Money Matters

The effects of relative price changes, margin changes and productivity changes on inflation and the price level.

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23-03-2000

(I) Introduction

The general price level is a *relative* price, the price of (a representative bundle of) goods (and services) in terms of money. It is therefore no surprise that this relative price is influenced both by the demand for and supply of money, and by the demand for and supply of goods. The excess demand for money and the excess demand for goods depend on a number of exogenous and endogenous variables, some of which affect both.

The statement that inflation is a monetary phenomenon is not terribly illuminating. It is on a par with the statement that the price of bananas is a banana phenomenon. It is also at best a half-truth. The relative price of money and goods is a monetary *and* goods phenomenon. It is still a long way from that recognition to a coherent analysis of the impact of real and nominal shocks on the general price level.

In modern economies, money is intrinsically worthless fiat money. Even rather weak concepts of rationality therefore imply that choices about real variables, and their equilibrium values, are homogenous of degree zero in nominal prices and exogenous or predetermined nominal stocks and flows. When money prices are flexible, at least in the long run, money will be neutral in the long run, although not necessarily super-neutral. Short-run non-neutrality of money plays an important part in the dynamic response of the price level, and other economy-wide aggregates, to both real and nominal shocks. These non-neutralities can be due to transitory nominal rigidities in price and wage setting or to the kind of financial market segmentation emphasized by 'limited participation models'. In what follows, I will only consider short-term nominal rigidities because the market segmentation underlying the limited participation models is not relevant to economies with reasonably well-developed financial markets and institutions.

This note looks at the implications of a number of changes in economic structure, or supply-side shocks, for the behaviour of the general price level. Specifically, I consider

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productivity shocks, 'margin' shocks (the reflection of changes in the competitive environment) and changes in the target growth rate of real wages (proxying changes in the natural rate of unemployment).

To gauge the implications for the general price level of such 'New Economy' shocks, we must trace their implications for the demand for and supply of money, the demand for and supply of goods and for the rest of the transmission mechanism. While the long-run effects of these shocks on the rate of inflation and the price level are relatively straightforward and non-controversial, the dynamic response to these shocks depends, among other things, on the details of the dynamic processes determining money wages and prices. In what follows I adopt a framework that implies a simple accelerationist Phillips curve. Superior, but more complex, specifications, based on partly forward-looking and partly backward-looking wage setting, or aggregate price formation built up from the explicit aggregation of micro-price setting in the presence of menu costs, could result in substantially different short-run dynamic responses.

The response of the inflation rate and the price level depends crucially on the monetary rule adopted by the authorities. The sky is the limit where possible or even plausible monetary rules are concerned, but this paper considers two classes of rules, with two sub-cases in each class. The first class of rules is a simple Taylor rule for the short nominal rate of interest. The nominal rate depends on a constant, the ouput gap and the deviation of inflation from target. We consider both the case where the intercept term in the Taylor rule is constant, and where the intercept term adjusts to support the target rate of inflation in steady state. The second class of rules takes the growth rate of the nominal money stock to be the policy instrument. We consider both the case where the growth rate of the nominal money stock is constant and the case where it adjusts to support the target rate of inflation in steady state.

(II) A simple ad-hoc model of a closed economy

(II.1) Monetary Equilibrium

Let M be the nominal money stock, V the income velocity of circulation of money, P the general price level and Y real GDP.

$$MV = PY \tag{2.1}$$

Lower-case characters represent the natural logarithms of the corresponding upper-case characters. The equation of exchange can therefore be rewritten as:

$$m + v = p + y \tag{2.2}$$

(II.2) Money demand

Velocity is an increasing function of the opportunity cost of holding money, identified for simplicity with the short nominal interest rate, i. For a broad monetary aggregate, the opportunity cost would be $i - i^M$, where i^M would be the own nominal interest rate on money.

$$v = v(i) \qquad v > 0;\ v' > 0 \tag{2.3}$$

For simplicity, we assume velocity to be log-linear in the short nominal rate of interest.

$$\begin{gathered} v = l_0 + l_1 i \\ l_1 > 0 \end{gathered} \tag{2.4}$$

(II.3) Aggregate supply and demand

Let p_j be the level of cyclically-adjusted productivity in sector *j* and p average (that is, economy-wide) cyclically-adjusted productivity. Potential output in industry *j* is y_j^n and y^n is aggregate potential output. There are *N* sectors or industries. Ignoring labour force growth and capital accumulation, potential output in industry *j* evolves according to:

$$\Delta y_j^n = \Delta p_j \tag{2.5}$$

where Δ is the backward difference operator, that is, $\Delta x(t) = x(t) - x(t-1)$.
Aggregate potential output is given by:

$$\begin{gathered} \Delta y^n = \sum_{j-1}^{N} a_j \Delta p_j = \Delta p \\ \sum_{j=1}^{N} a_j = 1;\ 0 \le a_j \le 1 \end{gathered} \tag{2.6}$$

The weights a_j used to construct the sectoral averages can be interpreted as the share of sector *j* in aggregate GDP. The same weights are used to construct an aggregate price index and a measure of aggregate output growth.

Real aggregate demand depends negatively on the real interest rate and positively on the stock of real money balances relative to GDP.

$$\begin{gathered} y = h_0 - h_1 r + h_2 (m - p - y) \\ h_1 > 0;\ 1 > h_2 \ge 0 \end{gathered} \tag{2.7}$$

$$r(t) = i(t) - \Delta p(t+1) \tag{2.8}$$

The aggregate demand equation includes an attempt to include a simple, tractable approximation to permanent-income influences on demand in an otherwise rather Keynesian ad-hoc model. The impact of permanent after-tax labour income and of ownership claims on real assets is represented by h_0, which consists of an exogenous constant component, e, and a 'permanent income' component y^p.

$$h_0 = e + y^p \tag{2.9}$$

I will carry permanent income as a parameter in what follows, but impose the following structure on it:

$$\sum_{i=0}^{\infty} \frac{\partial y^p(t)}{\partial p(t+i)} = 1$$

$$0 \leq \frac{\partial y^p(t)}{\partial p(t+i)} \leq 1 \tag{2.10}$$

$$\frac{\partial \Delta \bar{y}^p(t)}{\partial \Delta \bar{p}(t)} = 1$$

The first restriction in (2.10) is that, in response to an immediate, permanent shock to the *level* of productivity, permanent income rises in line with productivity.

The second restriction in (2.10) is that, in response to an immediate temporary increase in the level of productivity, permanent income increases by less than the current level of productivity, that is, $0 \leq \frac{\partial y^p(t)}{\partial p(t+i)} \leq 1$.

The third restriction in (2.10) is that, in steady state, the permanent income component grows at the steady-state growth rate of economy-wide productivity. Steady-state values of variables are denoted by *overbars.*

Equations (2.9) and (2.10) amount to a version of 'Say's Law' – supply creates its own demand. It is the simplest way of allowing for the effect of current and future productivity on current aggregate demand, through such channels as current real wage income, permanent income and the stock market. A change in profit margins and a change in target real wage growth, two determinants of wages and prices considered below, are assumed not to affect aggregate demand at a given real interest rate, real money stock and permanent income, y^p. Such changes redistribute income between profits and wages. I assume that such redistributions are aggregate-demand-neutral.

In an open economy, aggregate demand would also depend on the real exchange rate, and domestic and foreign interest rates would be linked by UIP relationships, possibly modified for risk premia. Such an extension is considered briefly in Section V.

(II.4) Wage and price determination

This note will use the simplest possible wage-price specification that has the long-run natural rate property and short-term price level and inflation persistence.

Target real wage growth, $\Delta[w(t+1)-p(t+1)]^*$ depends positively on the output gap. When there is a zero output gap, target real wage growth occurs at a rate Δp^{w}, treated as exogenous here. In fortunate economic systems, Δp^{w} will equal the growth rate of aggregate cyclically corrected productivity, Δp .

$$\Delta[w(t+1)-p(t+1)]^* = \Delta p^{w}(t) + d\,[y(t)-y^n(t)] \qquad (2.11)$$

$$d > 0$$

A very simple way to introduce nominal inertia into the model, is to assume that current money wage growth equals current target real wage growth plus last period's inflation. This gives the simplest accelerationist Phillips curve, which is all that is needed for present purposes.

$$\Delta w(t+1) = \Delta[w(t+1)-p(t+1)]^* + \Delta p(t) \qquad (2.12)$$

From (2.11) and (2.12) it follows that

$$\Delta w(t+1) = \Delta p^{w}(t) + d\,[y(t)-y^n(t)] + \Delta p(t) \qquad (2.13)$$

Each industry's price is a mark-up on normal unit labour cost, defined as labour cost per unit of potential output. The mark-up is treated as exogenous, again for expositional simplicity only, as is the growth rate of cyclically adjusted productivity.

$$p_j = m_j + w - y_j^n \qquad (2.14)$$

Industry *j*'s mark-up has an exogenous component $\tilde{m}_j$ and a cyclical component:

$$m_i = \tilde{m}_i + q_i\left(y_i - y_j^n\right) \qquad (2.15)$$

$$q_i \geq 0$$

Equation (2.15), with is pro-cyclical mark-up, can be rationalized as a simple version of the ‘customer market model’. Strategic firms lower their mark-up when current output is low relative to future expected profits, foregoing current profits in order to capture future market share. This pro-cyclical behaviour of profit margins is the opposite of that implied by markets characterised by ‘implicit collusion’, where firms lower their mark-ups when current output is high relative to expected future output, in order to lower the incentives to undercut the implicit cartel. Briton, Larson and Small [2000] argue that only the customer market model generates predictions consistent with UK evidence.

Equations (2.13), (2.14) and (2.15) give the following inflation equation for industry *i*.

$$\Delta p_j(+1) = \Delta\tilde{m}_j + q_j \Delta\left(y_j - y_j^n\right) + \Delta p^w - \Delta p_j + d[y - y^n] + \Delta p \qquad (2.16)$$

The general price level is defined as follows:

$$p = \sum_{j=1}^{N} a_j p_j \qquad (2.17)$$

We also define the following aggregates.

Average margin growth rate:

$$\Delta m = \sum_{j=1}^{N} a_j \Delta m_j \qquad (2.18)$$

Average growth rate of the exogenous margin component:

$$\Delta\tilde{m} = \sum_{j=1}^{N} a_j \Delta\tilde{m}_j \qquad (2.19)$$

Average cyclical responsiveness of margins:

$$q = \sum_{j=1}^{N} a_j q_j \qquad (2.20)$$

Aggregating (2.16) over all industries yields the following accelerationist price-Phillips curve:

$$\Delta p(+1) = \Delta\tilde{m} + \sum_{i=1}^{N} a_i q_i (\Delta y_i - \Delta y_j^n) + \Delta p^w - \Delta p + d[y - y^n] + \Delta p \qquad (2.21)$$

The effect of the cycle on the average mark-up can be written as follows:

$$\sum_{i=1}^{N} a_i q_i (\Delta y_i - \Delta y_i^n) = q(\Delta y - \Delta y^n) + \sum_{i=1}^{N} a_i (q_i - q)(\Delta y_i - \Delta y_i^n) \qquad (2.22)$$

The term $q(\Delta y - \Delta y^n)$ measures the contribution of the average, economy-wide output gap and the average cyclical responsiveness of margins to the mark-up. The term $\sum_{i=1}^{N} a_i (q_i - q)(\Delta y_i - \Delta y_i^n)$ will be positive (negative) if a sector with above-average cyclical responsiveness of margins is typically also a sector with above-average (below-average) growth of its sectoral output gap.

I will assume that changes in sectoral output gaps, $\Delta y_i - \Delta y_i^n$, are independent of deviations of sectoral cyclical margin responsiveness from their average value, $q_i - q$, that is, $\sum_{i=1}^{N} a_i (q_i - q)(\Delta y_i - \Delta y_i^n) = 0$. This is a useful benchmark and greatly simplifies the algebra.

$$\Delta p(+1) = \Delta\tilde{m} + \Delta p^w - \Delta p + d[y - y^n] + q(\Delta y - \Delta y^n) + \Delta p \qquad (2.23)$$

The change in the rate of inflation therefore depends on the change in the average, economy-wide exogenous margin component, on the excess of the growth rate of target real wages over the average, economy-wide, growth rate of cyclically adjusted productivity, and on the level and rate of change of the economy-wide output gap. Note that the 'speed limit effect' in the price-Phillips curve comes from the pro-cyclical behaviour of margins, not from the labour market.

(II.5) Relative price changes

The previous section implies that relative prices change only as a result of differential margin growth or productivity growth between sectors. Differential margin growth can be the result either of different rates of growth of the exogenous margin components or of differences in the cyclical margin components. Cyclical differences in sectoral margin behaviour can reflect either differences between the cyclical positions of sectors, $q_k[\Delta y_j - \Delta y_j^n - (\Delta y_k - \Delta y_k^n)]$, or differences in the cyclical sensitivity of margins among sectors, $(q_j - q_k)(\Delta y_j - \Delta y_j^n)$. Note that we can have wide-spread and large relative price changes without any of this impacting on the general price level, as long as average margin growth and average trend productivity growth are not affected.

$$\begin{aligned}\Delta[p_j - p_k] = \Delta[\tilde{m}_j - \tilde{m}_k] + q_k[\Delta y_j - \Delta y_j^n - (\Delta y_k - \Delta y_k^n)] + (q_j - q_k)(\Delta y_j - \Delta y_j^n) \\ - [\Delta p_j - \Delta p_k]\end{aligned} \qquad (2.24)$$

The structural margins or mark-ups are mark-ups on *normal* unit labour cost. Conventional mark-ups, m^c, are calculated as mark-ups on actual unit labour costs:

$$m_j^c = p_j - w - y_j \tag{2.25}$$

The change in the economy-wide average margin on actual until labour cost is therefore given by

$$\begin{aligned} \Delta m_j^c &= \Delta m_j + \Delta y_j - \Delta y_j^n \\ &= \Delta \tilde{m}_j + (1+q_j)(\Delta y_j - \Delta y_j^n) \end{aligned} \tag{2.26}$$

Conventionally calculated mark-ups are therefore more strongly pro-cyclical than the 'structural' mark-ups on normal unit labour cost.

(II.6) The monetary policy rule

I consider both a nominal interest rate rule and a rule for the growth rate of the nominal money stock.

(II.6a) A Taylor rule for the short nominal interest rate

The short nominal interest rate is the monetary instrument. For concreteness, assume a conventional Taylor rule. The target rate of inflation is denoted Δp^* :.

$$\begin{aligned} & i(t) = i^* + b[y(t) - y^n(t)] + g[\Delta p(t+1) - \Delta p^*] \\ & b \geq 0; g > 1 \end{aligned} \tag{2.27}$$

As will be shown below, we need $g > 1$ to ensure that the economy does not explode under out nominal interest rate rule. With $g > 1$ a higher rate of inflation raises the short nominal interest rate more than one-for-one. The short real rate of interest, $r = i - \Delta p$, therefore rises when the inflation rate increases and this will dampen demand and future inflation.

(II.6b) A constant growth rate rule for the nominal money stock

$$\Delta m = w^* \tag{2.28}$$

We could use a more activist monetary rule, in the spirit of McCallum, but the economy will be stable under fairly mild restrictions, even under a constant growth rate of the nominal money stock.

(III) Steady state

In steady state the inflation rate is constant and all stock-flow, flow-flow and stock-stock ratios are constant. Steady state values of variables are again denoted by *overbars*.

When the inflation rate is constant, the following must hold:

$$\bar{y} - \bar{y}^n = d^{-1}[\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}}] \tag{3.1}$$

Using (2.2). (2.4), (2.7), (2.8), (2.9) and (2.10), we obtain the following:

$$e + \bar{y}^p - h_1(\bar{i} - \Delta\bar{p}) - h_2(l_0 + l_1\bar{i}) = \bar{y}^n + d^{-1}(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}}) \tag{3.2}$$

Under the Taylor rule for the nominal interest rate, (2.27) we also have:

$$\bar{i} = i^* + bd^{-1}(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}}) + g\left(\Delta\bar{p} - \Delta p^*\right) \tag{3.3}$$

Under a constant growth rate for the nominal money stock we have instead:

$$\Delta\bar{p} = w^* - \Delta\bar{p} \tag{3.4}$$

(III.1) Steady-state analysis under a fixed Taylor rule

I first consider the case where all the coefficients of the Taylor rule are fixed. This implies that, when subjected to certain long-run shocks, the Taylor rule will not support an unchanged rate of inflation in steady state. Under the fixed Taylor rule, the steady state inflation rate, nominal interest rate, real interest rate and growth rate of nominal money are given by

$$\begin{aligned} \Delta\bar{p} = & -\frac{d^{-1}[1 + b(h_1 + h_2 l_1)]}{h_1(g-1) + h_2 l_1 g}\left(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}}\right) \\ & + \frac{1}{h_1(g-1) + h_2 l_1 g}\left(e + \bar{y}^p - \bar{y}^n - h_2 l_0\right) \\ & - \left(\frac{h_1 + h_2 l_1}{h_1(g-1) + h_2 l_1 g}\right)(i^* - g\Delta p^*) \end{aligned} \tag{3.5}$$

$$\bar{i} = -\left(\frac{d^{-1}\left(h_1 b + g\right)}{h_1\left(g-1\right) + h_2 l_1 g} \right)\left(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}}\right) + \frac{g}{h_1\left(g-1\right) + h_2 l_1 g}\left(e + \bar{y}^p - \bar{y}^n - h_2 l_0\right) - \frac{h_1}{h_1\left(g-1\right) + h_2 l_1 g}\left(i^* - g\Delta p^*\right) \quad (3.6)$$

$$\bar{r} = -\frac{d^{-1}\left(g - 1 - h_2 b l_1\right)}{h_1\left(g-1\right) + h_2 l_1 g}\left(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}}\right) + \left(\frac{g-1}{h_1\left(g-1\right) + h_2 l_1 g} \right)\left(e + \bar{y}^p - \bar{y}^n - h_2 l_0\right) - \frac{h_2 l_1}{h_1\left(g-1\right) + h_2 l_1 g}\left(i^* - g\Delta p^*\right) \quad (3.7)$$

$$\bar{y} = \bar{y}^n + d^{-1}\left(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}}\right) \quad (3.8)$$

$$\Delta\bar{m} = \Delta\bar{p} + \frac{d^{-1}\left[1 + \left(h_1 + h_2 l_1\right) b\right]}{h_1\left(g-1\right) + h_2 l_1 g}\left(\Delta\bar{\tilde{m}} + \Delta\bar{p}^w - \Delta\bar{p}\right) + \frac{1}{h_1\left(g-1\right) + h_2 l_1 g}\left(e + \bar{y}^p - \bar{y}^n - h_2 l_0\right) - \left(\frac{h_1 + h_2 l_1}{h_1\left(g-1\right) + h_2 l_1 g} \right)\left(i^* - g\Delta p^*\right) \quad (3.9)$$

The first thing to note is that, with a fixed Taylor rule, the steady state rate of inflation is affected only by changes in the steady-state values of the exogenous variables, the steady state growth rate of the economy-wide average margin, $\Delta\bar{\tilde{m}}$, the steady-state growth rate of economy-wide productivity, $\Delta\bar{p}$, the steady-state growth rate of the target real wage, $\Delta\bar{p}^w$, the steady-state level of the 'autonomous output gap', $e + \bar{y}^p - \bar{y}^n - h_2 l_0$, the target rate of inflation, Δp^*, and the exogenous intercept term in the Taylor rule, i^*. I maintain the assumption in what follows that the *level* of the steady state gap between permanent income and potential output, $\bar{y}^p - \bar{y}^n$, is not affected by any of the structural changes under considerations. Equation (2.10) only asserts that their steady-state growth rates are the same.

The growth rate of the economy-wide mark-up must be zero in steady state. Any positive growth rate would cause the mark-up to grow to infinite levels. Any negative growth rate would imply that the mark-up converges to zero. Therefore

$$\Delta\bar{\tilde{m}} = 0 \qquad (3.10)$$

The presentation of the results is simplified if we impose the restriction that there is no real balance effect on aggregate demand, that is, $h_2 = 0$. When this condition is satisfied, money is 'super-neutral' in the long run: higher steady-state monetary growth and associated higher inflation do not affect any steady-state real variable. I will make this assumption in what follows. This simplification does not affect any of the substantive conclusions.

With a fixed Taylor rule, a higher steady-state growth rate of economy-wide cyclically corrected productivity, without a matching increase in the growth rate of the target real wage, reduces the steady state inflation rate, raises the steady-state output gap and lowers the steady state real rate of interest. The steady-state nominal rate of interest falls. The effect on the steady-state growth rate of the nominal money stock is ambiguous.

With a fixed Taylor rule, a higher steady-state growth rate of the target real wage, without a matching increase in the steady-state growth rate of economy-wide cyclically corrected productivity, increases the steady state inflation rate, lowers the steady-state output gap and raises the steady state real rate of interest. The steady-state nominal rate of interest rises. The steady-state growth rate of the nominal money stock rises.

With a fixed Taylor rule, a higher steady-state growth rate of economy-wide cyclically corrected productivity, *with* a matching increase in the growth rate of the target real wage, leaves unchanged the steady state inflation rate, the steady-state output gap and the steady-state nominal and real interest rates. The steady-state growth rate of the nominal money stock increases by the same amount as steady-state productivity growth and steady-state target real wage growth.

Any transitory changes in the levels or growth rates of cyclically corrected productivity, target real wage growth and economy-wide margins have no effect on the steady-state rate of inflation, output gap, real and nominal interest rate and growth rate of the nominal money stock. Any relative price changes, transitory or permanent, that are not associated with permanent changes in the growth rate of economy-wide cyclically corrected productivity or in the target growth rate of real wages will not affect steady-state inflation, output gap, nominal and real interest rates or growth rate of the nominal money stock.

(III.2) Steady state analysis when the Taylor rule adjusts to support an unchanged inflation target in steady state.

For the policy rule to support the target rate of inflation in steady state, it must be the case that, in steady state,

$$i^* = \bar{r} + \Delta p^* + b(\bar{y} - \bar{y}^n) \qquad (3.11)$$

This implies that, for this simple model, the following restriction must be applied to the interest rate rule:

$$\begin{aligned} i^* &= \left(\frac{h_1}{h_1 + h_2 l_1}\right)\Delta p^* \\ &+ \left(\frac{1}{h_1 + h_2 l_1}\right)\left[e + \bar{y}^p - \bar{y}^n - h_2 l_0 - d^{-1}[1 + b(h_1 + h_2 l_1)](\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}})\right] \end{aligned} \tag{3.12}$$

When $h_2 = 0$, this simplifies to

$$i^* = \Delta p^* + \left(\frac{1}{h_1}\right)\left[e + \bar{y}^p - \bar{y}^n - d^{-1}(1 + bh_1)](\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}})\right] \tag{3.13}$$

In this simple linear model without uncertainty, the monetary authority could achieve the inflation target exactly each period, since there is no reason for them not to know the true model exactly and to observe perfectly all current and past values of the exogenous and endogenous variables. Under the Taylor rule this would involve period-by-period changes in i^*. Under the monetary rule, this would involve period-by-period changes in w^*. Since it is only the excessive simplicity of the model that permits this policy Nirvana, I will not work out this case. Instead I restrict the analysis to two policy rules that support the inflation target in steady state and achieve convergence to the steady state.

Assuming that the interest rate rule supports the inflation target in steady state, that is, assuming that i^* satisfies (3.12) or (3.13), we get

$$\Delta\bar{p} = \Delta p^* \tag{3.14}$$

$$\bar{i} = \left(\frac{1}{h_1 + h_2 l_1}\right)\left[h_1\Delta p^* + e + \bar{y}^p - \bar{y}^n - h_2 l_0 - d^{-1}(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}})\right] \tag{3.15}$$

when $h_2 = 0$ (1.38) simplifies to

$$\bar{i} = \Delta p^* + \left(\frac{1}{h_1}\right)\left[e + \bar{y}^p - \bar{y}^n - d^{-1}(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}})\right] \tag{3.16}$$

$$\bar{r} = \left(\frac{1}{h_1 + h_2 l_1}\right)\left[-h_2 l_1\Delta p^* + e + \bar{y}^p - \bar{y}^n - h_2 l_0 - d^{-1}(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}})\right] \tag{3.17}$$

$$\bar{y} = \bar{y}^n + d^{-1}(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}}) \tag{3.18}$$

$$\Delta\bar{m} = \Delta p^* + \Delta\bar{p} \qquad (3.19)$$

We again impose the condition that $\Delta\bar{\tilde{m}} = 0$: profit margins are constant in steady state.

Since the long-run inflation rate is, by construction, equal to its constant target value under the flexible Taylor rule, the interesting questions concern the effect of changes in the structure of the economy on the level of the nominal and real interest rates and on the growth rate of the nominal money stock that support this constant long-run rate of inflation.

With a flexible Taylor rule (that is, a Taylor rule whose intercept adjusts to support an unchanged rate of inflation in steady state), the steady state nominal and real interest rates, the steady-state output gap and the steady-state growth rate of the nominal money stock are affected only by changes in the steady-state values of the exogenous variables, the steady state growth rate of the economy-wide average margin, $\Delta\bar{\tilde{m}}$, the steady-state growth rate of economy-wide cyclically corrected productivity, $\Delta\bar{p}$, the steady-state growth rate of the target real wage, $\Delta\bar{p}^w$, the steady-state level of the 'autonomous output gap', $e + \bar{y}^p - \bar{y}^n - h_2 l_0$, and the target rate of inflation, Δp^*.

With a flexible Taylor rule, a hiker steady-state growth rate of economy-wide cyclically corrected productivity, without a matching increase in the growth rate of the target real wage, raises the steady-state output gap and lowers the steady state real rate of interest. The steady-state nominal rate of interest falls by the same amount as the steady-state real interest rate, as the inflation rate is constant. The steady-state growth rate of the nominal money stock increases by the same amount as the steady-state growth rate of economy-wide cyclically corrected productivity.

With a flexible Taylor rule, a higher steady-state growth rate of the target real wage, without a matching increase in the steady-state growth rate of economy-wide cyclically corrected productivity, lowers the steady-state output gap and raises the steady state real and nominal rates of interest by the same amount. The steady-state growth rate of the nominal money stock is unchanged.

With a flexible Taylor rule, a higher steady-state growth rate of economy-wide cyclically corrected productivity, *with* a matching increase in the growth rate of the target real wage, leaves unchanged the steady-state output gap and the steady-state nominal and real interest rates. The steady-state growth rate of the nominal money stock increases by the same amount as steady-state productivity growth and steady-state target real wage growth.

Any transitory changes in the levels or growth rates of cyclically corrected productivity, target real wage growth and economy-wide margins have no effect on the steady-state output gap, on steady-state real and nominal interest rate and on the steady-state growth rate of the nominal money stock. Any relative price changes, transitory or permanent, that are not associated with permanent changes in the growth rate of economy-wide cyclically adjusted productivity, or in the target growth rate of real wages, will not affect the steady-state output gap, nominal and real interest rates or growth rate of the nominal money stock.

(III.3) Steady state analysis under a fixed monetary growth rule

When the growth rate of the nominal money stock is exogenously fixed at w^*, the following steady-state conditions apply:

$$\Delta\bar{p} = w^* - \Delta\bar{p} \tag{3.20}$$

$$\begin{aligned}\bar{i} = &\frac{h_1}{h_1 + h_2 l_1}\left(w^* - \Delta\bar{p}\right) \\ &- \frac{d^{-1}}{h_1 + h_2 l_1}\left(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}}\right) \\ &+ \frac{1}{h_1 + h_2 l_1}\left(e + \bar{y}^p - \bar{y}^n - h_2 l_1\right)\end{aligned} \tag{3.21}$$

$$\begin{aligned}\bar{r} = &\frac{-h_2 l_1}{h_1 + h_2 l_1}\left(w^* - \Delta\bar{p}\right) \\ &- \frac{d^{-1}}{h_1 + h_2 l_1}\left(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}}\right) \\ &+ \frac{1}{h_1 + h_2 l_1}\left(e + \bar{y}^p - \bar{y}^n - h_2 l_1\right)\end{aligned} \tag{3.22}$$

$$\bar{y} = \bar{y}^n + d^{-1}(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}}) \tag{3.23}$$

$$\overline{m - p} = \bar{y} - l_0 - l_1\bar{i} \tag{3.24}$$

The effect of long-run structural change on the output gap is the same under the fixed monetary rule as under the fixed and flexible Taylor rules. When there is no real balance effect on aggregate demand, $h_2 = 0$, the effect of long-run structural change on the real interest rate is the same under the fixed monetary rule as under the fixed and flexible Taylor rules.

Under the fixed monetary growth rule, the only long-term structural change that affects the steady-state rate of inflation is a change in the long-run growth rate of economy-wide cyclically adjusted productivity. A higher long-run growth rate of cyclically adjusted productivity lowers long-run inflation one-for-one, regardless of whether there is a matching increase in the long-run growth rate of the target real wage. If only the long-run growth rate of cyclically corrected productivity increases, the long-run output gap increases and the long-run real interest falls. The long-run nominal interest falls by more, because the inflation rate also falls.

Under the fixed monetary growth rule, a higher long-run growth rate of cyclically adjusted productivity matched by an equal increase in the long-run growth rate of the target real wage, lowers the nominal interest rate. If there is no real balance effect, the nominal interest rate is reduced one-for-one, the same as the growth rate of the nominal money stock. The real interest rate and the output gap are unchanged.

Under the fixed monetary growth rule, a higher long-run growth rate of the target real wage has no effect on the long-run rate of inflation. The output gap falls and the real interest rate rises. With the inflation rate constant, the nominal interest rate increases by the same amount as the real interest rate.

Any transitory aggregate structural changes do not affect any steady –state endogenous variables. Relative price changes, transitory or permanent, that are not associated with changes in the long-run growth rate of economy-wide cyclically corrected productivity, or in the long-run growth rate of the target real wage, likewise have no effect on any of the long-run endogenous variables.

Note, from (3.24) that even a permanent change in the level of the mark-up will not have any lasting effect on the general price level. With real output and the nominal interest rate unchanged in steady state, an unchanged path of the nominal money stock implies an unchanged steady-state path for the price level.

(III.4) Steady state analysis when the monetary growth rate adjusts to support an unchanged inflation target in steady state.

For the monetary growth rule to support the target rate of inflation in steady state, the following must be true:

$$w^* = \Delta p^* + \Delta\overline{p} \tag{3.25}$$

The remaining steady-state conditions are:

$$\Delta\overline{p} = \Delta p^* \tag{3.26}$$

$$\bar{i} = \frac{h_1}{h_1 + h_2 l_1} \Delta p^* - \frac{d^{-1}}{h_1 + h_2 l_1}\left(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\bar{m}}\right) + \frac{1}{h_1 + h_2 l_1}\left(e + \bar{y}^p - \bar{y}^n - h_2 l_1\right) \tag{3.27}$$

(3.28)

$$\bar{r} = \frac{-h_2 l_1}{h_1 + h_2 l_1} \Delta p^* - \frac{d^{-1}}{h_1 + h_2 l_1}\left(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\bar{m}}\right) + \frac{1}{h_1 + h_2 l_1}\left(e + \bar{y}^p - \bar{y}^n - h_2 l_1\right) \tag{3.29}$$

$$\bar{y} = \bar{y}^n + d^{-1}(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\bar{m}}) \tag{3.30}$$

It follows from (3.24), that there is only one 'supply-side' or structural change that alters the growth rate of the nominal money stock consistent with an unchanged target rate of inflation in steady state. This is a permanent change in the growth rate of economy-wide cyclically corrected productivity, $\Delta\bar{p}$. The required change in the steady-state growth rate of the nominal money stock equals the change in the steady state growth rate of economy-wide cyclically corrected productivity. This long-run effect of higher economy-wide cyclically corrected productivity growth on the steady-state growth rate of the nominal money stock consistent with a constant rate of inflation is the same regardless of whether the increase in cyclically corrected productivity growth is matched by an increase in the trend rate of growth of real wages.

A higher growth rate of economy-wide cyclically adjusted productivity growth will, if it is matched by an equal increase in the trend growth rate of real wages, leave the output gap and the real interest rate unchanged. This results also holds under the fixed monetary growth rule and under the two Taylor rules. Since the long-run inflation rate is unchanged, the long-run nominal interest rate is also unchanged.

If the increase in the economy-wide growth rate of cyclically corrected productivity is not matched by an equal increase in the growth rate of the target real wage, the steady-state output gap, increases. In this case the long-run real interest rate will fall. So will the long-run nominal interest rate, and by the same amount.

Any transitory aggregate structural changes, and any relative price changes, transitory or permanent, not associated with a change in the growth rate of economy-wide cyclically corrected productivity or with a change in the growth rate of the target real wage do not affect the long-run growth rate of money, the long-run output gap or the long-run nominal and real rates of interest.

(IV) Transitional dynamics and steady-state effects on the price level and nominal money stock

(IV.1) Transitional dynamics with a fixed Taylor rule

The effect on the steady state price *level* path of changes in margins, productivity growth or whatnot, cannot be derived from the steady state conditions of the model alone. The nominal price level sequence, the money wage sequence and the nominal money stock sequence are all hysteretic or path-dependent. The behaviour of the inflation sequence, the price level sequence and the nominal money stock sequence can, however, be obtained easily period-by-period, given initial conditions for, say, $p(0)$ and $p(-1)$, and for $\bar{y}^n(0) - y^n(0)$, by talking one's way though equations (4.1), (4.2), (4.3) and (4.4).

$$\Delta p(+1) = \Delta\tilde{m} + \Delta p^w - \Delta p + d[y - y^n] + q[\Delta y - \Delta y^n] + \Delta p \qquad (4.1)$$

$$y = e + y^p - h_2 l_0 - (h_1 + h_2 l_1)\, i + h_1 \Delta p(+1) \qquad (4.2)$$

$$i = i^* + b[y - y^n] + g[\Delta p(+1) - \Delta p^*] \qquad (4.3)$$

$$m - p = y - l_0 - l_1 i \qquad (4.4)$$

Under the fixed Taylor rule, the inflation rate evolves as follows:

$$\begin{aligned} \Delta p(+1) = {} & \Gamma_1 \Delta p \\ & -\Gamma_2[\Delta p - \Delta p^w - \Delta\tilde{m}] \\ & +\Gamma_3 \begin{bmatrix} 1 & -(h_1 + h_2 l_1) & d^{-1}q \end{bmatrix} \begin{bmatrix} e + y^p - y^n - h_2 l_0 \\ i^* - g\Delta p^* \\ \Delta y^p - \Delta p \end{bmatrix} \end{aligned} \qquad (4.5)$$

where

$$\Gamma_1 = \Phi^{-1}\left([1 + b(h_1 + h_2 l_1) + q[h_1(g - 1) + h_2 l_1 g]\right) \qquad (4.6)$$

$$\Gamma_2 = \Phi^{-1}\left[1 + b(h_1 + h_2 l_1)\right] \qquad (4.7)$$

$$\Gamma_3 = \Phi^{-1} d \qquad (4.8)$$

$$\Phi = 1 + b\,(h_1 + h_2\,l_1) + \left(d + q\right)[h_1(g-1) + h_2\,l_1 g\,] > 0 \tag{4.9}$$

The reduced-form expressions for output and the nominal and real interest rate are

$$\begin{aligned}
y - y^n = & -\left(\frac{[h_1(g-1) + h_2\,l_1 g\,]\Gamma_1}{1 + b\,(h_1 + h_2\,l_1)}\right)\Delta p \\
& + \left(\frac{[h_1(g-1) + h_2\,l_1 g\,]\Gamma_2}{1 + b\,(h_1 + h_2\,l_1)}\right)(\Delta p - \Delta p^{\,w} - \Delta\tilde{m}) \\
& + \left(\frac{1 - [h_1(g-1) + h_2\,l_1 g\,]\Gamma_3}{1 + b\,(h_1 + h_2\,l_1)}\right)(e + y^p - y^n - h_2\,l_0) \\
& + \left(\frac{\left(h_1 + h_2\,l_1\right)\left(1 - [h_1(g-1) + h_2\,l_1 g\,]\Gamma_3\right)}{1 + b\,(h_1 + h_2\,l_1)}\right)(i^* - g\Delta p^*) \\
& - \left(\frac{[h_1(g-1) + h_2\,l_1 g]q}{\left[1 + b\,(h_1 + h_2\,l_1)\right]\Phi}\right)(\Delta y^p - \Delta p\,)
\end{aligned} \tag{4.10}$$

and

$$\begin{aligned}
i = & \left(\frac{(\,bh_1 + g\,)\Gamma_1}{1 + b\,(h_1 + h_2\,l_1)}\right)\Delta p \\
& - \left(\frac{(\,bh_1 + g\,)\Gamma_2}{1 + b\,(h_1 + h_2\,l_1)}\right)(\Delta p - \Delta p^{\,w} - \Delta\tilde{m}) \\
& + \left(\frac{b + (\,bh_1 + g\,)\Gamma_3}{1 + b\,(h_1 + h_2\,l_1)}\right)(e + y^p - y^n - h_2\,l_0) \\
& + \left(\frac{1 - (\,bh_1 + g\,)(h_1 + h_2\,l_1)\Gamma_3}{1 + b\,(h_1 + h_2\,l_1)}\right)(i^* - g\Delta p^*) \\
& + \frac{q\,(\,bh_1 + g\,)}{\left(1 + b\,(h_1 + h_2\,l_1)\right)\Phi}(\Delta y^p - \Delta p\,)
\end{aligned} \tag{4.11}$$

$$
\begin{aligned}
r = & \left(\frac{(g - 1 - h_2 b\, l_1)\Gamma_1}{1 + b\,(h_1 + h_2\, l_1)} \right) \Delta p \\
& - \left(\frac{(g - 1 - h_2 b\, l_1)\Gamma_2}{1 + b\,(h_1 + h_2\, l_1)} \right) (\Delta p - \Delta p^{w} - \Delta \tilde{m}) \\
& + \left(\frac{b + (g - 1 - h_2\, l_1)\Gamma_3}{1 + b\,(h_1 + h_2\, l_1)} \right) (e + y^{p} - y^{n} - h_2\, l_0) \\
& + \left(\frac{1 - (h_1 + h_2\, l_1)(g - 1 - h_2\, l_1)\Gamma_3}{1 + b\,(h_1 + h_2\, l_1)} \right) (i^{*} - g \Delta p^{*}) \\
& + \frac{q\,(g - 1 - h_2 b\, l_1)}{[1 + b\,(h_1 + h_2\, l_1)]\Phi} \left(\Delta y^{p} - \Delta p \right)
\end{aligned}
\tag{4.12}
$$

The endogenous rate of growth of the nominal money stock is governed by

$$\Delta m = \Delta p + \Delta y - l_1 \Delta i \tag{4.13}$$

Because of the Taylor rule ($g > 1$), this inflation process is stationary, since $0 < \Gamma_1 < 1$. Also, $0 < \Gamma_2 < 1$, so the effect on the price level of a one-period increase in the growth rate of margins, the trend real wage or trend productivity (that is, a permanent increase in the level of the mark-up, the level of the trend real wage or the level of trend productivity), will build up gradually.

In principle, we could consider the dynamic responses to the following shocks.

(1) A permanent increase in the *level* of economy-wide cyclically corrected productivity, that is, a one-period increase in Δp .

(2) A temporary increase in the *level* of economy-wide cyclically corrected productivity, that is, a one-period increase in Δp , followed the next period by an equal decrease in Δp .

(3) A permanent increase in the *growth rate* of economy-wide cyclically adjusted productivity, that is, a permanent increase in Δp .

(4) A permanent increase in the *level* of the target real wage, that is, a one-period increase in Δp^{w} .

(5) A temporary increase in the *level* of the target real wage, that is, that is, a one-period increase in Δp^{w} , followed the next period by an equal decrease in Δp^{w} .

(6) A permanent increase in the *growth rate* of the target real wage, that is, a permanent increase in Δp^{w} .

(7) A permanent increase in the *level* of the average, economy-wide mark-up, that is, a one-period increase in $\Delta \tilde{m}$.

(8) A temporary increase in the *level* of the average economy-wide mark-up, that is, that is, a one-period increase in $\Delta \tilde{m}$, followed the next period by an equal decrease in $\Delta \tilde{m}$.

An exhaustive taxonomy would, however, be both tedious and time-consuming. Instead I will focus on a few representative examples.

Note that productivity growth affects price behaviour through three channels. First, given nominal earnings, a higher level of productivity lowers unit labour costs. Given the mark-up, this means a lower price level. Second, higher productivity raises potential output. Other things being equal, this lowers the output gap. Third, to the extent that the increase in productivity raises permanent income, y^p, it will boost aggregate demand and raise the output gap. The 'permanent income' effect on aggregate demand is probably best interpreted as operating through private consumption. If higher productivity (current or future) were to boost domestic capital formation, at least temporarily, there could be further effects on aggregate demand. Formally, out model could capture this through an effect of productivity on e, the exogenous component of aggregate demand, but this possibility is not investigated further in what follows.

A permanent increase in the level of economy-wide cyclically-corrected productivity has a temporary negative effect on the inflation rate and permanently lowers the path of the price level. The decline of the price level is gradual. The effect of productivity shocks on the price level comes in principle from two sources. First, the direct effect of higher productivity on unit costs. Second, the effect of the productivity shock on the output gap. In this case, the increase in permanent income, and therefore in demand, is the same as the increase in potential output, and the second channel is not present.

A temporary increase in the level of economy-wide cyclically-corrected productivity has no long-run effect either on the rate of inflation or on the price level. In period 1, the price level falls, through the reduction in unit costs and because potential ouput has increased by more than permanent income, which is unchanged. The price level therefore falls more initially than when the productivity improvement is permanent. In period 2, the productivity increase is reversed and the price level returns gradually to its initial level.

A permanent increase in the growth rate of productivity, without a matching increase in the growth rate of the target real wage raises the long-run equilibrium output gap (lowers the natural rate of unemployment) and, with a fixed Taylor rule, permanently lowers the rate of inflation. The approach to the lower inflation equilibrium is gradual and monotone.

A permanent increase in the growth rate of productivity matched by an equal increase in the target growth rate of real wages has no effect on inflation or on anything else (output gap, nominal or real interest rates).

A permanent reduction in the level of the mark-up has the same effect on the rate of inflation and on the price level as a permanent increase in the level of productivity: the price level falls gradually to a lower path and the long-run rate of inflation is unaffected.

A temporary reduction in the level of the mark-up temporarily lowers the price level. In the long run the price level returns to its original path. Note that the effect on the price level is not identical to that brought about by a temporary increase in the level of productivity. This is because the margin change, by assumption, does not affect either potential supply or aggregate demand (through permanent income).

(IV.2) Transitional dynamics when the Taylor rule adjusts to support a constant target rate of inflation in steady state

When the Taylor rule is constrained to support the target rate of inflation in steady state, the equations of motion are given by (4.14).

$$
\begin{aligned}
\Delta p(+1) = {} & \Gamma_1 \Delta p \\
& -\Phi^{-1} b (h_1 + h_2 l_1)\Big[\big(\Delta p - \Delta\bar{p}\big) - \big(\Delta p^{w} - \Delta\bar{p}^{w}\big) - \big(\Delta\tilde{m} - \Delta\bar{\tilde{m}}\big)\Big] \\
& +\Phi^{-1} d\,[y^{p} - \bar{y}^{p} - (y^{n} - \bar{y}^{n})] \\
& +\Phi^{-1} d\,\big[h_1(g-1) + h_2 l_1 g\big]\Delta p^{*} \\
& +\Phi^{-1} q\,(\Delta y^{p} - \Delta p)
\end{aligned}
\tag{4.14}
$$

$$
\begin{aligned}
i = {} & \frac{(bh_1 + g)\Gamma_1}{1 + b(h_1 + h_2 l_1)}\Delta p \\
& -\left(\frac{h_1(g-1) + h_2 g\, l_1}{h_1 + h_2 l_1}\right)\left(\frac{1 - (bh_1 + g)(h_1 + h_2 l_1)\Gamma_3}{1 + b(h_1 + h_2 l_1)}\right)\Delta p^{*} \\
& +\left(\frac{b + (bh_1 + g)\Gamma_3}{1 + b(h_1 + h_2 l_1)}\right)[y^{p} - \bar{y}^{p} - (y^{n} - \bar{y}^{n})] \\
& +\frac{1}{h_1 + h_2 l_1}(e + \bar{y}^{p} - \bar{y}^{n} - h_2 l_0) \\
& +d^{-1}(bh_1 + g)\Gamma_3[\Delta\bar{p} - \Delta p - (\Delta\bar{p}^{w} - \Delta p^{w}) - (\Delta\bar{\tilde{m}} - \Delta\tilde{m})] \\
& -\frac{d^{-1}}{h_1 + h_2 l_1}(\Delta\bar{p} - \Delta\bar{p}^{w} - \Delta\bar{\tilde{m}}) \\
& +\frac{q(bh_1 + g)}{[1 + b(h_1 + h_2 l_1)]\Phi}(\Delta y^{p} - \Delta p)
\end{aligned}
\tag{4.15}
$$

With a flexible Taylor rule, which successfully targets an unchanged long-run rate of inflation, only deviations of the growth rates of the exogenous variables from their steady-state values will have any effect on the rate of inflation, short-run or long-run. The constant term in the nominal interest rate equation adjusts immediately to support the unchanged target rate of inflation in the long run, as can be seen from (3.12) and (4.15).

With a permanent increase in the level of productivity, for instance, the price level will fall on impact because the constant term in the Taylor rule does not adjust as there is no long-run change in the growth rate of productivity. With a permanent increase in the growth rate of productivity, unmatched by an increase in the growth rate of the target real wage, the nominal interest rate falls and inflation remains unchanged. With a permanent increase in the growth rate of productivity that is matched by an equal increase in the growth rate of the target real wage, neither the rate of inflation nor the short nominal interest rate change, short-run or long-run.

A permanent fall in the level of the will have a temporary negative effect on the inflation rate and a permanent negative effect on the price level. The constant term in the Taylor rule again does not change, because there is no change in the steady-state growth rate of margins (which is zero, by assumption).

A permanent reduction in the average margin (modeled by a one period increase in $\Delta\tilde{m}$), will have an initial negative effect on the aggregate price level, but less than proportional. Through the nominal persistence built into the model, the long-run effect on the price level path will be equal to the one-off change in m. The same holds for a permanent increase in the level of productivity, unmatched by an equal increase in the level of the target real wage.

A temporary reduction in the average margin (say a one period reduction in m followed the next period by an equal increase in m, will have no long-run effect on the price level.

A permanent increase in the growth rate of average productivity growth matched by an equal increase in the growth rate of the target real wage will have no effect on the path of prices and inflation. Nor will it have any effect on the path of nominal interest rates.

Relative price changes that are not associated with economy-wide margin changes or economy-wide changes in productivity will have no effect on the price level or the rate of inflation.

(IV.3) Transitional dynamics with a constant growth rate of the nominal money stock

Let the logarithm of the stock of real money balances as a fraction of potential GDP be denoted ℓ, that is,

$$\ell = m - p - y^n \tag{4.16}$$

The transitional dynamics are governed by the following simultaneous system of 2 first-order linear difference equations:

$$\begin{bmatrix}\Delta p(+1)\\ \ell\end{bmatrix} = \Omega^{-1}\begin{bmatrix} h_1 + l_1(1+h_2) - q\,l_1 h_1 & d(h_1 + h_2 l_1) \\ -\left[h_1 + l_1(1+h_2) - q\,l_1 h_1\right] & \begin{Bmatrix}(1+q)(h_1 + h_2 l_1) \\ + l_1 - (d+q)\,l_1 h_1\end{Bmatrix}\end{bmatrix}\begin{bmatrix}\Delta p \\ \ell(-1)\end{bmatrix}$$
$$+\Omega^{-1}\begin{bmatrix}-\left[h_1 + l_1(1+h_2)\right] & d\,l_1 \\ h_1 + l_1(1+h_2) & -d\,l_1\end{bmatrix}\begin{bmatrix}\Delta p - \Delta p^{w} - \Delta\tilde{m} \\ e + y^{p} - y^{n} + h_1 l_1^{-1} l_0\end{bmatrix} \qquad (4.17)$$
$$+\Omega^{-1}\begin{bmatrix}(d+q)(h_1 + h_2 l_1) & (d+q)(h_1 + h_2 l_1) + q\,l_1 \\ \begin{Bmatrix}h_1 + l_1(1+h_2) \\ -(d+q)\,l_1 h_1\end{Bmatrix} & \begin{Bmatrix}h_1 + l_1(1+h_2) \\ -(d+q)\,l_1 h_1 - q\,l_1\end{Bmatrix}\end{bmatrix}\begin{bmatrix}w^{*} - \Delta\bar{p} \\ \Delta y^{p} - \Delta p\end{bmatrix}$$

where

$$\Omega = h_1 + h_2 l_1 + l_1 + (d+q)[h_1(1-l_1) + h_2 l_1] \qquad (4.18)$$

The system will be stable *iff* the following two conditions are satisfied:

$$\Omega^{-1}[(d+q)(h_1 + h_2 l_1) - d\,h_1 l_1] > 0 \qquad (4.19)$$

$$(1+d+q)(h_1 + h_2 l_1) - (d+q)h_1 l_1 > 0 \qquad (4.20)$$

The first of these two stability conditions means that a higher rate of inflation during the previous period does not increase current period inflation more than one-for one. Holding current output constant, higher past inflation increases current inflation one-for-one, a reflection of the simple accelerationist Phillips curve we are using. For current inflation to rise less than one-for-one when last period's inflation rises, higher past inflation must reduce current output. Through the output gap, $d(y - y^{n})$ and through the speed limit effect, $q(\Delta y - \Delta y^{n})$, this will lower current period inflation.

With a constant growth rate of the nominal money stock, higher past inflation has two opposing effects on current output. These effects are more easily understood if one assumes that higher past inflation raises current inflation. In that case, higher past inflation reduces the real stock of money balances. This will cause the nominal interest rate to rise and may further depress demand if there is a real balance effect on aggregate demand. This money stock channel, or Pigou channel (which works through the higher price *level* associated with a higher rate of inflation) is counteracted by the (expected) inflation channel, or Wicksell channel, which boosts demand because, at a given nominal interest rate, the real interest rate is lower when inflation is higher. For the economy to be stable, it is necessary that the Pigou channel dominates the Wicksell channel.

If the stability conditions are satisfied, the convergence to steady state can be either monotone or cyclical. For concreteness, I assume in what follows that $\Omega > 0$.

A permanent reduction in the level of the mark-up will have a temporary negative effect on the price level and the rate of inflation. In the long run, both the price level and the rate of inflation return to their initial levels. The reason the price level will return to its initial level is that the path of the nominal money stock is unchanged. The lower price level associated initially with a lower mark-up means a higher stock of real money balances. This lowers the nominal interest rate and the short real interest rate. Along with a possible real balance effect, this boosts spending and brings the price level back to its level prior to the margin cut.

A permanent increase in the growth rate of economy-wide productivity, matched by an equal increase in the target growth rate of real wages, will gradually reduce the rate of inflation. The steady-state reduction in the rate of inflation equals the increase in productivity growth. By assumption, there is no downward 'cost-push' pressure on inflation: productivity growth and target real wage growth go up by the same amount. Neither is there any direct effect of the higher productivity growth on the output gap. By assumption, the growth rate of permanent income, and thus of demand, goes up by the same amount as the increase in the permanent growth rate of productivity. The reason inflation comes down is that the higher growth rate of actual and potential output raises the growth rate of the demand for real money balances. With an unchanged growth rate of the nominal money stock, there is downward pressure on nominal and real interest rates. This reduces demand, brings down the rate of inflation and generates the desired increase in the growth rate of the real money stock.

Any changes in relative prices that are not associated with a change in economy-wide margins, productivity or target real wage growth will not have any effect on the price level or the rate of inflation.

(IV.4) Transitional dynamics when the growth rate of the nominal money stock adjusts to support a constant inflation target in the long run

When the growth rate of the nominal money stock adjusts to support the inflation target in steady state, the dynamics of the economy are as follows:

$$\begin{aligned}\begin{bmatrix}\Delta p(+1)\\ \ell\end{bmatrix} &= \Omega^{-1}\begin{bmatrix} h_1 + l_1(1+h_2) - q\,l_1 h_1 & d\,(h_1 + h_2 l_1) \\ -\left[h_1 + l_1(1+h_2) - q\,l_1 h_1\right] & \begin{Bmatrix}(1+q)(h_1 + h_2 l_1)\\ + l_1 - (d+q)\,l_1 h_1\end{Bmatrix}\end{bmatrix}\begin{bmatrix}\Delta p\\ \ell(-1)\end{bmatrix} \\ &+ \Omega^{-1}\begin{bmatrix} -\left[h_1 + l_1(1+h_2)\right] & d\,l_1 \\ h_1 + l_1(1+h_2) & -d\,l_1\end{bmatrix}\begin{bmatrix}\Delta p - \Delta p^{w} - \Delta\tilde{m}\\ e + y^{p} - y^{n} + h_1 l_1^{-1} l_0\end{bmatrix} \\ &+ \Omega^{-1}\begin{bmatrix}(d+q)(h_1 + h_2 l_1) & (d+q)(h_1 + h_2 l_1) + q\,l_1 \\ \begin{Bmatrix}h_1 + l_1(1+h_2)\\ -(d+q)\,l_1 h_1\end{Bmatrix} & \begin{Bmatrix}h_1 + l_1(1+h_2)\\ -(d+q)\,l_1 h_1 - q\,l_1\end{Bmatrix}\end{bmatrix}\begin{bmatrix}\Delta p^{*}\\ \Delta y^{p} - \Delta p\end{bmatrix}\end{aligned} \qquad (4.21)$$

$$w^* = \Delta p^* + \Delta \overline{p} \quad (4.22)$$

The only point worth making about this case is that, when the economy is hit by a permanent increase in the growth rate of economy-wide productivity (and a matching increase in the growth rate of the target real wage), there now is no effect on the rate of inflation. Instead, the growth rate of the nominal money stock increases by the exact amount of the increase in the productivity growth rate.

(V) An open economy extension

I briefly consider an open economy extension of the model. The purpose is not to go exhaustively through the whole list of issues considered in the earlier sections. Instead I will focus on the long-run implications for the real exchange rate of supply-side improvements in a small open economy.

When the economy is open to trade in real goods and services and financial claims, the story does not change much.

The mark-up pricing equation (2.12) still applies, but only to the GDP deflator, p^v.

$$p_j^v = m_j + w - y_j^n \quad (5.1)$$

The price index for 'domestically produced goods', p^d, depends on the prices of imported intermediate and raw materials inputs used in their production, that is,

$$\begin{aligned} p^d &= u_1 p^v + (1-u_1)(s + p^f) \\ 0 &\leq u_1 \leq 1 \end{aligned} \quad (5.2)$$

Here u_1 is the share of import cost in total unit variable cost, *s* is the nominal spot exchange rate. For simplicity, productivity changes in the use of intermediate and raw materials imports are ignored. p^f is the exogenous world price of intermediate and raw materials goods (in foreign currency).

The consumer price index is a weighted average of the price of'domestically produced' goods and imported consumer goods and services.

$$p = u_2 p^d + (1-u_2)(s + p^f) \quad (5.3)$$

Here u_2 is the weight of domestically produced goods in the domestic consumption bundle and we assume for simplicity that the price of imported consumption goods is the same as the price of imported intermediates and raw materials.

Let the real exchange rate, c , be defined as the ratio of the foreign to the domestic price level, expressed in a common currency, that is,

$$c = s + p^f - p \tag{5.4}$$

We can write the open-economy price Phillips curve as follows,

$$\begin{aligned} \Delta p(+1) &= \Delta p + \Delta\tilde{m} + \Delta p^{w} - \Delta p + d\,(y - y^n) + q\,(\Delta y - \Delta y^n) \\ &+ \left(\frac{1 - u_1 u_2}{u_1 u_2} \right) \Delta c(+1) \end{aligned} \tag{5.5}$$

This open economy price-Phillips curve can be contrasted with the closed economy version in (2.23).

Aggregate demand now also depends (positively) on the real exchange rate, $c = s + p^f - p$, and on world demand, y_f^p. World demand is taken to be exogenous. For brevity's sake, I omit the real balance effect on aggregate demand, that is, I assume $h_2 = 0$.

$$\begin{gathered} y = h_0 - h_1 r + h_3 c \\ h_3 > 0 \end{gathered} \tag{5.6}$$

$$\begin{gathered} h_0 = e + s\, y^p + (1 - s\,)\, y_f^p \\ 0 < s < 1 \end{gathered} \tag{5.7}$$

Equation (5.7) builds a 'home bias' into aggregate demand: a given proportional increase in domestic permanent income will lead to a smaller proportional increase in demand for domestic goods.[1] Equal proportional increases in domestic permanent income and in world permanent income will raise the demand for domestic goods by the same proportion.

There is an efficient international financial market. The domestic short nominal interest rate is linked to the exogenous short foreign nominal rate of interest, i^f, through uncovered interest parity (UIP) and exogenous risk premia, r

$$i(t) = i^f(t) + s(t+1) - s(t) + r(t) \tag{5.8}$$

The rest of the model is unchanged from the closed economy case. For reasons of space, I will only consider the fixed Taylor rule.

[1] See Buiter [1989].

(V.1) Steady state

There can only be a steady state if the long-run growth rate of world potential output is the same as the long-run growth rate of domestic potential output. I assume that this condition is satisfied in what follows.

$$\bar{y} = \bar{y}^n + d^{-1}(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}}) \tag{5.9}$$

$$\bar{r} = \bar{i}^f + \bar{r} - \Delta\bar{p}^f \tag{5.10}$$

$$\Delta\bar{p} = \frac{1}{1-g}\left[i^* - g\Delta p^* - (\bar{i}^f + \bar{r} - \Delta\bar{p}^f) + bd^{-1}\left(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}}\right)\right] \tag{5.11}$$

$$\bar{i} = \frac{1}{1-g}\left[i^* - g\Delta p^* - g(\bar{i}^f + \bar{r} - \Delta\bar{p}^f) + bd^{-1}\left(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}}\right)\right] \tag{5.12}$$

$$c = \frac{1}{h_3}\left\{\bar{y}^n - \bar{y}^p + (1-s)(\bar{y}^p - \bar{y}^p_f) - e + d^{-1}(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}}) + h_1(\bar{i}^f + \bar{r} - \Delta\bar{p}^f)\right\} \tag{5.13}$$

Consider the effect on the long-run real exchange rate of a favourable supply shock in the home country. We can model this most simply by considering a reduction in the growth rate of the target real wage, $\bar{p}^w$. This will lover the natural rate of unemployment, proxied in our simple model by the output gap, $\bar{y} - \bar{y}^n = d^{-1}(\Delta\bar{p} - \Delta\bar{p}^w - \Delta\bar{\tilde{m}})$, which rises when $\bar{p}^w$ falls. Domestic output goes up. We assume this represents increased income to domestic residents (the assets producing GDP are owned by residents) and that $\bar{y}^p$ rises in line with $\bar{y}^n$. Domestic residents spend more both on domestic goods and on imports. Since only part of the increase in domestic income is spent on domestic goods, there is not excess supply of domestic output at any given real interest rate and real exchange rate. In the long run, the domestic real interest rate is equal to the risk-adjusted foreign interest rate, $\bar{r}^f = \bar{i}^f + \bar{r} - \Delta\bar{p}^f$. Equilibrium in the world market for domestic goods therefore requires a decline in the relative price of domestic goods, that is, a real depreciation or an increase in c.

We cannot analyze the steady-state effects of an increase in domestic productivity growth, $\Delta\bar{p}$, because no non-trivial steady state exists unless the rest of the world grows at the same rate as the home country, that is unless $\Delta\bar{p} = \Delta\bar{p}^f$. We can, however, consider the effect of a permanent increase in the *level* of home country productivity relative to that in the rest of the world. In equation (5.13), that would be represented by an increase in $\bar{y}^p - \bar{y}^p_f$. We again assume that domestic permanent income rises in line with potential steady-state output. Not surprisingly, this increase in the relative supply of domestic and foreign goods also requires a long-run real depreciation, that is, an increase in c.

In the short-run, a positive supply shock need not be associated with a depreciation of the real exchange rate. In the short run, output can deviate from its natural level, and the real interest rate can depart from the (risk-adjusted) world real interest rate if the real exchange rate is not expected to remain constant since

$$r(t) = r^f(t) + \Delta c(t+1) \qquad (5.14)$$

In the short run, a supply-side improvement could well boost aggregate demand by more than it boosts potential output. In the model under consideration, this would be the case, for instance, if the economy were hit by news about future improvements in productivity or a future lowering of the natural rate of unemployment. Current aggregate demand, from (5.5) is given by

$$y = e + s\, y^p + (1-s)\, y_f^p - h_1 r + h_3 c$$

This depends on our proxy for permanent income, $\bar{y}^p$, and could well rise by more in the short run than current capacity output, y^n. Indeed, if the productivity shock improves the rate of return to capital formation, investment could be boosted in the short run, before the anticipated future supply-side improvements have had time to materialise. In our model, this could be represented by e, autonomous aggregate demand, being made a function of anticipated future productivity levels.

It is sometimes argued with reference to the UK economy and the recent strength of sterling, that supply-side improvements in the UK relative to its major trading partners may have strengthened the equilibrium real exchange rate. It is clear that the model considered here does not support this proposition in the long run. A positive supply-side shock is associated with a lower relative price of domestic to foreign goods, that is, with a real depreciation In the short run, of course, current or anticipated future supply side improvements may boost aggregate demand by more than aggregate supply, making for a short-run real appreciation.

One possible interpretation of the proposition that supply-side improvements may be associated with a stronger long-run real exchange rate involves unrecorded quality improvements or changes in product mix. If, say, the UK had shifted the composition of its exportable goods towards higher-quality and more highly priced goods, the conventional relative price indices could overstate the true, quality-adjusted, relative price of UK goods, because these indices do not fully recognise the quality improvements that have taken place. It could then be the case that the ‘true’, hedonic price indices would show a fall in the relative price of (quality-adjusted) UK goods, while the official indices record a relative price increase.

Finally, note that even a permanent change in the level of the mark-up does not affect competitiveness in the long run. This is because it affects neither the long-run real interest rate nor the long-run output gap.

(VI) Conclusion

It is impossible to make informative statements about the likely effects of structural changes or supply-side shocks on the price level or the rate of inflation, without recourse to a dynamic model that explicitly incorporates both the real and the nominal aspects of the transmission mechanism. A cost-plus (or cost-minus) approach to the inflationary impact of, say, productivity shocks, margin shocks or natural rate shocks is bound to fail, because such real shocks are in principle consistent with a wide variety of nominal outcomes. Their effect on the price level and the rate of inflation depend on the details of the monetary policy rule, even in steady state. For instance, a permanent increase in the growth rate of economy-wide productivity will, under plausible conditions, have no effect on the path of nominal interest rates that supports a given inflation target. Equivalently, for a given Taylor rule, such an increase in productivity growth will have no effect on the rate of inflation. However, the growth rate of the nominal money stock that supports an unchanged inflation target will rise one-for-one with the growth rate of productivity.

The dynamic response in real time of the price level and the rate of inflation also depend on the details of the monetary transmission mechanism, including the sources of short-run nominal rigidities in price and wage setting behaviour.

Relative price changes and other microeconomic shocks that do not affect average or economy-wide margins, economy-wide productivity growth or the natural rate of unemployment, will have no effect on inflation (for given monetary policy rules) or on the paths of the monetary instruments that support an unchanged inflation target. Inflation is a monetary and real phenomenon.

References

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