Cross-Border Tax Externalities: Are Budget Deficits Too Small?[*]

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Abstract

In a dynamic optimising model with costly tax collection, a tax cut by one nation creates positive externalities for the rest of the world if initial public debt stocks are positive. By reducing tax collection costs, current tax cuts boost the resources available for current private consumption, lowering the global interest rate. This pecuniary externality benefits other countries because it reduces the tax collection costs of current and future debt service. In the non-cooperative equilibrium, nationalistic governments do not allow for the effect of lower domestic taxes on debt service costs abroad. Taxes are too high and government budget deficits too low compared to the global cooperative equilibrium. Even in the cooperative equilibrium complete tax smoothing is not optimal: current taxes will be lower than future taxes.

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1 Introduction

Does one country's borrowing in an integrated global financial market impose externalities on other countries? If so, are these spillovers welfare enhancing or welfare reducing? This issue figures prominently in the debates about the European Union's Stability and Growth Pact and the merits of G3 policy coordination. In this paper we consider cross-border externalities associated with the transmission of national public debt policies through their effect on the global risk-free real interest rate.[1]

We provide a dynamic equilibrium model with optimising households and governments, in which public debt and the government's intertemporal budget constraint provide a link between current and future tax decisions. Such models are analytically difficult, especially if they do not exhibit Ricardian equivalence or debt neutrality. Thus, we initially focus on the simplest possible supply side for the national economies (a perishable endowment), a representative infinite-lived consumer with log-linear preferences and a simple source of Ricardian nonequivalence, or absence of debt neutrality: fiscal transfer costs. We then extend this benchmark model to allow for CES preferences and a production technology. We assume that there are increasing and strictly convex real resource costs of administering and collecting taxes.[2,3] With negative taxes, or subsidies, resource costs result from private rent-seeking behaviour.

We focus on real interest rate cross-border spillovers that occur in the absence of sovereign default risk and without strategic interactions between a national fiscal authority and a national or supranational monetary authority. We model a non-monetary economy in which every national fiscal authority satisfies its intertemporal budget constraint. Government spending on goods and services is exogenous. We assume that each government

[1] Another type of externality is that associated with either sovereign debt default or with actions undertaken by the debtor country or others to prevent sovereign defaults.

[2] Slemrod and Yitzhaki (2000) report that the administrative cost of the US tax system is 0.6 cents per dollar of revenue raised. Slemrod (1996) estimates compliance costs to be about 10 cents per dollar collected.

[3] Barro (1979) pioneered these strictly convex fiscal transfer costs in a closed-economy setting. He assumed that the government minimises the discounted sum of these costs, rather than maximises the discounted welfare of households.

can commit to a path of taxes, taking other governments' taxes as given. Thus, there is commitment, but no international cooperation. We show that this noncooperative behaviour results in inefficient global equilibria.

We assume that resources are always fully utilised and financial capital is perfectly mobile across countries. International transmission of national fiscal policy is only through interest rates. Taxes are lump-sum; their incidence cannot be altered by changing private behaviour, but because of the strict convexity of the fiscal transfer costs, the timing of taxes matters in this model, just as it would with conventional distortionary taxes on labour income or asset income in models with endogenous labour supply and capital accumulation. We model the tax collection costs as borne by the public sector. Allowing for compliance costs borne by the private sector would merely add notational complexity without changing our qualitative conclusions.

Without fiscal transfer costs our model, with its representative private agent, would exhibit Ricardian equivalence: any sequence of lump-sum taxes and debt that satisfies the intertemporal budget constraints would support the same equilibrium for any given sequence of public spending on goods and services. There would be no international spillovers. This is true even if a country is large in the world capital market and exploits its monopoly power.

If we had assumed overlapping generations, instead of a representative agent, then alternative rules for financing a given public spending programme would cause pure *pecuniary* externalities if there were no fiscal transfer costs and taxes were lump sum. Even with symmetric countries, there could be distributional effects between generations, but as long as dynamic inefficiency does not occur, any feasible sequence of lump-sum taxes and debt supports a Pareto efficient equilibrium.[4]

In the one-country special case of our model, fiscal transfer costs do not cause ineffi-

[4]See Buiter and Kletzer (1991). If there is dynamic inefficiency, then fiscal policy that causes redistribution from the young to the old can lead to a Pareto improvement. With asymmetric countries, alternative deficit-financing policies would have international, as well as intergenerational, distributional implications.

ciency if one assumes that resource transfers between the private and public sector in the counterfactual command economy are subject to the same fiscal transfer costs as in our market economy. Inefficiency arises when there are multiple countries and each country affects other countries' choice sets in a way that is not adequately reflected in market prices.

With symmetric countries and representative, infinite-lived consumers, symmetric tax policies have no distributional effects. However, they can have welfare consequences if they change the world interest rate. If, for example, countries have outstanding stocks of debt and a change in policy causes the interest rate to rise, then higher debt service means that countries must raise taxes, now or in the future, and fiscal transfer costs increase. National governments that maximise their own representative resident's welfare do not internalise the cost of the higher fiscal transfer costs to other countries. Thus, a national government's financing decision that raises the world interest rate inflicts a negative externality on the rest of the world. This is in line with conventional wisdom. Where our model departs radically from conventional wisdom is through the mechanism by which financing choices affect interest rates. It is conventional to associate deficit financing of public spending with 'financial crowding out'. That is, for a given public spending programme, larger bond-financed deficits brought about by lower taxes are assumed to raise interest rates. In our thoroughly neoclassical intertemporal model the opposite is true. Lower taxes and larger deficits early on result in a lower global rate of interest.

We show that if a government is too small to affect the global interest rate, it minimises the costs of collecting taxes by smoothing them over time. If it is able to influence the interest rate and has positive initial debt, then it sets a lower tax in the initial period than in future periods. This is because lower fiscal transfer costs in the initial period than in later periods imply higher aggregate consumption in the initial period than in later periods. Thus, the real interest rate in the initial period is lower than with perfectly smooth taxes and this lowers the interest payment on the government's outstanding debt and, hence, lowers future fiscal transfer costs.

Relative to the global (cooperative) optimum, noncooperative countries tax too much and issue too little debt in the initial period. Reducing current taxes has a positive welfare spillover, even though it requires issuing more debt. Lowering the current interest rate by lowering current taxes lowers the cost of servicing all countries' outstanding debt and thus reduces all countries' need to collect costly taxes. In a noncooperative equilibrium, countries do not take into account this benefit to other countries and they tax too much in the initial period.

Our conclusion that lack of international cooperation leads to taxes that are initially too high and public deficits that are initially too small seems to contradict the presumption reflected in the debt and deficit ceilings of the Stability and Growth Pact that deficits are apt to be too large. However, we do not want to make too much of the size of the externalities associated with alternative tax and borrowing policies of national governments in EMU; even the larger EMU countries are small fish in the global financial pond. Our analysis is more relevant to interaction between the United States, the European Union as a whole, possibly Japan and soon China.

Our paper analysing the welfare economics of international interest rate spillovers from the tax and borrowing strategies of national governments using a dynamic optimising equilibrium model is related to the vast literature on other international fiscal policy linkages. It is also related to the more modest literature on the political economy of the timing of taxes in closed economies and the sizable literature on the optimal timing of multiple distortionary taxes in a closed economy.

The literature on the international transmission of fiscal policy has two main strands. First, there is the work on the transmission of government-expenditure shocks with lump-sum taxes and without fiscal transfer costs. Examples are Frenkel and Razin (1985, 1987), Buiter (1987, 1989) and Turnovsky (1988); Turnovsky (1997) provides a survey. The papers in this vein are in sharp contrast to ours. We take government expenditure as exogenous and ask how the financing matters when there are fiscal transfer costs. Second, there are papers on the transmission of tax shocks in models with distortionary taxes in a balanced-budget setting. There is a sizable literature – going back to Hamada (1966) – on the strategic taxation of capital income in a world economy. In this literature, a capital-exporting (importing) country can increase its national income by acting as a monopolist (monopsonist) and restricting capital movements. The result is a Nash equilibrium where nations want to tax capital flows. Other papers consider issues of the feasibility of different tax regimes in an integrated world economy, tax harmonisation and tax competition. Examples of such papers are Sinn (1990) and Bovenberg (1994).

In the closed-economy political economy literature, excessive public deficits and debt may result from a political party's desire to tie the hand's of a possible successor (Persson and Svensson (1989) and Alesina and Tabellini (1990)), an incumbent government's incentive to signal its competency prior to an election (Rogoff and Sibert (1988)), or a war-of-attrition game over the allotment of the costs of fiscal adjustment (Alesina and Drazen (1991)). Drazen (2000) provides a discussion of this literature. In this paper, we abstract from political economy concerns; governments are able to commit to policies which maximise national welfare.

Chamley (1981, 1986) pioneered the normative study of dynamic optimal taxation in a closed economy setting when the government can borrow or lend. He focuses on the choice between distortionary capital and labour income taxation and does not consider fiscal transfer costs of the kind studied here. The state's optimal policy is to impose the maximum possible capital levy on the private sector's initial, predetermined stocks of capital and public debt and then to switch permanently to a zero capital income tax rate.[5] Incorporating fiscal transfer costs of the kind considered here would render Chamley's highly uneven time profile of tax receipts suboptimal.

In section 2 we present the model with perishable endowments and log-linear preferences. In section 3 we extend the model to consider CES preferences and small changes in the households' intertemporal elasticity of substitution. We show that as this elasticity

[5] See also Lucas (1988).

falls, the deviation between cooperative and noncooperative taxes rises. In section 4 we consider a production economy and CES preferences. We show that if the world economy is at a steady state with positive debt, a coordinated reduction in the current tax financed by higher future taxes improves welfare. Section 5 concludes.

2 The Model

The model comprises $N \geq 1$ countries, each inhabited by a representative infinite-lived household and a government. Each period, each household receives an endowment of the single tradeable, non-storable consumption good and each government purchases an exogenous amount of the good. Governments finance their purchases by issuing debt or by taxing their resident households. We assume that the tax system is costly to administer; governments use up real resources collecting taxes. All savings are in the form of privately or publicly issued real bonds. We assume that the households' preferences and endowments and the governments' purchases are constant over time and identical across countries. The households' initial asset holdings and the governments' initial stocks of debt or credit are the same across countries. There is perfect international integration of the national financial markets, and hence, a common world interest rate.

2.1 The households

The country-i household, $i = 1, ..., N$, has preferences over its consumption path given by

$$u^i = \sum_{t=0}^{\infty} \beta^t \ln c_t^i, \tag{1}$$

where c_t^i is its period-t consumption and $\beta \in (0, 1)$ is its discount factor.

The household's period-t budget constraint is

$$c_t^i + a_{t+1}^i = W - \tau_t^i + R_t a_t^i, \; t = 0, 1, \ldots, \tag{2}$$

where a_t^i is the household's stock of assets in the form of real bonds at the start of period t, R_t is (one plus) the interest rate between period $t-1$ and period t, $W > 0$ is the household's per-period endowment of the good and τ_t^i is its time-t tax bill. The household's initial assets, a_0, are given.

In addition to satisfying its within-period budget constraint, the household must satisfy the long-run solvency condition that the present discounted value of its assets be non-negative as time goes to infinity. The transversality condition associated with its optimisation problem ensures that the present discounted value of its assets is not strictly positive. Thus,

$$\lim_{t\to\infty} a_{t+1}^i / \prod_{s=0}^{t} R_s = 0. \tag{3}$$

Equations (2) and (3) imply that the present discounted value of the household's consumption equals the present discounted value of its (after-tax) income plus its initial assets:

$$a_0 + \sum_{t=0}^{\infty} \left(W - \tau_t^i\right) / \prod_{s=0}^{t} R_s = \sum_{t=0}^{\infty} c_t^i / \prod_{s=0}^{t} R_s. \tag{4}$$

The household chooses its consumption path to maximise its utility function (1) subject to its intertemporal budget constraint (4). The solution satisfies the Euler equation

$$c_{t+1}^i = \beta R_{t+1} c_t^i, \; t = 0, 1, \dots . \tag{5}$$

Solving the difference equation (5) yields the household's time-t consumption as a function of its initial consumption and the t-period interest factor:

$$c_t^i = \beta^t \left(\prod_{s=1}^{t} R_s\right) c_0^i, \; t = 0, 1, \dots . \tag{6}$$

Substituting equation (6) into equation (4) yields the household's initial consumption as a function of its taxes and the interest factors:

$$c_0^i = (1-\beta) R_0 \left[a_0 + \sum_{t=0}^{\infty} \left(W - \tau_t^i\right) / \prod_{s=0}^{t} R_s\right]. \tag{7}$$

Substituting equation (6) into equation (1) yields that the household's indirect utility as a function of initial consumption and the interest factors:

$$u^i = \ln c_0^i + (1-\beta) \sum_{t=1}^{\infty} \beta^t \ln \left(\prod_{s=1}^{t} R_s \right), \tag{8}$$

where constants that do not affect the household's optimisation problem are ignored.

2.2 The government

The country-i, $i = 1, ..., N$, government's period-t budget constraint is

$$\tau_t^i - (\phi/2) \left(\tau_t^i\right)^2 + b_{t+1}^i = G + R_t b_t^i, \ t = 0, 1, ..., \tag{9}$$

where b_t^i is the government's outstanding debt at the start of period t and $G > 0$ is its per-period purchase of the good. The fiscal transfer cost associated with a tax (or surplus, if negative) τ is $(\phi/2)\tau^2$, where $\phi > 0$. The government's initial debt (or credit, if negative), b_0, is given. We restrict the model's parameters so that satisfying equation (9) is feasible; the restrictions are detailed later in this section.

In addition to satisfying its within-period budget constraint, the government also satisfies

$$\lim_{t \to \infty} b_{t+1}^i / \prod_{s=0}^{t} R_s = 0. \tag{10}$$

As with the household, this is an implication of the long-run solvency constraint and the transversality condition associated with the government's optimisation problem.

Equations (9) and (10) imply that the present discounted value of the government's purchases, plus its initial debt, equals the present discounted value of its tax stream, net of collection costs:

$$\sum_{t=0}^{\infty}\left[\tau_t^i-(\phi/2)\left(\tau_t^i\right)^2-g_t\right]/\prod_{s=0}^{t}R_s=0, \tag{11}$$

$$\text{where } g_t=\begin{cases} G+R_0b_0 \text{ if } t=0 \\ G \text{ otherwise.}\end{cases}$$

2.3 Market clearing

Market clearing requires that the sum of the N households' asset holdings equals the sum of the N governments' debt. Thus,

$$a_t=b_t,\ t=0,1,\dots, \tag{12}$$

where variables without a superscript denote global averages.

The global resource constraint requires that the sum of average household consumption, average government purchases and average fiscal transfer costs equals the average endowment. Thus,

$$c_t=W-G-\frac{\phi}{2N}\sum_{j=1}^{N}\left(\tau_t^j\right)^2,\ t=0,1,\dots. \tag{13}$$

Equation (13) is, of course, also implied by equations (2), (9) and (12).

Averaging both sides of the Euler equation (6) over the N countries yields

$$\prod_{s=1}^{t}R_s=c_t/\left(\beta^t c_0\right),\ t=1,2,\dots. \tag{14}$$

Equations (13) and (14) imply that in equilibrium the interest rate between periods 0 and t is solely a function of time-0 and time-t taxes.

Lower time-zero taxes financed by higher time-one taxes *lower* the interest rate between period zero and period one. It is interesting to ask whether the timing of taxes affects the global risk-free interest rate in the manner that this model predicts. There is a large body of empirical work attempting to quantify the relationship between interest rates and government budget deficits.[6] However, it is problematic and the results are hard to interpret for several reasons.

First, both taxes and interest rates are endogenous and an apparent relationship between them may be due to the influence of other variables. For example, automatic stabilisers cause tax revenue to be lower and deficits to be higher during recessions. At the same time, an expansionary monetary policy (not considered in this paper) may temporarily lower real interest rates. Thus, the role of monetary policy over the business cycle may cause deficits and real interest rates to be negatively correlated. Second, while lowering taxes may lower the global risk-free interest rate, it may also increase sovereign-default risk premia. If the effect on the national sovereign risk premium is larger than the effect on the global risk-free interest rate, it will cause tax decreases to be associated with higher measured market interest rates. Third, if tax cuts in a country include lower capital taxes, then the country's marginal product of capital may fall to equate after-tax returns. This causes lower taxes to be associated with lower (before-tax) interest rates. Fourth, the scenario here has lower taxes in the current period accompanied by higher future taxes and unchanged government spending. However, households may interpret lower taxes today as a signal of a change in the government's attitude toward public expenditure and they may expect lower taxes in the medium-run as well. In empirical studies, it is difficult to control for the public's expectation of future tax policy and its beliefs about future spending.

Substituting equation (14) into equation (8) gives the country-i household's indirect utility as a function of its initial consumption and the path of aggregate consumption:

$$u^i = \ln c_0^i - \beta \ln c_0 + (1-\beta) \sum_{t=1}^{\infty} \beta^t \ln c_t. \tag{15}$$

Substituting equations (12) and (14) into equation (7) gives the household's initial

[6] A recent example is Laubach (2003).

consumption as a function of the path of taxes, with the predetermined value of initial government debt entering as a parameter:

$$c_0^i = (1-\beta)\, c_0 \sum_{t=0}^{\infty} \beta^t \left(w_t^i - \tau_t^i\right)/c_t, \tag{16}$$

$$\text{where } w_t = \begin{cases} W + R_0 b_0 \text{ if } t = 0 \\ W \text{ otherwise.} \end{cases}$$

Substituting equation (16) into the indirect utility function (equation (15)) yields

$$u^i = \ln\left[\sum_{t=0}^{\infty} \beta^t \left(w_t - \tau_t^i\right)/c_t\right] + (1-\beta) \sum_{t=0}^{\infty} \beta^t \ln c_t. \tag{17}$$

Substituting equation (14) into equation (11) yields

$$\begin{aligned} B^i &\equiv \sum_{t=0}^{\infty} \beta^t s_t^i = 0, \qquad (18) \\ \text{where } s_t^i &\equiv \left[\tau_t^i - (\phi/2)\left(\tau_t^i\right)^2 - g_t\right]/c_t. \end{aligned}$$

The variable s_t^i in the above proposition is the period-0 value of the government's time-t budget surplus (or deficit, if negative), divided by c_0. For $t = 0$, this surplus is the total surplus; for periods $t > 0$ it is the primary surplus. We will refer to s_t^i as country i's *discounted time-t surplus.*

Substituting the global resource constraint (equation (13)) into equations (17) and (18) would allow the household's indirect utility and the government's budget constraint to be expressed solely as functions of the paths of the taxes in the N countries.

2.4 Taxes and Revenues

We impose further restrictions on the parameter space to ensure an equilibrium exists:

$$g_0 > 0,\ \max\{G, g_0\} \leq 1/(2\phi),\ W - G \geq 2/\phi,\ \min\{W, w_0\} > 25/(12\phi). \tag{19}$$

We allow for negative taxes, or subsidies. In this case the collection cost is the cost of administering and disbursing the surplus. We rule out, however, the empirically implausible case of an initial stock of credit (negative public debt) that is so large that the government can achieve a balanced budget (including interest payments) in period zero with a subsidy; this is the first inequality in assumption (19).

The net tax revenue function, $\tau - (\phi/2)\,\tau^2$, looks like a Laffer curve with a maximum of $1/(2\phi)$ at $\tau = 1/\phi$, although its shape is the result of tax collection costs and not the distortions associated with non-lump sum taxes. The second inequality in assumption (19) ensures that (exogenous) government spending is not so large that it cannot be financed with the revenue-maximising tax. The time-t budget is balanced at $\tau_t = \tau_t^- \equiv \left(1 - \sqrt{1 - 2\phi g_t}\right)/\phi$ or at $\tau_t = \tau_t^+ \equiv \left(1 + \sqrt{1 - 2\phi g_t}\right)/\phi$. The tax τ_t^- is on the "right", or upward-sloping part of the time-t net tax revenue curve; the tax τ_t^+ is on the "wrong" or downward-sloping part. There is a conventional government budget surplus in country i in period 0 if and only if $\tau_0^i \in \left[\tau_0^-, \tau_0^+\right]$ and a primary (that is, net of interest payments) surplus in period t if and only if $\tau_t^i \in \left[\tau_t^-, \tau_t^+\right]$.

In a symmetric outcome, a tax is feasible if consumption is strictly positive. Thus, by the global resource constraint (13), $\bar{\tau} \equiv \sqrt{(2/\phi)\,(W - G)}$ is the least upper bound on feasible taxes in a symmetric equilibrium. The third inequality in assumption (19) ensures that $\bar{\tau} > \tau_t^+$; hence, in a symmetric outcome any period-zero tax tax that yields a conventional budget surplus and any time-t, $t > 0$, tax that yields a primary budget surplus is feasible.[7]

In the next proposition we show that the discounted revenue curves, s_t^i, also have a Laffer-curve shape on $\left[\tau_t^-, \tau_t^+\right]$, with a discounted-revenue-maximising tax τ_t^{i*} located to the right of $1/\phi$. The proof of this and all other propositions is in the Appendix.

[7]Given the first three inequalities in assumption (19), the fourth inequality is sufficient, but not necessary, to ensure that the second-order conditions of the government's optimisation problem are satisfied.

Proposition 1 *For $t \geq 0$, let $\tau_t^j \in (-\bar{\tau}, \bar{\tau})$, $j \neq i$, be given. Then discounted revenue s_t^i is maximised at $\tau_t^{i*} > 1/\phi$ and is strictly increasing on $\left[\tau_t^-, \tau_t^{i*}\right)$ and strictly decreasing on $\left(\tau_t^{i*}, \tau_t^+\right]$.*

The above Proposition suggests that a rational government with market power may set taxes on the "wrong" side of the net tax revenue curve, that is, at a tax higher than the net revenue maximising tax $1/\phi$. To see this, suppose that $\tau_t^i = 1/\phi$. With market power, a country can influence global interest rates. Holding the other countries' taxes constant, a marginal rise in τ_t^i causes aggregate period-t consumption to fall. As net revenues are insensitive to taxes at $1/\phi$, they are unaffected by a marginal increase in the tax at this point. Thus, a marginal increase in τ_t^i above $1/\phi$ causes the discounted time-t surplus to rise.

3 Dynamic Optimal Taxation

We assume that at time zero, the government in country i can commit to a tax plan $\{\tau_t^i\}_{t=0}^{\infty}$. It takes the tax plans of the other governments as given and maximises the indirect utility of its household (equation (17)) subject to its budget constraint (equation (18)). Let τ_t^* be the time-t discounted-revenue-maximising tax when countries act symmetrically. By the definition of s_t^i, given in equation (18), $\tau_t^* = \tau^*$, $t > 0$.

Proposition 2 *A sequence $\{\tau_t\}_{t=0}^{\infty}$ is a symmetric Nash equilibrium if and only if it satisfies $\tau_0 \in [0, \tau_0^*]$, $\tau_t = \tau \in [0, \tau^*]$, $t > 0$, and*[8]

$$\frac{\tau_0}{1 + \phi \tau_0 s_0 / N} = \frac{\tau}{1 + \phi \tau s / N} \tag{20}$$

$$(1 - \beta) s_0 + \beta s = 0, \text{ where } s \equiv \left(\tau - (\phi/2) \tau^2 - G\right) / c_t. \tag{21}$$

To highlight the intuition, we use a graphical approach to prove that a unique tax sequence satisfying the conditions in the above Proposition exists and to analyse the properties of the equilibrium. We graph equations (20) and (21) for $(\tau, \tau_0) \in [0, \tau^*] \times [0, \tau_0^*]$ in Figure 1.

[8] In deriving equation (20), both sides were divided by $1/\phi$. If $\phi = 0$, the timing of taxes is irrelevant as long as the government satisfies its intertemporal budget constraint.

The feasibility condition (21) is represented by the curves F_{-}, F, and F_{+}. Curve F represents the case of no initial debt; curve F_{+} represents the case of positive initial debt; curve F_{-} represents the case of initial credit. Recall that τ_t^- is the smaller of the two taxes that balance the time-t budget; $\tau_t^- = \tau^-$, $t > 0$.

Proposition 3 *When $(\tau, \tau_0) \in [0, \tau^*] \times [0, \tau_0^*]$, the feasibility curves slope down with F_{+} lying above F and F_{-} lying below F.*

The feasibility curves slope down because an increase in the future tax allows the government to reduce the current tax and still balance its budget.[9] With no initial debt, $\tau_0^- = \tau^-$ and curve F goes through the point $A = (\tau^-, \tau^-)$. With positive initial debt, $\tau_0^- > \tau^-$ and curve F_{+} passes through the point $\left(\tau^-, \tau_0^-\right)$, which in this case lies above point A. Likewise, with negative initial debt, $\tau_0^- < \tau^-$ and curve F_{-} passes through $\left(\tau^-, \tau_0^-\right)$, which is below A in this case.

The curves representing the optimality condition, equation (20), in Figure 1 are represented by the upward-sloping curves passing through the origin.[10] Curve O represents either the case of no initial debt or the case where $N \to \infty$. Curves $O_{+}^{N'}$ and $O_{+}^{N''}$ represent cases where there is strictly positive initial debt and there are N' and N'' countries, respectively, where $1 \leq N'' \leq N' \leq \infty$. Curves $O_{-}^{N'}$ and $O_{-}^{N''}$ represent cases of initial credit when there are N' and N'' countries, respectively.

Proposition 4 *When $(\tau, \tau_0) \in [0, \tau^*] \times [0, \tau_0^*]$, the curves representing the optimality condition have the following properties:*

(i) Curve O is the 45-degree line.

(ii) All of the optimality curves are upward sloping and pass through the origin.

(iii) Curves $O_{-}^{N'}$ and $O_{-}^{N''}$ lie above curve O; curves $O_{+}^{N'}$ and $O_{+}^{N''}$ lie below curve O.

(iv) Curve $O_{+}^{N'}$ lies above curve $O_{+}^{N''}$ when $\tau > \tau^-$ and $\tau_0 < \tau_0^-$; curve $O_{-}^{N'}$ lies below curve $O_{-}^{N''}$ when $\tau < \tau^-$ and $\tau_0 > \tau_0^-$.

The intuition behind the optimality curves in Figure 1 is that the government trades off two objectives. First, it wants to smooth consumption by smoothing fiscal transfer

[9] The curves are drawn as convex to the origin. This is true if N is sufficiently large, but need not be true otherwise.

[10] Only curve O is a straight line, as is represented in Figure 1.

costs over time. If this were its sole objective, optimality would be represented by curve O. Second, it wants to lower the discounted value of the fiscal transfer costs through its influence on the global interest rate. If it is an initial debtor, it does this by lowering initial taxes and raising future taxes. Through the global resource constraint (equation (13)) this raises initial consumption and lowers future consumption, thus lowering the interest rate between periods zero and one. Thus, its required tax revenue falls. Likewise, if it is an initial creditor it can lower its required discounted tax revenue, and thus its tax collection costs, by raising initial taxes and lowering future taxes, thus raising the interest rate between periods zero and one.

This second objective means that the curve representing the optimality condition in Figure 1 is flatter than curve O when there is initial debt and it is steeper than curve O when there is an initial surplus. The more market power a country has (that is, the smaller is N) the greater is its ability to affect the global interest rate and the more important this second motive becomes. Thus, as the number of countries falls, the optimality curve becomes flatter if the country is an initial debtor and steeper if the country is an initial creditor. When $N \to \infty$ countries have no market power. Only the first objective matters and the optimality equation is represented by curve O.

Equilibrium occurs at the intersection of the relevant feasibility and optimality curves. We show that a unique intersection must occur in $[0, \tau^*] \times [0, \tau_0^*]$.

Proposition 5 *A unique symmetric equilibrium exists*

Different equilibria are represented by the points A - G in Figure 1. Point A is the equilibrium when there is no initial debt. In this case there is complete tax smoothing and the budget is balanced each period. Points B, C and D represent equilibria when there is positive initial debt. If $N \to \infty$, the equilibrium is represented by point B and there is complete tax smoothing. Points C and D lie below the 45-degree line; hence, if there is positive initial debt, $\tau > \tau_0$. As N falls, the negative slope of curve F_+ ensures that the initial tax declines and the future tax rises.

Likewise, points E, F and G represent equilibria when there is negative initial debt. If $N \to \infty$ (point G), there is complete tax smoothing. Points E and F lie above the 45-degree line; hence, if there is negative initial debt, $\tau < \tau_0$. As N falls, the initial tax rises and the future tax falls. These results are summarised below.

Proposition 6 *If countries have no market power* ($N \to \infty$) *or if initial government debt is zero, then there is complete tax smoothing. Otherwise, the initial tax is strictly less (greater) than the subsequent taxes if there is strictly positive (negative) initial government debt.*

When countries have no market power, we have Barro's (1979) result. Taxes result in resource losses because they are costly to collect. If these costs are convex, then an optimising government smooths them over time. If, however, the government can affect the interest rate and is an initial debtor, then it lowers the discounted value of its required tax revenue by reducing initial taxes and raising future taxes. If it is an initial creditor, it raises its return to its savings by increasing the initial tax and lowering future taxes.

The case of $N = 1$ corresponds to the social planner's outcome. Hence, we have the following result.

Proposition 7 *Suppose that* $N > 1$. *If there is positive (negative) initial government debt, then the initial tax is too high (low) relative to the social optimum. The subsequent tax is too low (high) relative to the social optimum.*

With positive initial debt, lowering initial taxes causes a positive externality by decreasing all countries' borrowing costs. Countries do not take into account this social benefit and they do not lower initial taxes enough. The outcome is furthest from the optimal outcome when the number of countries goes to infinity and countries lose their market power. This is in stark contrast to the result in "beggar-thy-neighbour" policy games where nations attempt to exploit their market power to gain at the expense of other countries. In such papers, as the number of countries goes to infinity and nations lose their market power, the noncooperative outcome converges to the cooperative outcome.[11]

[11] This would occur for, for example, in Hamada (1966).

The result that the distortion does not vanish, but increases when nations lose their market power, is similar in spirit to that in Kehoe (1987), although the economic mechanism is quite different. In his paper governments balance their budgets each period and do not fully take into account the negative effect on world capital accumulation of taxing workers to provide current government spending. Here, governments do not take into account the positive effect of current tax reductions and larger budget deficits on the world interest rate. In both papers, as the number of countries increases and the effect of any country on global variables declines, the failure of countries to take into account the effect of their actions on the world economy becomes more severe.

The results in this section, and in the rest of the paper, depend on the assumption that the government can commit to its path of planned taxes. If, for example, the government is an initial debtor, then Proposition 6 says that the inital tax is lower than subseqent taxes. This implies that the government enters period one with a strictly positive stock of debt. Thus, if the government could re-optimise in period one, Proposition 6 implies that it would set a lower tax in period one than in later periods. This implies that the equilibrium, which features constant taxes from period one on, is not time consistent unless there is no initial debt or countries have no market power. The time inconsistency arises because the initial debt is taken as exogenous, and hence, unaffected by taxes. We conjecture that with strictly positive initial debt, the time-consistent solution features taxes that are rising over time.[12]

4 CES Preferences

The log-linear preference specification of the previous section is the special case of CES preferences for an elasticity of intertemporal substitution equal to one. In this section, we look at how small changes in the value of the elasticity of substitution in the neighbourhood of one affect the results of the last section. We restrict attention to the case of

[12]In three-period numerical experiments with strictly positive initial debt, we found that with commitment the government sets $\tau_0 < \tau_1 = \tau_2$. Without commitment, the government sets $\tau_0 < \tau_1 < \tau_2$.

$R_0 b_0 > 0$. Let

$$u^i = \frac{1}{1-\theta} \sum_{t=0}^{\infty} \beta^t \left[\left(c_t^i \right)^{1-\theta} - 1 \right], 0 < \beta < 1, 0 < \theta \neq 1, \tag{22}$$

where θ is the reciprocal of the elasticity of intertemporal substitution. As $\theta \to 1$, the above preferences become the logarithmic specification of the previous sections. We assume that θ is arbitrarily close to one.[13]

Let $\hat{s}_t^i \equiv \left[\tau_t^i - (\phi/2) \left(\tau_t^i \right)^2 - g_t \right] / c_t^{\theta}$. Let $\hat{\tau}_0^*$ be the period-zero tax which maximises $\hat{s}_0^i$ when countries act symmetrically and let $\hat{\tau}^*$ be the period-t tax which maximises $\hat{s}_t^i$ when countries act symmetrically, $t > 0$.

Proposition 8 *A sequence $\{\tau_t\}_{t=0}^{\infty}$ is a symmetric Nash equilibrium if it satisfies $\tau_0 \in [0, \hat{\tau}_0^*]$, $\tau_t = \tau \in [0, \hat{\tau}^*]$, $t > 0$, and*

$$\frac{\tau_0}{1 + \theta \phi \tau_0 s_0 / N} = \frac{\tau}{1 + \theta \phi \tau s / N} \tag{23}$$

$$(1-\beta)\, \hat{s}_0 + \beta \hat{s} = 0. \tag{24}$$

The feasibility constraint and the optimality condition are represented graphically in Figure 2. Curves F^k and O^k represent the feasibility and optimality conditions, respectively, when $\theta = \theta^k$, $k = 0, 1$, where $\theta^0 < \theta^1$. The properties of the curves on $[0, \hat{\tau}^*] \times [0, \hat{\tau}_0^*]$ are summarised in the following proposition.

Proposition 9 *Curves F^0 and F^1 are downward sloping and intersect at $\left(\tau^-, \tau_0^-\right)$ and on the 45-degree line. Curve F^0 lies above curve F^1 above point $\left(\tau^-, \tau_0^-\right)$ and below the 45-degree line; it lies below curve F^1 elsewhere. Curves O^0 and O^1 are upward sloping, pass through the origin and lie below the 45-degree line. Curve O^0 lies above curve O^1.*

To see the properties of the feasibility curves, first suppose that there were no initial debt. Then increasing θ improves the government's tradeoff over feasible current and future taxes. To see this, suppose that the government sets lower taxes in period zero than in period one, borrowing from the households to finance the period-zero deficit.

[13] We do not reproduce the "necessary" part of Proposition 2 for $\theta \neq 1$ although one can use continuity arguments to extend the results to θ within a neighbourhood of one.

Then consumption is higher in period zero than in period one and households smooth their consumption by lending to the government. The higher is θ, the greater is their incentive to smooth consumption and the lower is the equilibrium interest rate. By a similar argument, if taxes are higher in period zero than in period one, the higher is θ, the higher is the interest rate that the government receives on its period-zero lending.

With strictly positive initial debt, the government's tradeoff is more favourable with a higher value of θ than with a lower value of θ if consumption is higher in the period in which the government runs a deficit. With positive initial debt, however, it is possible for the government to run a deficit in period zero, even though consumption is lower in period zero than in period one. In Figure 2, this corresponds to the parts of the curves between the two intersecting points. Consumption is made less smooth by the government's borrowing and the higher is θ, the more the government must pay to borrow.

With strictly positive initial debt, the optimality curves lie below the 45-degree line in Figure 2 for the same reason that they did in Figure 1 in the previous section: the government can reduce the interest rate it pays on period-zero borrowing by lowering period-zero taxes and raising future taxes. Here, the larger is θ, and the more households want to smooth their consumption, the greater is the effect of a period-zero tax cut financed by a future tax rise on the period-one interest rate. Thus, the larger is θ the flatter is the optimality curve.

Given Proposition 9 we have the following.

Proposition 10 *Suppose that $N > 1$. If there is positive initial government debt, then the initial tax is too high relative to the social optimum and subsequent taxes are too low relative to the social optimum. An increase in θ causes the socially optimal value of the initial tax to fall.*

As well as considering marginal changes in θ around one, we can consider the polar cases where θ goes to zero and to infinity. In the limit as θ falls to zero, consumers do not care about smoothing their consumption and a reduction in period-zero taxes financed by future tax increases has no effect on the period-one interest rate. Thus, the socially optimal and uncoordinated outcomes coincide and taxes are smoothed over time. In the limit as θ goes to infinity, indifference curves for current and future consumption become right angles and only the minimum consumption matters. With strictly positive initial debt, cooperating governments should choose an initial tax that is marginally higher than the one that balances their primary budget (which is constant over time) and future taxes which exactly balance their primary budget. Then, as period-zero consumption is lower than period-one consumption, the (gross) interest rate between periods zero and one is zero and governments can borrow marginally less than their initial debt in period zero without having to repay it in period one. Thus, minimum consumption occurs in period zero and can be made arbitrarily close to $W - G - (\phi/2)(\tau^{-})^2$.

5 Production and Capital Accumulation

An important simplifying feature of the model is that varying the timing of taxes, and thus fiscal transfer costs, over time is the only way to transfer real resources across periods. In equilibrium, net global private and public saving is always zero because the good is perishable. Reducing taxes in any given period increases the resources available that period and increases private consumption. In our benchmark model, the interest rate on current savings falls as a result of the current tax cut. In this section, we allow for production. With capital formation, real resources can be transferred across periods not only by changing the path of taxes, but also by capital formation.

Formally, we assume that the household has CES preferences and that the single good in the model is both a capital and a consumption good. The representative households each supply one unit of labour inelastically each period and save both bonds and the output of the current good in the form of capital. The savings of capital are loaned to the firms to be used in the next-period's production process. The firms transform capital and labour into output via a Cobb-Douglas production function where output per unit of labour is $f(k) = Ak^{\alpha}$, where k is the capital-labour ratio, $A > 0$ and $\alpha \in (0, 1)$. We suppose that labour is immobile across countries, capital is perfectly mobile and capital depreciates completely. Then perfect mobility of capital and perfect competition imply that capital-labour ratios and wages are equalised across countries and $k_t = k(R_t) = [A(1-\alpha)/R_t]^{1/\alpha}$.

A symmetric equilibrium is characterised by the Euler equation and the government budget constraint for the case of CES preferences (equations (37) and (42) in the appendix) and the global resource constraint

$$f(k(R_t)) - G - (\phi/2)\tau_t^2 - c_t - k_{t+1} = 0. \tag{25}$$

The model with capital is far more difficult to analyse than the one without. To obtain an analytical result, we restrict ourselves to a simple experiment. Imagine that the world is at a symmetric steady state with constant taxes and a positive initial stock of debt. Can policy makers raise welfare with a coordinated symmetric marginal tax cut?

Proposition 11 *Suppose countries are at a symmetric steady state with constant taxes and strictly positive debt. Then it is possible to increase welfare with a coordinated marginal tax cut in the current period.*

The proof demonstrates that welfare can be improved with a current tax cut financed by future tax rises that leave consumption constant from period one on. Lowering the current tax and raising future taxes raises current consumption and lowers future consumption, thus lowering the current interest rate as in the previous sections. This lowers the cost of servicing the debt and reduces future tax collection costs. To see that the interest rate must fall, suppose that it did not. Then next period's marginal product of capital rises and current capital accumulation falls. With lower tax collection costs and fixed current output, this implies current consumption rises. This is inconsistent with the interest rate falling in the current period unless next period's, and hence every future period's, consumption rises by more than current consumption. However, with lower current capital accumulation and higher future tax collection costs this is impossible. Thus, we have a contradiction.

6 Conclusion

We have demonstrated that, in our baseline model, optimising governments will perfectly smooth taxes if they have no market power or if they have no initial debt or credit. If countries are large enough to affect the world interest rate, then governments will set lower taxes in the current period than in the future if there is positive initial government debt and higher taxes in the current period and lower taxes in the future if there is negative initial government debt.

We show that if initial government debt is positive then, relative to the first-best co-operative outcome, goverments set current taxes too high. Thus, relative to the optimum, initial budget deficits are too low. Similarly, if initial government debt is negative, initial budget deficits are too high.

We extend our baseline model with its log-linear preferences, to the case of CES preferences. We show that a marginal fall in the intertemporal elasticity of substitution increases the deviation between the uncoordinated outcome and the first-best outcome; a marginal rise decreases the deviation. We also consider the case of production and capital accumulation. We show that if there is a steady state with constant taxes and strictly positive debt, then it is possible to increase welfare with a coordinated cut in current taxes.

7 Appendix

Proof of Proposition 1. The first-order condition associated with maximising s_t^i with respect to τ_t^i is $ds_t^i/d\tau_t^i = (1 - \phi\tau_t^i + \phi\tau_t^i s_t^i\ /N)/c_t = 0$. Clearly a maximum must be an element of $\left(\tau_t^-, \tau_t^+\right)$ and have $s_t^i > 0$. We have $\tau_t^- < 1/\phi < \tau_t^+$; hence, $ds_t^i/d\tau_t^i > 0$ at τ_t^- and $ds_t^i/d\tau_t^i < 0$ at τ_t^+. Thus, there is a solution to the first-order condition in $\left(\tau_t^-, \tau_t^+\right)$. At a solution, $d^2 s_t^i/d\tau_t^{i2} = -\left(\tau_t^i c_t\right)^{-1} < 0$; hence the solution is unique, $ds_t^i/d\tau_t^i$ cannot change signs on $\left[\tau_t^-, \tau_t^{i*}\right)$ or $\left(\tau_t^{i*}, \tau_t^+\right]$ and the second-order condition is satisfied. It is clear from the first-order condition that $\tau_t^{i*} > 1/\phi$.

Proof of Proposition 2. We first show that the conditions of the Proposition are sufficient for a symmetric equilibrium. The first-order conditions associated with maximising (17) subject to (18) are (18) and $\left(\partial u^i/\partial \tau_t^i\right)/\left(\partial u^i/\partial \tau_s^i\right) = \left(\partial B^i/\partial \tau_t^i\right)/\left(\partial B^i/\partial \tau_s^i\right)$, $s, t = 0, \ldots$. Differentiating (17) and (18), using (13), yields

$$\begin{aligned} \partial u^i/\partial \tau_t^i &= \beta m_t^i,\ \partial B^i/\partial \tau_t^i = \beta n_t^i \text{ where} && (26) \\ m_t^i &= \frac{-1 + \frac{\phi \tau_t^i}{N}\frac{w_t - \tau_t^i}{c_t}}{c_t \sum_{s=0}^{\infty} \beta^s \frac{w_s - \tau_s}{c_s}} - \frac{(1-\beta)\,\phi \tau_t^i}{N c_t};\ n_t^i = \frac{1 - \phi \tau_t^i + \phi \tau_t^i s_t^i / N}{c_t}. && (27) \end{aligned}$$

At a symmetric solution with $\tau_t = \tau$, $t > 0$, (27) is

$$m_t^i = \begin{cases} m_0 \equiv \frac{-(1-\beta)(1+\phi\tau_0 s_0/N)}{c_0}, & \text{if } t = 0 \\ m_1 \equiv \frac{-(1-\beta)(1+\phi\tau s/N)}{c}, & \text{if } t > 0 \end{cases};\ n_t^i = \begin{cases} n_0 \equiv \frac{1-\phi\tau_0+\phi\tau_0 s_0/N}{c_0}, & \text{if } t = 0 \\ n_1 \equiv \frac{1-\phi\tau+\phi\tau s/N}{c}, & \text{if } t > 0 \end{cases}. \quad (28)$$

Substituting (28) into the first-order conditions yields (20) and (21). We now demonstrate that this solution satisfies the second-order condition for a maximum. The proof requires the following lemma.

Lemma 1. If the symmetric optimum satisfies the conditions of Proposition 2, then $n_t > 0$, $t = 0, 1$.

Proof of Lemma 1. By (26) - (28) and $\tau_t > 0$, it is sufficient to show that $s_0 < 1$ and $s < 1$. By the definition of s_0 and s, this is true if $\tau_0 < w_0$ and $\tau < W$. This is true if $\tau_0 < \tau_0^{*1} \equiv w_0 - \sqrt{w_0^2 - \bar{\tau}^2}$ and $\tau < \tau_1^{*1} \equiv W - \sqrt{W^2 - \bar{\tau}^2}$ where τ_t^{*1} is τ_t^* when $N = 1$. Thus, it is sufficient to show that $\tau_t^* \leq \tau_t^{*1}$. We have $ds_t^i/d\tau_t^i = -\phi \tau_t^{*1} s_t^i (N-1)/(N c_t)$ at asymmetric outcome when $\tau_t = \tau_t^{*1}$. As this derivative is negative, Proposition 1 implies $\tau_t^* \leq \tau_t^{*1}$.■

By (26) - (27),

$$d^2u^i = \sum_{t=0}^{\infty} \beta^t m_t^i d^2\tau_t^i + \sum_{t=0}^{\infty}\sum_{s=0}^{\infty} \beta^t \frac{\partial m_t^i}{\partial \tau_s^i} d\tau_t^i \, d\tau_s^i \text{ where} \tag{29}$$

$$\sum_{t=0}^{\infty} \beta^t n_t^i d^2\tau_t^i + \sum_{t=0}^{\infty} \beta^t \frac{\partial n_t^i}{\partial \tau_t^i} \left(d\tau_t^i\right)^2 = 0. \tag{30}$$

Solving (30) for $d^2\tau_0^i$, substiuting the result into (29) and using the first-order conditions and Lemma 1 yields that the second variation is strictly negative if and only if

$$n_0^i \sum_{t=0}^{\infty}\sum_{s=0}^{\infty} \beta^t \frac{\partial m_t^i}{\partial \tau_s^i} d\tau_t^i \, d\tau_s^i - m_0^i \sum_{t=0}^{\infty} \beta^t \frac{\partial n_t^i}{\partial \tau_t^i} \left(d\tau_t^i\right)^2 < 0. \tag{31}$$

At a symmetric optimum, the second-order derivatives are

$$\partial n_t^i / \partial \tau_t^i = \begin{cases} n_{00} \equiv \frac{\phi}{Nc_0}\left(s_0 - N + 2\tau_0 n_0\right), \ t = 0 \\ n_{11} \equiv \frac{\phi}{Nc}\left(s - N + 2\tau n_1\right), \ t > 0 \end{cases} \tag{32}$$

$$\frac{\partial m_t^i}{\partial \tau_t^i} = \begin{cases} q_{00} - q_0^2, \text{ where } q_{00} \equiv -\frac{(1-\beta)\phi}{Nc_0}\left[s_0 + 2\tau_0 n_0 + \frac{(2N-1)\phi\tau_0^2}{Nc_0}\right], \ t = 0 \\ q_{11} - \beta^t q_1^2, \text{ where } q_{11} \equiv -\frac{(1-\beta)\phi}{Nc}\left[s + 2\tau n_1 + \frac{(2N-1)\phi\tau^2}{Nc}\right] \ t > 0 \end{cases} \tag{33}$$

$$\frac{\partial m_t^i}{\partial \tau_s^i} = \begin{cases} -\beta^s q_0 q_1, \ t = 0 \\ -q_0 q_1, \ s = 0 \\ -\beta^s q_1^2, \text{ otherwise} \end{cases}, \ s \neq t, \tag{34}$$

where

$$q_0 \equiv -(1-\beta)\left[n_0 + \frac{\phi(N-1)\tau_0}{Nc_0}\right], \ q_1 \equiv -(1-\beta)\left[n_1 + \frac{\phi(N-1)\tau}{Nc}\right], \ t > 0. \tag{35}$$

Substituting (32) - (35) into (31) yields that (31) holds at the symmetric solution with constant taxes after period zero iff

$$\left[n_0\left(q_{00}-q_0^2\right)-m_0n_{00}\right]\left(d\tau_0^i\right)^2-2n_0q_0q_1\left(\sum_{t=1}^{\infty}\beta^t d\tau_t^i\right)d\tau_0^i-n_0q_1^2\left(\sum_{t=1}^{\infty}\beta^t d\tau_t^i\right)^2+$$
$$\left(n_0q_{11}-m_0n_{11}\right)\sum_{t=1}^{\infty}\beta^t\left(d\tau_t^i\right)^2<0. \tag{36}$$

Using $\sum_{t=0}^{\infty}\left(\partial B^i/\partial\tau_t^i\right)d\tau_t^i=0$ and (26) - (28) to solve for $\sum_{t=1}^{\infty}\beta^t d\tau_t^i$, (36) holds if $n_0q_{11}-m_0n_{11}<0$ and $\left(n_0q_{00}-m_0n_{00}\right)n_1^2-n_0\left(n_1q_0-n_0q_1\right)^2<0$. Thus, it is sufficient to show that $n_0q_{tt}-m_0n_{tt}<0$, $t=0,1$. Using (28), (32), (33) and (35) this is true if $\left(1-\phi\tau_0+\phi\tau_0s_0\right)\phi\tau_0^2<c_0$ and $\left(1-\phi\tau+\phi\tau s\right)\phi\tau^2<c$. I show the latter is true; the former follows by a similar argument. The latter inequality is true if and only if $(W-G)^2-2\phi\tau^2(W-G)-\phi^2\tau^4/4+\phi^2\tau^3W>0$. The left-hand side is minimised in G at $W-G=\phi\tau^2$. Substituting this into the inequality, it is sufficient to show that $\tau<4W/5$. This is true if $\tau_1^{*1}<4W/5$, where τ_1^{*1} is defined in the proof of Lemma 1. This requires $(24/25)W^2-(2/\phi)W+2G/\phi>0$, which is true if $W>25/(12\phi)$ which is true by (19).

We now show that the conditions of the Proposition are necessary. We first rule out equilibria with $\tau_t>\tau_t^*$. Let $\tau_t^j\in(-\bar{\tau},\bar{\tau})$, $j\neq i$. Let $\bar{\tau}^i>0$ be such that $c_t=0$ when $\tau_t^i=\bar{\tau}^i$ and denote the value of s_t^i at τ_t^{i*} by s_t^{i*}. Suppose to the contrary that the government of country i chooses $\tau_t^i=\tau^W>\tau_t^{i*}$ and let $s_t^W\in(-\infty,s_t^{i*})$ denote the value of s_t^i when $\tau_t^i=\tau^W$. We have that s_t^i is continuous in τ_t^i on $(-\bar{\tau}^i,\tau_t^{i*})$ with $s_t^i\to-\infty$ when $\tau_t^i\searrow-\bar{\tau}^i$ and $s_t^i\to s_t^{i*}$ when $\tau_t^i\nearrow\tau_t^{i*}$; hence there exists $\tau^R\in(-\bar{\tau}^i,\tau_t^{i*})$ such that $s_t^i=s_t^W$ when $\tau_t^i=\tau^R$.

By equation (18), a switch from $\tau_t^i=\tau^W$ to $\tau_t^i=\tau^R$ has no effect on the government's intertemporal budget constraint and, hence, does not require a change in any other tax. The government prefers $\tau_t^i=\tau^R$ to $\tau_t^i=\tau^W$ if its indirect utility, given by (17), is higher at $\tau_t^i=\tau^R$ than at $\tau_t^i=\tau^W$. Let c_t evaluated at $\tau_t^i=\tau^K$ be denoted by $c_t^K, K=W,R$. Then this is the case if $c_t^R>c_t^W$ and $\left(W-\tau^R\right)/c_t^R>$

$\left(W-\tau^K\right)/c_t^K$. The first inequality follows from $\left(\tau^W\right)^2 > \left(\tau^R\right)^2$. This is clearly true when $\tau^R > 0$. When $\tau^R < 0$ this follows from $ds_t^i/d\tau_t^i > 0$ when $\tau_t^i < 0$ and $s_t^W > s_t^i$ when $\tau_t^i = -\tau^W$. The second inequality follows from $\left(W-\tau^K\right)/c_t^K = 1 - s_t^W + (\phi/2)\left[(N-1)/N\right]\left(d-\left(\tau^k\right)^2\right)/\left\{W-g-d+\left[d-\left(\tau^k\right)^2\right]/N\right\}, K = W, R$ where $d \equiv [1/(N-1)]\sum_{j\neq i}\left(\tau_t^j\right)^2$ and $\left(\tau^W\right)^2 > \left(\tau^R\right)^2$.

We now show that there cannot exist a symmetric equilibrium with subsidies and (possibly) time-varying taxes after period 0. Suppose to the contrary that there exists $t \geq 0$ such that $\tau_t < 0$. Then $s_t < 0$ and assumption (19) ensures that there exists s such that $\tau_s > 0$ and $s_s > 0$. The first-order conditions $\left(\partial u^i/\partial\tau_t^i\right)/\left(\partial u^i/\partial\tau_s^i\right) = \left(\partial B^i/\partial\tau_t^i\right)/\left(\partial B^i/\partial\tau_s^i\right)$, (26) and (27) imply that $\tau_s - \tau_t = \phi\tau_s\tau_t\left(s_s - s_t\right)$. As the left-hand side of this expression is strictly positive and the right-hand side is strictly negative, this is a contradiction.

Finally, we demonstrate that there is no symmetric equilibrium with time-varying taxes after period zero. The first-order conditions $\left(\partial u^i/\partial\tau_0^i\right)/\left(\partial u^i/\partial\tau_s^i\right) = \left(\partial B^i/\partial\tau_0^i\right)/\left(\partial B^i/\partial\tau_s^i\right)$, (26) and (27) imply that $\left(1+\phi\tau_t s/N\right)/\tau_t = \left(1+\phi\tau_0 s_0/N\right)/\tau_0$, $t > 0$. Thus, is is sufficient to show that $\left(1+\phi\tau_t s/N\right)/\tau_t$ is strictly decreasing in τ_t on $[0,\tau_t^{*1}]$. This is true if $1 > \left(\phi\tau_t^2/N\right)\left(W-G-\phi\tau_t W+\phi\tau_t^2/2\right)/c_t$ on $[0,\tau_t^{*1}]$. If the right-hand side of the inequality is negative, this is true. If it is positive, it is true if it is true when $1 > \phi\tau_t^2\left(W-G-\phi\tau_t W+\phi\tau_t^2/2\right)/c_t$. This is true if and only if $H\left(\tau_t\right) = \left(\bar{\tau}^2-\tau_t^2\right)^2 - 2\tau_t^2\left(\bar{\tau}^2-2W\tau_t+\tau_t^2\right) > 0$. We have $H'\left(\tau_t\right) = 0$ and $H''\left(\tau_t\right) > 0$ implies $\tau_t = (3W - \sqrt{9W^2-8\bar{\tau}^2})/2 > \tau_t^{*1}$; hence H has no interior minimum on $[0,\tau_t^{*1}]$. $H(0) = \bar{\tau}^4 > 0$ and $H\left(\tau_t^{*1}\right) = \left[\bar{\tau}^2-\left(\tau_t^{*1}\right)^2\right]^2 > 0$; hence $H > 0$ on $[0,\tau_t^{*1}]$.

Proof of Proposition 3. By Lemma 1, $ds_t/d\tau_t > 0$ when $N = 1$. This ensures that the feasibility curves slope down. Clearly no two of the curves can intersect and at $\tau = \tau^-$, F_+ lies above F and F lies above F_-. This yields the rest of the proof.

Proof of Proposition 4. Let $(\tau,\tau_0) \in [0,\tau^*] \times [0,\tau_0^*]$. The right-hand side of (20) is $h(\tau;N) \equiv \tau/\left(1+\phi\tau s/N\right)$; the left-hand side is $h_0\left(\tau_0;N\right) \equiv \tau_0/\left(1+\phi\tau_0 s_0/N\right)$. The functions h and h_0 have the following properties:

(a) $h(0; N) = h_0(0; N) = 0$.

(b) $dh(\tau; N)/d\tau > 0; dh_0(\tau_0; N)/d\tau_0 > 0$. This follows from the argument in Proposition 2.

(c) $h(\tau; N') > (=, <)\ h(\tau; N'')$ when $\tau > (=, <)\ \tau^-$ and $h_0(\tau_0; N') > (=, <)$ $h_0(\tau_0; N'')$ when $\tau_0 > (=, <)\ \tau_0^-$.

(d) $h(\tau; N) > (=, <)\ h_0(\tau; N)$ when $R_0 b_0 > (=, <)\ 0$.

Result (i) follows from (d) and $h_0(\tau_0; N) \rightarrow h(\tau; N)$ when $N \rightarrow \infty$. Result (ii) follows from (a) and (b). Result (iii) follows from (b) and (d). Result (iv) follows from (b) and (c).

Proof of Proposition 5. Uniqueness follows from the strictly negative slopes of the feasibility curves and the strictly positive slopes of the optimality curves.

Suppose that $R_0 b_0 \geq 0$. An equilibrium fails to exist if and only if the F_+ curve lies above O_+^N at the largest τ that is consistent with equilibrium, τ^*. By the algebra in the proof of Proposition 1, τ^* satisfies $1 - \phi\tau^* + \phi\tau^* s/N = 0$. Thus, $h(\tau^*; N) = 1/\phi$. Likewise, $h_0(\tau_0^*; N) = 1/\phi$. Thus, (τ^*, τ_0^*) is the point on O_+^N when $\tau = \tau^*$. At this point, $s > 0$ and $s_0 > 0$; hence there is a surplus in both periods and this point must lie above F_+ and an equilibrium must exist. The proof for $R_0 b_0 < 0$ is similar.

Proof of Proposition 8. When preferences are represented by (22) the Euler equation of the consumer's optimisation problem becomes

$$c_{t+1}^i = (\beta R_{t+1})^{1/\theta} c_t^i, \ t = 0, 1, \dots . \tag{37}$$

Solving the difference equation (37) yields

$$c_t^i = \beta \left(\prod_{s=1}^{t} R_s \right)^{1/\theta} c_0^i, \ t = 0, 1, \dots . \tag{38}$$

Averaging both sides of (38) across countries yields

$$\prod_{s=1}^{t} R_s = \left(1/\beta^t\right)\left(c_t/c_0\right)^{\theta}, \ t = 1, 2, \dots . \tag{39}$$

Substituting (38) and (39) into the household's budget constraint yields

$$\frac{c_0^i}{c_0} = \sum_{t=0}^{\infty} \beta^t \frac{w_t^i - \tau_t^i}{c_t^{\theta}} / \sum_{t=0}^{\infty} \beta^t c_t^{1-\theta}. \tag{40}$$

Substituting (38) - (40) into (22) and ignoring constants that are unimportant to the optimisation problem yields the indirect utility function

$$\frac{1}{1-\theta}\left(\sum_{t=0}^{\infty} \beta^t \frac{w_t^i - \tau_t^i}{c_t^{\theta}}\right)^{1-\theta}\left(\sum_{t=0}^{\infty} \beta^t c_t^{1-\theta}\right)^{\theta}. \tag{41}$$

Substituting (39) into (11) yields

$$\sum_{t=0}^{\infty} \beta^t \hat{s}_t^i = 0. \tag{42}$$

It is straightforward to show that the first-order condiditons associated with maximising (41) subject to (42) satisfy (23) and (24); by a continuity argument, the second-order conditions continue to hold for θ arbitrarily close to one.

Proof of Proposition 9. Using the argument in the proof of Proposition 1 and a continuity argument, $\hat{s}_t$ is increasing in τ_t on $[0, \hat{\tau}_t]$ for θ in a neighbourhood of one. Hence, F^0 and F^1 slope down. Clearly (24) is satisfied at $\left(\tau^-, \tau_0^-\right)$ for all $\theta > 0$ and if (24) is satisfied at (τ, τ) for some $\theta > 0$ it is satisfied for all $\theta > 0$. Thus, F^0 and F^1 intersect at $\left(\tau^-, \tau_0^-\right)$ and on the 45-degree line. Suppose that F^0 passes through (τ', τ_0'). Then when $\tau_0 = \tau_0'$, $\tau = \tau'$ and $\theta = \theta^1$, $(1-\beta)\hat{s}_0 + \beta\hat{s} > 0$ if and only if $\tau' > \tau^-$ and $\tau_0' < \tau'$ or $\tau' < \tau^-$ and $\tau_0' > \tau'$. This and $\hat{s}_t$ increasing in τ_t establish the last property of F^0 and F^1.

Redefine h and h_0 in the proof of Proposition 4 as $h(\tau; N) = \tau/(1 + \theta\phi\tau s/N)$ and

$h_0(\tau_0; N) = \tau_0 / (1 + \theta\phi\tau_0 s_0/N)$ for $(\tau, \tau_0) \in [0, \hat{\tau}^*] \times [0, \hat{\tau}_0^*]$. It is easy to establish that property (b) in the proof of Proposition 4 continues to hold for θ sufficiently close to one. In addition, property (d) holds and h is decreasing in θ when $\tau > \tau^-$ and h_0 is decreasing in θ when $\tau_0 < \tau^-$. These properties ensure that O^0 and O^1 have the stated properties.

Proof of Proposition 10. Suppose that the initial tax is less than $1/\phi$, the tax that maximises (undiscounted) revenue, and hence is on the "right" side of the revenue curve. If this were not true, then with constant taxes welfare could be improved by moving to the lower revenue-equivalent tax. Let the initial period be denoted by $t = 0$. Suppose that the coordinated marginal fall in the initial tax $d\tau_0 < 0$ is financed by a sequence of future tax changes $\{d\tau_t\}_{t=1}^{\infty}$ such that $dc_t = dc$, $t > 0$. Differentiating (22) and evaluating at the initial steady state yields

$$du^i = \frac{dc_0}{c^{1-\theta}} + \frac{\beta}{1-\beta}\frac{dc}{c^{1-\theta}} > 0 \Longleftrightarrow dc_0 + \frac{\beta dc}{1-\beta} > 0. \tag{43}$$

Differentiating (42) and evaluating at a steady state yields

$$(1-\phi\tau)\sum_{t=0}^{\infty}\beta^t d\tau_t - \left(s - \frac{R_0 b_0}{c}\right) dc_0 - \frac{\beta s dc}{1-\beta} = 0. \tag{44}$$

At a steady state, the gross interest rate must be equal to $1/\beta$. Thus evaluating (22) at the steady state yields $s = (1-\beta) b_0 / (\beta c)$. Substituting this into (44) yields

$$(1-\phi\tau)\sum_{t=0}^{\infty}\beta^t d\tau_t + b_0 dc_0/c - b_0 dc/c = 0. \tag{45}$$

Differentiating (37) and evaluating at the steady state yields

$$dR_1 = \theta(dc - dc_0) / (\beta dc), \; dR_t = 0, \; t = 2, 3, \ldots . \tag{46}$$

Differentiating (25), employing $f'(k_t) = R_t$ and $dk_t/dR_t = -k_t/(\alpha R_t)$, substituting in (25) and (45) and evaluating at a steady state yields

$$\begin{aligned} \phi\tau d\tau_0 &= \theta\left(dc - dc_0\right) k_1 / \left(\alpha c\right) - dc_0 \\ \phi\tau d\tau_1 &= \theta\left(dc - dc_0\right) k_1 / \left(\alpha\beta c\right) - dc \\ \phi\tau d\tau_t &= -dc,\ t = 2, 3, \dots . \end{aligned} \tag{47}$$

Substituting (47) into (43) and using $b_0 > 0$ yields that utility rises if and only if

$$\frac{1-\phi\tau}{\phi\tau}\frac{\beta}{1-\beta} + \frac{b_0}{c} > 0. \tag{48}$$

This is true because $\tau < 1/\phi$. This gives our result.

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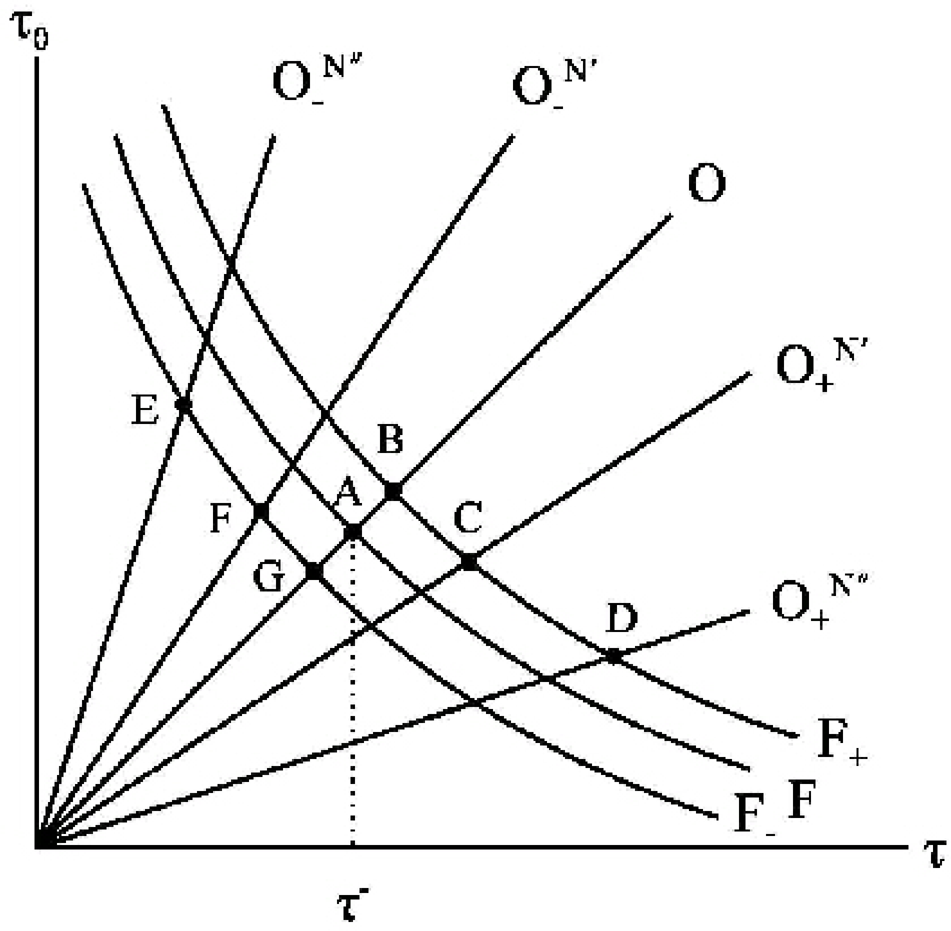

Figure 1: Equilibrium Taxes

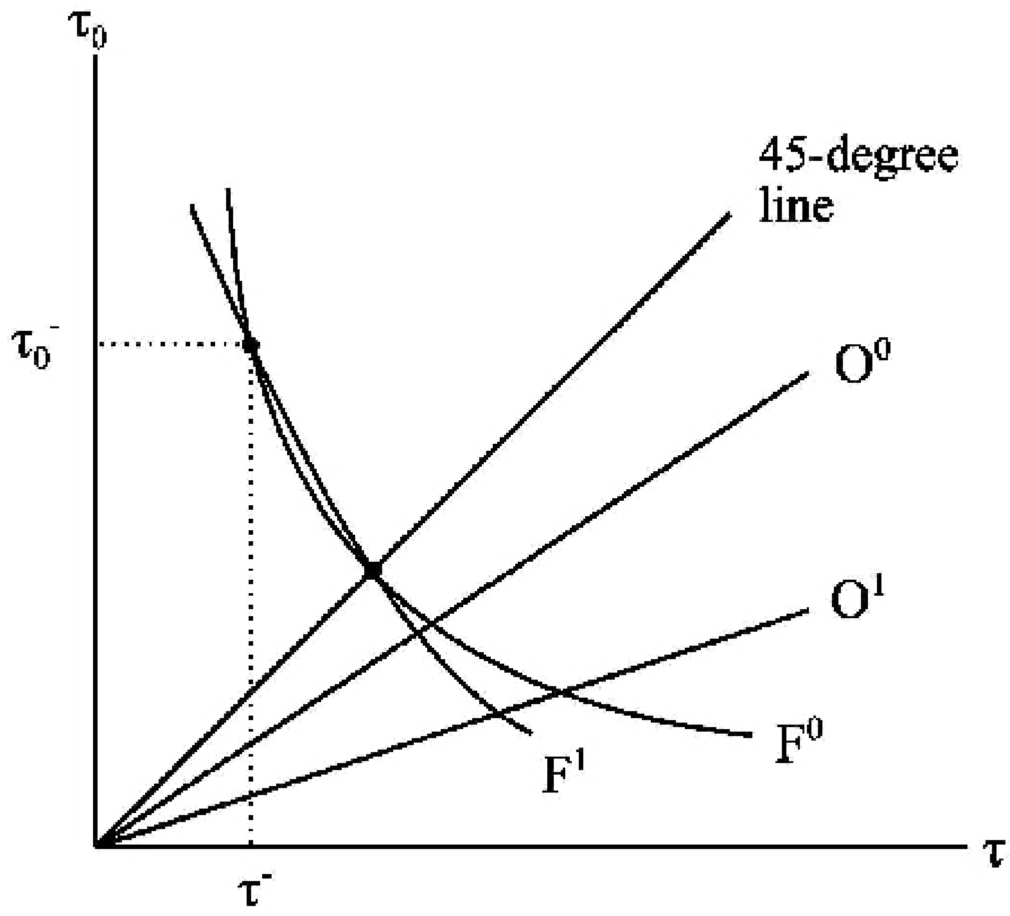

Figure 2: Equilibrium with CES Preferences and Initial Debt