Helicopter Money
Irredeemable fiat money and the liquidity trap
Or: is money net wealth after all?

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Abstract

The paper provides a formalisation of the monetary economics folk proposition that government fiat money is an asset of the holder (the private sector) but not a liability of the issuer (the state). Money is 'net wealth' in the limited sense that, after consolidation of the intertemporal budget constraints of the private and public sectors, the present value of the terminal money stock remains a component of comprehensive household wealth. The issuance of irredeemable fiat money can therefore affect consumption demand through a weak 'real balance or Pigou effect', if it can alter the present value of the terminal stock of money. With irredeemable fiat money, weak restrictions on the monetary policy rule suffice to rule out liquidity trap equilibria (that is, equilibria in which all current and future short nominal interest rates are at their lower bound) that are also rational expectations equilibria. Liquidity trap equilibria that are not (long run) rational expectations equilibria can exist if and for as long as the private sector has incorrect but irrefutable expectations that the monetary authorities will ultimately reverse, in present value, any current expansion of the monetary base. If 'quantitative easing' is never reversed, in present value terms, and never expected to be reversed, liquidity trap equilibria cannot occur

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1 Introduction

> *"Let us suppose now that one day a helicopter flies over this community and drops an additional $1000 in bills from the sky, ... Let us suppose further that everyone is convinced that this is a unique event which will never be repeated," (Friedman (1969, pp 4-5).*

The aim of this paper is to provide a rigorous analysis of Milton Friedman's famous parable of the 'helicopter' drop of money. This is achieved through a reformulation of the *real balance effect* - the wealth effect of a change in the stock of government-issued fiat money. A related objective is to show that even when the economy is in a liquidity trap, that is, when all current and future short nominal interests rates are at their lower bounds, a helicopter drop of money will stimulate demand, but a helicopter drop of government bonds will not.

The argument involves several steps. First, using an optimising dynamic general equilibrium model with money, I provide a formal statement of the 'folk proposition' that government fiat money (or base money) is an asset to the private holder but not a liability of the issuer. This is because government fiat money is *irredeemable* or *inconvertible,* which motivates an asymmetric specification of the solvency constraints of the households and the government. For the household, solvency requires that the present discounted value of the household's terminal net financial wealth (the sum of its money holdings and its net non-monetary financial assets) be non-negative. For the government, solvency requires that the present discounted value of its terminal *non-monetary* net financial debt be non-positive. The government does not view its monetary debt as a liability that will eventually have to be redeemed for something else of equal value. This is the central new idea in the paper.[1]

The next step is to consolidate the intertemporal budget constraints of the private and public sectors and demonstrate formally the sense in which irredeemable government fiat money is 'net wealth'. This paper's representative agent model exhibits debt-neutrality or Ricardian equivalence: holding constant the sequences of real public spending on goods and services and of base money issuance, changes in the timing of lump-sum taxes that keep constant their present discounted value affects neither real nor nominal equilibrium values.

Weil (1991), who uses a similar model but without irredeemable government fiat money, shows that, in sharp contrast to our approach, consolidation of the government's and the private sector's intertemporal budget constraints eliminates not only government bonds (non-monetary debt) as a determinant of private consumption, but also government fiat money. Neither government bonds nor government fiat money are 'net wealth'.

Here the irredeemability of government fiat money restores a weak version of the real balance effect: the present value of the terminal stock of government fiat money *is* net wealth. This has implications for the existence of liquidity trap equilibria. With irredeemable fiat money, liquidity trap equilibria cannot exist as rational expectations equilibria, provided the monetary policy rule satisfies some simple and intuitive properties. This is demonstrated both for a model with flexible prices and for a model with a New-Keynesian Phillips curve.[2] These same restrictions on the monetary policy rule also rule out the existence of deflationary bubbles even when money is not the only financial liability issued by the government (see Buiter and Sibert (2003)). Milton Friedman's optimal quantity of money (OQM) rule remains feasible and optimal even with irredeemable money.

The formalisation of the notion that government fiat money is an asset to the private sector but not a liability to the public sector, is new. Whether and in what way 'outside' money is net wealth to the private sector or to the consolidated private sector and public sector was the subject of much discussion in some of the great treatises on monetary economics of the 1950s and 1960s, including Patinkin (1956/1965), Gurley and Shaw (1960) and Pesek and Saving (1967). The discussions in Hall (1983), Stockman (1983), King (1983), Fama (1983), Sargent and Wallace (1984) and Sargent (1987) of outside money, private money and the payment of interest on money ask some of the same questions as this paper, but do not offer the same answer. Sargent (1987, Chapter 4) and many other authors use the term '*inconvertible* currency' or '*unbacked* paper currency' to refer to a monetary asset that is essentially the same as the irredeemable fiat base money of this paper. However, these authors do not develop the implications of the inconvertibility of money for private and public sector intertemporal budget constraints.

Sims (2000, 2003), Buiter (2003a,b), Eggertson (2003) and Eggertsson and Woodford (2003) all stress that to boost demand in a liquidity trap, base money increases should not be, or expected to be, reversed. None of these papers recognised that even a permanent increase in the stock of base money will not have an expansionary wealth effect unless money is irredeemable in the sense developed here; without this, there is no real balance effect.

In what follows, 'money' means government fiat money or base money – the monetary liabilities of the state. Today, real world fiat base money is currency (notes and coin which typically bear a zero nominal interest rate) plus commercial bank balances held with the central bank (which can be interest-bearing). In the model, base money is the only money and all base money is assumed to have an exogenous government-determined nominal rate of interest.[3]

The property that base money is an *irredeemable final means of settlement* for obligations of the government to the private sector is related to, but not the same as the *legal tender* status of base money. A private agent's acceptance of payment by the government in base money constitutes a final settlement between that private agent and the government. The private agent has no further claim on the government. The popular definition of 'legal tender' is a means of payment than cannot be refused if it is offered (tendered) to settle a debt or other financial obligation.[4] This definition goes beyond the irredeemability property of base money that is central to this paper, in that it relates to the settlement of *all* claims, including claims among private parties. On the other hand, legal tender can be redeemable. Confederate fiat money (Confederate Treasury notes), for example, was convertible into Confederate bonds.[5]

Irredeemability is clearly a property of government-issued currency. This is nicely illustrated in the phrase *"...promise to pay the bearer the sum of ..."* found on all Bank of England notes. It means that the Bank of England will pay out the face value of any genuine Bank of England note no matter how old. The promise to pay stands good for all time but simply means that the Bank will always be willing to exchange one £10 Bank of England note for one £ 10 Bank of England note. Because it promises only money in exchange for money, this 'promise to pay' is a statement of the irredeemable nature of Bank of England notes.

With irredeemable base money, the proper combination of current monetary and fiscal policies ought to boost aggregate demand. The qualification *ought* reflects the existence of two possible exceptions. First, there is the possibility that perverse *future* policies (future reversals of current increases in the stock of base money) could, through the *present* rational anticipation of these policies, negate what would otherwise be the expansionary effect of the current policy measures. Second, there is the possibility that *expectations* of future contractionary monetary policy could undo the expansionary impact of current increases in the money stock, even when the authorities, unbeknownst to the private sector, will in fact *not* engage in such future behaviour. The expectations that matter concern the behaviour of the terminal money stock as the time horizon goes to infinity. Such incorrect expectations that cannot be refuted by observing actual monetary policy over any finite time interval will be called *incorrect but irrefutable* (*IBI*) expectations.[6]

A further implication of the approach adopted in this paper is that, provided the government always satisfies its intertemporal budget constraint, the effect of an increase in the stock of base money is the same, regardless of whether it is brought about by an open market purchase of public debt or through a cut in lump-sum taxes. Thus, holding constant the sequence of real public spending on goods and services, 'quantitative easing' has the same effect when the stock of base money expands through the financing of the government deficit as when it expands through an open market purchase of government debt held by the public.[3]

2. Modeling household and government behaviour

I consider a discrete-time infinite-horizon dynamic competitive equilibrium model of an endowment economy with a representative private agent and a government sector consisting of a consolidated fiscal authority and monetary authority (the Treasury and Central Bank).

2.1 Household behaviour

There is a unit interval of households. In each period *t, t* = 1, 2,... each household receives a strictly positive exogenous endowment y_t of the single perishable good. It consumes $c_t \geq 0$ and pays a lump-sum real tax τ_t. There are two stores of value: government-issued fiat money and nominal bonds. The household's within-period budget constraint in period *t* is

$$y_t - c_t - \tau_t + \frac{(1+i_t^M)M_{t-1}^h + (1+i_t)B_{t-1}^h}{P_t} \equiv \frac{M_t^h + B_t^h}{P_t}, \qquad \textbf{(1)}$$

where M_t^h and B_t^h are the nominal stocks of money and bonds carried by the household into period t+1, respectively, i_{t+1}^M and i_{t+1} are the nominal interest rates on money and bonds held between *t* and *t+1,* respectively, and $P_t \geq 0$ is the period *t* money price of the good. The government has a monopoly of base money issuance so $M_t^h \geq 0$. Rewriting the within-period budget constraint yields

$$(1+r_{t+1})(y_t - \tau_t - c_t + F_t/P_t) \equiv F_{t+1}/P_{t+1} + (i_{t+1}^M - i_{t+1})M_t^h/P_{t+1}, \qquad \textbf{(2)}$$

[3] 'Quantitative easing' refers to policies implemented by the central bank to achieve a monetary base growth target either in addition to the standard interest rate target or, when the nominal interest rate is at its lower bound, instead of it. The expression has been widely used to characterise Japanese monetary policy since March 2001. Three common ways of hitting a monetary base growth target are (1) open market purchases by the central bank from the public; (2) non-sterilised foreign exchange market intervention and (3); outright financing of government purchases of goods and services, transfer payments or tax cuts.

where $F^h_{t+1} \equiv (1+i_{t+1})B^h_t + (1+i^M_{t+1})M^h_t$ is the nominal value of household financial wealth, including interest, at the start of period t+1 and $1+r_{t+1} \equiv (1+i_{t+1})P_t/P_{t+1}$ is one plus the real interest rate. Let $I_t \equiv \Pi^t_{s=2}(1+i_s)^{-1}$, $t > 1$; $I_1 \equiv 1$ be the nominal market discount factor between periods t and 1, and $R_t \equiv \Pi^t_{s=2}(1+r_s)^{-1}$, $t > 1$; $R_1 \equiv 1$ the corresponding real discount factor. Let $m^h_t \equiv M^h_t/P_t, b^h_t \equiv B^h_t/P_t$ and $f^h_t \equiv F^h_t/P_t$. Solving the within- period budget constraint (2) forward yields

$$f^h_1 \equiv \sum_{t=1}^{\infty} R_t(c_t + \tau_t - y_t) + \sum_{t=1}^{\infty} R_t(i_{t+1} - i^M_{t+1})(1+i_{t+1})^{-1}m^h_t + \lim_{T\to\infty} R_T f^h_T. \qquad \textbf{(3)}$$

The household solvency constraint requires the present discounted value of terminal financial wealth to be non-negative in the limit as the terminal date goes to infinity:

$$\lim_{T\to\infty} R_T f^h_T \equiv (1/P_1)\lim_{T\to\infty} I_T[(1+i^M_T)M^h_{T-1} + (1+i_T)B^h_{T-1}] \geq 0. \qquad \textbf{(4)}$$

This no-Ponzi game condition in equation (4) incorporates the important assumption that both base money and bonds are assets of the household, even in the (infinitely) long run. The household solvency constraint holds 'identically', that is, for all feasible values of the variables entering the household's intertemporal budget constraint given by equations (3) and (4).

The household objective function is given in equation (5):

$$v = \sum_{t=1}^{\infty} (1+\rho)^{-t} u(c_t, m^h_t), \; \rho > 0; \; c_t, m^h_t \geq 0. \qquad \textbf{(5)}$$

The period felicity function u is increasing, concave and twice continuously differentiable. I also will assume that $u_c > 0$ for finite values of c. There is satiation in real money balances as a finite value of the stock of real money balances. To permit the derivation of closed-form consumption rules, or consumption functions, and thus to enhance the transparency of the argument, the period felicity function is, in most of what follows, assumed to have a constant elasticity of intertemporal substitution (CIS), σ (equation (6)), while the preferences governing static substitution between consumption and real money balances are assumed to be homothetic (equation (7)).[7]

$$u(c,m) \equiv \begin{cases} \dfrac{\sigma}{\sigma - 1} z^{\frac{\sigma-1}{\sigma}}, & 1 \neq \sigma > 0 \\ \ln z, & \sigma = 1, \end{cases} \qquad \textbf{(6)}$$

$$\begin{aligned} & z(c,m) = c v(m/c) \\ & v(m/c) - v'(m/c) m/c > 0 \; ; \; v'(m/c) > 0 \text{ for } m/c < \eta ; \; \eta > 0 \\ & v'(m/c) = 0 \text{ for } m/c \geq \eta ; \; \lim_{m/c \to 0} v'(m/c) = +\infty \\ & v''(m/c) < 0 \text{ for } m/c < \eta ; \; v''(m/c) = 0 \text{ for } m/c \geq \eta \end{aligned} \qquad \textbf{(7)}$$

The period utility function has satiation in real money balances at $m/c = \eta$, where η is positive but finite.[8] This means not only that $v(m/c)$ is bounded from above, but that it reaches its upper bound at a finite value of *m/c*. All key results in the paper go through even if there is no satiation in real money balances at a finite value of *m/c*, or even when there is no satiation in real money balances at all. The reasons for considering utility functions with satiation in real money balances are, first, that it is a-priori plausible and second, that it makes it possible to support Friedman's OQM monetary rule without worrying about implementing an infinite stock of real money balances.

Money is held in every period *t*, even when $i_t > i_t^M$, because end-of-period real money balances are an argument in the period *t* felicity function. All key propositions in this paper go through for more general functional forms and for most alternative ways of introducing money into the model including ‘money in the shopping function’, ‘money in the production function’ and ‘cash-in-advance’ models (Clower (1967), Lucas (1980, 1982)). A cash-in-advance model is considered in a longer version of this paper (Buiter (2003a)).[9] Initial financial asset stocks are predetermined:

$$B_0^h = \bar{B}_0;\ M_0^h = \bar{M}_0 > 0. \tag{8}$$

Obviously there exists no equilibrium where $i_t^M > i_t$ for some t; hence I only consider equilibria where $i_t^M \leq i_t$ for all t.

The consumer maximises lifetime utility given in equation (5), subject to the intertemporal budget constraint (equations (3) and (4)), and the initial asset stocks given in (8). It takes taxes as given. The optimum programme is characterised, for $t \geq 1$, by

$$u_c(c_t, m_t^h) = (1 + r_{t+1})(1 + \rho)^{-1} u_c(c_{t+1}, m_{t+1}^h). \tag{9}$$

$$u_m(c_t, m_t^h) = (1 + i_{t+1})^{-1}(i_{t+1} - i_{t+1}^M) u_c(c_t, m_t^h) \tag{10}$$

$$\lim_{T \to \infty} (1 + \rho)^{-T} u_c(c_T, m_T^h) f_T^h = 0. \tag{11}$$

The Euler equation (9) implies $u_c(c_t, m_t^h) = (1 + \rho)^{t-1} R_t u_c(c_1, m_1^h)$, $t \geq 1$. The transversality condition (10) can therefore also be written as $u_c(c_1, m_1^h) \lim_{T \to \infty} R_T f_T^h = 0$. As long as $u_c > 0$, this implies that

$$(1/P_1) \lim_{T \to \infty} I_T F_T^h \equiv \lim_{T \to \infty} R_T f_T^h \equiv 0. \tag{12}$$

For the CIS/homothetic period utility function in (6) and (7), the optimality conditions (9), (10) together with (2) yield:[10 11]

$$\frac{c_{t+1}}{c_t} = \left(\frac{1 + r_{t+1}}{1 + \rho}\right)^{\sigma}\frac{\Omega_t}{\Omega_{t+1}}; \; \Omega_t \equiv v(m_t^h/c_t)[(v(m_t/c_t) - v'(m_t^h/c_t)m_t^h/c_t]^{-\sigma} \tag{13}$$

$$\begin{gathered} m_t^h \geq \mathcal{R}(\mathrm{g}_{t+1}^M)c_t \\ m_t^h = \mathcal{R}(\mathrm{g}_{t+1}^M)c_t \quad \text{if } i_{t+1} > i_{t+1}^M; \\ \mathcal{R}(0) = \eta; \;\; \mathcal{R} > 0 \text{ for } i_{t+1} \geq i_{t+1}^M; \\ \mathcal{R}' = \frac{v(m^h/c) - v'(m^h/c)m^h/c}{[1 + (i - i^M)(1+i)^{-1}m^h/c]v''(m^h/c)} < 0 \text{ if } i_{t+1} > i_{t+1}^M; \end{gathered} \tag{14}$$

$$\mathrm{g}_{t+1}^M \equiv (i_{t+1} - i_{t+1}^M)(1 + i_{t+1})^{-1} \tag{15}$$

The existence and uniqueness of a continuously differentiable 'velocity function' $\mathcal{R}$ follows from the implicit function theorem. If there is satiation, the real stock of money balances can exceed η, because in that case the pecuniary opportunity cost of holding money rather than bonds for one period, $\mathrm{g}_{t+1}^M \equiv (i_{t+1} - i_{t+1}^M)/(1 + i_{t+1})$ is zero and money may be held in excess of η just as a store of value. When $m_t/c_t > \eta$, $\Omega_t = v(\eta)[v(\eta) - v'(\eta)\eta]^{-\sigma}$. It follows that

$$\Omega_t \equiv v\left(\mathcal{R}(\mathrm{g}_{t+1}^M)\right)\left[v\left(\mathcal{R}(\mathrm{g}_{t+1}^M)\right) - v'\left(\mathcal{R}(\mathrm{g}_{t+1}^M)\right)\mathcal{R}(\mathrm{g}_{t+1}^M)\right]^{-\sigma} \tag{16}$$

When (12) holds, the household's intertemporal budget constraint becomes:

$$\frac{(1 + i_1^M)M_0^h}{P_1} + \frac{(1 + i_1)B_0^h}{P_1} = \sum_{t=1}^{\infty} R_t(c_t + \tau_t - y_t) + \sum_{t=1}^{\infty} R_t\frac{i_{t+1} - i_{t+1}^M}{1 + i_{t+1}}m_t. \tag{17}$$

The optimality conditions (13), to (16) and the household intertemporal budget constraint (17), imply

$$c_t = \mu_t w_t, \text{ where} \tag{18}$$

$$w_t \equiv f_t^h + \sum_{s=0}^{\infty} (R_{t+s}/R_t)(y_{t+s} - \tau_{t+s}) \tag{19}$$

$$\mu_t \equiv \left\{ \sum_{s=0}^{\infty} \left(1+\gamma^M_{t+1+s} \mathrm{R}(\gamma^M_{t+1+s})\right) \frac{(R_{t+s}/R_t)^{\sigma-1}}{(1+\rho)^{\sigma s}} \left(\frac{\Omega_{t+1}}{\Omega_{t+1+s}} \right) \right\}^{-1}. \tag{20}$$

The variable w_t is interpreted as comprehensive household wealth in period t. It is the sum of real household financial wealth held at the beginning of period *t*, and the present discounted value of current and future after-tax endowment income. The key point to note about the marginal propensity to consume out of comprehensive wealth, μ_t, is that, from (15), (16) and (20), it is a function only of current and future real and nominal interest rates.[12]

2.2 The government

The government's decision rules are exogenously given. Like the household sector, it is subject to a solvency constraint. The government solvency constraint is the requirement that the present discounted value of its *non-monetary* debt (bonds) must be non-positive in the limit as the time horizon goes to infinity. Unlike bonds, government fiat money, by assumption, does not have to be redeemed *ever* by the government. This means that, while money is in a formal, legal sense a liability of the government, it does not represent an effective liability of the government: there is no obligation for the issuer ever to redeem it or convert it into anything else.

The stocks of money and bonds supplied by the government at the end of period t (and the beginning of period $t+1$) are M_t^g and B_t^g respectively. The aggregate financial liability of the government at the beginning of period t (principal plus interest), F_t^g is $\mathcal{F}_t^g \equiv (1+i_t^M)M_{t-1}^g + (1+i_t)B_{t-1}^g$ and $f_t^g \equiv F_t^g/P_t$. The government's single-period budget identity for $t \geq 1$ is

$$f_{t+1}^g \equiv (1+r_{t+1})(f_t^g + g_t - \tau_t) + (i_{t+1}^M - i_{t+1})(P_t/P_{t+1})m_t^g. \tag{21a}$$

or, equivalently,

$$b_t^g \equiv (1+r_t)b_t^g + g_t - \tau_t - \frac{M_t - (i_{t+1}^M)M_{t-1}}{P_t} \tag{21b}$$

The government's no-Ponzi game condition or solvency constraint is

$$\lim_{T\to\infty} R_{T-1}b_T^g \leq 0. \tag{22}$$

We assume that the government would not have strictly negative non-monetary debt in the long run, hence equation (22) is assumed to hold with equality. From (21a) and (22), assumed to hold with equality, we obtain the intertemporal budget constraint for the government:[13]

$$f_1^g \equiv \sum_{t=1}^{\infty} R_t(\tau_t - g_t) + \sum_{t=1}^{\infty} (R_t P_t/P_{t+1})(i_{t+1} - i_{t+1}^M)m_t^g + (1/P_1)\lim_{T\to\infty} I_T(1+i_T^M)M_{T-1}^g. \tag{23}$$

Without irredeemability of money, the natural specification of the government's solvency constraint involves symmetry with the private sector solvency constraint and therefore requires that the present discounted value of the terminal aggregate monetary and non-monetary government debt should be non-positive:[14]

$$\lim_{T \to \infty} R_T f_T^g \equiv (1/P_1) \lim_{T \to \infty} I_T [(1 + i_T^M) M_{T-1}^g + (1 + i_T) B_{T-1}^g] \leq 0. \qquad \textbf{(24)}$$

Consider the case where the government-determined nominal interest rate on base money and the nominal interest rate on non-monetary financial claims are zero, now and in the future ($i_t^M = i_t = 0,\ t \geq 1$). The government solvency constraint implies that, in this case, the net non-monetary government debt has to be retired in the long run: $\lim_{N \to \infty} B_{N-1}^g \leq 0$. Thus, while both money and bonds have the same (zero) interest rate, bonds, unlike base money, must have their principal redeemed, even in the long run. Irredeemability of money is the key feature that makes a helicopter drop of money a potential means of boosting demand in a liquidity trap, while a helicopter drop of bonds does nothing. This differential impact of a money drop and a bond drop disappears only if households expect that the government will, in the long run, *choose* to redeem (retire) its stock of base money, including any increases in the money stock it may engineer in the short run. Reducing the nominal money stock to zero in the long run may seem like odd behaviour by the monetary authority, but it characterises the monetary rule that supports Milton Friedman's Optimal Quantity of Money (OQM) equilibrium: $\frac{M_{t+1}}{M_t} = \frac{1 + i_{t+1}^M}{1 + \rho};\ t \geq 1$ when $i_{t+1}^M = 0;\ t \geq 1$.[15]

The fiscal-financial-monetary programme of the government, specified below, supports the government's solvency constraint (22) identically, that is, for all feasible values of the variables entering the government's intertemporal budget constraint.

2.3 Financial equilibrium and the consolidation of accounts

Actual stocks of nominal financial wealth, money and bonds are denoted F_t, M_t and B_t respectively, and $f_t \equiv F_t/P_t$; $m_t \equiv M_t/P_t$ and $b_t \equiv B_t/P_t$. Financial market clearing requires for $t \geq 1$:

$$
\begin{aligned}
f_t^g &= f_t^h = f_t \\
b_t^g &= b_t^h = b_t \\
m_t^g &= m_t^h = m_t
\end{aligned}
\tag{25a}
$$

$$
\lim_{T\to\infty} \frac{R_T P_{T-1}(1 + i_T^M) m_{T-1}^g}{P_T} = \lim_{T\to\infty} \frac{R_T P_{T-1}(1 + i_T^M) m_{T-1}}{P_T} = \frac{1}{P_1} \lim_{T\to\infty} I_T (1 + i_T^M) M_{T-1}. \tag{25b}
$$

Clearly, the present value of the terminal money stock, given in equation (25b) need not equal zero. In particular, suppose that there is a liquidity trap with a constant nominal interest rate $\bar{i}^M$ on both money and bonds. Suppose further that $M_{t+1}/M_t = 1 + \nu$. Then $(1/P_1)\lim_{T\to\infty} I_T (1 + i_T^M) M_{T-1} = (M_0/P_1) \lim_{T\to\infty} [(1 + \nu)/ (1 + \bar{i}^M)]^{T-1} > 0$ if and only if $\bar{\nu} > \bar{i}^M$. If $\bar{i}^M = 0$, then any positive long-run growth rate of the nominal money stock will cause the present discounted value of the terminal money stock to be positive. The only liquidity trap equilibria that have a zero present discounted value of the terminal money stock are those for which the terminal money stock goes to zero. This is the case for Friedman's OQM equilibrium, but not for any deflationary bubble equilibrium. This suggests a simple design feature for monetary policy rules to rule out non-OQM liquidity traps as rational expectations equilibria, as shown in Section 3..

We can use the household's intertemporal budget constraint (17), the government's intertemporal budget constraint (23), the financial market clearing conditions (25a) and equation (25b) to obtain the following 'consolidated' intertemporal budget constraint:

$$
\sum_{s=0}^{\infty} \frac{R_{t+s} c_{t+s}}{R_t} \equiv \sum_{s=0}^{\infty} \frac{R_{t+s}(y_{t+s} - g_{t+s})}{R_t} + \lim_{T\to\infty} \frac{I_{t+1+T}(1 + i_{t+1+T}^M) m_{t+T}}{I_t P_t}. \tag{26}
$$

Equation (26) holds for *any* utility function. For the special case of the utility function in equations (7) and (8), we can solve equations (13) to (16) and (26) to yield[16]

$$c_t = \hat{\mu}_t \hat{w}_t \tag{27}$$

$$\hat{w}_t = \sum_{s=0}^{\infty} \frac{R_{t+s}}{R_t}(y_{t+s} - g_{t+s}) + \lim_{T\to\infty} \frac{I_{t+1+T} P_{t+T}}{I_t P_t}(1 + i^M_{t+1+T}) m_{t+T} \tag{28}$$

$$\hat{\mu}_t \equiv \left[\sum_{s=0}^{\infty} \frac{(R_{t+s}/R_t)^{\sigma-1}}{(1+\rho)^{\sigma s}} \left(\frac{\Omega_{t+1}}{\Omega_{t+1+s}} \right) \right]^{-1} > 0 \tag{29}$$

Equations (27) to (29) (with Ω_t defined in equations (15) and (16)) differ from the consumption function that would be obtained without irredeemable base money because of the presence of the present discounted value of the terminal money stock, $\lim_{T\to\infty}[(I_{t+1+T}P_{t+T})/(I_tP_t)](1 + i^M_{t+1+T})m_{t+T} \equiv \lim_{T\to\infty}[(R_{t+1+T}P_{t+T})/(R_tP_{t+1+T})](1 + i^M_{t+1+T})m_{t+T}$ in the expression for the intertemporal budget constraint of the consolidated private and government sectors. Irredeemable base money is net wealth to the consolidated private and public sectors in the limited sense that the present discounted value of the terminal stock of base money is perceived by the private sector to be part of the consolidated resource base for private consumption, alongside the present discounted value of the sequence of real endowments net of real government spending, $\sum_{s=0}^{\infty} (R_{t+s}/R_t)(y_{t+s} - g_{t+s})$.

Definition* 1**. *Monetary policy is said to have a* ***pure wealth effect *on household consumption demand if changes in the sequence of current and future nominal money stocks can change consumption demand, holding constant the initial financial asset stocks, the sequences of current and future values of nominal and real interest rates, the initial price level, real government spending on goods and services, and before-tax endowments.*

From the consumption function of equations (15), (16) and (18) to (20), it follows that monetary policy can have a *pure wealth effect* on private consumption if and only if it can change, *ceteris paribus*, the present discounted value of current and future taxes. In this representative agent model, which exhibits debt neutrality or Ricardian equivalence, monetary policy cannot do so, unless base money is irredeemable.[17] Without the 'irredeemability' assumption, this representative agent model would have the property, noted in Weil (1991), that money is not net wealth, just as government debt is not net wealth.

Equations (15), (16) and (27) to (29) do not say that, if $\lim_{N\to\infty} R_{t+1,N}(1 + i_N^M)(M_{N-1}/P_N) = 0$, monetary policy cannot affect real consumption demand. If the present value of the terminal base money stock is zero, monetary policy can still affect *real* consumption demand if it can affect, directly or indirectly, either current or anticipated future real endowment income, or current or anticipated real and nominal interest rates.

2.4 The Fiscal-Financial-Monetary Programme (FFMP) of the government

Real public spending on goods and services is assumed constant:

$$0 \leq g_t = \bar{g} \qquad\qquad t \geq 1 \tag{30}$$

Real lump-sum taxes are assumed to vary endogenously to keep constant the real value of the stock of non-monetary government debt.[18] As long as the long-run real interest rate is positive, this tax rule ensures that (22), the government's solvency constraint when money is irredeemable, always holds with equality. Using (21b) and $(1+r_{t+1})b_t = (1+r_t)b_{t-1} = (1+r_1)b_0,\ t \geq 1$, it follows that:

$$\tau_t = \bar{g} + \frac{r_{t+1}}{1+r_{t+1}}(1+r_1)b_0 - [m_t - (1+i_t^M)\frac{P_{t-1}}{P_t}m_{t-1}] \tag{31}$$

The nominal interest rate on base money is exogenous and constant:

$$i_t^M = \bar{i}^M, \qquad t \geq 1 \tag{32}$$

I consider two monetary policy rules. The first is a constant growth rate for the nominal stock of base money:

$$\begin{aligned} &\text{If } 1+\pi_t = (1+\bar{i}^M)(1+\rho)^{-1} \ \forall\, t \geq 1,\ M_{t+1}/M_t = (1+\bar{i}^M)(1+\rho)^{-1} \\ &\text{If } 1+\pi_t \neq (1+\bar{i}^M)(1+\rho)^{-1} \ \forall\, t \geq 1,\ M_{t+1}/M_t = 1+\nu > 1+\bar{i}^M. \end{aligned} \tag{33}$$

This is the monetary rule considered for the flexible price level model. The first line of (33) ensures that the monetary rule supports Friedman's stationary OQM equilibrium. The second line of (33) rules out any liquidity trap equilibrium other than the OQM equilibrium.

The second monetary rule is the combination of a (simplified) Taylor rule for the short nominal interest rate (when application of the Taylor rule does not violate the lower bound $i \geq i^M$) and, when the application of the Taylor rule would violate this lower bound, a constant growth rate of the nominal money stock given by equation (33).[19] The nominal interest rate in that case is kept at the lower bound. Again, this ensures that the stationary liquidity trap equilibrium corresponding to Friedman's OQM rule is supported. This *augmented Taylor rule*, given in (34), is used for the New-Keynesian variant.

$$\begin{aligned} &\text{If } 1+\pi^* + \gamma(\pi_{t+1}-\pi^*) > (1+\bar{i}^M)(1+\rho)^{-1},\quad 1+i_{t+1} = (1+\rho)[1+\pi^* + \gamma(\pi_{t+1}-\pi^*)] \ ;\ \gamma > 1 \\ &\text{If } 1+\pi^* + \gamma(\pi_{t+1}-\pi^*) \leq (1+\bar{i}^M)(1+\rho)^{-1},\quad 1+i_{t+1} = 1+\bar{i}^M \text{ and equation (33) applies.} \end{aligned} \tag{34}$$

Equation (34) says that, as long as the lower bound on the short nominal interest rate is not binding, the short nominal interest rate rises more than one-for-one ($\gamma > 1$) with the (expected) inflation rate. The parameter π^* can be interpreted as the long-run target rate of inflation. When $1+\pi_{t+1} > \gamma^{-1}[(1+\bar{i}^{M})(1+\rho)^{-1}+(\gamma-1)(1+\pi^{*})]$ (in the 'normal' region) the nominal money stock is endogenously determined through the money demand function:

$$M_t/P_t \equiv m_t = R(g^{M}_{t+1})\, c_t, \qquad i_{t+1} \geq \bar{i}^{M} \tag{35}$$

The behaviour of the aggregate stock of monetary and non-monetary public debt follows from the constancy of $(1+r_{t+1})b_t$ and the behaviour of the endogenous stock of real money balances:

$$f_{t+1} \equiv (1+r_1)b_0 + (1+\bar{i}^{M})m_t\ , \ t \geq 0 \tag{36}$$

3. Irredeemable money in general equilibrium

I consider two alternative supply-side specifications: a flexible price or New-Classical model with an exogenous and constant level of capacity output, $y^{*} > 0$, and a New-Keynesian Phillips curve, with a predetermined general price level but a non-predetermined, forward-looking rate of inflation. For both models, actual output always equals demand, so $y_t = c_t + g_t,\ t \geq 1$. The economy-wide real resource constraint is

$$y_t \leq y^{*}. \tag{37}$$

3.1 A flexible price level

With a flexible price level actual output always equals capacity output:

$$y_t = y^{*} \tag{38}$$

An equilibrium exists only if public spending is less than capacity output, $0 \leq g_t = \bar{g} < y^{*}$. An equilibrium must satisfy, for $t \geq 1$, equations (8), (15), (16), (29) and (39) to (42).

$$c_t = y^* - \bar{g} \quad \textbf{(39)}$$

$$c_t = (y^* - \bar{g})\hat{\mu}_t \sum_{s=0}^{\infty} \frac{R_{t+s}}{R_t} + \hat{\mu}_t P_t^{-1} \lim_{T \to \infty} \frac{I_{t+T}}{I_t}(1 + \bar{i}^M) M_{t+T-1} \quad \textbf{(40)}$$

$$1 + r_{t+1} = (1 + \rho)\left(\frac{\Omega_{t+1}}{\Omega_t}\right)^{\frac{1}{\sigma}} \quad \textbf{(41)}$$

$$\begin{gathered} M_t/P_t = L\Big((1+r_{t+1})P_{t+1}P_t^{-1}[(1+r_{t+1})P_{t+1}P_t^{-1} - (1+\bar{i}^M)]^{-1}\Big)(y^* - \bar{g}) \\ 1+i_{t+1} \equiv (1+r_{t+1})P_{t+1}P_t^{-1} \geq 1+\bar{i}^M \end{gathered} \quad \textbf{(42)}$$

Consider the case where the authorities fix the growth rate of the nominal stock of base money (equation (33)). When $1 + \nu > (1 + \bar{i}^M)(1 + \rho)^{-1}$ there exists a stationary equilibrium, the *fundamental* equilibrium, with $\pi = \nu$. The stock of real money balances is constant and finite. We call this stationary equilibrium a *fundamental* equilibrium because all exogenous variables are stationary (constant) and there are only non-predetermined state variables Ω and m). The only other fundamental equilibrium is the non-monetary or barter equilibrium, with $m_t = 0,\ t \geq 1$.

In the stationary equilibrium, the constant real interest rate is positive ($r = \rho > 0$), and the discounted value of the finite terminal money stock is therefore zero. Does there exist a non-stationary equilibrium that converges to a liquidity trap, that is, does there exist a period t_0 such that equations (8), (15), (16), (29), (33) and (39) to (42) are satisfied for all $t \geq t_0 \geq 1$ with $1 + i_t = 1 + \bar{i}^M$? If this is possible only in the limit as $t_0 \to \infty$, convergence to a liquidity trap equilibrium is said to be asymptotic.

Proposition 1.

> *A liquidity trap equilibrium does not exist, even asymptotically, in the flexible price model if the growth rate of the stock of nominal base money is equal to or greater than the nominal interest rate on base money, that is, if* $\nu \geq \bar{i}^M$.

Proof: Assume a liquidity trap equilibrium exists, beginning in period $t_1 \geq 1$. By definition, in a liquidity trap the opportunity cost of holding money is zero ($g_t^M = 0,\ t \geq t_1$) and therefore the nominal interest rate is constant ($i_t = \bar{i}^M,\ t \geq t_1$). From (15) and (16), when the short nominal interest rate is constant, Ω_t is constant. This implies, from (41), that the real interest rate is constant: $r_t = \rho$. From (29) it then follows that the marginal propensity to consume out of consolidated comprehensive wealth is constant and given by $\hat{\mu}_t = \dfrac{\rho}{1+\rho}$. Equation (40) becomes $c_t = y^* - \bar{g} + \dfrac{\rho}{1+\rho} P_t^{-1} \lim_{T \to \infty} \dfrac{I_{t+T}}{I_t}(1+\bar{i}^M) M_{t+T-1}$. This is consistent with (39) only if $P_t^{-1} \lim_{T \to \infty} \dfrac{I_{t+T}}{I_t}(1+\bar{i}^M) M_{t+T-1} = 0$. Since a liquidity trap is defined as an equilibrium with $i_t = \bar{i}^M,\ t \geq t_1$, it follows that in a liquidity trap, (39) and (40) can both hold only if $(1+\nu)^t P_t^{-1} \lim_{T \to \infty} \left(\dfrac{1+\nu}{1+\bar{i}^M} \right)^{(T-1)} \bar{M}_0 = 0$, for all $t \geq t_1$. Since $\bar{M}_0 > 0$ and $\nu \geq \bar{i}^M$, it follows that $(1+\nu)^t P_t^{-1} \lim_{T \to \infty} \left(\dfrac{1+\nu}{1+\bar{i}^M} \right)^{(T-1)} \bar{M}_0 \geq (1+\nu)^t P_t^{-1} \bar{M}_0 = (1+\nu)^t M_t / P_t$. Equations (39) and (40) can then both be satisfied only if $M_t/P_t = 0,\ t \geq t_1$.[20] If $M_t/P_t = 0$, we are in the (stationary) barter equilibrium (and would stay in it forever after). When $g_t^M = 0$, it follows from (14), that $R(0) = \eta$. Since $\eta > 0$, the demand for real money balances is positive in period $t \geq t_1$. The monetary equilibrium condition (42) is violated. □

For Proposition 1 to hold, all that is required is that the demand for real money balances be positive when $i_t = \bar{i}^M$. The proposition holds, of course, if the demand for real money balances is unbounded when $i_t = \bar{i}^M$, but does not require this. Proposition 1 also applies when money derives from a cash-in-advance constraint (see Buiter (2003a)).

Friedman's OQM rule for the case where the interest rate on base money can be non-zero - $1+\nu = (1+\bar{i}^M)(1+\rho)^{-1}$ - supports a stationary (fundamental) equilibrium with $\pi_t = \nu$ and $i_t = \bar{i}^M$. When $\bar{i}^M = 0$, the growth rate of the nominal stock of base money under the OQM rule is negative and the nominal stock of base money goes to zero in the long run.

Proposition 1 has the following implications for monetary policy in the practically relevant case where the nominal interest rate on base money is zero.

Corollary 1

With irredeemable money, when the nominal interest rate on base money is zero, there can be a liquidity trap equilibrium in the flexible price level model only if, in the long run, the authorities (are expected to) reduce the nominal stock of base money to zero. Any monetary rule that does not lead (and is not expected to lead) to eventual demonetisation of the economy precludes a liquidity trap equilibrium. Friedman's OQM rule supports a stationary liquidity trap equilibrium in which the nominal stock of money goes to zero in the long run.

Proposition 1 also has obvious implications for the existence of deflationary bubbles in the flexible price level model.

Corollary 2

In the flexible price level model, deflationary bubbles do not exist when base money is irredeemable, even though base money is not the only financial liability of the government. Without the irredeemability of base money, deflationary bubbles can exist in models in which the government issues both monetary and non-monetary financial liabilities, under the FFMP given by equations (30)-(33).

Proof: see Buiter and Sibert (2003) (see also. Obstfeld and Rogoff (1983, 1986, 1996) and Woodford (2003)). The intuition is simple. Consider the classical deflationary bubble problem with $\bar{i}^M = 0$ and a positive constant nominal money stock. Do there exist equilibria in which the price level goes to zero? As the stock of real money balances grows without bound, the nominal interest rate goes to zero and the present value of the terminal stock of real money balances becomes unbounded. Private consumption demand (see equation (40)) becomes unbounded and violates the economy-wide real resource constraint (37). Equivalently, for the household transversality condition in equation (11) to be satisfied when the present discounted value of total available resources is unbounded requires $u_c(c, m) = 0$. This requires unbounded consumption and violates the aggregate resource constraint (37).

3.2 The helicopter drop of money

The neutral monetary operation described by Friedman in the introductory quote to this paper as a helicopter drop of money can be represented either as an a-historical ('parallel universes') counterfactual or as a real-time (calendar-time) policy action. The a-historical counterfactual compares, starting in period *t=1,* two alternative economies, I and II, that are identical in all but two respects.[21] First, in economy II the path of the nominal stock of base money lies $\kappa > 0$ percent above that in economy I, that is, $M_t^{II} = (1 + \kappa)M_t^I,\ t \geq 0$. As the growth rates of the nominal money stocks are the same in the two economies ($\nu^I = \nu^{II}$), we only require their initial (predetermined) nominal money stocks to differ ($\bar{M}_0^{II} = (1+\kappa)\bar{M}_0^I$).[22] Second, the bond and real tax sequences are endogenous (except for the initial stock of debt, $\bar{B}_0$ which is the same for both economies) and can therefore be different in the two economies. The governments in both economies choose sequences for bond issuance and lump-sum taxes that satisfy their intertemporal budget constraints. An example would be the tax rule (31).[23]

Consider an equilibrium for the benchmark economy (I) given by $\{P_t^I;\ i_t^I;\ r_t^I;\ c_t^I; t \geq 1\}$. If current and future nominal interest rates are not at their lower bounds ($i_t^I > \bar{i}^M$, $t \geq 1$), it is clear by inspection of the equilibrium conditions ((8), (15), (16), (29) and (39) to (42)), that there exists an equilibrium in economy (II) given by $\{P_t^{II} = (1+\kappa)P_t^I;\ i_t^{II} = i_t^I;\ r_t^{II} = r_t^I;\ c_t^{II} = c_t^I;\ t \geq 1\}$: money is neutral.[24]

In general, an equiproportional increase in current and future nominal prices will reduce not only the real value of the initial nominal money stock, but also the real value of any net initial (pre-determined) non-monetary nominally denominated financial assets or liabilities the private and public sectors may have. In the model this will be the case if $\bar{B}_0^I = \bar{B}_0^{II} \neq 0$. Because of the government's intertemporal budget constraint and the debt-neutrality inherent in the representative agent model, any difference between the real values of the common initial stock of nominal bonds will be matched by a difference of equal present discounted value between the sequences of current and expected future lump sum taxes in economies I and II.

The interpretation of Friedman's helicopter drop of money as an event taking place in calendar time formalises it as an unanticipated, temporary (one period) tax cut in period $t_0 \geq 1$, financed through an increase in the period t_0 nominal stock of base money. Think of it as the tax function in equation (31) having an extra term $\lambda < 0$ added to it just for period t_0, with $\Delta M_{t_0} = (1+\nu)\bar{M}_{t_0-1} - \lambda P_{t_0}$. After the unanticipated increase in the period t_0 stock of base money, the stock of nominal base money again grows at the rate ν. I consider only equilibria where the lower bound on the nominal interest rate does not bind. In this real-time counterfactual, there exists an equilibrium in which money is neutral, even though the initial, predetermined stock of money, $\bar{M}_{t_0-1}$, is not increased when the period t_0 nominal money stock is increased unexpectedly.

It is clear, however, that, if $t_0 = 1$, the equilibrium price sequence and the equilibrium sequences of real and nominal interest rates and consumption are the same as in the a-historical counterfactual. Money is neutral despite the fact that the unexpected κ percent increase in the period $t \geq t_0$ price levels reduces the real value of the initial, predetermined, stock of base money, $\bar{M}_{t_0-1}/P_{t_0}$,in addition to changing the real value of the initial, predetermined, stock of nominal government bonds, $\bar{B}_{t_0-1}/P_{t_0}$. Neither the initial money stock nor the initial stock of bonds figures in the equilibrium conditions ((8), (15), (16), (29) and (39) to (42)). They only play a role 'in the background' through the government's and the household sector's intertemporal budget constraints and the government's FFMP. With the tax rule given in (31), current and future lump-sum taxes and bond issuance adjust to absorb any impact on the intertemporal budget constraints of a lower real value of the inherited stocks of nominal base money and nominal bonds.

This neutrality result assumed that the economy was not in a liquidity trap. The following proposition, however, holds true both outside and in a liquidity trap:

Proposition 2:

> *In the representative agent model, it does not matter how money gets into the system: Because of Ricardian equivalence, unanticipated money-financed tax cuts (real-time helicopter money drops) have the same effect on real and nominal equilibrium prices and quantities as unanticipated open market purchases.*

Compare again two economies, indexed by superscripts I and II. Initial conditions are identical. In both economies the government overrides the FFMP (equations (31) and (33)) for one period (period 1, say). In both economies the government unexpectedly increases the stock of base money in period 1 by an amount $-\lambda P_1$ over and above the amount given by the monetary rule (33). In economy I the unexpected increase in the stock of money is financed by a cut in period 1 real taxes (over and above the amount given by (31)) by an amount λ. In economy II the same unexpected increase in period 1 base money is achieved through the purchase in period 1 of an amount $-\lambda P_1$ of nominal bonds by the government (a so-called 'open market purchase'). In economy II the government sticks to the tax rule in (31) in each period, including period 1. The sequences of real public spending on goods and services are the same in the two economies, and so are the nominal money stock sequences for $t > 1$. The government satisfies its intertemporal budget constraint in economy I by applying the tax rule (31) after period 1. It follows that the equilibrium sequences for all nominal and real endogenous variables are the same in the two equilibria, except the sequences of real values of period 1 and later bond stocks and the real value of current and future lump-sum taxes - although the present value of current and future real taxes is the same in both economies.

Proposition 2 is a direct implication of debt neutrality or Ricardian equivalence, and many versions of it are around (see e.g. Wallace (1981) and Sargent (1987)). Proof is by inspection of the equilibrium conditions (equations (8), (15), (16), (29) and (39) to (42)), the government's intertemporal budget constraint (23) and the financial equilibrium conditions (24). Debt neutrality or Ricardian equivalence means that a helicopter drop of government bonds makes no difference to any real or nominal equilibrium values, except of course for the present value of current and future lump-sum taxes. Bonds are redeemable, so the present value of the terminal stock of bonds is zero. Because $(1+i_t)B_{t-1}/P_t + \lim_{T\to\infty}\sum_{s=0}^{T} R_{t+s}R_t^{-1}[B_{t+s}-(1+i_{t+s})B_{t+s-1}]/P_{t+s} \equiv \lim_{T\to\infty} R_{t+T}R_t^{-1} b_{t+T}$, and $\lim_{T\to\infty} R_{t+T}R_t^{-1} b_{t+T} = 0$, it follows that $(1+i_t)B_{t-1}/P_t + \lim_{T\to\infty}\sum_{s=0}^{T} R_{t+s}R_t^{-1}[B_{t+s}-(1+i_{t+s})B_{t+s-1}]/P_{t+s} = 0$: the ability to issue bonds does not relax the government's intertemporal budget constraint in any way. Because of debt neutrality, the timing of lump-sum taxes does not matter, only their present value. Therefore, issuing money by lowering taxes today by an amount λ has the same effect on the real and nominal equilibrium as issuing the same amount of money today by purchasing (retiring) non-monetary debt today and cutting future taxes by λ in present value. Quantitative easing has the same effect regardless of whether the additional base money is put in the system through a current cut in lump-sum taxes or through an open market purchase of public debt and a cut in future lump-sum taxes of equal present discounted value.

3.3 A New-Keynesian Phillips curve

The result that liquidity trap equilibria can be ruled out when some mild restrictions are imposed on permissible monetary policies is not restricted to the flexible price level model, as the following New-Keynesian example makes clear. Let $\pi_t \equiv \frac{P_t}{P_{t-1}} - 1$ be the period t rate of inflation. Calvo's (1983) New-Keynesian Phillips curve is given by:

$$\pi_{t+1} = \pi_t + \eta_0 - \eta_1\left(y^* - y + \eta_1\eta_0^{-1}\right)^{-1}, \qquad \eta_0, \eta_1 > 0 \tag{43}$$

Equation (43) implies there is a finite limit, $\bar{y} \equiv y^* + \eta_1\eta_0^{-1}$, to the amount of output that can be produced with existing, finite resources. This is *a-priori* plausible**,** and resonates well with the Woodford's (1996) version of the New-Keynesian Phillips, which is based on monopolistically competitive mark-ups over marginal cost (see also Chadha and Nolan (2002)). It amounts to the assumption that the marginal cost curve becomes vertical at $\bar{y}$**.**

A complete qualitative analysis of even the simplest version (with $\sigma = 1$) of the New-Keynesian model using the convenient two-dimensional phase diagram is possible only for the continuous time version of the model.[25] I also assume that the homothetic period utility function has the following form:

$$cv(m/c) = c^{\alpha}m^{1-\alpha}, \; 0 < \alpha < 1 \tag{44}$$

This period utility function does not exhibit satiation in real money balances at a finite level of the stock of real money balances; instead $\eta = +\infty$. The resulting monetary equilibrium condition would be $m_t = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1 + i_{t+1}}{i_{t+1} - \bar{i}^M}\right)c_t$. With $\sigma = 1$ and (44), the state-space representation of the continuous time dynamic system consists of two first-order non-linear differential equations, (46) and (47), in consumption, c, and inflation, $\pi \equiv \dot{P}/P$, the first of which has a regime switch when the short nominal interest rate of the Taylor rule hits the lower bound $\bar{i}^M$.[26] This happens when π crosses a threshold value, $\hat{\pi}$, given by

$$\hat{\pi} = \gamma^{-1}[(\gamma-1)\pi^* + \bar{i}^M - \rho] \tag{45}$$

$$\begin{aligned} \dot{c} &= (\gamma - 1)(\pi - \pi^*)c \qquad &\text{if } \pi > \hat{\pi} \\ \dot{c} &= (\bar{i}^M - \pi - \rho)c \qquad &\text{if } \pi \le \hat{\pi} \end{aligned} \tag{46}$$

$$\dot{\pi} = \eta_0 - \eta_1\left(y^* + \eta_1\eta_0^{-1} - g - c\right)^{-1} \tag{47}$$

We also need the consumption function given in (48), where $r \equiv i - \pi$,

$$c(t) = \rho\left(\int_t^{\infty} e^{-\int_t^s r(u)du}[y(s)-g(s)]ds + P(t)^{-1}\lim_{s\to\infty} e^{-\int_t^s i(u)du} M(s)\right), \tag{48}$$

and a supplement to the Taylor rule (analogous to (33) in the discrete time model):

$$\begin{aligned} &\text{If } \pi \le \hat{\pi}, \text{ then } \ i = \bar{i}^M \text{ and} \\ &\dot{M}/M = \bar{i}^M - \rho \quad \text{if } \pi(t) = \bar{i}^M - \rho, \ \forall t \ge 0 \\ &\dot{M}/M = \bar{\nu} > \bar{i}^M \quad \text{if } \pi(t) \ne \bar{i}^M - \rho, \ \forall t \ge 0 \end{aligned} \tag{49}$$

The economy-wide real resource constraint implies that

$$c \le \bar{y} - \bar{g} \tag{50}$$

The steady states are $c = y^{*} - \bar{g}$, and either $\pi = \pi^{*}$ (the 'target' rate of inflation implicit in the Taylor rule) or $\pi = \bar{i}^{M} - \rho$ (the liquidity trap steady state - Friedman's OQM equilibrium). I assume that $\pi^{*} > \bar{i}^{M} - \rho$, or, equivalently, that $\hat{\pi} < \pi^{*}$. Both state variables, c and π are non-predetermined, but the price level, P, is predetermined. The dynamic system in (46) and (47), graphed in Figures 1 and 2, does not incorporate the constraint on consumption implied by (48), (49) and (50) (another interpretation is that Figures 1 and 2 incorporate the assumption that the *expectation* of $P(t)^{-1}\lim_{s\to\infty} e^{-\int_t^s i(u)du} M(s) = 0$). Figure 1 shows the system near the steady states**.**

FIGURE 1 HERE

The direction of motion along the solution orbits to (46) and (47) is counter-clockwise. Any solution that starts on an orbit outside the closed orbit, centered on Ω^{N} that just touches Ω^{L} from the right (the *normal* solution region, shown as the shaded area), will sooner or later end up in and forever after stay inside, the region where the lower bound on the nominal interest rate is binding (to the left of $\hat{\pi}$).[27] Figure 2 shows the global behaviour of the system.

FIGURE 2 HERE

When the implication for private consumption of the irredeemability of base money is added as a constraint on permissible solution trajectories, it is clear that none of the solution orbits that start outside the normal solution region are permissible. At some point, say at time $t = t'$, the nominal interest rate sequence along such an explosive orbit would reach the lower bound and stay there forever after. Base money growth at a rate no less than the nominal interest rate at the lower bound (that is, $v > \bar{i}^{M}$) would ensure that the present discounted value of the nominal money stock would grow without bound. With the price level predetermined at $t = t'$, real consumption demand would be unbounded and would exceed the physical upper bound $\bar{y}$. Such solutions (henceforth *non-OQM liquidity trap solutions*) therefore do not satisfy the aggregate resource constraint (50).[28] This leads to the following proposition:

Proposition 3.

> *In the New-Keynesian model, the augmented Taylor rule (given in (34)), suffices to rule out non-OQM liquidity trap equilibria that are also rational expectations equilibria.*

Corollary.

> *If the nominal interest rate on base money is zero, any monetary rule that prescribes a positive growth rate of the nominal stock of base money when the nominal interest rate is at its zero lower bound, suffices to rule out liquidity trap equilibria that are also rational expectations equilibria.*

The analysis also suggests how an economic system may find itself on a non-OQM liquidity trap solution trajectory when following the Taylor rule, despite the authorities' intent to follow a monetary rule such as (33) when the nominal interest rate is at its lower bound.

Proposition 4[29]:

> *When the interest rate on money is zero, perverse expectations, that is, expectations that the authorities will demonetise the economy in the long run (that is, $P(t')^{-1}\lim_{s\to\infty} e^{-(s-t')\bar{i}^M}M(s) = 0$) can cause the economy to be on a non-OQM liquidity trap solution trajectory at time $t = t'$.*

The argument that the adoption of the augmented Taylor rule, given in (34) would preclude liquidity trap equilibria relied on the effect on current consumption of private agent's expectations concerning the behaviour of the nominal money stock in the very long run, should the economy land in a liquidity trap. If households know and believe (34), a liquidity trap cannot be a rational expectations equilibrium. If households do not know or believe (34), a liquidity trap solution trajectory can be a long-lasting, non-rational but 'not empirically refuted' expectations equilibrium - an equilibrium supported by 'incorrect but irrefutable' (*IBI*) expectations . If the private sector expects that any past or present increase in the stock of nominal base money will eventually be reversed, the long-run expected value of the nominal money stock could be too low to rule out persistent non-OQM liquidity trap equilibria.

Presumably, if the authorities persist in their expansionary monetary policies, sooner or later, learning would occur and eventually expectations concerning the long-run behaviour of the nominal money stock would be revised upwards. This could, in the long run, provide an exit from the non-OQM liquidity trap trajectory. However, because the anticipated reductions in the money stock relate to the (infinitely) distant future, households may, for a long time, not take the absence of evidence to be evidence of absence of long-run contractionary monetary policy. The economy could be stuck for some time on a non-OQM liquidity trap trajectory.

3.4 Irredeemable money in overlapping generations models

In many ways, the asymmetric perception of base money by the private and public sectors is more easily rationalised in the overlapping generations (OLG) model (without operative intergenerational gifts) than in the representative agent model. In the finite and certain horizon OLG models of Allais (1947) and Samuelson (1958), each generation will aim for a life-time consumption profile that implies zero financial wealth at the end of life. All generations currently alive will base their consumption decisions on the individually and collective rational assumption that they all can reduce their end-of-life holdings of net financial assets (bonds *plus* base money) to zero. They can do so because new generations are born every period that will absorb the financial assets divested by the old. These unborn generations are not present in today's markets, however. When this OLG model of household consumption is juxtaposed with an infinite-lived state that views money as irredeemable, the aggregate consumption function (of all generations currently alive) will have the present discounted value of the terminal stock of base money as an argument in exactly the manner proposed for the representative agent model in this paper.(see Buiter (2003a)).[30]

4. Conclusion

The paper provides a formalisation of the monetary folk proposition that government fiat money is an asset of the private holder but not a liability of the public issuer. It shows how the irredeemable nature of the monetary liabilities of the state can be incorporated into otherwise conventional approaches to monetary economics.

Fiat base money *is* net wealth to households and influences consumption through a real balance or Pigou effect, in the restricted sense that, when the households' and government's intertemporal budget constraints are consolidated, the present value of the (infinitely distant) terminal stock of fiat government money is part of perceived consolidated comprehensive wealth. The issuance of irredeemable base money can therefore have a pure wealth effect on consumption (holding constant all prices, endowments and real public spending). In equilibria that are not liquidity traps, this paper's asymmetric treatment of the solvency constraints of the private sector and the state has no implications for the behaviour of nominal or real equilibrium prices and quantities. It plays a role whenever, with symmetric treatment of private and public solvency constraints, the economy would be in a liquidity trap.

When the state views money as irredeemable but the private sector views it as a realizable financial asset, simple restrictions on the monetary policy rule suffice to rule out non-OQM liquidity trap equilibria as rational expectations equilibria. If the interest rate on base money is zero, the flexible price level model supports a liquidity trap equilibrium only if the monetary authorities are expected to reduce the money stock to zero in the long run. In the New-Keynesian variant, a liquidity trap equilibrium is ruled out whenever the authorities are expected to keep the nominal stock of base money above some finite threshold level in the long run. Any positive long-run expected growth rate for the nominal stock of base money is sufficient to rule out a liquidity trap equilibrium. Liquidity trap equilibria are therefore possible as rational expectations equilibria only if monetary policies are strongly contractionary in the long run. With non-rational expectations - e.g. the incorrect belief that the monetary authorities will, in the long run, reverse and undo any past and present increases in the stock of base money - liquidity trap equilibria can exist for as long as these *incorrect but irrefutable* expectations persist.

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Figures

Figure 1

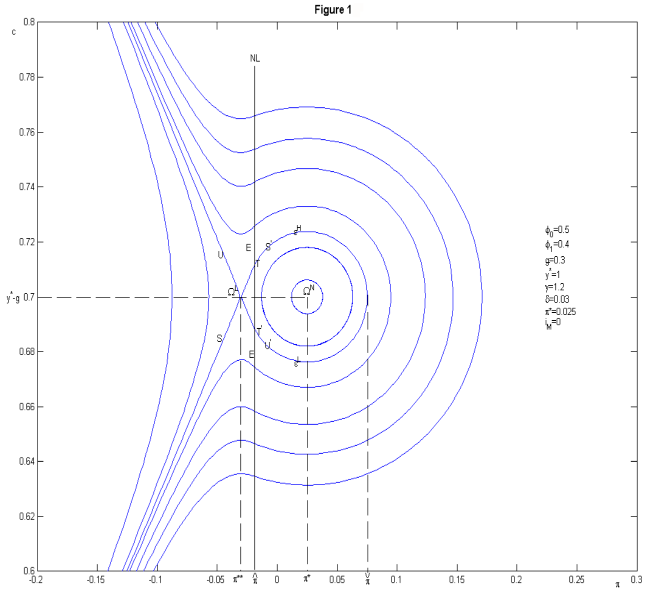
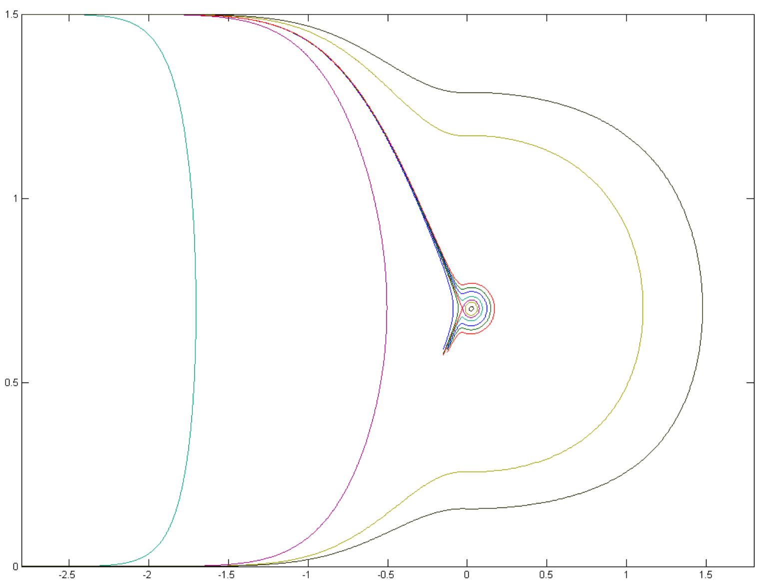


Figure 2



Footnotes

1.The Merriam-Webster Collegiate Dictionary defines *convertible* as: ‘capable of being exchanged for a specified equivalent (as another currency or security) (a bond *convertible* to 12 shares of common stock)’. *Inconvertible* is defined as ‘not redeemable for money in coin: *inconvertible paper currency’,* or ‘not able to be legally exchanged for another currency’.

2. It would also hold with an Old-Keynesian supply side, e.g. an accelerationist Phillips curve.

3.See Hall (1983) and Buiter and Panigirtzoglou (2001, 2003) for examples of models where the nominal interest rate on money is determined by a simple rule.

4.The folk definition is the one found in generalist dictionaries. For instance, the Concise Oxford Dictionary defines *legal tender* as "currency that cannot legally be refused in payment of debt (usually up a limited amount for baser coins, etc.)". This folk definition of legal tender is stronger than the legal definition (see Buiter (2003a)).

5. Three successive monetary reforms encouraged holders of Confederate Treasury notes to exchange these notes for Confederate bonds by imposing deadlines on their convertibility (see Burdekin and Weidenmier (2003)).

6. The expression ‘irrational but irrefutable’ expectations is due to Anne Sibert.

7. The convenient CES form for the sub-utility function *z* given below, where η denotes the static elasticity of substitution between consumption and real money balances,

$$z \equiv \begin{cases} \left[\alpha^{\frac{1}{\eta}} c^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} m^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, & 1 \neq \eta > 0 \\ c^{\alpha} m^{1-\alpha}, & \eta = 1. \end{cases}$$

does not have satiation at a finite value of *m/c*.

8. It is also assumed that $v(\eta)$ is sufficiently large to ensure an interior solution for consumption in each period.

9.A cash-in-advance version of the representative agent model can also be found in http://www.nber.org/wbuiter/heliap1.pdf.

10. As long as $u_c > 0$, the household solvency constraint (4) binds and equation (13) is part of the optimal programme. Under the conditions imposed on the household optimisation programme, the transversality condition (11) or (12) is sufficient (together with the other optimality conditions (9) and (10) or (15) to (17)) for an optimum. There is an extensive literature on the necessity and sufficiency of the transversality condition in infinite horizon optimisation problems, see e.g. Arrow and Kurz (1970), Weitzman (1973), Araujo and Scheinkman (1983), Stokey and Lucas with Prescott (1989), Michel (1990) and Kamihigashi (2001, 2002).

11. For the double CES specification of equation (6) and footnote 7, the counterparts to equations (13) and (14) are $\frac{c_{t+1}}{c_t} = \left(\frac{1+r_{t+1}}{1+\rho}\right)^{\sigma}\left(\frac{\Lambda_{t+2}}{\Lambda_{t+1}}\right)^{\frac{\sigma-\eta}{\eta-1}}$, $\Lambda_t \equiv 1 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+i_t}{i_t - i_t^M}\right)^{\eta-1}$ and

$c_t = \frac{\alpha}{1-\alpha}\left(\frac{i_{t+1} - i_{t+1}^M}{1+i_{t+1}}\right)^{\eta} m_t^h$, $i_{t+1} \geq i_{t+1}^M$. There are some special cases. When $\sigma = \eta \neq 1$, $c_{t+1}/c_t = \left((1+r_{t+1})/(1+\rho)\right)^{\sigma}$. When $\sigma = 1$, $c_{t+1}/c_t = \left((1+r_{t+1})/(1+\rho)\right)\Lambda_{t+1}/\Lambda_{t+2}$. When $\eta = 1$, $c_{t+1}/c_t = \left((1+r_{t+1})/(1+\rho)\right)^{\sigma}\left((1+i_{t+1})(1+i_{t+2})^{-1}(i_{t+2}-i_{t+2}^M)(i_{t+1}-i_{t+1}^M)^{-1}\right)^{(1-\sigma)(1-\alpha)}$.

12. For the double CES specification of equation (6) and footnote 7, we have $c_t = \tilde{\mu}_t w_t$, where w_t is the same

as in (19) and $\tilde{\mu}_t = \left[\sum_{j=t}^{\infty} \Lambda_{j+1} \frac{(R_t/R_j)^{\sigma-1}}{(1+\rho)^{\sigma(j-t)}} \left(\frac{\Lambda_{j+1}}{\Lambda_{t+1}}\right)^{\frac{\sigma-\varphi}{\varphi-1}}\right]^{-1}$. This expression for the marginal propensity to consume out of comprehensive wealth simplifies when future real and nominal interest rates are expected to be constant. In that case, we get: $\mu = \Lambda^{-1}[(1+\rho)^{\sigma}-(1+r)^{\sigma-1}](1+\rho)^{-\sigma}$. From this equation, the *steady state* marginal propensity to consume out of comprehensive wealth is independent of the nominal interest rate only if $\varphi = 1$. In that case the steady-state marginal propensity to consume becomes

$\mu = \alpha[(1+\rho)^{\sigma}-(1+r)^{\sigma-1}](1+\rho)^{-\sigma}$. However, $\varphi = 1$ is not sufficient for the marginal propensity to consume to be independent of the sequence of current and future nominal interest rates outside steady state. For that to be true we require both φ and σ, the intertemporal substitution elasticity, to be equal to unity (see Fischer (1979a,b) and Buiter (2003a)). When $\varphi = \sigma = 1$, the marginal propensity to consume out of comprehensive wealth simplifies to the expression given below. It is now also independent of the real interest rate:

$$\mu = \alpha\rho(1+\rho)^{-1}.$$

13. The government's intertemporal budget constraint, holding with equality, can equivalently be written as:

$$(1+i_1)B_0 P_1^{-1} \equiv \sum_{t=1}^{\infty} R_t\left(\tau_t - g_t + [M_t - (1+i_t^M)M_{t-1}]P_t^{-1}\right).$$

14. Without irredeemable money, the last term on the RHS of (23), $(1/P_1)\lim_{T\to\infty} I_T(1+i_T^M)M_{T-1}^g$, would equal zero in equilibrium. This is because with symmetric solvency constraints, the government's solvency constraint (24) would hold with equality in equilibrium.

15. This statement applies to the case where the nominal interest rate on base money, i^M, equals zero.

16. For the double CES specification of equation (6) and footnote 7, the counterpart to equations (27) to (29) is $c_t = \tilde{\mu}_t \hat{w}_t$, where $\hat{w}_t$ is defined in (28) and $\tilde{\mu}_t \equiv \left[\sum_{j=t}^{\infty} \frac{(R_t/R_j)^{\sigma-1}}{(1+\rho)^{\sigma(j-t)}} \left(\frac{\Lambda_{j+1}}{\Lambda_{t+1}}\right)^{\frac{\sigma-\varphi}{\varphi-1}}\right]^{-1}$.

17. In overlapping generations models without operative intergenerational gift or bequest motives, postponing taxes while keeping their present discounted value constant will boost consumption demand if this fiscal action redistributes resources from households with long remaining time horizons (the young and the unborn) to households with short remaining time horizons (the old). See Buiter (1991, 2003a).

18. Without irredeemable money, the FFMP would have to ensure that the present value of the terminal stock of aggregate monetary and non-monetary debt is always non-positive. A simple rule that ensures this almost surely (whenever the long-run real interest rate is positive), is given below. It keeps constant the real value of the aggregate monetary and non-monetary public debt constant, that is, $f_{t+1} = f_t = f_0,\ t \geq 0$. Taxes in that case would be given by : . $\tau_t = \bar{g} + \frac{r_{t+1}}{1+r_{t+1}} f_0 + \left(\frac{i_{t+1}^M - i_{t+1}}{1+i_{t+1}}\right) m_t = \bar{g} + \frac{r_{t+1}}{1+r_{t+1}} f_0 + \left(\frac{\alpha-1}{\alpha}\right)\left(\frac{i_{t+1}-i_{t+1}^M}{1+i_{t+1}}\right)^{1-\varphi} c_t$

19. It is a simplified Taylor rule, because the nominal interest rate does not respond to the output gap, $y_t - y^*$. Nothing significant depends on this simplification.

20. Note that this proof works even if convergence to the liquidity trap is only asymptotic: t_1 could be infinite.

21. Real public spending $\bar{g}$, endowments y^*, the growth rate of base money, ν, parameters ρ, α, σ, φ, $\bar{i}^M$ and the initial non-monetary debt stock, B_0 are the same.

22. It is the fact that the initial, predetermined nominal money stocks differ that makes this an a-historical comparative statics exercise.

23.They would be the same if there were no nominally denominated non-monetary debt.

24.There may exist other equilibria also, including non-monetary and sunspot or bubble equilibria.

25. The details of the derivation of the continuous time version of the model are available at http://www.nber.org/~wbuiter/heliap2.pdf.

26. The continuous time Taylor rule is given by $i = \rho + \pi^* + \gamma(\pi - \pi^*)$ when $\pi > \hat{\pi}$, $i = \bar{i}^M$ otherwise, and equation (49) applies.

27. Note that from a welfare point of view, these explosively deflationary solutions are not obviously undesirable: consumption ends up at the physical maximum and there is satiation with real money balances.

28.A growth rate of the nominal stock of base money *permanently* higher than the nominal interest rate on base money is sufficient but not necessary for this result. For instance, if $\bar{i}^M = 0$, as long as the long-run nominal stock of base money expected at $t = t'$ (when a solution to (46) and (47) crosses permanently into the liquidity trap region) satisfies $\lim_{s\to\infty} M(s) > P(t')[\bar{y} - \bar{g} - \rho \int_t^\infty e^{-\int_t^v r(u)du}(y(v) - \bar{g})dv]$, it cannot be a rational expectations solution to (46) to (50).

29.The extension to the case where $\bar{i}^M \neq 0$ is obvious.

30. Buiter (2003a) also considers the Yaari-Blanchard OLG model. The absence of debt neutrality in this model means that, for instance, postponing taxes by borrowing while keeping their present value constant boosts consumption demand, as long as the birth rate is positive. The irredeemability of money, however, makes for a quite distinct wealth effect, which affects aggregate consumption in exactly the same way as it does in the representative agent model.