Appendix 1: A cash-in-advance model

The money-in-the-direct utility function used in most of the paper has the property that the demand for real base money (relative to private consumption) goes to infinity as the opportunity cost of holding real money balances goes to zero from above. The liquidity trap-preventing properties of irredeemable money do not depend on this particular feature of the money demand function. This is readily demonstrated by considering a simple cash-in-advance alternative. The details of the trading technology are from Lucas (1980, 1982) (see also Sargent (1987, Chapter 5, pp. 156-162)).

The unit period, *t*, say, is partitioned into three distinct sub-periods, each of which contains one trading session. Households/portfolio holders are divided into shoppers and workers who do not communicate until the third sub-period. No household can consume its own endowment. Purchases of consumption goods by households and by the government are subject to a Clowerian cash-in-advance constraint (Clower (1967)). In sub-period one, only securities are traded. During that sub-period, the consumption good cannot be traded and firms cannot pay out the period-*t* endowments to households. The securities traded in sub-period one include asset stocks, base money and interest-bearing non-monetary debt or bonds, carried over from the previous period. Without affecting any of the results of this paper, the asset menu could be extended to include tradable equity - ownership claims to the endowment streams - although in our formal model, equity is not included in the tradable asset menu. The endowment is therefore rather like labour time or labour services. There is a market for current labour services but, in the absence of slavery, no market in ownership claims to future labour services. Each household therefore owns a non-traded store of value – the technology that generates its current and future labour endowments. The government announces its taxes, public spending, net new money issuance and net new debt issuance for period *t* at the beginning of the period, before the securities markets open and pays interest and principal due on its outstanding stocks of debt instruments. Both the household sector and the government are subject to a cash-in-advance constraint on their purchases of the perishable consumption good. In sub-period one, when the financial markets are open, the household sector and the government have acquire the money balances each needs to purchase period *t*'s planned consumption.

For the household sector, the cash-in-advance constraint is

$$\begin{aligned} z_t^h &= P_t c_t \\ &= P_t c_t \quad \text{if } i_{t+1} > i_{t+1}^M \end{aligned} \tag{A1.1}$$

For the government we assume that the cash-in-advance constraint always holds strictly:

$$z_t^g = P_t g_t \tag{A1.2}$$

In sub-period two, the financial markets are closed. Each household shopper and the government purchases consumption goods with the money it has acquired in sub-period one. Each worker sells the perishable period *t* endowment of its household to the shoppers of the other households or to the government. The money received by a worker from the sale of its household endowment to other shoppers and/or the government in the second sub-period is handed over to its household's shopper in sub-period three, when both the goods market and the financial markets are closed. The household therefore has to hold the money it has received during the goods trading period, $P_t(c_t + g_t) + z_t^h - P_t c_t = P_t y_t + z_t^h - P_t c_t$, till the beginning of period *t+1,* when it will result in a pay out of $(1+i_t^M)(P_t y_t + z_t^h - P_t c_t)$ units of money. The government's period budget identity, solvency constraint and intertemporal budget constraint are again given by (28), (29) and (30) or, equivalently, by (35), (36) and (37).

For the representative household, we have the following equilibrium condition for the asset market trading during the first sub-period of period t:

$$z_t^h P_t^{-1} + \tau_t + B_t^h P_t^{-1} \leq f_t^h \qquad \textbf{(A1.3)}$$

With period felicity increasing in consumption, (A1.3) will hold with equality. The value of the financial assets held by the representative household at the beginning of period $t+1$ (including interest due) is given by:

$$f_{t+1}^h \equiv (1+i_{t+1}^M)[P_t y_t + (z_t^h - P_t c_t)]P_{t+1}^{-1} + (1+i_{t+1})B_t P_{t+1}^{-1} \qquad \textbf{(A1.4)}$$

Equations (A1.3) holding with equality), (A1.4) and (4) imply that

$$f_{t+1}^h \equiv (1+r_{t+1})(f_t^h + y_t - \tau_t - c_t) + (i_{t+1}^M - i_{t+1})(P_t y_t + z_t^h - P_t c_t)P_{t+1}^{-1} \qquad \textbf{(A1.5)}$$

The first term on the RHS of equation represents sum of the monetary receipts from the sale of the endowment in period t, which had to be carried over into period $t+1$ (plus any interest earned) and the 'excess cash balances' carried by the household into period $t+1$, plus any interest earned. The term $(i_{t+1}^M - i_{t+1})(z_t^h - P_t c_t)P_{t+1}^{-1}$ will be zero both when $i_{t+1}^M < i_{t+1}$ and when $i_{t+1}^M = i_{t+1}$. From the point of view of the household's optimization programme, the sequence $\{(i_{t+1}^M - i_{t+1})P_t P_{t+1}^{-1} y_t;\ t \geq 1\}$ is exogenous. The household's monetary choice variable in period t is z_t^h.

Formally, we get our cash-in-advance model by setting $\alpha = 1$ in equation (3) and adding the cash-in-advance constraint (A1.1). The solvency constraint of the household is again given by equation (7). The household intertemporal budget constraint is:

$$f_t^h = \sum_{j=t}^{\infty} R_{t+1,j}[c_j + \tau_j - y_j + (i_{j+1} - i_{j+1}^M)(z_j^h - P_j c_j + P_j y_j) P_{j+1}^{-1}] \qquad \textbf{(A1.6)}$$

The Euler equation for private consumption is

$$c_{t+1}/c_t = [(1+r_{t+1})/(1+\rho)]^{\sigma} \qquad \textbf{(A1.7)}$$

The consumption function for the cash-in-advance model is

$$c_t = \tilde{\mu}_t[w_t + \sum_{j=t}^{\infty} R_{t+1,j}(i_{j+1}^M - i_{j+1}) P_j P_{j+1}^{-1} y_j] \qquad \textbf{(A1.8)}$$

where comprehensive private wealth, w_t, is defined in equations (22), (23) and (24). The marginal propensity to consume out of comprehensive wealth for the cash-in-advance model is:

$$\tilde{\mu}_t = [\sum_{j=t}^{\infty} (1+\rho)^{-\sigma(j-t)} \prod_{s=t+1}^{j} (1+r_s)^{\sigma-1}]^{-1} \qquad \textbf{(A1.9)}$$

Two equilibrium conditions that have to apply regardless of the details of the 'supply side' of the model are monetary equilibrium:

$$\begin{aligned} M_t &\geq P_t(c_t + g_t) \\ &= P_t(c_t + g_t) \quad \text{if } i_{t+1} > i_{t+1}^M \end{aligned} \qquad \textbf{(A1.10)}$$

and output equals expenditure or demand, given in equation (51). This permits us to rewrite the consumption function as

$$c_t = \bar{\mu}_t[w_t + \sum_{j=t}^{\infty} R_{t+1,j}(i_{j+1}^M - i_{j+1})P_j P_{j+1}^{-1} g_j] \qquad \textbf{(A1.11)}$$

where

$$\bar{\mu}_t = \left\{ \sum_{j=t}^{\infty} \left[1 + \left(\frac{i_j - i_j^M}{1+\pi_j} \right) \left(\frac{1+\rho}{1+r_j} \right)^{\sigma} \right] \prod_{s=t+1}^{j} \left[\frac{(1+r_s)^{\sigma-1}}{(1+\rho)^{\sigma}} \right] \right\}^{-1} \qquad \textbf{(A1.12)}$$

Again making the assumption that $f_t^g = f_t^h = f_t,\ t \geq 0$, we obtain, by substituting the government's intertemporal budget constraint into the household consumption function and using $M_j = P_j y_j$, the consumption function given in (A1.13). This has the same implications for monetary policy and the liquidity trap as the consumption function for the money-in-the-direct-utility function in equation

$$c_t = \tilde{\mu}_t \hat{w}_t \qquad \textbf{(A1.13)}$$

where $\hat{w}_t$ is given in equation (41).