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Seigniorage

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Abstract:

Governments through the ages have appropriated real resources through the monopoly of the 'coinage'. In modern fiat money economies, the monopoly of the issue of legal tender is generally assigned to an agency of the state, the Central Bank, which may have varying degrees of operational and target independence from the government of the day.

In this paper I analyse four different but related concepts, each of which highlights some aspect of the way in which the state acquires command over real resources through its ability to issue fiat money. They are (1) seigniorage (the change in the monetary base), (2) Central Bank revenue (the interest bill saved by the authorities on the outstanding stock of base money liabilities), (3) the inflation tax (the reduction in the real value of the stock of base money due to inflation and (4) the operating profits of the central bank, or the taxes paid by the Central Bank to the Treasury.

To understand the relationship between these four concepts, an explicitly intertemporal approach is required, which focuses on the present discounted value of the current and future resource transfers between the private sector and the state. Furthermore, when the Central Bank is operationally independent, it is essential to decompose the familiar consolidated 'government budget constraint' and consolidated 'government intertemporal budget constraint' into the separate accounts and budget constraints of the Central Bank and the Treasury. Only by doing this can we appreciate the financial constraints on the Central Bank's ability to pursue and achieve an inflation target, and the importance of cooperation and coordination between the Treasury and the Central Bank when faced with financial sector crises involving the need for long-term recapitalisation or when confronted with the need to mimick Milton Friedman's helicopter drop of money in an economy faced with a liquidity trap.

JEL: E4, E5, E6, H6
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I. Introduction

Seigniorage refers historically, that is, in a world with commodity money, to the difference between the face value of a coin and its costs of production and mintage. In fiat money economies, the difference between the face value of a currency note and its marginal printing cost are almost equal to the face value of the note – marginal printing costs are effectively zero. Printing fiat money is therefore a highly profitable activity – one that has been jealously regulated and often monopolized by the state.

While the profitability of printing money is widely recognized, the literature on the subject contains a number of different measures of the revenue appropriated by the state through the use of the printing presses. In this paper, I discuss five of them. There also is the empirical institutional regularity, that the state tends to assign the issuance of fiat money to a specialized agency, the Central Bank, which has some (variable) degree of independence from the other organs of the state and from the government administration of the day. This institutional arrangement has implications for the conduct of monetary policy that cannot be analysed in the textbook macroeconomic models, which consolidate the Central Bank with the rest of the government.

In the next five Sections, the paper addresses the following five questions. (1) What revenue does the state obtain from seigniorage, that is, its monopoly of the issuance of base money (currency and commercial bank balances with the Central Bank)? (2) What inflation rate would result if the monetary authority were to try to maximise its revenues? (3) Who ultimate appropriates and benefits from these resources, the Central Bank or the Treasury/ministry of finance? (4) Does the Central Bank have adequate financial resources to pursue its monetary policy and financial stability mandate, and more specifically for inflation-targeting Central

Banks, is the inflation target financeable? (4) What is the relevance for monetary policy of the fact that the central bank's fiat money liabilities are irredeemable - a given amount of base money gives the holder no other claim on the issuer than for that same amount of base money)? The first two questions receive simple preliminary answers in Section II of the paper, confirming results that can be found e.g. in Walsh (2003) and Romer (2006). The second half of Section II contains an analysis of the relationship between three of base money issuance revenue measures (seigniorage, central bank revenue and the inflation tax) in real time, that is, outside the steady state and without the assumptions that the Fisher hypothesis holds and that the velocity of circulation of base money is constant over time. It derives the 'intertemporal seigniorage identity' relating the present discounted value of seigniorage and the present discounted value of central bank revenue.

The government's period budget constraint and its intertemporal budget constraint are familiar components of dynamic macroeconomic models at least since the late 1960s (see e.g. Christ (1968), Blinder and Solow (1973) and Tobin and Buiter (1976)). The 'government' in question is invariably the consolidated general government (central, state and local, henceforth the 'Treasury') and Central Bank. When the Central Bank has operational independence, it is useful, and at times even essential, to disaggregate the general government accounts into separate Treasury and Central Bank accounts. Section III of the paper presents an example of such a decomposition, adding to the work of Walsh (2003). In Section IV, a simple dynamic general equilibrium model with money is presented, which incorporates the Treasury and Central Bank whose accounts were constructed in Section III. It permits all four questions to be addressed. Section V raises two further issues prompted by the decomposition of the government's accounts into separate Central Bank and Treasury accounts: the need for fiscal resources to recapitalise an financially stretched or even insolvent Central Bank and the institutional modalities of helicopter drops of money. In Section V of the paper I work out the formal implications of irredeemability of base money. I argue that this means that base money is perceived as an asset by the holder bunt not as a liability by the issuer. This means that in a liquidity trap, both helicopter drops of money (money-financed tax cuts) and open market purchases will stimulate consumption demand.

The systematic analysis of the sources of Central Bank revenue or seigniorage is part of a tradition that is both venerable and patchy. It starts (at least) with Thornton (1802) and includes such classics as Bresciani-Turroni (1937) and Cagan (1956). Milton Friedman (1971), Phelps (1973), Sargent (1982, 1987) and Sargent and Wallace (1981) have made important contributions. Empirical investigations include King and Plosser (1985), Dornbusch and Fischer (1986), Anand and van Wijnbergen (1989), Kiguel and Neumeyer (1995) and Easterly, Mauro and Schmidt-Hebbel (1995). Recent theoretical investigations include Sims (2004, 2005) and Buiter (2004, 2005). Modern advanced textbooks/treatises such as Walsh (2003 and Romer (2006) devote considerable space to the issue. The explicitly multi-period or intertemporal dimension linking the various notions of seigniorage has not, however, been brought out and exploited before.

II. Three faces of seigniorage

There are two common measures of 'seigniorage', the resources appropriated by the monetary authority through its capacity to issue zero interest fiat money. The first is the change in the monetary base, $S_{1,t} = \Delta M_t = M_t - M_{t-1}$, where M_t is the stock of nominal base money outstanding at the end of period t and the beginning of period $t-1$. The term *seigniorage* is sometimes reserved for this measure (see e.g. Flandreau (2006), and Bordo (2006)) and I shall follow this convention, although usage is not standardised. The second measure is the interest earned by investing the resources obtained though the past issuance of base money in interest-bearing assets: $S_{2,t} = i_t M_{t-1}$, where i_t is the risk-free nominal interest rate on financial instruments other than base money between periods *t-1* and *t*. Flandreau refers to this as *Central Bank revenue* and again I shall follow this usage.

It is often helpful to measure seigniorage and Central Bank revenue in real terms or as a share of GDP. Period *t* seigniorage as a share of GDP, $s_{1,t}$, is defined as $s_{1,t} = \frac{\Delta M_t}{P_t Y_t}$ and period *t* Central Bank revenue as a share of GDP, $s_{2,t}$, as $s_{2,t} = i_t \frac{M_{t-1}}{P_t Y_t}$, where P_t is the period *t* price level and Y_t period *t* real output.

A distinct but related concept to seigniorage and Central Bank revenue is the inflation tax. The inflation tax is the reduction in the real value of the stock of base money caused by inflation.[1] Let $\pi_t = \frac{P_t}{P_{t-1}} - 1$ be the rate of inflation between periods *t-1* and *t*, then the period *t* inflation tax is $S_{3,t} = \pi_t M_{t-1}$. The inflation tax as a share of GDP will be denoted $s_{3,t} = \pi_t \frac{M_{t-1}}{P_t Y_t}$.

Let $\gamma_t = \frac{Y_t}{Y_{t-1}} - 1$ be the growth rate of real GDP between periods *t-1* and *t*. The real interest rate between periods *t-1* and *t* is denoted r_t where

$$(1 + r_t)(1 + \pi_t) = 1 + i_t \tag{1}$$

[1] This is sometimes called the ‘anticipated inflation tax’, to distinguish it from the ‘unanticipated inflation tax’, the reduction in the real value of outstanding fixed interest rate nominally denominated debt instruments caused by an unexpected increase in the rate of inflation which causes their price and real value to decline.

The growth rate of the nominal stock of base money between periods *t-1* and *t* is denoted $\mu_t = \frac{M_t}{M_{t-1}} - 1$. Finally, let the ratio of the beginning-of-period base money stock to nominal GDP in period *t* be denoted $m_t = \frac{M_{t-1}}{P_t Y_t}$.

Steady-state seigniorage

Assume that in a deterministic steady state, the ratio of base money to nominal GDP is constant, that is,

$$1+\bar{\mu} = (1+\bar{\pi})(1+\bar{\gamma}) \tag{2}$$

where variables with overbars denote deterministic steady-state values. In steady state,

$$\begin{aligned} \bar{s}_1 &= \bar{\mu}\bar{m} \\ \bar{s}_2 &= \bar{i}\bar{m} \\ \bar{s}_3 &= \bar{\pi}\bar{m} \end{aligned}$$

or, using (1) and (2)

$$\begin{aligned} \bar{s}_1 &= \left((1+\bar{\pi})(1+\bar{\gamma})-1\right)\bar{m} \\ \bar{s}_2 &= \left((1+\bar{\pi})(1+\bar{r})-1\right)\bar{m} \\ \bar{s}_3 &= \bar{\pi}\bar{m} \end{aligned} \tag{3}$$

In what follows I will only consider steady-state money demand functions $\bar{m} = \ell(\bar{\pi}),\ \ell' < 0$ that have the property that $\bar{s}_i,\ i = 1,2,3$ is continuously differentiable, increasing in $\bar{\pi}$ when $\bar{\pi} = \bar{r} = \bar{\gamma} = 0$ and has a unique maximum.[2] Such unimodal long-run seigniorage Laffer curves are consistent with the available empirical evidence (see Cagan (1956), Anand and van Wijnbergen (1989), Easterly, Mauro and Schmidt-Hebbel (1995) and Kiguel and Neumeyer

[2] For $\bar{s}_1$ this means that for $\bar{\pi} < \hat{\bar{\pi}}_1$, $\left((1+\bar{\pi})(1+\bar{\gamma})-1\right)\eta(\bar{\pi}) < 1+\bar{\gamma}$ and for $\bar{\pi} > \hat{\bar{\pi}}_1$, $\left((1+\bar{\pi})(1+\bar{\gamma})-1\right)\eta(\bar{\pi}) > 1+\bar{\gamma}$. For $\bar{s}_2$ this means that for $\bar{\pi} < \hat{\bar{\pi}}_2$, $\left((1+\bar{\pi})(1+\bar{r})-1\right)\eta(\bar{\pi}) < 1+\bar{r}$ and for $\bar{\pi} > \hat{\bar{\pi}}_2$, $\left((1+\bar{\pi})(1+\bar{r})-1\right)\eta(\bar{\pi}) > 1+\bar{r}$. For $\bar{s}_3$ this means that for $\bar{\pi} < \hat{\bar{\pi}}_3$, $\bar{\pi}\eta(\bar{\pi}) < 1$ and that for $\bar{\pi} > \hat{\bar{\pi}}_3$, $\bar{\pi}\eta(\bar{\pi}) > 1$, the familiar microeconomic condition that when price falls total revenue increases (decreases) if and only if the price elasticity of demand is less than (greater than) one.

(1995)). Let $\eta(\bar{\pi}) \equiv -\frac{\ell'(\bar{\pi})}{\ell(\bar{\pi})}$ be the semi-elasticity of long-run money demand with respect to the inflation rate. I will also assume that the long-run money demand function has the property that the semi-elasticity of long-run money demand with respect to the inflation rate is non-decreasing: $\eta'(\bar{\pi}) \geq 0$; this is, again, a property shared by the empirically successful base money demand functions. A familiar example is the semi-logarithmic long-run base money demand function, made popular by Cagan's studies (Cagan (1956)) of hyperinflations, with its constant semi-elasticity of money demand ($\eta(\bar{\pi}) = \eta$):

$$\begin{aligned} &\ln \bar{m} = \alpha - \eta\bar{\pi} \\ &\eta > 0 \end{aligned} \tag{4}$$

Taking steady state output growth as exogenous, the constant inflation rate that maximises steady-state seigniorage as a share of GDP is given by:

$$\hat{\bar{\pi}}_1 = \arg\max \left((1+\bar{\pi})(1+\bar{\gamma})-1\right)\ell(\bar{\pi}) = \frac{1}{\eta(\hat{\bar{\pi}}_1)} - \frac{\bar{\gamma}}{1+\bar{\gamma}} \tag{5}$$

Taking the steady-state real rate of interest as given, the constant inflation rate that maximises steady-state Central Bank revenue as a share of GDP is given by:

$$\hat{\bar{\pi}}_2 = \arg\max \left((1+\bar{\pi})(1+\bar{r})-1\right)\ell(\bar{\pi}) = \frac{1}{\eta(\hat{\bar{\pi}}_2)} - \frac{\bar{r}}{1+\bar{r}} \tag{6}$$

The constant inflation rate that maximises steady-state inflation tax revenue as a share of GDP is given by

$$\hat{\bar{\pi}}_3 = \arg\max \bar{\pi}\ell(\bar{\pi}) = \frac{1}{\eta(\hat{\bar{\pi}}_3)} - 1 \tag{7}$$

Proposition 1:

> *Assume that the long-run seigniorage Laffer curve is increasing at $\bar{\pi} = 0$ and unimodal and that the semi-elasticity of money demand with respect to the inflation rate is non-decreasing in the inflation rate. The inflation rate that maximises steady-*

state seigniorage as a share of GDP is lower than the inflation rate that maximises steady state Central Bank revenue as a share of GDP if and only if the growth rate of real GDP is greater than the real interest. The inflation rate that maximises the inflation tax as a share of GDP is greater than the inflation rate that maximises seigniorage as a share of GDP (Central Bank revenue as a share of GDP) if and only if the growth rate of real GDP (the real interest rate) is positive.[3]

Corollary 1:

The ranking of the maximised values of $\bar{s}_1$, $\bar{s}_2$ *and* $\bar{s}_3$ *is the same as the ranking of the magnitudes of* $\hat{\bar{\pi}}_1$, $\hat{\bar{\pi}}_2$ *and* $\hat{\bar{\pi}}_3$.

Seigniorage in real time

I shall generalise these three measures of Central Bank resource appropriation to allow for a non-zero risk-free nominal interest rate on base money; i_t^M is the own rate of interest on base money between periods *t-1* and *t*. The generalised seigniorage measure, denoted , $S_{1,t}$, is defined by $S_{1,t} = M_t - (1+i_t^M)M_{t-1}$ and the generalised measure of Central Bank revenue, denoted $S_{2,t}$, is defined by $S_{2,t} \equiv \left(i_t - i_t^M\right)M_{t-1}$

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[3] It suffices to show that $\hat{\bar{\pi}}_1$ is decreasing in $\bar{\gamma}$. Since $\frac{d\hat{\bar{\pi}}_1}{d\bar{\gamma}} = -\left(\frac{1}{1+\bar{\gamma}}\right)^2 \left(\frac{\left(\eta(\hat{\bar{\pi}}_1)\right)^2}{\eta'(\hat{\bar{\pi}}_1) + \left(\eta(\hat{\bar{\pi}}_1)\right)^2}\right)$, $\eta' \geq 0$ is sufficient but not necessary for the result. This result applies to a large number of empirically plausible base money demand functions. For the linear demand function found e.g. in Sargent and Wallace's Unpleasant Monetarist Arithmetic model (Sargent and Wallace (1981)) $m = \alpha - \beta(1+\pi),\ \beta > 0,\ m > 0$, for instance, we have

$\hat{\bar{\pi}}_1 = \arg\max\left((1+\bar{\pi})(1+\bar{\gamma})-1\right)\left(\alpha - \beta(1+\bar{\pi})\right) = \frac{1}{2}\left(\frac{\alpha}{\beta} + \frac{1}{1+\bar{\gamma}}\right) - 1$,

$\hat{\bar{\pi}}_2 = \arg\max\left((1+\bar{\pi})(1+\bar{r})-1\right)\left(\alpha - \beta(1+\bar{\pi})\right) = \frac{1}{2}\left(\frac{\alpha}{\beta} + \frac{1}{1+\bar{r}}\right) - 1$ and

$\hat{\bar{\pi}}_3 = \arg\max \bar{\pi}\left(\alpha - \beta(1+\bar{\pi})\right) = \frac{1}{2}\left(\frac{\alpha}{\beta} + 1\right) - 1$. Proposition 1 applies here also, with

$\eta(\bar{\pi}) = \frac{\beta}{\alpha - \beta(1+\bar{\pi})}$.(see Buiter (1990)).

Expressed as shares of GDP, these two seigniorage measures become:

$$s_{1,t} \equiv \frac{M_t - (1+i_t^M)M_{t-1}}{P_t Y_t}$$

and

$$s_{2,t} \equiv (i_t - i_t^M)\frac{M_{t-1}}{P_t Y_t}$$

The following notation will be needed to define the appropriate intertemporal relative prices or stochastic discount factors: I_{t_1,t_0} is the nominal stochastic discount factor between periods t_1 and t_0, defined recursively by

$$\begin{aligned} I_{t_1,t_0} &= \prod_{k=t_0+1}^{t_1} I_{k,k-1} \quad \text{for } \ t_1 > t_0 \\ &= 1 \qquad\qquad\quad \text{for } \ t_1 = t_0 \end{aligned}$$

The interpretation of I_{t_1,t_0} is the price in terms of period t_0 money of one unit of money in period $t_1 \geq t_0$. There will in general be many possible states in period t_1, and period t_1 money has a period t_0 (forward) price for each state. Let E_t be the mathematical expectation operator conditional on information available at the beginning of period t. Provided earlier dated information sets do not contain more information than later dated information sets, these stochastic discount factors satisfy the recursion property

$$E_{t_0}\left(I_{t_1,t_0} E_{t_1} I_{t_2,t_1}\right) = E_{t_0} I_{t_2,t_0} \ \text{ for } t_2 \geq t_1 \geq t_0$$

Finally, the risk-free nominal interest rate in period *t*, i_t, that is the money price in period *t* of one unit of money in every state of the world in period *t+1* is defined by

$$\frac{1}{1+i_t} = E_t I_{t+1,t} \tag{8}$$

For future reference I also define recursively the real stochastic discount factor between periods t_0 and t_1, R_{t_1,t_0}. Let the inflation factor between period t_0 and t_1, Π_{t_1,t_0}, be defined by

$$\begin{aligned} \Pi_{t_1,t_0} &= \frac{P_{t_1}}{P_{t_0}} = \sum_{k=t_0+1}^{t_1} (1+\pi_k) && \text{for } t_1 > t_0 \\ &= 1 && \text{for } t_1 = t_0 \end{aligned}$$

The real stochastic discount factor is defined by

$$R_{t_1,t_0} = I_{t_1,t_0}\Pi_{t_1,t_0}$$

It is easily checked that it has the same recursive properties as the nominal discount factor:

$$\begin{aligned} R_{t_1,t_0} &= \prod_{k=t_0+1}^{t_1} R_{k,k-1} && \text{for } t_1 > t_0 \\ &= 1 && \text{for } \mathrm{t}_1 = t_0 \end{aligned}$$

$$E_{t_0}\left(R_{t_1,t_0}E_{t_1}R_{t_2,t_1}\right) = E_{t_0}R_{t_2,t_0} \text{ for } t_2 \geq t_1 \geq t_0$$

The risk-free real rate of interest between periods t and $t+1$, r_{t+1}, is defined as

$$\frac{1}{1+r_t} = E_t R_{t+1,t}$$

Note that the growth-corrected discount factors satisfy:

$$E_{t_0}\left[R_{t_1,t_0}Y_{t_1,t_0}E_{t_1}\left(R_{t_2,t_1}Y_{t_2,t_1}\right)\right] = E_{t_0}\left(R_{t_2,t_0}Y_{t_2,t_0}\right) \text{ for } t_2 \geq t_1 \geq t_0$$

The Intertemporal Seigniorage Identity

Acting in real time, the monetary authority will be interested in the present discounted value of current and future seigniorage, rather than in just its current value or its steady-state value. A focus on the current value alone would be myopic and an exclusive concern with steady state seigniorage would not be a appropriate if the traverse to the steady state is non-instantaneous and could involve transitional seigniorage revenues that could be different from their steady state values. The present discounted value of the nominal value of seigniorage ($S_{1,t}$) is given by:

$$PDV_{t-1}(S_1) \equiv E_t \sum_{j=t}^{\infty} I_{j,t-1}\left(M_j - (1+i_j^M)M_{j-1}\right) \tag{9}$$

The present discounted value of nominal Central Bank revenue ($S_{2,t}$) is given by:

$$\begin{aligned} PDV_{t-1}(S_2) &\equiv E_t \sum_{j=t}^{\infty} I_{j,t-1}\left(\frac{i_{j+1}-i_{j+1}^M}{1+i_{j+1}}\right)M_j \\ &= E_t \sum_{j=t}^{\infty}\left(I_{j,t-1} - I_{j+1,t-1}(1+i_{j+1}^M)\right)M_j \end{aligned} \tag{10}$$[4]

Through the application of brute force (or in continuous time, through the use of the formula for integration by parts), and using the second equality in (10), it is easily established that the following relationship holds identically (see Buiter (1990)):

$$\begin{aligned} E_t \sum_{j=t}^{\infty} \mathrm{I}_{j,t-1}\left(M_j - (1+i_j^M)M_{j-1}\right) \equiv E_t \sum_{j=t}^{\infty} I_{j,t-1}\left(\frac{i_{j+1}-i_{j+1}^M}{1+i_{j+1}}\right)M_j - \left(\frac{1+i_t^M}{1+i_t}\right)M_{t-1} \\ + \lim_{N\to\infty} E_t I_{N,t-1} M_N \end{aligned} \tag{11}$$

I will refer to (11) as the intertemporal seigniorage identity or ISI.

If we impose the boundary condition that the present value of the terminal base money stock is zero in the limit as the terminal date goes to infinity, that is,

$$\lim_{N\to\infty} E_t I_{N,t-1} M_N = 0 \tag{12}$$

the ISI becomes

[4] The equality of the last two expressions in (10) is established as follows. For $j \geq t+1$,
$E_t\left(I_{j-1,t} - I_{j,t}(1+i_j^M)\right)M_{j-1} = E_t I_{j-1,t}\left(1 - I_{j,j-1}(1+i_j^M)\right)M_{j-1}$. Therefore,
$E_t I_{j-1,t}\left(1 - I_{j,j-1}(1+i_j^M)\right)M_{j-1} = E_t I_{j-1,t}\left(1 - E_{j-1} I_{j,j-1}(1+i_j^M)\right)M_{j-1}$, and therefore,
$E_t I_{j-1,t}\left(1 - E_{j-1} I_{j,j-1}(1+i_j^M)\right)M_{j-1} = E_t I_{j-1,t}\left(1 - (1+i_j)^{-1}(1+i_j^M)\right)M_{j-1}$.

$$E_t \sum_{j=t}^{\infty} \mathrm{I}_{j,t} \left(M_j - (1+i_j^M) M_{j-1} \right) = E_t \sum_{j=t}^{\infty} I_{j,t} \left(\frac{i_{j+1} - i_{j+1}^M}{1+i_{j+1}} \right) M_j - \left(1+i_t^M\right) M_{t-1}$$

or (13)

$$PDV_t(S_1) = PDV_t(S_2) - \left(1+i_t^M\right) M_{t-1}$$

There are no additional interesting relationships that can be established between the inflation tax and the other two monetary resource appropriation measures – seigniorage and Central Bank revenue, beyond the familiar identity that seigniorage revenue as a share of GDP equals the inflation tax plus the 'real growth bonus' plus (the increase in the demand for real money balances associated, cet. par. with real income growth) plus the change in the ratio of base money to GDP:

$$\frac{\Delta M_t}{P_t Y_t} = \pi_{t+1} m_{t+1} + \left(1+\gamma_{t+1}(1+\pi_{t+1})\right) m_{t+1} + \Delta m_{t+1} \qquad (14)^5$$

Using real GDP units as the numéraire rather than money, equation (13) becomes

[5] Consider, for sake of brevity, the continuous time analogue of (14): $\mu m = (\pi + \gamma) m + \dot{m}$.

Taking present discounted values on both sides of this relationship yields:

$\int_t^{\infty} e^{-\int_t^s [r(u)-\gamma(u)]du} \mu(s) m(s) ds = \int_t^{\infty} e^{-\int_t^s [r(u)-\gamma(u)]du} [\pi(s)+\gamma(s)] m(s) + \int_t^{\infty} e^{-\int_t^s [r(u)-\gamma(u)]du} \dot{m}(s) ds$. Applying integration by parts to the second term on the r.h.s. of the last equation yields

$\int_t^{\infty} e^{-\int_t^s [r(u)-\gamma(u)]du} \mu(s) m(s) ds = \int_t^{\infty} e^{-\int_t^s [r(u)-\gamma(u)]du} [\pi(s)+r(s)] m(s) - m(t)$. With

$r(s) + \pi(s) = i(s)$, this is simply the continuous time version of the ISI.

$$E_t\sum_{j=t}^{\infty}R_{j,t-1}\Gamma_{j,t-1}\left(\frac{M_j-(1+i_j^M)M_{j-1}}{P_jY_j}\right)=E_t\sum_{j=t}^{\infty}R_{j,t-1}\Gamma_{j,t-1}\left(\frac{i_{j+1}-i_{j+1}^M}{1+i_{j+1}}\right)m_j-\left(\frac{1+i_t^M}{1+i_t}\right)m_t$$

or (15)

$$\sigma_{1,t}=\sigma_{2,t}-\left(\frac{1+i_t^M}{1+i_t}\right)m_t$$

Where

$$\begin{aligned}\sigma_{1,t}&=PDV_{t-1}(s_1)=E_t\sum_{j=t}^{\infty}R_{j,t-1}\Gamma_{j,t-1}\left(\frac{M_j-(1+i_j^M)M_{j-1}}{P_jY_j}\right)\\ \sigma_{2,t}&=PDV_{t-1}(s_2)=E_t\sum_{j=t}^{\infty}R_{j,t-1}\Gamma_{j,t-1}\left(\frac{i_{j+1}-i_{j+1}^M}{1+i_{j+1}}\right)m_j\end{aligned} \tag{16}$$

From equation (13) it is clear that maximizing the present discounted value of current and future nominal seigniorage ($M_j-M_{j-1}(1+i_j^M)$) according to the S_1 definition, is equivalent to maximizing the present discounted value of current and future nominal Central Bank revenues according to the S_2 definition ($\left(\frac{i_j-i_j^M}{1+i_j}\right)M_{j-1}$). The two differ only by the inherited value of the nominal stock of base money gross of interest on base money, $(1+i_t^M)M_{t-1}$, which is not a choice variable in period *t.* I summarise this as **Proposition 2.**

Proposition 2:

Acting in real time, and therefore treating the initial nominal stock of base money as predetermined, maximising the present discounted value of current and future nominal seigniorage is equivalent to maximising the present discounted value of current and future nominal Central Bank revenue.

The same result cannot be inferred quite as readily for either the present discounted values of current and future real seigniorage $\frac{M_j-(1+i_j^M)M_{j-1}}{P_j}$ and future real Central Bank revenues $\left(\frac{i_j-i_j^M}{1+i_j}\right)\frac{M_{j-1}}{P_j}$, or for the present discounted value of current and future seigniorage as a share of

GDP $\frac{M_j - (1+i_j^M)M_{j-1}}{P_jY_j}$ and future Central Bank revenues as a share of GDP $\left(\frac{i_j - i_j^M}{1+i_j}\right)\frac{M_{j-1}}{P_jY_j}$.

The reason is that both the initial level of real GDP, Y_t, and the initial value of the general price level, P_t, are, in principle, endogenous and could be choice variables of or influenced by the monetary authority. This suggests Corollaries 2 and 3:

Corollary 2:

Acting in real time, maximising the present discounted value of current and future real seigniorage is equivalent to maximising the present discounted value of current and future real Central Bank revenue if and only if the current price level is given.

Classes of models for which the current general price level is predetermined, exogenous or constant for other reasons include the following: (1) Old-Keynesian and New-Keynesian models, for which price level is predetermined); (2) any model of a small open economy with only traded goods, all of which obey the law of one price.

Corollary 3:

Acting in real time, maximising the present discounted value of current and future seigniorage as a share of GDP is equivalent to maximising the present discounted value of current and future Central Bank revenue as a share of GDP if and only if the current value of nominal GDP is given.

I will assume that a level of nominal GDP that is predetermined, exogenous or constant for other reasons requires both a general price level that is predetermined, exogenous or constant for other reasons and a level of real GDP that is predetermined, exogenous or constant for other reasons. The New Keynesian model has a predetermined price level. The current value of real GDP can be shown to be invariant to the policy actions under consideration in this paper provided only the price level but not the rate of inflation is predetermined. I show this formally in Section IV of the paper. Most Old-Keynesian models have both a predetermined price level and a predetermined rate of inflation, so maximising σ_1 in real time will not be equivalent to maximising σ_2 in real time. The equivalence result applies also for any model of a small open economy with only traded goods, all of which obey the law of one price, and an exogenous level of real GDP.

It is important to note that maximising, in real time, the present discounted value of current and future seigniorage when the inflation rate determined in the current period and in all other future periods is constant, and current and future real interest rates and real growth rates are constant, is not the same as maximising the present discounted value of steady state seigniorage. To clarify the difference, consider for simplicity an economy that, starting in period *t*, is in steady state, although the initial ratio of base money to GDP, m_t, need not be the same as the subsequent steady-state values. When the system is in a deterministic steady state starting from period *t*, the following hold for $j \geq t+1$:

$$(1+\pi_j)(1+\gamma_j) = (1+\bar{\pi})(1+\bar{\gamma}) = (1+\bar{\mu}) = 1+\mu_j \tag{17}$$

$$(1+r_j)(1+\pi_j) = (1+\bar{r})(1+\bar{\pi}) = 1+\bar{i} = 1+i_j \tag{18}$$

For simplicity, assume that the nominal interest rate on base money is zero. For simplicity I also assume that $\mu_t = \mu$. It does not follow, however, that $(1+\pi_t)(1+\gamma_t) = 1+\mu_t = (1+\bar{\mu}) = (1+\bar{\pi})(1+\bar{\gamma})$. The ISI now simplifies to (19):

$$\begin{gathered}\bar{\mu}\left(m_t + \bar{m}\left(\frac{1+\bar{\gamma}}{\bar{r}-\bar{\gamma}}\right)\right) = \bar{i}\bar{m}\left(\frac{1+\bar{\gamma}}{\bar{r}-\bar{\gamma}}\right) - m_t \\ \textbf{or} \\ \sigma_{1,t} = \sigma_{2,t} - m_t\end{gathered} \tag{19}$$

If the the monetary authority cannot choose or influence the initial ratio of money to GDP, maximizing the present discounted value of current and future s_1 is equivalent to maximising $\left[(1+\bar{r})(1+\bar{\pi})-1\right]\left(\frac{1+\bar{\gamma}}{r-\bar{\gamma}}\right)\bar{m}$, which is the present discounted value of present and future s_2. If the initial value of the money-GDP ratio could be chosen, subject to the constraint that it is equal to the steady-state value of the ratio of the stock of base money to GDP from period t onward, and if (18) also holds for $j=t$, then the two maximization problems are not equivalent. When the initial value of base money velocity is a choice variable, in the sense that it can be set to equal to steady state value of velocity for period *t* and beyond, the following holds:

$$\begin{aligned}\bar{\sigma}_1 &= \left[\left(1+\bar{\gamma}\right)\left(1+\bar{\pi}\right)-1\right]\left(\frac{1+\bar{r}}{\bar{r}-\bar{\gamma}}\right)\bar{m} \\ \bar{\sigma}_2 &= \left[\left(1+\bar{r}\right)\left(1+\bar{\pi}\right)-1\right]\left(\frac{1+\bar{\gamma}}{\bar{r}-\bar{\gamma}}\right)\bar{m}\end{aligned} \tag{20}$$

Consider again the semi-logarithmic base money demand function in (4), or any long-run money demand function that results in a well-behaved unimodal long-run seigniorage Laffer curve. It is clear that, if the steady state growth rate of GDP and the steady-state real rate of interest are independent of monetary policy, maximising $\bar{\sigma}_1$ subject to (4) yields the same result as maximising $\bar{s}_1$, and maximising $\bar{\sigma}_2$ subject to (4) yields the same result as maximising $\bar{s}_2$. It is also obvious that maximising the present discounted value of the inflation tax $\bar{\sigma}_3 = \bar{\pi}\left(\frac{1+\bar{\gamma}}{\bar{r}-\bar{\gamma}}\right)\bar{m}$ subject to (4) yields the same result as maximising $\bar{s}_3$. However, because the steady-state present discounted values in (20) only exist if $\bar{r} > \bar{\gamma}$, the case where the inflation rate that maximises the steady-state value of seigniorage as a share of GDP is below the inflation rate that maximises the steady state value of Central Bank revenue as a share of GDP ($\hat{\bar{\pi}}_1 < \hat{\bar{\pi}}_2$) has no counterpart in the maximisation of the present discounted values of steady state seigniorage as a share of GDP and of steady state Central Bank revenue as a share of GDP.

The main message of this section is, however, that maximisation of seigniorage, Central Bank revenue and the inflation tax should be viewed from an explicitly intertemporal and real-time perspective.

III. The intertemporal budget constraints of the Central Bank and the Treasury

To obtain a full understanding of the constraints the Central Bank is subject to in the conduct of monetary policy in general and in its use of seigniorage in particular, it is essential to have a view of the Central Bank as an economic agent with a period budget constraint and an intertemporal budget constraint or solvency constraint. This requires us to decompose the Government's financial accounts and solvency constraint into separate accounts and solvency constraints for the Central Bank and the Treasury (see also Buiter (2004), Sims (2004), (2005) and Ize (2005)).[6] In this Section, I therefore introduce a stylized set of accounts for a small open economy. Separate period budget constraints for the Central Bank and Treasury are also considered in Walsh (2003) and in Buiter (2003, 2004 and 2005). The latter also considers the solvency constraints and intertemporal budget constraints of the two state sectors separately. Walsh leaves out the payments made by the Central Bank to the Treasury. While this does not, of course, affect the options available to the consolidated Government, it does prevent the consideration of how the Treasury can, through its fiscal claims on the Central Bank, facilitate or prevent the Central Bank from implementing its monetary and supervisory mandates.

[6] The term 'government' as used in 'government budget constraint' refers to the consolidated general government and central bank. 'State' would be a better term, to avoid confusion with the particular administration in office at a point in time. The unfortunate usage is, however, to0 firmly ensconced to try to dislodge it here.

The Central Bank has only the monetary base $M \geq 0$ on the liability side of its financial balance sheet.[7] On the asset side it has the stock of international foreign exchange reserves, R^f, earning a risk-free nominal interest rate in terms of foreign currency i^f and the stock of domestic credit, which consists of Central Bank holdings of nominal, interest-bearing Treasury bills, D, earning a risk-free domestic-currency nominal interest rate i, and Central Bank claims on the private sector, L, with domestic-currency nominal interest rate i^L.[8] The stock of Treasury debt (all assumed to be denominated in domestic currency) held outside the Central Bank is B; it pays the risk-free nominal interest rate i; T^p is the real value of the tax payments by the domestic private sector to the Treasury; it is a choice variable of the Treasury and can be positive or negative; T^b is the real value of taxes paid by the Central Bank to the Treasury; it is a choice variable of the Treasury and can be positive or negative; $T^g = T^p + T^b$ is the real value of total Treasury tax receipts; H is the real value of the transfer payments made by the Central Bank to the private sector ('helicopter drops'). I assume H to be a choice variable of the Central Bank It is true that in most countries the Central Bank is not a fiscal agent. I can neither tax nor make transfer payments. While I shall deny the Central Bank the power to tax, $H \geq 0$, I will until further notice allow it to make transfer payments. This is necessary for 'helicopter drops of money' to be implementable by the Central Bank on its own, without Treasury support. Total real taxes net of transfer payments received by the Government, that is, the consolidated Treasury and Central Bank are $T = T^p - H$,; e is the value of the spot nominal exchange rate (the domestic currency price of foreign exchange); $C^g \geq 0$ is the real value of Treasury spending on goods and

[7] In the real world this would be currency plus commercial bank reserves with the Central Bank. In many emerging markets and developing countries, the central bank also has non-monetary interest-bearing liabilities. These could be added easily to the accounting framework.
[8] For simplicity, I consider only short maturity bonds. Generalisations to longer maturities, index-linked debt or foreign-currency denominated debt are straightforward.

services and $C^b \geq 0$ the real value of Central Bank spending on goods and services. Public spending on goods and services is assumed to be public consumption only.

Equation (21) is the period budget identity of the Treasury and equation (22) that of the Central Bank.

$$\frac{B_t + D_t}{P_t} = C_t^g - T_t^p - T_t^b + (1+i_t)\left(\frac{B_{t-1} + D_{t-1}}{P_t}\right) \tag{21}$$

$$\begin{aligned}\frac{M_t - D_t - L_t - e_t R_t^f}{P_t} &= C_t^b + T_t^b + H_t \\ &+ \frac{(1+i_t^M)M_{t-1} - (1+i_t)D_{t-1} - (1+i_t^L)L_{t-1} - (1+i_t^f)e_t R_{t-1}^f}{P_t}\end{aligned} \tag{22}$$

The solvency constraints of, respectively, the Treasury and Central Bank are given in equations (23) and (24):

$$\lim_{N\to\infty} E_t I_{N,t-1}\left(B_N + D_N\right) \leq 0 \tag{23}$$

$$\lim_{N\to\infty} E_t I_{N,t-1}\left(D_N + L_N + e_N R_N^f\right) \geq 0 \tag{24}$$

When there exist complete contingent claims markets, and the no-arbitrage condition is satisfied, these solvency constraints, which rule out Ponzi finance by both the Treasury and the Central Bank, imply the following intertemporal budget constraints for the Treasury (equation (25)) and for the Central Bank (equation (26)).

$$B_{t-1} + D_{t-1} \leq E_t \sum_{j=t}^{\infty} I_{j,t-1} P_j (T_j^p + T_j^b - C_j^g) \tag{25}$$ [9]

$$D_{t-1} + L_{t-1} + e_{t-1}R_{t-1}^f \leq E_t \sum_{j=t}^{\infty} I_{j,t-1}\left(P_j\left(C_j^b + T_j^b + H_j + Q_j\right) - \left(M_j - (1+i_j^M)M_{j-1}\right)\right) \tag{26}$$

where

[9] Note that $E_t E_{t-1} I_{t,t-1} = E_{t-1} I_{t,t-1} = \frac{1}{1+i_t}$.

$$P_j Q_j = (i_j - i_j^L)L_{j-1} + \left(1 + i_j - (1+i_j^f)\frac{e_j}{e_{j-1}}\right)e_{j-1}R_{j-1}^f \tag{27}$$

The expression Q in equation (27) stands for the real value of the quasi-fiscal implicit interest subsidies made by the Central Bank. If the rate of return on government debt exceeds that on loans to the private sector, there is an implicit subsidy to the private sector equal in period t to $\left(i_t - i_t^L\right)L_{t-1}$. If the rate of return on foreign exchange reserves is less than what would be implied by Uncovered Interest Parity (UIP), there is an implicit subsidy to the issuers of these reserves, given in period t by $\left(1 + i_t - (1+i_t^f)\frac{e_t}{e_{t-1}}\right)e_{t-1}R_{t-1}^f$.

The solvency constraint of the Central Bank only requires that the present discounted value of its net *non-monetary* liabilities be non-positive in the long run. Its monetary liabilities are liabilities only in name, as they are *irredeemable*: the holder of base money cannot insist at any time on the redemption of a given amount of base money into anything else other than the same amount of itself (base money).

Summing (21) and (22) gives the period budget identity of the Government (the consolidated Treasury and Central Bank), in equation (28); summing (23) and (24) gives the solvency constraint of the Government in equation (29) and summing (25) and (26) gives the intertemporal budget constraint of the Government in equation (30).

$$\begin{aligned} M_t + B_t - L_t - e_t R_t^f \equiv\ & P_t(C_t^g + C_t^b - T_t) \\ & +(1+i_t^M)M_{t-1} + (1+i_t)B_{t-1} - (1+i_t^L)L_{t-1} - e_t(1+i_t^f)R_{t-1}^f \end{aligned} \tag{28}$$

$$\lim_{N\to\infty} E_t I_{N,t-1}\left(B_N - L_N - e_N R_N^f\right) \le 0 \tag{29}$$

$$B_{t-1} - L_{t-1} - e_{t-1}R_{t-1}^f \le E_t \sum_{j=t}^{\infty} I_{j,t-1}\left(P_j\left(T_j - Q_j - C_j^g - C_j^b\right) + M_j - (1+i_j^M)M_{j-1}\right) \tag{30}$$

Consider the conventional financial balance sheet of the Central Bank in Table 1, that of the Treasury in Table 2, and that of the Government in Table 3. Loans to the private sector and international reserves are valued at their notional or face values.[10]

Table 1
Central Bank Conventional Financial Balance Sheet

Assets	**Liabilities**
D	M
L	
eR^f	
	W^b

[10] If the outstanding stock of (one-period maturity) loans to the private sector were marked-to-market, its fair value would be $L_t\left(\frac{1+i^L_{t+1}}{1+i_{t+1}}\right)$, the fair value of the (one-period maturity) international reserves would be $e_t R^f_t\left(\frac{(1+i^f_{t+1})e_{t+1}/e_t}{i_{t+1}}\right)$. It might be thought that the fair value of the stock of base money would be $M_t\left(\frac{1+i^M_{t+1}}{1+i_{t+1}}\right)$. However, currency is not a one-period maturity store of value. As a store of value, base money is a perpetuity paying $1+i^M_j$ in each period $j>t$ for each unit of money acquired in period t. The marked-to-market or fair value of a unit of base money acquired in period *t* (ex-dividend, that is, after period *t* interest due has been paid) is therefore $E_t\sum_{t+1}^{\infty} I_{j,j-1} i^M_j$. In the deterministic case, this becomes $\sum_{j=t+1}^{\infty}\prod_{k=t+1}^{j}\frac{i^M_j}{1+i_k}$. If follows that, as a store of value, the fair value of currency, which has a zero interest rate, is zero, as it is effectively a consol with a zero coupon.

Table 2 **Treasury Conventional Financial Balance Sheet**	
Assets	**Liabilities**
	D
	B
	W^t

Table 3 **Government Conventional Financial Balance Sheet**	
Assets	**Liabilities**
L	B
eR^f	M
	W^g

The Central Bank's financial net worth, $W^b \equiv D + L + eR^f - M$, is the excess of the value of its financial assets, Treasury debt, D, loans to the private sector, L and foreign exchange reserves, eR^f, over its monetary liabilities, M. The Treasury's conventional financial net worth is denoted W^t, the Government's by W^g.

To make the relationship between the intertemporal budget constraints of the Treasury and the Central Bank and their conventional balance sheets more apparent, it is helpful to use the ISI, (assuming $\lim_{N\to\infty} E_t I_{N,t-1} M_N = 0$) given in equation (13), to rewrite the intertemporal budget constraint of the Central Bank (26) as in equation (31):

$$\left(\frac{1+i_t^M}{1+i_t}\right)M_{t-1}-\left(D_{t-1}+L_{t-1}+e_{t-1}R_{t-1}^f\right)$$
$$\leq E_t\sum_{j=t}^{\infty}I_{j,t-1}\left[P_j\left(-C_j^b-T_j^b-H_j-Q_j\right)+\left(\frac{i_{j+1}-i_{j+1}^M}{1+i_{j+1}}\right)M_j\right] \tag{31}$$

III.1 Can Central Banks survive with 'negative equity'?

On the left-hand side of (31) we have (minus) the equity of the Central Bank – the excess of its monetary liabilities over its financial assets. On the right-hand side of (31) we have, $E_t\sum_{j=t}^{\infty}I_{j,t-1}\left(\frac{i_{j+1}-i_{j+1}^M}{1+i_{j+1}}\right)M_j$, the present discounted value Central Bank revenue, that is, of the future interest payments saved by the Central Bank because of its ability to issue monetary liabilities bearing an interest rate i_j^M. The difference between these two terms, is, from the ISI, the present discounted value of future seigniorage, $E_t\sum_{j=t}^{\infty}I_{j,t-1}\left(M_j-(1+i_j^M)M_{j-1}\right)$, provided the present discounted value of the terminal money stock is zero: $\lim_{N\to\infty}E_tI_{N,t-1}M_N=0$ (see equations (11), (12) and (13)).

It should be noted that order to obtain the Central Bank's intertemporal budget constraint (26), I imposed the no-Ponzi game terminal condition $\lim_{N\to\infty}E_tI_{N,t-1}\left(D_N+L_N+e_NR_N^f\right)\geq 0$, that is, the present value of the terminal net *non-monetary* liabilities had to be non-negative. I did not impose the condition $\lim_{N\to\infty}E_tI_{N,t-1}\left(D_N+L_N+e_NR_N^f-M_N\right)\geq 0$, that is, that the present value of the terminal total net liabilities, *monetary and non-monetary*, had to be non-negative. The reason is that the monetary 'liabilities' of the Central Bank are not in any meaningful sense liabilities of the Central Bank. The owner (holder) of currency notes worth *X* units of currency have a claim on the Central Bank for currency notes worth *X* units of currency – nothing more. The monetary liabilities of the Central Bank are irredeemable or inconvertible into anything other than the same amount of itself. While in most well-behaved economies, $\lim_{N\to\infty} E_t I_{N,t-1} M_N = 0$, this will not be the case, for instance, in a permanent liquidity trap where $\lim_{N\to\infty} E_t I_{N,t-1} M_N = \lim_{N\to\infty} E_t M_N > 0$ unless the monetary authorities adopt a policy of (asymptotically) demonetising the economy in nominal terms. Such asymptotic demonetization (in nominal terms) characterises the efficient stationary liquidity trap equilibrium of the Bailey-Friedman Optimal Quantity of Money rule (Bailey (1956), Friedman (1969)), when the interest rate on base money is zero and the risk-free nominal interest rate on non-monetary assets is kept at zero throughout. The nominal stock of base money shrinks at a proportional rate equal to the real interest rate and the rate of time preference.

Even if the conventionally defined net worth or equity of the Central Bank is negative, that is, if $W^b_{t-1} \equiv D_{t-1} + L_{t-1} + e_{t-1}R^f_{t-1} - M_{t-1} < 0$, the Central Bank can be solvent provided $W^b_{t-1} \geq E_t \sum_{j=t}^{\infty} I_{j,t-1}\left[P_j\left(C^b_j + T^b_j + H_j + Q_j\right) - \left(\frac{i_{j+1} - i^M_{j+1}}{1+i_{j+1}}\right) M_j \right]$. Conventionally defined financial net worth or equity excludes the present value of anticipated or planned future non-contractual outlays and revenues (the right-hand side of equation (31). It is therefore perfectly possible, for the central bank to survive and thrive with negative financial net worth. This might, however, require the central bank to raise so much seigniorage in real terms, $\frac{M_j - (1+i^M_j)M_j}{P_j}$, through current and future nominal base money issuance, that, given the demand function for real base money, unacceptable rates of inflation would result.

The financial net worth of the Treasury, $W^t = -(B+D)$ is negative in most countries. The financial net worth of the Government, that is, the consolidated Treasury and Central Bank $W^g \equiv W^t + W^b = eR^f + L - M - B$, is also likely to be negative for most countries. None of this need be a source of concern, unless the gap between the outstanding contractual non-monetary debt of the state and the present discounted value of the future primary (non-interest) surpluses of the state, $T_j - C_j^g - C_j^b - Q_j$, $j \geq t$ is so large, that it either cannot be filled at all at all (the maximum value of the discounted future real seigniorage stream is too low) and the state defaults, or can only be closed at unacceptably high rates of inflation.

The only intertemporal budget constraint that ought to matter, that is, the only one that would matter in a well-managed economy, is that of the consolidated Treasury and Central Bank, given in equation (30). Its breakdown into the Treasury's intertemporal budget constraint (equation (25)) and the Central Bank's intertemporal budget constraint (equation (26)) is without macroeconomic interest, unless there is a failure of cooperation and coordination between the monetary and fiscal authorities, that is, between the Central Bank and the Treasury. Operational independence for central banks has probably raised the risk of such mishaps occurring.

The separation of the accounts of the Treasury and the Central Bank allows us to recognise a fourth measure of the revenues extracted by the state through its monopoly of the issuance of base money. This is the conventionally measured operating profits of the Central Bank (before payment of taxes to the Treasury), which will be denoted $S_{4,t}$. It consists of its net interest income minus its operating expenses:

$$S_{4,t} = i_t D_{t-1} + i_t^L L_{t-1} + e_t i_t^f R_{t-1}^f - i_t^M M_{t-1} - P_t H_t - P_t C_t^b \tag{32}$$

From equation (22) it follows that

$$\Delta M_t - \Delta D_t - \Delta L_t - e_t \Delta R_t^f \equiv P_t T_t^b - S_{4,t}$$

If we make the further assumption that the operating profits of the Central Bank are paid in taxes to the Treasury[11], that is,

$$P_t \mathrm{T}_t^b = S_{4,t}\,, \tag{33}$$

then, and only then, does the textbook identity hold that the change in the stock of base money, ΔM_t, equals domestic credit expansion, $\Delta D_t + \Delta L_t$, plus the value of the increase in the stock of foreign exchange reserves, $e_t \Delta R_t^f$:

$$\Delta M_t = \Delta D_t + \Delta L_t + e_t \Delta R_t^f \tag{34}$$

A little rearranging of the identities in (31) and (32) yields:

$$\begin{aligned} E_t \sum_{j=t}^{\infty} I_{j,t-1} S_{4,j} \leq & \left(\frac{1+i_t^M}{1+i_t}\right) M_{t-1} - (D_{t-1} + L_{t-1} + e_{t-1} R_{t-1}^f) \\ & + E_t \sum_{j=t}^{\infty} I_{j,t-1} P_j T_j^b \\ & + E_t \sum_{j=t}^{\infty} I_{j,t-1} i_j \left(D_{j-1} + L_{j-1} + e_{j-1} R_{j-1}^f - \left(\frac{1+i_j^M}{1+i_j}\right) M_{j-1} \right) \end{aligned} \tag{35}$$

So if the Treasury always taxes away all the operating profits of the Central Bank (equation (33) holds, then

$$\begin{aligned} \left(\frac{1+i_t^M}{1+i_t}\right) M_{t-1} - (D_{t-1} + L_{t-1} + e_{t-1} R_{t-1}^f) \leq & E_t \sum_{j=t}^{\infty} I_{j,t-1} i_j \left(D_{j-1} + L_{j-1} + e_{t-1} R_{j-1}^f - \left(\frac{1+i_j^M}{1+i_j}\right) M_{j-1} \right) \\ & + E_t \sum_{j=t}^{\infty} I_{j,t-1} i_j \left(e_{j-1} - e_{t-1} \right) R_{j-1}^f \end{aligned}$$

From (34) it then follows that

$$D_{j-1} + L_{j-1} + e_{t-1} R_{j-1}^f - \left(\frac{1+i_j^M}{1+i_j}\right) M_{j-1} = D_{t-1} + L_{t-1} + e_{t-1} R_{t-1}^f - \left(\frac{1+i_t^M}{1+i_t}\right) M_{t-1} \quad \text{for } j \geq t \text{, so}$$

[11] The profits of the Bank of England (after Corporation Tax) are split fifty-fifty between the Treasury and additions to the Bank of England's reserves, but this arrangement can at any time be altered by the Treasury.

$$E_t \sum_{j=t}^{\infty} I_{j,t-1} i_j \left(e_{j-1} - e_{t-1}\right) R^f_{j-1} \geq 0$$

So unless the Central Bank experiences, on average, capital gains (through currency appreciation) rather than capital losses (through currency appreciation) on its foreign exchange reserves, the Central Bank's solvency constraint will be violated when the Treasury taxes away its operating profits.

Regardless of the tax rule the Treasury imposes on the Central Bank, it is always the case that the present discounted value of the taxes paid by Central Bank to the Treasury can be written as

$$\begin{aligned}
PDV_{t-1}(PT^b) &\equiv E_t \sum_{j=t}^{\infty} I_{j,t-1} P_j T^b_j \\
&= D_{t-1} + L_{t-1} + e_{t-1} R^f_{t-1} - \left(\frac{1+i^M_t}{1+i_t}\right) M_{t-1} \\
&\quad + E_t \sum_{j=t}^{\infty} I_{j,t-1} \left[P_j \left(-C^b_j - H_j - Q_j\right) + \left(\frac{i_{j+1} - i^M_{j+1}}{1+i_{j+1}}\right) M_j \right]
\end{aligned}$$

So unless the Central Bank can influence the present discounted value of its primary (non-interest) deficits (before taxes paid to the Treasury), maximising the present discounted value of the profits of the Central Bank, $PDV_{t-1}(S_4)$ is equivalent to maximising the present discounted value of central bank revenues, $PDV_{t-1}(S_2)$ and of the present discounted value of seigniorage, $PDV_{t-1}(S_1)$.

We now turn to the consideration of the question as to whether the Central Bank has the financial resources to successfully pursue its inflation target.

IV. Is the inflation target independently financeable by the Central Bank?

I consider here whether and under what conditions the inflation target is consistent with the Central Bank's intertemporal budget constraint. Consider a closed economy model of an endowment economy, whose Treasury and Central Bank can be represented by a simplified version of the accounting framework developed in the previous Section. There are no international reserves, $R_t^f = 0$, no Central Bank loans to the private sector, $L_t = 0$, and therefore no quasi-fiscal subsidies by the Central Bank, $Q_t = 0$.

The intertemporal budget constraints of the Treasury remains as in equation (25), those of the Central Bank, respectively the Government (the consolidated Central Bank and Treasury) are given below:

$$-D_{t-1} \le \sum_{j=t}^{\infty} E_t I_{j,t-1} \left(P_j \left(-C_j^b - T_j^b - H_j \right) + M_j - (1+i_j^M) M_{j-1} \right)$$

and

$$B_{t-1} \le \sum_{j=t}^{\infty} E_t I_{j,t-1} \left(P_j \left(T_j - C_t^g - C_t^b \right) + M_j - (1+i_j^M) M_{j-1} \right)$$

Let the ratio of the stock of domestic credit to GDP be $d_t = \frac{D_{t-1}}{P_t Y_t}$, and let

$$c_t^b = \frac{C_t^b}{Y_t},\ c_t^g = \frac{C_t^g}{Y_t},\ \tau_t^b = \frac{T_t^b}{Y_t},\ \tau_t^p = \frac{T_t^p}{Y_t},\ h_t = \frac{H_t}{Y_t};\ \tau_t = \frac{T_t}{Y_t} = \tau_t^p - h_t$$

We can re-write the intertemporal budget constraints of the Treasury, the Central Bank and the consolidated Government as, respectively:

$$b_t + d_t \le E_t \sum_{j=t}^{\infty} R_{j,t-1} \Gamma_{j,t-1} (\tau_j^p + \tau_j^b - c_j^g) \tag{36}$$

$$-d_t \le \sum_{j=t}^{\infty} E_t R_{j,t-1} \Gamma_{j,t-1} \left(-c_j^b - \tau_j^b - h_j + (\mu_j - i_j^M) m_j \right) \tag{37}$$

$$b_t \le \sum_{j=t}^{\infty} E_t R_{j,t-1} \Gamma_{j,t-1} \left(\tau_j - c_t^g - c_t^b + (\mu_j - i_j^M) m_j \right). \tag{38}$$

The period budget constraint of the representative household is given in (39) and its solvency constraint in (40); A_t is the nominal value of its non-monetary assets (inclusive of period *t* interest or similar payments): The nominal value of total household financial wealth is denoted W_t where

$$W_t \equiv A_t + (1+i_t^M) M_{t-1}$$

$$\begin{aligned} &E_t I_{t+1,t} W_{t+1} \equiv W_t - \left(\frac{i_{t+1} - i_{t+1}^M}{1+i_{t+1}} \right) M_t + P_t (Y_t - \mathrm{T}_t - C_t) \\ &C_t \ge 0 \end{aligned} \qquad (39)^{12}$$

$$\lim_{N \to \infty} E_t I_{N,t} W_N \ge 0 \tag{40}$$

Note that while the Central Bank does not, in its solvency constraint (24), view irredeemable base money as an effective liability, households do view base money as an asset in their solvency constraint. This asymmetry is the formal expression of the view that fiat money is an asset of the holder but not a liability of the issuer.

This implies the following intertemporal budget constraint for the household:

$$W_t \ge E_t \sum_{j=t}^{\infty} I_{j,t} \left[P_j (C_j + T_j - Y_j) + \left(\frac{i_{j+1} - i_{j+1}^M}{1+i_{j+1}} \right) M_j \right] \tag{41}$$

The household optimizes the following utility function:

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$$\begin{aligned} E_t \left(I_{t+1,t} A_{t+1} \right) + M_t &= E_t \left[I_{t+1,t} \left(A_{t+1} + (1+i_{t+1}^M) M_t \right) \right] + \left[1 - E_t \left(I_{t+1,t} (1+i_{t+1}^M) \right) \right] M_t \\ &= E_t \left[I_{t+1,t} \left(A_{t+1} + (1+i_{t+1}^M) M_t \right) \right] + \left(\frac{i_{t+1} - i_{t+1}^M}{1+i_{t+1}} \right) M_t \end{aligned}$$

$$U_t = E_t \sum_{j=t}^{\infty} \left(\frac{1}{1+\delta} \right)^{j-t} u(C_j, \frac{M_j}{P_{j+1}};.)$$
$$\delta > 0 \tag{42}$$

where $u(C_t, \frac{M_t}{P_{t+1}};.) = v\left(\frac{C_t}{Y_t}\right) + w\left(\frac{M_t}{P_{t+1}Y_{t+1}}\right)$ is twice continuously differentiable, increasing in consumption, increasing in real money balances for low values of the stock of real money balances, strictly concave and satisfies the Inada conditions for consumption. Preferences are assumed separable in consumption and real money balances and homothetic in consumption, real money balances and the exogenous level of real output, so as to permit the existence of a steady state with non-zero real growth. Let $c_t = C_t / Y_t$. For expositional simplicity I will use the following parametric example: $v(c_t) = \ln(c_t)$ and $w(m_{t+1}) = (m_{t+1} - \theta)\ln(\theta - m_{t+1}) - m_{t+1}; \ \ \theta > m_{t+1} + 1$. These yield a money demand function close to the textbook semi-logarithmic one (I assume that the value of the parameter θ is sufficiently large to ensure an interior solution for the stock of real money balances, in the range where the marginal utility of real money balances is positive).[13] The interior optimality conditions are:

$$u_m(c_t, m_{t+1}) = w'(m_{t+1}) = \left(\frac{i_{t+1} - i_{t+1}^M}{1+i_{t+1}} \right) u_c(c_t, m_{t+1}) = \left(\frac{i_{t+1} - i_{t+1}^M}{1+i_{t+1}} \right) v'(c_t) \tag{43}$$

$$\frac{1}{1+\delta} E_t \left(R_{t,t+1} \frac{u_c(c_{t+1}, m_{t+2})}{u_c(c_t, m_{t+1})} \right) = \frac{1}{1+\delta} E_t \left(R_{t,t+1} \frac{v'(c_{t+1})}{v'(c_t)} \right) = 1 \tag{44}$$

For the specific functional forms chosen for the sub-utility functions for consumption and real money balances, (43) and (44) become:

[13] In discrete time money-in-the-utility function models, a choice has to be made as to whether the end-of-period stock of nominal money balances is to be deflated by this period's price level (the backward looking opportunity cost approach, M_t / P_t) or next period's price level, when these money balances will actually available (the forward-looking purchasing power approach, M_t / P_{t+1}). Little of substance depends on this choice, but the algebra is a beat neater with the forward-looking approach, which is adopted in this paper.

$$\ln(\theta - m_{t+1}) = \left(\frac{i_{t+1} - i^M_{t+1}}{1+i_{t+1}}\right)c_t^{-1} \quad (45)$$

$$\frac{1}{1+\delta}E_t\left(R_{t,t+1}\frac{c_t}{c_{t+1}}\right) = 1 \quad (46)$$

Output is demand-determined, so

$$1 = c_t + c_t^g + c_t^b \quad (47)$$

Financial asset market equilibrium requires that

$$A_t = (1+i_t)B_{t-1} \quad (48)$$ [14]

Pricing behaviour is given by slightly modified New-Keynesian Phillips curve in (49)

$$\begin{aligned}&\pi_t - \omega_t = \varphi E_{t-1}(Y_t - Y_t^*) + \frac{1}{1+\delta}E_{t-1}(\pi_{t+1} - \omega_{t+1})\\ &\varphi > 0\end{aligned} \quad (49)$$

Here $Y_t^* > C_t^g + C_t^b$ is the exogenously given level of capacity output or potential output. Its proportional growth rate is denoted $\gamma_t^* = \frac{Y_t^*}{Y_{t-1}^*} - 1$.

The Phillips curve in (49) combines Calvo's model of staggered overlapping nominal contracts with the assumption that even those price setters who are free to set their prices have to do so one period in advance (see Calvo (1983) and Woodford (2003)).[15] The current price level, P_t is therefore predetermined. The variable ω_t is the inflation rate chosen in period *t-1* for period *t* by those price setters who follow a simple behavioural rule or heuristic for setting prices.

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[14] The household solvency constraint (40) and the consolidated Government solvency constraint government intertemporal budget constraint (29) (with $R_j^f = 0$ for the closed economy special case) together with $A_t = B_t$ and $W_t = A_t + (1+i_t^M)M_{t-1}$ imply that $E_t I_{j+1,t} M_{j+1} \geq 0$, which, when holding with equality, was the assumption made to obtain the version of the ISI given in (13).

[15] Without the assumption that the optimising price setters have to set prices one period in advance, the Phillips curve would be $\pi_t - \omega_t = \varphi(Y_t - \bar{Y}_t) + \frac{1}{1+\delta}E_t(\pi_{t+1} - \omega_{t+1})$. Although prices would not be fully flexible, unless $\pi_t = \omega_t$ for all *t*, there can be some response of the period *t* price level to events and news in period *t*.

In the original Calvo (1983) model, $\omega_t = 0$. I will assume that the period *t* inflation heuristic is the deterministic steady state rate of inflation of the model expected at time *t-1*:

$$\omega_t = E_{t-1}\overline{\pi} \tag{50}$$

Thus, while the price level in period *t*, P_t, is predetermined, the rate of inflation in period *t*, π_{t+1} and in later periods in flexible. It is therefore possible to achieve an immediate transition to a different rate of inflation without any effect on real output, provided the change in monetary policy is unexpected, immediate and permanent.

Economic decisions are made and equilibrium is established for periods $t \geq 1$. Initial financial asset stocks, M_0, D_0 and M_0, B_0, D_0 are given. Central Bank instruments are i_t^M, h_t, c_t^b and μ_t.[16] Fiscal policy instruments are c_t^g, τ_t^b and τ_t^p.

It is clear that in the model developed here, as in any model with a predetermined price level, Corollary 2 holds: maximising the present discounted value of current and future real seigniorage is equivalent to maximising the present discounted value of future real Central Bank revenues. However, in the special case of the fully flexible price level (when, in the Calvo model, the fraction of price setters each period that are constrained to follow simple ad-hoc rules is zero), the initial price level is not predetermined. The analysis of the fully flexible price model involves setting $\pi_{t+1} = \omega_{t+1}$ for all *t* in the New-Keynesian Phillips curve (49) or, equivalently, replacing (49) by $Y_t = Y_t^*$ for all *t*.

The transition to the new steady state, when there is an unanticipated immediate and permanent change in the growth rate of nominal base money is an instantaneous transition to the

[16] It would be more descriptively realistic to make i_t a monetary policy instrument rather than μ_t. None of the results of this paper depend on this choice of monetary policy instrument and for expositional simplicity an exogenous growth rate of the nominal money stock is best here.

new steady state. In the deterministic special case of the model, given in equations (51) to (57), it is clear by inspection that with a fully flexible price level the ‘real time analysis’ is equivalent to the steady-state seigniorage analysis of Section II, and the ranking of the present discounted value of the alternative seigniorage concepts (as shares of GDP) is the same as their steady-state ranking.

In the Neo-Keynesian model, the actual level of current output is demand-determined and can therefore be influenced by past, present and anticipated future policy. In what follows I will consider the deterministic special case of the model developed here. All exogenous variables and policy instruments are constant. In period 0 the system starts off in a deterministic steady state. Then, in period, $t=1$, the monetary authorities announce a constant growth rate for the nominal money stock, $\mu_{t+1}=\bar{\mu}$, which they will adhere to forever afterwards. If this growth rate for the nominal money stock is different from the growth rate of the nominal money stock that supported the original deterministic steady state, the announcement is unexpected but fully credible. For this policy experiment to support an immediate transition to the new steady state, despite the predetermined price level, the nominal money stock held at the end of period 1 (the beginning of period *2*) has to be set at the level that supports monetary equilibrium in period 1 with the new steady-state stock of real money balances. This will, in general require a growth rate of the nominal money stock in period 1, μ_1 that is different from the subsequent steady state growth rate of the nominal money stock $\bar{\mu}$. This would certainly be the case if the demand for real money balances in period t were to be defined in terms of M_t / P_t. It may also be required when instead, as in the present paper, it is defined in terms of M_t / P_{t+1}.

The stationary equilibrium is characterised by the following conditions for $t \geq 1$:

$$c_t = 1 - c^g - c^b \tag{51}$$

equilibrium:

$$r_{t+1} = \rho \tag{52}$$

$$1+\pi_{t+1} = \frac{1+\bar{\mu}}{1+\gamma^*} \tag{53}$$

$$1+i_{t+1} = (1+\rho)(1+\pi_{t+1}) \tag{54}$$

$$v'(m_{t+1}) = \left(\frac{i_{t+1}-i^M}{1+i_{t+1}}\right) v'\left(c_t\right)$$

or (55)

$$m_{t+1} = \theta - e^{\left(\frac{i_{t+1}-i^M}{1+i_{t+1}}\right)c_t^{-1}}$$

$$Y_t = Y_t^* \tag{56}$$

$$\omega_{t+1} = \pi_{t+1} = \bar{\pi} \tag{57}$$

I am only considering equilibria where $i \geq i^M$ and $\rho > \gamma^*$.

I want to consider which constant rate(s) of inflation, $\pi = \bar{\pi}$, this economy can support, with a Central Bank whose intertemporal budget constraint is given by equation (37). With the economy in steady state from period *1*, it follows that the Central Bank's intertemporal budget constraint can be rewritten as follows:

$$-d_t + \left(\frac{1+\gamma^*}{\rho-\gamma^*}\right)\left(c^b + \tau^b + h\right) \leq \left(\frac{1+\gamma^*}{\rho-\gamma^*}\right)\left(\mu - i^M\right)m = \bar{\sigma}_1(\pi) \tag{58}$$

where

$$m = \ell(\pi; c, \rho, i^M);\ \ell_\pi = \frac{v'(c)}{w''(m)}\left(\frac{1+i^M}{(1+\rho)(1+\pi)^2}\right) < 0;\ \ell_c = \left(\frac{i-i^M}{1+i}\right)\frac{v''(c)}{w''(m)} > 0 \tag{59}$$

For the specific functional form $m = \theta - e^{\left(\frac{(1+\rho)(1+\pi)-(1+i^M)}{(1+\rho)(1+\pi)c}\right)}$, we have

$$\frac{d\bar{\sigma}_1}{d\pi} = \left(\frac{1+\gamma^*}{\rho-\gamma^*}\right)\left[(1+\gamma^*)m - \left((1+\pi)(1+\gamma^*)-(1+i^M)\right)\left(\theta-m\right)\left(\frac{1+i^M}{(1+\rho)(1+\pi)^2 c}\right)\right]$$

Consider the case where the nominal interest rate on base money is zero, so

$$\frac{d\bar{\sigma}_1}{d\pi}=\left(\frac{1+\gamma^*}{\rho-\gamma^*}\right)\left[(1+\gamma^*)\left(\theta-e^{\left(\frac{(1+\rho)(1+\pi)-1}{(1+\rho)(1+\pi)c}\right)}\right)-e^{\left(\frac{(1+\rho)(1+\pi)-1}{(1+\rho)(1+\pi)c}\right)}\left(\frac{(1+\pi)(1+\gamma^*)-1}{(1+\rho)(1+\pi)^2c}\right)\right] \tag{60}$$

Assume both the long-run nominal interest rate and the long-run growth rate of nominal GDP are non-negative. Then $\frac{d\bar{\sigma}_1}{d\pi}>0$ when $\pi=0$ provided the demand for real money balances is sufficiently large at a zero rate of inflation. A sufficiently large value of steady state private consumption $c=1-c^b+c^g$ as a share of GDP will ensure that. I assume this condition is satisfied. The long-run seigniorage Laffer curve has a single peak at

$$m=\hat{m}_1=\frac{\left((1+\hat{\pi}_1)(1+\gamma^*)-(1+i^M)\right)(1+i^M)\theta}{\left((1+\rho)(1+\hat{\pi}_1)^2c\right)(1+\gamma^*)+\left((1+\hat{\pi}_1)(1+\gamma^*)-(1+i^M)\right)(1+i^M)}$$

where

$$\hat{\pi}_1=\arg\max\bar{\sigma}_1$$

Let $\pi^b_{\min}$ be the lowest constant inflation rate that is consistent with the Central Bank's intertemporal budget constraint, given in (58), for given values of $d_t, c^b\geq 0, \tau^b$ and $h\geq 0$.[17] If there is a long-run Seigniorage Laffer curve, $\pi^b_{\min}$ may not exist: there may be no constant inflation rate that would generate enough real seigniorage to satisfy (58). If the value of the inflation target, π^*, is less than the value of the lowest, then the Central Bank cannot achieve the inflation target, because doing so would bankrupt it. The most it could do would be to set both c^b and h equal to zero: there would be no Central Bank-initiated helicopter drops of money and Central Bank staff would not get paid. If that is not enough to cause the weak inequality in (58) to be satisfied with $\pi=\bar{\pi}=\pi^*$, I will call this a situation where the inflation target is not *independently financeable* by the Central Bank. The value of the Central Bank's

[17] That is, $\pi^b_{\min}$ is the lowest value of π that solves $-d_t+\left(\frac{1+\gamma^*}{\rho-\gamma^*}\right)\left(c^b+\tau^b+h\right)=\bar{\sigma}_1(\pi)$.

holdings of Treasury debt, d_t, is determined by history; the net tax paid by the Central Bank to the Treasury, τ^b is determined unilaterally by the Treasury. I summarise this as follows:

Proposition 3:

If either $\pi^b_{\min}$ does not exist or $\pi^* < \pi^b_{\min}$, the inflation target is not independently financeable by the Central Bank.

If the Treasury decides to support the Central Bank in the pursuit of the inflation objective, the inflation target is *jointly financeable* by the Central Bank and the Treasury, as long as the consolidated intertemporal budget constraint of the Treasury and the Central Bank can be satisfied with the seigniorage revenue generated by the implementation of the inflation target. The intertemporal budget constraint of the Treasury and of the consolidated Government for this simple economy are given by, respectively:

$$b_t + d_t \leq \left(\frac{1+\gamma^*}{\rho-\gamma^*}\right)\left(\tau^p + \tau^b - c^g\right) \tag{61}$$

$$b_t + \left(\frac{1+\gamma^*}{\rho-\gamma^*}\right)\left(c^g + c^b - \tau\right) \leq \sigma_1(\pi) \tag{62}$$

Let $\pi^g_{\min}$ be the lowest constant inflation rate that is consistent with the intertemporal budget constraint of the consolidated Government, given in (62), for given values of $b_t, c^g \geq 0, c^b \geq 0$ and τ. Again, $\pi^g_{\min}$ could either not exist or could exceed the inflation target π^*. This suggests the following:

Proposition 4:

If either $\pi^g_{\min}$ does not exist or if $\pi^* < \pi^g_{\min}$, the inflation target is not financeable, even with cooperation between Treasury and Central Bank. The inflation target in that case is not feasible.

If (62) is satisfied with $\pi = \pi^*$, the inflation target is financeable by the consolidated Treasury and Central Bank – that is, the inflation target is feasible with cooperation between

Treasury and Central Bank. It may of course (if (58) is satisfied as well as (62)), also be independently financeable by the Central Bank. Note that the feasibility condition for the inflation target, equation (62), is independent of τ^b (which is a transfer payment within the consolidated Treasury and Central Bank) and of d_t which is an internal liability/asset within the consolidated Treasury and Central Bank. What matters is the net debt of the consolidated Treasury and Central Bank, b_t, and the taxes net of transfers of the consolidated Treasury and Central Bank, τ. If the feasibility condition (62) is satisfied, the Treasury can always provide the Central Bank with the resources it requires to implement the inflation target. All it has to do is reduce taxes on the Central Bank (or increase transfer payments to the Central Bank), in an amount sufficient to ensure that equation (58) is also satisfied.[18]

If (62) is satisfied with $\pi = \pi^*$, but (58) is not, then the inflation target is only financeable by the Treasury and Central Bank jointly, not independently by the Central Bank. Note that this can only happen if the Treasury has 'surplus' resources, that is, (61) holds as a strict inequality. In that case, a reduction in τ^b can permit the Central Bank's intertemporal budget constraint (58) to be satisfied without violating the Treasury's intertemporal budget constraint (62). I summarise this as follows:

Corollary 4:

> *If* $\pi^b_{\min} < \pi^* < \pi^g_{\min}$*, the inflation target is only cooperatively financeable by the Central Bank and the Treasury jointly.*

This discussion provides an argument in support of the view that the Central Bank should not have *operational target independence* (freedom to choose a quantitative inflation target) even when it has *operational independence* (the freedom to set the short nominal interest rate as it sees

[18] This could be achieved through a one-off capital transfer rather than through a sequence of current transfers.

fit). The reason is that if the political authorities choose the operational target, there is less of a risk of 'mandating without funding'. On its own, the Central Bank cannot be guaranteed to have the right degree of financial independence. Without Treasury support, there can be no guarantee that the minimal amount of seigniorage required to ensure the solvency of the Central Bank is supported by the inflation target. Only the Treasury can make sure that the Central Bank has enough resources, other than seigniorage, to make the inflation target financeable by the Central Bank. The Treasury, through its ability to tax the Central Bank, is effectively constrained only by the consolidated intertemporal budget constrained in (62), even though formally it faces the intertemporal budget constraint given in equation (61).

Proposition 4 and Corollary 4, which deal with the consolidated Treasury and Central Bank, that is, with the Government, are straightforward implications of results established over a quarter of a century ago by Sargent and Wallace (1981). Of course, their analysis predates modern inflation targeting, which was 'invented' in New Zealand in 1989, so it did not address the financeability of an inflation target but rather the closely related question as to whether, with a given Government primary surplus as a share of GDP and for a given ratio of non-monetary Government debt to GDP, seigniorage would be sufficient to ensure Government solvency.

V. Other aspects of necessary co-operation and co-ordination between Central Bank and Treasury

Even if the Treasury supports the Central Bank's inflation target and provides it with the financial resources to implement it, there are at least two other economic contingencies for which active Central Bank and Treasury co-ordination and co-operation is desirable.

V.1 Recapitalizing the central bank

The first case occurs when the (threat of) a serious banking crisis or financial crisis with systemic implications forces the Central Bank to act as a lender of last resort, and the problem turns out to be (or becomes), for a significant portion of the banking/financial system, a solvency crisis as well as a liquidity crisis. It could happen that recapitalising the insolvent banks or financial institutions with only the financial resources of the Central Bank (including a given sequence of net payments to the treasury, T^b) would require the Central Bank to engage in excessive base money issuance, which would result in unacceptable rates of inflation. As long as the resources of the consolidated Treasury and Central Bank are sufficient, the Treasury should either recapitalise the Central Bank (if the Central Bank recapitalised the private banking/financial system in the first instance), or the Treasury should directly recapitalise the banking/financial system. In the accounts set out above, recapitalising the Central Bank would amount to one or more large negative realisations of T^b, with as counterparts an increase in Central Bank holdings of Treasury debt, D (see Ize (2005)).

Special problems occur when the insolvency of (part of) the financial system is due to an excess of foreign-currency liabilities over foreign-currency assets. In that case the Treasury, in order to recapitalise the Central Bank (or some other part of the financial sector directly), has to be able to engineer both an internal fiscal transfer and an external transfer of resources of the required magnitude. If the external credit of the state is undermined, this may only be possible gradually, if and as the state can lay claim to (part of) the current and future external primary surpluses of the nation.

In the usual nation state setting, a single treasury or national fiscal authority stands behind a single central bank. Unique complications arise in the EMU, where each national fiscal authority stands financially behind its own national central bank (NCB), but no fiscal authority stands directly behind the ECB. The lender of last resort function in the EMU is assigned to the NCB members of the ESCB (see Padoa-Schioppa (2004) and Goodhart (2002)). This will work fine when a troubled or failing bank or other financial institution deemed to be of systemic importance has a clear nationality, as most Eurozone-domiciled banks and other financial institutions do today. Likewise, banks that are subsidiaries of institutions domiciled outside the EMU will be the responsibility of their respective Central Bank (be it the Bank of England, the Federal Reserve System or the Bank of Japan) and of the national fiscal authority that stands behind each of these Central Banks.

Trouble arises as and when Eurozone-domiciled banks emerge that do not have a clear national identity, say banks incorporated solely under European Law. As there is no fiscal authority, national or supranational, standing behind the ECB, who would organise and fund the bail-out and recapitalisation of such a 'European bank'? Whether this potential vulnerability will in due course be remedied by the creation of a serious supra-national fiscal authority at the EMU level that would stand behind the ECB, or by implicit or explicit agreements between the ECB, the NCBs (the shareholders of the ECB) and the national fiscal authorities is as yet unclear.

V.2 Helicopter drops of money

The second set of circumstances when cooperation and coordination between the monetary and fiscal authorities is essential is when an economy is confronting the need to avoid unwanted deflation or, having succumbed to it, to escape from it. In principle, the potential benefits from cooperation between the monetary and fiscal authority apply to stabilisation policy in general, that is to counter-inflationary as well as to counter-deflationary policies. The issue is particularly urgent, however, when deflation is the enemy and conventional monetary policy has run out of steam.

Faced with deflation, the Central Bank on its own can cut the short nominal interest rate - the primary monetary policy instrument in most economies with a floating exchange rate. It can engage in sterilised foreign exchange market operations. If there are reserve requirements imposed on commercial banks or other financial institutions, these can be relaxed, as can the collateral standards in Repos and the eligibility requirements that must be met by potential counterparties.

Once the short nominal interest rate is at the zero floor, conventional monetary policy is effectively exhausted. The Central Bank can then engage in generalised open market purchases, monetising the outstanding stock of non-monetary public debt, of all maturities, nominally denominated or index-linked, held outside the Central Bank. Once all outstanding public debt has been absorbed by the Central Bank, it could turn its attention to the purchase and monetisation of private securities, from foreign currency-denominated securities, to stocks and shares, land, property or contingent claims. Clearly, such socialisation of private wealth would be subject to all kinds of moral hazard, adverse selection and governance problems.

Should this too fail to boost aggregate demand and end deflation, the monetary authority on its own has one remaining exotic instrument and the combined monetary and fiscal authorities have one conventional but truly effective instrument. The unconventional instrument is to lower the zero floor on nominal interest rates (which is a result of the zero nominal interest rate paid on currency and often on all base money), by paying a negative nominal interest rate on base money. For commercial banks' reserves with the central bank, paying a negative nominal interest rate is technically and administratively trivial. Imposing a 'carry tax' on currency is administratively cumbersome and intrusive, but not impossible. Silvio Gesell (1916) recommended it many years ago, and as great an economist as Irving Fisher (1933) thought the proposal had merit (see also Goodfriend (2000) and Buiter and Panigirtzoglou (2001, 2003)).

There is, however, a very conventional policy alternative. Milton Friedman referred to it as (base) money dropped from a helicopter (Friedman (1969, p. 4)). If the recipients of this largesse do not expect it to reversed (in present discounted value terms) in the future, that is, if they do not expect the helicopter drop of money to be followed by a vacuum cleaner sucking up the currency notes again, this would, at a given price level, represent an increase in the real net wealth of the private sector (see Buiter (2003)). Because base money does not have to be redeemed *ever*, it does not constitute an effective liability of the state. The increase in net private wealth is also in the most liquid form possible.[19]

In the context of the simplified closed-economy version of the model, the solvency constraint and intertemporal budget constraint of the consolidated central bank and treasury are (from (29), (30), (41), and $W_t = (1+i_t^M)M_{t-1} + (1+i_t)B_{t-1}$)

$$\lim_{N\to\infty} E_t I_{N,t-1} B_N \le 0 \tag{63}$$

$$B_{t-1} \le E_t \sum_{j=t}^{\infty} I_{j,t-1}\left(P_j\left(T_j - C_j^g - C_j^b\right) + M_j - (1+i_j^M)M_{j-1}\right) \tag{64}$$

The household intertemporal budget constraint and solvency constraint are

$$B_{t-1} + \left(\frac{1+i_t^M}{1+i_t}\right)M_{t-1} \ge E_t \sum_{j=t}^{\infty} I_{j,t-1}\left[P_j(C_j + T_j - Y_j) + \left(\frac{i_{j+1} - i_{j+1}^M}{1+i_{j+1}}\right)M_j\right] \tag{65}$$

$$\lim_{N\to\infty} E_t I_{N+1,t} W_{N+1} = \lim_{N\to\infty} E_t I_{N,t}\left(\left(\frac{1+i_{N+1}^M}{1+i_{N+1}}\right)M_N + B_N\right) \ge 0 \tag{66}$$

[19] In this case, the boundary condition $E_t I_{N,t} M_N \ge 0$ would hold as a strict inequality.

Subtracting (65) from (64), assuming that the household intertemporal budget constraint holds with equality (an implication of optimising behaviour) and using the ISI (11) yields:

$$E_t \sum_{j=t}^{\infty} I_{j,t} \left[P_j (C_j + C_j^g + C_j^b - Y_j) \right] = E_t \lim_{N\to\infty} I_{N,t} \left(\frac{1+i_{N+1}^M}{1+i_{N+1}} \right) M_N \geq 0 \tag{67}$$

The key asymmetry in the perception of government-issued fiat money – an asset to the private sector but not, in an economically meaningful sense, a liability to the government, accounts for the fact that the conventional present value intertemporal real resource constraint of the economy can, in principle, be violated if $E_t \lim_{N\to\infty} I_{N,t-1}(1+i_{N+1}^M)M_N > 0$. For this to happen, we require that the household solvency constraint holds with equality:

$\lim_{N\to\infty} E_t I_{t+N+1,t-1} W_{N+1} = \lim_{N\to\infty} E_t I_{t+N,t-1} \left(\left(\frac{1+i_{N+1}^M}{1+i_{N+1}} \right) M_N + B_N \right) = 0$ but the consolidated government solvency constraint holds as a strict inequality: $\lim_{N\to\infty} E_t I_{t+N,t-1} B_N < 0$. For such an economy, the government has monetary liabilities to the private sector and also lends to the private sector (or holds non-monetary claims on the private sector, so

$$\lim_{N\to\infty} E_t I_{t+N,t-1} \left(\frac{1+i_{N+1}^M}{1+i_{N+1}} \right) M_N = -\lim_{N\to\infty} E_t I_{t+N,t-1} B_N > 0 \tag{68}$$

We can rewrite (67) as

$$E_t \sum_{j=t}^{\infty} R_{j,t} C_j = E_t \sum_{j=t}^{\infty} R_{j,t} \left[Y_j - (C_j^g + C_j^b) \right] + \frac{1}{P_t} E_t \lim_{N\to\infty} I_{N,t} \left(\frac{1+i_{N+1}^M}{1+i_{N+1}} \right) M_N \tag{69}$$

In the deterministic version of the model, the Euler equation for private consumption implies that $C_j = \frac{R_{t,j}}{(1+\delta)^{j-t}} C_t$, and therefore

$$C_t = \delta \left[E_t \sum_{j=t}^{\infty} R_{j,t} \left[Y_j - (C_j^g + C_j^b) \right] + \frac{1}{P_t} E_t \lim_{N\to\infty} I_{N,t} \left(\frac{1+i_{N+1}^M}{1+i_{N+1}} \right) M_N \right] \tag{70}$$

The 'permanent income consumption function' after consolidating the household and government intertemporal budget constraints makes consumption in each period a function of the present discounted value of the terminal money stock. It follows that there can be no liquidity trap equilibrium if the government is expected, in the long run, to have a growth rate of the nominal money stock at least equal to the nominal interest rate on money. Assume that there is a liquidity trap, that is $i_t^M = i_t$ for all t and the government cannot influence the price level, nominal and real interest rates and real activity. The consumption function in (70) becomes:

$$C_t = \delta \left[E_t \sum_{j=t}^{\infty} R_{j,t} \left[Y_j - (C_j^g + C_j^b) \right] + \frac{1}{P_t} E_t \lim_{N \to \infty} I_{N,t}^M M_N \right] \tag{71}$$

If the growth rate of the nominal money stock exceeds the nominal interest rate on money, the term $I_{N,t}^M M_N$ will grow without bound. Therefore either private consumption grows without bound or the price level rises to offset the increase in $I_{N,t}^M M_N$. Either outcome is inconsistent with there being a liquidity trap equilibrium. When the nominal interest rate on money is zero, there can be no liquidity trap equilibrium if the long-run growth rate of the nominal money stock is strictly positive. I summarise this as Proposition 5.

Proposition 5.

> *When government fiat money is perceived as an asset by the private sector but not as a liability by the government, there can be no liquidity trap equilibrium if the long-run growth rate of the nominal money stock exceeds the nominal interest rate on money balances.*

It is clear from the consolidated intertemporal budget constraint (67), that this result does not depend on the absence of uncertainty. The only property of the utility function necessary for the result is that utility is strictly increasing in consumption.

An example of a helicopter drop, in the UK context, would be for the Governor of the Bank of England issue a £1,000 cheque, drawn upon the Bank of England, to every man, woman and child in the country. On the balance sheet of the Bank this would show up as an increase in the stock of base money and a corresponding reduction in the financial net worth of the Bank. In its budget constraint it would be a one-off transfer payment to the private sector (h in our notation).

Would it work? If the money rain is not expected to be reversed in present value, it surely would. It does not rely on the strength of the intertemporal substitution effect in private consumption or on the interest sensitivity of private investment demand. All that it requires is that aggregate consumption today is a normal good. If the wealth effect is weak and the £1,000.00 cheque does not do the job, the Governor can add zeros in front of the decimal point on the cheque until the private consumer surrenders and goes out and spends.

Even if the economic mechanism of the helicopter drop of money is straightforward, its practical implementation cannot be done by the Central Bank alone. The reason is that in reality central banks do not have an instrument like H in their arsenals. Making transfer payments to the private sector is not something Central Banks are legally permitted to do, because they are not fiscal agents of the state. So the economically equivalent action has to be coordinated between the Treasury and the Central Bank. The treasury will implement a tax cut or increase in transfer payments (a cut in T^p) and will finance this by selling debt to the Central Bank (increasing D). The acquisition of Treasury debt by the central bank is financed through the issuance of base money, an increase in M.

VI. Conclusion

Governments through the ages have appropriate real resources through the monopoly of the 'coinage'. In modern fiat money economies, the monopoly of the issue of legal tender is generally assigned to an agency of the state, the Central Bank, which may have varying degrees of operational and target independence from the government of the day.

In this paper I analyse four different but related concepts, each of which highlights some aspect of the way in which the state acquires command over real resources from its ability to issue fiat money. They are (1) seigniorage (the change in the monetary base), (2) Central Bank revenue (the interest bill saved by the authorities on the outstanding stock of base money liabilities), (3) the inflation tax (the reduction in the real value of the stock of base money due to inflation and (4) the taxes paid by the Central Bank to the Treasury.

To understand the relationship between these four concepts, an explicitly intertemporal approach is required, which focuses on the present discounted value of the current and future resource transfers involved. Furthermore, when the Central Bank is operationally independent, it is essential to decompose the familiar consolidated 'government budget constraint' and consolidated 'government intertemporal budget constraint' into the separate accounts and budget constraints of the Central Bank and the Treasury. Only by doing this can we appreciate the financial constraints on the Central Bank's ability to pursue and achieve an inflation target, or the importance of cooperation and coordination between the Treasury and the Central Bank when faced with financial sector crises involving the need for long-term recapitalisation or when confronted with the need to mimick Milton Friedman's helicopter drop of money in an economy faced with a liquidity trap.

References

Anand, Ritu and Sweder van Wijnbergen (1989), "Inflation and the Financing of Government Expenditure: an Introductory Analysis with an Application to Turkey", *World Bank Economic Review*, Vol. 3, No. 1, pp. 17-38. *Further information in IDEAS/RePEc*

Bailey, Martin J. (1956), "The Welfare Costs of Inflationary Finance," *Journal of Political Economy*, vol. 64, no. 2, April, pp. 93-110.

Blinder, Alan S. and Robert M. Solow (1973), "Does fiscal policy matter?", *Journal of Public Economics*, 2, pp. 319-337. *Further information in IDEAS/RePEc*

Bordo, Michael (2006), "Comment on Marc Flandreau, 'Pillars of Globalization: A History of Monetary Policy Targets, 1797-1997'", Prepared for the Fourth ECB, Monetary Policy conference, Frankfurt November 9-10 2006.
http://www.ecb.int/events/pdf/conferences/cbc4/Discussion_Bordo.pdf

Bresciani-Turroni, Constantino (1937), *The Economics of Inflation: A Study of Currency Depreciation in Post-War Germany*. London, Allen and Unwin. *Further information*

Buiter, Willem H. (1990), Principles of Budgetary and Financial Theory, MIT Press, Cambridge, Massachusetts. *Further Information*

Buiter, Willem H. (2003), "Helicopter Money: Irredeemable Fiat Money and the Liquidity Trap", NBER Working Paper No. W10163, December. *Further information in IDEAS/RePEc*

Buiter, Willem H. (2004), "Two naked emperors? Concerns about the Stability and Growth Pact and second thoughts about Central Bank independence", *Fiscal Studies*, Vol. 25(3), pp. 249-77.
Further information in IDEAS/RePEc

Buiter, Willem H. (2005), "New Developments in Monetary Economics: two ghosts, two eccentricities, a fallacy, a mirage and a mythos", Royal Economic Society 2004 Hahn Lecture, *The Economic Journal*, Conference Papers, Vol. 115, No. 502, March 2005, pp. C1-C31.
Further information in IDEAS/RePEc

Buiter, Willem H. and Nikolaos Panigirtzoglou (2001), "Liquidity Traps: How to Avoid Them and How to Escape Them", with Nikolaos Panigirtzoglou, in *Reflections on Economics and Econometrics, Essays in Honour of Martin Fase*, edited by Wim F.V. Vanthoor and Joke Mooij, 2001, pp. 13-58, De Nederlandsche Bank NV, Amsterdam.
Further information in IDEAS/RePEc

Buiter, Willem H. and Nikolaos Panigirtzoglou (2003), "Overcoming the Zero Bound on Nominal Interest Rates with Negative Interest on Currency: Gesell's Solution", *Economic Journal*, Volume 113, Issue 490, October 2003, pp. 723-746.
Further information in IDEAS/RePEc

Cagan, Philip (1956), "Monetary Dynamics of Hyperinflation", in Milton Friedman, Editor, *Studies in the Quantity Theory of Money*, University of Chicago Press, Chicago, Illinois.

Calvo, Guillermo (1983), "Staggered Contracts in a Utility-Maximizing Framework", *Journal of Monetary Economics*, September.
Christ, Carl (1968), "A Simple Macroeconomic Model with a Government Budget Restraint," *Journal of Political Economy* 76, No. 1, January/February, pp. 53-67.

Dornbusch, Rudiger, and Staley Fischer (1986), "Stopping Hyperinflations Past and Present", *Weltwirtschaftliches Archiv* 122, pp. 1-47. Further information in IDEAS/RePEc

Easterly, William R., Paolo Mauro, Klaus Schmidt-Hebbel (1995), **"**Money Demand and Seigniorage-Maximizing Inflation", *Journal of Money, Credit and Banking***,** Vol. 27, No. 2 (May), pp. 583-603. Further information in IDEAS/RePEc

Flandreau, Marc (2006), "Pillars of Globalization: A history of monetary policy targets, 1797-1997", Revised Draft : November 2, 2006, *Paper prepared for the Fourth ECB Central Banking Conference,* Frankfurt-Am-Main, November 9-10 2006. Further information

Friedman, Milton (1968), "The Role of Monetary Policy." *American Economic Review* 58, no. 1, pp. 1-17.

Friedman, Milton (1969), "The Optimum Quantity of Money", Chapter 1 in Milton Friedman, *The Optimum Quantity of Money and other Essays*, Aldine Publishing Company, Chicago, pp. 1-50.

Friedman, M.(1971), "Government Revenue from inflation", *Journal of Political Economy*, vol. 79, N° 4, pp. 846-56. Further information in IDEAS/RePEc

Gesell, Silvio (1916, 1949), *Die Natuerliche Wirtschaftsordnung*, Rudolf Zitzman Verlag, available in English as *The Natural Economic Order,* Peter Owen Ltd, London, 1958.
Further information

Goodfriend, Marvin (2000), "Overcoming the Zero Bound on Interest Rate Policy, *Journal of Money, Credit and Banking*, Vol. 32, No. 4, Pt. 2, November, pp. 1007-1035.
Further information in IDEAS/RePEc

Goodhart, Charles (2002), "Recent Developments in Central Banking." In *Monetary Policy, Capital Flows and Exchange Rates: Essays in Honour of Maxwell Fry*. Edited by Fry, M. J.; Dickinson, D. G.; Allen, B. Routledge,
Further information

Ize, Alain (2005), "Capitalising Central Banks: A Net Worth Approach", IMF Working Paper, WP/05/15 January 2005 Further information in IDEAS/RePEc

Kiguel, Miguel A. and Pablo A. Neumeyer (1995), "Seigniorage and Inflation: The Case of Argentina", *Journal of Money, Credit and Banking*, Vol. 27, No. 3 (Aug.), pp. 672-682.
Further information in IDEAS/RePEc

King, Robert G. and C. I. Plossser (1985), "Money, Deficits and Inflation", *Carnegie-Rochester*

Conference Series on Public Policy, 22, Spring, pp. 147-196.

Padoa-Schioppa, Tommaso (2004). The Euro and Its Central Bank, The MIT Press, Cambridge, Massachusetts. *Further information*
Phelps, Edmund (1973), Inflation in the Theory of Public Finance", *Swedish Journal of Economics*, 75, March, pp. 67-82.

Romer, David (2006), *Advanced Macroeconomics*, Third Edition, Chapter 10. McGraw-Hill, New York. *Further information*

Sargent, Thomas J. (1982), "The End of Four Big Inflations", in Robert E. Hall, ed., *Inflation*, pp. 41-98, University of Chicago Press, Chicago Illinois. *Further information in IDEAS/RePEc*

Sargent, Thomas J. (1987), *Dynamic Macroeconomic Theory*, Harvard University Press, Cambridge, Massachusetts. *Further information*

Sargent, Thomas J. and Neil Wallace (1981), "Some unpleasant monetarist arithmetic", Federal Reserve Bank of Minneapolis Quarterly Review, 5(3), pp. 1-17.
Further information in IDEAS/RePEc

Sims, C. A. (2004): "Fiscal Aspects of Central Bank Independence," Chapter 4, p.103-116, in *European Monetary Integration*, Hans-Werner Sinn, Mika Widgrén, and Marko Köthenbürger, editors, MIT Press. *Further information in IDEAS/RePEc*

Sims, C. A. (2005), "Limits to Inflation Targeting", Chapter 7 in *The Inflation-Targeting Debate*, Ben S. Bernanke and Michael Woodford, editors, NBER Studies in Business Cycles Volume32, p. 283-310. *Further information*

Thornton, H. (1802), *An Enquiry into the Nature and Effects of the Paper Credit of Great Britain.*

Tobin, James and Willem H. Buiter (1976), " Long-run effects of fiscal and monetary policy on aggregate demand", in J. Stein ed. *Monetarism*, North Holland, Amsterdam, pp. 273-309.
Further information in IDEAS/RePEc

Walsh, Carl E. (2003), Monetary Theory and Policy, 2nd Edition, Chapter 4, pp. 135-197, The MIT Press, Cambridge Massachusetts. *Further information*

Woodford, Michael (2003), Interest & Prices; Foundations of a Theory of Monetary Policy, Princeton University Press, Princeton and Oxford. *Further information*