems, it may be preferable to look at the behavior (in the past and anticipated for the next few years) of the public sector non-monetary financial debt net of the public sector capital stock valued at current reproduction cost, $\bar{b}$, given in equations 16(15) and 17 rather than at the non-monetary financial debt b, given in equations 14 and 15.

4. PRIMARY GAPS.

For notational simplicity, we now define the augmented primary surplus-GDP ratio, $\tilde{s}$, as follows:

$$\tilde{s}_t \equiv s_t - \frac{[i_t^*(1+\epsilon_t)+\epsilon_t - i_t]}{(1+\pi_t)(1+g_t)}(b_{t-1}^* - \rho_{t-1}^*) + \sigma_t \tag{19}$$

The augmented primary surplus therefore adds to the conventional primary surplus both seigniorage (the resources appropriated by the government by printing non-interest-bearing base money) and the excess of the cost of borrowing by issuing domestic currency denominated debt over the cost of borrowing by issuing foreign currency denominated debt times the net stock of foreign debt.

Given the initial value of the total non-monetary government debt-GDP ratio at the beginning of period t, b_{t-1}, the target value of the debt-GDP ratio $N \geq 1$ periods later, b_{t-1+N} , the projected future one-period real interest rates during the next N periods, r_{t+j}, $j = 0, ..., N$, and the projected growth rates of real GDP during the next N periods, g_{t+j}, $j = 0, ..., N-1$, the constant augmented primary surplus to GDP ratio, $\tilde{s}_R^N$, that will achieve the target is given by:

$$\tilde{s}_R^N(b_{t-1} - b_{t-1+N}) = \left[\sum_{k=0}^{N-1}\prod_{j=0}^{k}\left(\frac{1+g_{t+j}}{1+r_{t+j}}\right)\right]^{-1}\left[b_{t-1} - \prod_{j=0}^{N-1}\left(\frac{1+g_{t+j}}{1+r_{t+j}}\right)b_{t-1+N}\right] \tag{20}$$

We shall refer to $\tilde{s}_R^N$ as the *required N-period* (augmented) *primary surplus-GDP ratio.* With constant real N-period interest rates r^N and constant N-period growth rates of real GDP g^N, the required N-period primary surplus-GDP ratio simplifies to:

$$\tilde{s}_R^N(b_{t-1} - b_{t-1+N}) = \frac{\left(r_t^N - g_t^N\right)}{(1+g_t^N)\left[1 - \left(\frac{1+g_t^N}{1+r_t^N}\right)^N\right]}\left[b_{t-1} - \left(\frac{1+g_t^N}{1+r_t^N}\right)^N b_{t-1+N}\right] \tag{21}$$

If the target debt-GDP ratio is the same as the initial debt-GDP ratio, equation the required

N-period primary surplus-GDP ratio simplifies to

$$\tilde{s}_R^N(0) = \left[\sum_{k=0}^{N-1}\prod_{j=0}^{k}\left(\frac{1+g_{t+j}}{1+r_{t+j}}\right)\right]^{-1}\left[1-\prod_{j=0}^{N-1}\left(\frac{1+g_{t+j}}{1+r_{t+j}}\right)\right]b_{t-1} \tag{22}$$

With a constant N-period real interest rate $r^N$and a constant N-period growth rate of real GDP g^N, the required N-period primary surplus-GDP ratio for this case becomes

$$\tilde{s}_R^N(0) = \frac{\left(r_t^N - g_t^N\right)}{1+g_t^N}b_{t-1} \tag{23}$$

When $N = 1$, equation 23 simplifies to

$$\tilde{s}_R^1(0) = \frac{(r_t - g_t)}{1+g_t}b_{t-1} \tag{24}$$

We also define the *actual N-Period (augmented) primary surplus-GDP ratio*, $\tilde{s}_A^N$, to be that constant augmented primary surplus-GDP ratio whose present discounted value over the next N periods is the same as the present discounted value of the actually planned or expected primary surplus-GDP ratio over the next N periods, that is,

$$\tilde{s}_A^N(t) = \left[\sum_{k=0}^{N-1}\prod_{j=0}^{k}\left(\frac{1+g_{t+j}}{1+r_{t+j}}\right)\right]^{-1}\left[1-\prod_{j=0}^{N-1}\left(\frac{1+g_{t+j}}{1+r_{t+j}}\right)\right]b_{t-1} \tag{25}$$

When the real interest rate and the real growth rate are constant, equation 25 simplifies to

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$$\tilde{s}_A^N = \frac{\left(r_t^N - g_t^N\right)}{(1+g_t^N)\left[1-\left(\frac{1+g_t^N}{1+r_t^N}\right)^N\right]}\sum_{k=1}^{N}\left(\frac{1+g_t^N}{1+r_t^N}\right)^k\tilde{s}_{t-1+k} \tag{26}$$

The N-period *primary gap* (see Blanchard et. al. [1990]) in period t, GAP_t^N , is defined as the excess of the rquired N-period (augmented) primary surplus-GDP ratio, $\tilde{s}_R^N$, over the actual N-period (augmented) primary surplus -GDP ratio, $\tilde{s}_A^N$:

$$GAP_t^N \equiv \tilde{s}_R^N(b_{t-1} - b_{t-1+N}) - \tilde{s}_A^N \tag{27}$$

Given an initial debt-GDP ratio and a target debt-GDP ratio for N periods hence, the N-period primary gap can be calculated using forecasts for real interest rates, real growth rates and actual planned or expected augmented primary surplus-GDP ratios.

Note that primary gaps defined with reference to the public sector non-monetary financial debt net of the value of the public sector capital stock, $\bar{b}$, can be defined exactly analogously. The relevant primary surplus would be $\tilde{s}^c$,the *augmented* current or consumption account primary surplus (as a fraction of GDP), defined by

$$\tilde{s}_t^c \equiv s_t^c - \frac{[i_t^*(1+\epsilon_t)+\epsilon_t - i_t]}{(1+\pi_t)(1+g_t)}(b_{t-1}^* - \rho_{t-1}^*) + \sigma_t \tag{28}$$

Two particularly simple special cases of the N-period primary gap provide the second and third fiscal indicators.

5. FISCAL SUSTAINABILITY INDICATOR #2: THE ONE-PERIOD PRIMARY GAP.

When $N = 1$and the initial debt-GDP ratio is the same as the target debt-GDP ratio at the end of the period, the primary gap calculation simplifies to:

$$GAP_t^1 \equiv \tilde{s}_R^1(0) - \tilde{s}_A^1 = \left(\frac{r_t - g_t}{1+g_t}\right) b_{t-1} - \tilde{s}_t \tag{29}$$

Note that the calculation of 29 does not require any forecasts other than those going into the calculation of the current real interest rate and current growth rate of real GDP. GAP_t^1, the one-period primary gap in period t is the excess of the augmented primary surplus-GDP ratio that stabilizes this period's debt-GDP ratio over the actual current augmented primary surplus-GDP ratio.

Apart from the problems arising from the treatment of public sector capital formation and privatization, the one-period primary gap may give a distorted picture of the amount of adjustment that would reasonably be required for two further reasons. The first is that the actual current primary surplus may be affected by transitory (e.g. cyclical) increases or reductions in public sector revenues and non-interest expenditures. A cyclical correction may therefore be in order. The second is that the current real interest rate and growth rate of real GDP may be unrepresentative of their respective long-run expected average values. This suggest need for the longer-run perspective adopted in the next section.

6. LONG-RUN SOLVENCY.

When the long-run interest rate is above the long-run growth rate of GDP, that is if $\lim_{N\to\infty}\prod_{j=0}^{N-1}\left(\frac{1+g_{t+j}}{1+r_{t+j}}\right) = 0$, the following government solvency constraint is assumed to hold for an economy without a finite terminal date

$$lim_{N\to\infty}\prod_{j=0}^{N-1}\left(\frac{1+g_{t+j}}{1+r_{t+j}}\right)b_{t-1+N}=0 \tag{30}$$

When equation 30 holds, the present value budget constraint given in 31 applies to the government. This states that the current face value of the debt cannot exceed the present discounted value of future primary surpluses and seigniorage.

$$b_{t-1}=lim_{N\to\infty}\sum_{k=0}^{N-1}\prod_{j=0}^{k}\left(\frac{1+g_{t+j}}{1+r_{t+j}}\right)\tilde{s}_{t+j} \tag{31}$$

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We can define the *required permanent (augmented) primary surplus-GDP ratio,* $\tilde{s}_R^\infty$ as follows:

$$\tilde{s}_R^\infty=lim_{N\to\infty}\left[\sum_{k=0}^{N-1}\prod_{j=0}^{k}\left(\frac{1+g_{t+j}}{1+r_{t+j}}\right)\right]^{-1}b_{t-1} \tag{32}$$

When the real interest rate and the growth rate of real GDP are constant, equation 32 becomes

$$\tilde{s}_R^\infty=\left(\frac{r_t^\infty-g_t^\infty}{1+g_t^\infty}\right)b_{t-1} \tag{33}$$

The required permanent (augmented) primary surplus-GDP ratio is the constant (augmented) primary surplus GDP ratio that, if maintained indefinitely, would ensure government solvency. It is also the constant primary surplus-GDP ratio that will ensure that, ultimately, the debt-GDP ratio does not exceed any finite upper limit (including its current value b_{t-1}).

7. FISCAL SUSTAINABILITY INDICATOR #3: THE PERMANENT PRIMARY GAP.

The *permanent primary gap*, GAP_t^∞, was proposed in Buiter [1983, 1985 and 1990a]. It measures the magnitude of the permanent correction required to be made to the actual current and future planned augmented primary surplus-GDP ratios in order to ensure government solvency. This measure was also proposed in Blanchard [1990] and in Blanchard et. al. [1990]. It is given by the excess of the required permanent primary surplus-GDP ratio over actual permanent primary surplus-GDP ratio:

[3] Note that, when the long-run interest rate exceeds the long-run growth rate, government solvency is consistent even with unbounded (and forever rising) debt-GDP ratios as long as, ultimately, the public debt grows at a rate less than the interest rate.

$$\begin{aligned} GAP_t^{\infty} &= \tilde{s}_R^{\infty} - \tilde{s}_A^{\infty} \\ &= lim_{N\to\infty}\left[\sum_{k=0}^{N-1}\prod_{j=0}^{k}\left(\frac{1+g_{t+j}}{1+r_{t+j}}\right)\right]^{-1}\left[b_{t-1}-\sum_{k=0}^{N-1}\prod_{j=0}^{k}\left(\frac{1+g_{t+j}}{1+r_{t+j}}\right)\tilde{s}_{t+j}\right] \end{aligned} \tag{34}$$

When the real interest rate and the growth rate of real GDP are constant, becomes

$$GAP_t^{\infty} = \left(\frac{r_t^{\infty}-g_t^{\infty}}{1+g_t^{\infty}}\right)[b_{t-1}-\sum_{k=1}^{N}\left(\frac{1+g_t^{\infty}}{1+r_t^{\infty}}\right)^k \tilde{s}_{t-1+k}] \tag{35}$$

The calculation of the permanent primary gap requires forecasts of the long-run real interest rate and the long-run real growth rate and of the future primary surpluses that would materialize under current spending and revenue-raising plans. The lazy man's (or myopic) alternative, measured by $MGAP_t^{\infty}$, substitutes the current augmented primary surplus-GDP ratio, $\tilde{s}_t$, for the actual permanent augmented primary surplus-GDP ratio, that is

$$MGAP_t^{\infty} = \left(\frac{r_t^{\infty}-g_t^{\infty}}{1+g_t^{\infty}}\right)b_{t-1}-\tilde{s}_t \tag{36}$$

This is the same as the one-period gap, except for the substitution of the long real interest rate for the current real interest rate and the substitution of the long-run growth rate of real GDP for the current growth rate of real GDP.

8. FISCAL SUSTAINABILITY CRITERION # 4: THE DISCOUNTED PUBLIC DEBT.

The government solvency constraint

$\lim_{N\to\infty}\prod_{j=0}^{N-1}\left(\frac{1+g_{t+j}}{1+r_{t+j}}\right)b_{t-1+N} \equiv \lim_{N\to\infty}\prod_{j=0}^{N-1}\left(\frac{1}{1+i_{t+j}}\right)B_{t-1+N}=0$,

suggests that the behavior of the discounted public debt $PDV(b_t)$ (the present discounted value of the public debt discounted to a fixed initial date, t_0, say), can serve as a useful indicator of potential trouble. Note that the discounted debt rises if and only if the augmented primary surplus is negative. If the discounted debt has been rising significantly and looks like continuing to do so in the foreseeable future, good reasons must be given why this upward trend will eventually be reversed.

The measure is given by

$$PDV(b_t) \equiv \prod_{j=0}^{t-t_o}\left(\frac{1}{1+i_{t_o+j}}\right)B_t \tag{37}$$

9. FISCAL SUSTAINABILITY CRITERION # 5: THE LONG-RUN INFLATION RATE IMPLIED BY THE FISCAL-FINANCIAL PLANS.

This is a standard financial programming exercise. For a reference, see *e.g.* Buiter [1993], Anand and van Wijnbergen [1989] and Buiter [1990b].

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