Liquidity Traps

How to Avoid Them and How to Escape Them

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31 March, 1999

This revision, 1 June 2001

[**] The views and opinions expressed are those of the authors. They do not necessarily reflect the views and opinions of the European Bank for Reconstruction and Development or of the Bank of England. We would like to thank Olivier Blanchard, Alec Chrystal, DeAnne Julius, Nobu Kiyotaki, David Laidler and Ronald McKinnon for helpful discussions on the subject. Special thanks are due to Anne Sibert for key technical assistance in Section II of the paper. We are also very much indebted to Ian Plenderleith, John Rippon and Rupert Thorne, for educating us on the subject of bearer bonds, and to Jenny Salvage and Nick Hanchard for assistance with the Figures. Finally we would like C.K. Folkertsma and other participants in the Seminar in Honour of Martin Fase, on 26 March 2001 at the Nederlandsche Bank, Amsterdam, for breathing life back into this paper.

Abstract

An economy is in a liquidity trap when monetary policy cannot influence either real or nominal variables of interest. A necessary condition for this is that the short nominal interest rate is constrained by its lower bound, typically zero. The paper develops a small analytical model to show how an economy can get into a liquidity trap, how it can avoid getting into one and how it can get out. We also consider the empirical likelihood of the UK economy hitting the zero nominal rate bound, by investigating the relationship between the level of the short nominal interest rate and its volatility. The empirical evidence on this issue is mixed.

To avoid the risk of falling into a liquidity trap, or to escape from one, the authorities can remove the zero nominal interest rate floor, by adopting an augmented monetary rule that systematically keeps the own nominal interest rate on currency below the nominal interest rate on non-monetary instruments. This involves paying interest, negative or positive, on government 'bearer bonds' -- coin and currency, that is 'taxing money', as advocated by Gesell. There are likely to be significant shoe leather costs associated with any scheme to tax currency.

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AEA-JEL Classification: B22, E41, E31, E32, E51, E52, E58, N12, N13, N14.
Key words: Liquidity trap; Gesell, stamp scrip, inflation targeting; multiple equilibria.

(I) Introduction

The liquidity trap used to be a standard topic in macro textbooks, but disappeared in the 1970s. Because of recent developments in Japan, where short nominal interest rates have been very close to zero for quite some time, liquidity traps are receiving attention again. The economy is said to be in a liquidity trap when the ability to use monetary policy to stimulate demand has vanished. The conditions that have to be satisfied for monetary policy to fail to affect both real and nominal variables depend on one's view on the monetary transmission mechanism. A necessary condition for monetary policy ineffectiveness is that monetary policy cannot affect the joint distribution of real and nominal rates of return on financial and real assets.[1] In very simple closed economy IS-LM-type models, where there is but one rate of return – a nominal interest rate of unspecified maturity - monetary policy in powerless when an increase in the nominal money stock cannot reduce this short nominal rate. In models with a more extended menu of financial and real assets, for monetary policy to be powerless, the yields on all non-monetary assets (short and long maturity, private and public, financial and real) must be at their lower bounds, not just the short nominal interest rate. With portfolio holders indifferent a regards the composition of their financial wealth between money and all non-money assets, changes in the supply of money cannot affect the spreads between money and non-money assets. When, as is institutionally more relevant, the short nominal interest rate is taken to be the monetary instrument, rather than some monetary aggregate, the argument is not changed in any essential way. When monetary policy also

[1] In an open economy, the relevant rates of return would include the expected rate of depreciation of the exchange rate. If, say, domestic nominal interest rates are linked to world interest rates through an uncovered interest parity condition, monetary policy will still be ineffective whenever the demand for money becomes infinitely interest-sensitive. When international interest rate differentials can be influenced by changes in the relative supplies of non-monetary government debt instruments denominated in different currencies, there is a further monetary transmission channel.

works through channels other than rates of return (say, through the availability as well as the cost of credit, or through the exchange rate), a liquidity trap is only operative if these additional liquidity, credit or exchange rate channels of monetary transmission too are blocked.[2]

The textbook treatment of liquidity traps, based on Hicks's [1936] interpretation of Keynes [1936], involves the assumption that the opportunity cost of holding money is a long nominal interest rate, and that the demand for money becomes infinitely sensitive to the current value of this long nominal yield because of regressive (what we now call 'mean reverting') expectations about the future behaviour of the long nominal yield (see e.g. Tobin [1958] and Laidler [1993]). In most modern theories, the short (riskless) nominal interest rate on government debt is the opportunity cost of holding currency. The nominal yield on short government debt is then related to yields on other assets through equilibrium asset pricing relationships such as the expectations theory of the term structure of interest rates, the CAPM model or other portfolio balance models.

The modern argument assumes explicitly (and the traditional theories assumed implicitly) that the pecuniary own rate of return on money was zero, an appropriate assumption for coin and currency, although not for the liabilities of private deposit-taking institutions that make up most of the broader monetary aggregates, which now typically have positive nominal returns. With the own rate of return on currency administratively fixed at

[2] A 'helicopter drop' of money will, unlike money injected through open market purchases, have a wealth effect on private consumption, for a given distribution of rates of return. Since the essence of this part of the monetary transmission mechanism is a transfer of wealth between the public and private sectors, we consider it to be fiscal rather than monetary policy. In the rest of this paper, monetary policy is interpreted as pertaining only to the composition of the government's financial liabilities between monetary and non-monetary claims. The magnitude of the government's aggregate stock of financial liabilities is the province of intertemporal fiscal policy.

zero, a floor for the spread between the non-monetary and monetary claims becomes a floor for the nominal yields on non-monetary financial instruments.

In Section II of the paper, we develop a small analytical model of a closed economy in which, under a conventional Taylor rule for the nominal interest rate, a liquidity trap can exist. We show how a simple augmentation of the Taylor rule can eliminate liquidity trap equilibria. The augmented rule may involve paying (negative as well as positive) interest on currency. Section III focuses on the empirical issue of the likelihood that the instantaneous risk-free nominal yield, the short nominal interest rate, would reach its zero floor.[3] We fully recognise that a zero short nominal rate is only a necessary, and not a sufficient, condition for the liquidity trap to be operative. The practicalities of paying interest on currency are reviewed in Section IV.

Even in the simple analytical model of a liquidity trap developed in Section II, monetary policy is powerless only if it cannot affect nominal yields at any maturity. While the demand for narrow money or base money in that model depends on just one opportunity cost variable: the current short nominal interest rate, aggregate demand is affected by current and anticipated future short rates. The economy is in a liquidity trap only if the entire yield curve is flat at a zero level (see also Orphanides and Wieland [1998]). In an open economy extension of this model, the same conclusion would apply if domestic and foreign nominal

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[3] With the nominal rate on currency fixed at zero, the nominal interest rate on other financial claims can be negative if the cost of holding and storing currency exceeds that of holding and storing these other claims, because of differential tax treatment of currency and other stores of value or because of different collateralizability properties (see Porter [1999]). From late 1938 to early 1941, weekly data on auctions of new Treasury bills in the US showed occasional negative yields. During this period, negative yields were also reported on US Treasury bonds with up to two years maturity (see Clouse et. al. [1999] and Cecchetti [1988]). In Japan, short-term interest rates on government debt and some inter-bank lending became slightly negative in late 1998. These exceptions to the zero nominal interest rate floor have been rare and unimportant quantitatively. For expositional simplicity, we proceed in what follows as if the floor were equal to the pecuniary rate of return on currency, that is, zero under historical practice.

interest rates were linked through an uncovered interest parity (UIP) condition. If domestic and foreign currency denominated non-monetary securities were imperfect substitutes, monetary policy might work through the exchange rate channel, even with the entire domestic yield curve flat at zero.

This liquidity trap used to be treated, in the mainstream accounts of the monetary transmission mechanism, as a theoretical curiosum without practical relevance.[4] The revival of interest in the liquidity trap is not surprising.

First, Japan is in a protracted economic slump. Short nominal interest rates there are near zero. Zero is the absolute nominal interest rate floor in Japan because yen notes and coin bear a zero nominal interest rate. Of course, the yields on longer-maturity government debt instruments remain positive (albeit at historically low levels), and the nominal yields on a variety of private and public financial and real assets also remain positive. While the strict conditions for a liquidity trap to be operative are therefore not satisfied, monetary policy in Japan currently appears to have a rather limited effect on aggregate demand. A number of observers have concluded that there is a liquidity trap at work (see e.g. Krugman [1998a,b,c,d; 1999], Ito [1998], McKinnon and Ohno [1999] and Svensson [2000]); for a view that liquidity traps are unlikely to pose a problem, see Meltzer [1999] and Hondroyiannis, Swamy and Tavlas [2000])[5].

Second, HICP inflation in Euroland averaged 1.1 percent per annum during 1999. The ECB's repo rate reached a trough of 2.5 percent during April 1999. At the time, this raised the question as to whether a margin of two hundred and fifty basis points provides

[4] [4]See e.g. Romer [1996], which covers the topic as half of an exercise at the end of the chapter 5, "Traditional Keynesian Theories of Fluctuations".

[5] Hondroyannis, Swamy and Tavlas [2000] argue that because their empirical study suggests that the interest rate elasticity of money demand is lower at lower rates of interest (and has declined in recent years), Japan cannot be in a liquidity trap. This is a non-sequitur. If all

enough insurance against a slump in aggregate demand. Demand could weaken to such an extent that a cut in the short nominal rate of more than two hundred and fifty points would be required to boost aggregate demand sufficiently.

For virtually all monetary authorities in developed market economies, the monetary instrument is a short nominal interest rate.[6] Monetary policy impacts aggregate demand primarily through its effect on real interest rates, short and long. The transmission of monetary policy through other real asset prices, including the real exchange rate, depends on the ability of the monetary authorities to influence real interest rates. For the monetary authority to affect real demand, changes in nominal interest rates have to be translated, at least temporarily, into changes in real interest rates. In a moderate or low-inflation environment, inflation and inflation expectations tend to move only gradually and sluggishly. This Keynesian feature of the economy gives monetary policy a temporary handle on the real economy.

If short nominal interest rates cannot fall any further, short real rates can only be pushed down through a rise in the expected rate of inflation. If the price stability gospel has been widely internalised by market participants, expected inflation is unlikely to rise to produce the required cut in real rates.

Once an economy is in such a situation, it is not possible to get out of it using the conventional monetary policy instruments - changes in the short nominal interest rates.

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interest rates are at the zero floor, monetary policy is ineffective, no matter how low the interest sensitivity of money demand.

[6]The argument could be recast in terms of the monetary authority using some monetary aggregate as the instrument, with the short nominal interest rate on risk-free non-monetary financial claims treated as endogenous. Taking the short nominal rate as the instrument has two advantages. First, the exposition in simpler. Second, it is what central banks actually do. Changes in reserve requirements, open market operations etc. are best viewed as ways of changing the interest rate. .In an open economy, the other institutionally relevant instrument of monetary policy is the nominal exchange rate. When capital mobility is limited, the short

Inflation expectations are not a policy instrument. Why would inflation expectations rise when monetary policy cannot stimulate demand?

Of course, in a liquidity trap, expansionary fiscal policy, or any other exogenous shock to aggregate demand, is supposed to be at its most effective. There are, however, conditions under which fiscal policy cannot be used to stimulate aggregate demand. Debt-financed lump-sum tax cuts could fail to stimulate aggregate demand if there is Ricardian equivalence or debt neutrality. Alternatively, the government's creditworthiness may be so impaired that it cannot borrow. Finally, there could be external, Maastricht Treaty or Stability and Growth Pact-like external constraints on a government's ability to use deficit financing.

If Ricardian equivalence holds, a temporary increase in exhaustive public spending will, even with a balanced budget, and in virtually any model of the economy, boosts aggregate demand. For this fiscal policy channel to be ineffective also, exhaustive public spending must be a direct perfect substitute for exhaustive private spending, say because public consumption is a perfect substitute for private consumption in private utility functions, and public investment is a perfect substitute for private investment in private production functions.[7] Recent theoretical analyses of liquidity traps include Wolman [1998], Buiter and Panigirtzoglou [1999], McCallum [2000, 2001], Christiano [2000], and Benhabib, Schmitt-Grohé and Uribe [1999a,b],

(II) A Simple Model of the Liquidity Trap

We model a simple, closed endowment economy with a single perishable commodity that can be consumed privately or publicly.

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nominal interest rate and the nominal exchange rate both can be instruments of policy, at any rate in the short run.

[7]See Buiter [1977].

Households

A representative infinite-lived, competitive consumer maximises for all $t \geq 0$ the utility functional given in (1) subject to his instantaneous flow budget identity (2), solvency constraint (3) and his initial financial wealth. We use the simplest money-in-the-direct-utility-function approach to motivate a demand for money despite it being dominated as a store of value. Instantaneous felicity therefore depends on consumption and real money balances. We define the following notation; c is real private consumption, y is real output, τ is real (lump-sum taxes), M is the nominal stock of base money (currency), B is the nominal stock of short (strictly zero maturity) non-monetary debt, i is the instantaneous risk-free nominal interest rate on non-monetary debt, i_M is the instantaneous risk-free nominal interest rate on money (or the own rate on money), p is the price level in terms of money, a is the real stock of private financial wealth, m is the stock of real currency and b the stock of real non-monetary debt.

$$\int_t^{\infty} e^{-\delta(v-t)}[\frac{1}{1+\eta}\ln c(v) + \frac{\eta}{1+\eta}\ln m(v)]dv$$

$$\eta > 0 \qquad \textbf{(1)}$$

$$\delta > 0$$

$$\dot{M} + \dot{B} \equiv p(y - \tau - c) + iB + i_M M$$

$$\textbf{(2)}$$

$$c \geq 0; M \geq 0$$

$$\lim_{v\to\infty} e^{-\int_t^v i(u)du}[M(v) + B(v)] \geq 0 \qquad \textbf{(3)}$$

$$M(0) + B(0) = \overline{A}(0) \qquad \textbf{(4)}$$

By definition,

$$a \equiv \frac{M + B}{P} \qquad \textbf{(5)}$$

The household budget identity (2) can be rewritten as follows

$$\dot{a} \equiv ra + y - \tau - c + (i_M - i)m \tag{6}$$

where *r*, the instantaneous real rate of interest on non-monetary assets, is defined by

$$r \equiv i - \pi \tag{7}$$

and $\pi \equiv \frac{\dot{p}}{p}$ is the instantaneous rate of inflation.

The household solvency constraint can now be rewritten as

$$\lim_{v \to \infty} e^{-\int_t^v r(u)du} a(v) \geq 0 \tag{8}$$

and the intertemporal budget constraint for the household sector can be rewritten as:

$$\int_t^\infty e^{-\int_t^v r(u)du} \left[c(v) + \tau(v) + [i(v) - i_M(v)]m(v) - y(v) \right] dv \leq a(t) \tag{9}$$

The first-order conditions for an optimum imply that the solvency constraint will hold with equality. Also,

$$\dot{c} = (r - \delta)c \tag{10}$$

and for $i \geq i_M$,

$$m = \left(\frac{\eta}{i - i_M} \right) c \tag{11}$$

If $i < i_M$, currency would dominate non-monetary financial assets ('bonds') as a store of value. Households would wish to take infinite long positions in money, financed by infinite short positions in non-monetary securities. The rate of return on the portfolio would be infinite. This cannot be an equilibrium.

If $i = i_d$, currency and bonds are perfect substitutes as stores of value. With flexible prices, this will, from the point of view of the household's utility functional, be the first-best equilibrium, characterised by satiation in real money balances. With the logarithmic utility function, satiation occurs only when the stock of money is infinite (relative to the finite consumption level). Provided the authorities provide government money and absorb private bonds in the right (infinite) amounts, this can be an equilibrium.

There is a continuum of identical consumers whose aggregate measure is normalised to *1*. The individual relationships derived in this section therefore also characterise the aggregate behaviour of the consumers.

Government

The budget identity of the consolidated general government and central bank is given in (12). The level of real public consumption is denoted $g \geq 0$.

$$\dot{M} + \dot{B} \equiv iB + i_M M + p(g - \tau) \qquad \textbf{(12)}$$

Again, the initial nominal value of the government's financial liabilities is predetermined, $M(0) + B(0) = \bar{A}(0)$.

This budget identity can be rewritten as

$$\dot{a} \equiv ra + g - \tau + (i_M - i)m \qquad \textbf{(13)}$$

The government solvency constraint is

$$\lim_{v \to \infty} e^{-\int_t^v r(u)du} a(v) \leq 0 \qquad \textbf{(14)}$$

Equations (13) and (14) imply the intertemporal government budget constraint:

$$\int_t^{\infty} e^{-\int_t^v r(u)du} \left[\tau(v) + [i(v) - i_M(v)]m(v) - g(v)\right] dv \geq a(t) \qquad \textbf{(15)}$$

Government consumption spending is exogenous. To ensure that public consumption spending does not exceed total available capacity resources, $\bar{y} > 0$, we therefore have to impose $g < \bar{y}$. With a representative consumer, this model will exhibit debt neutrality or Ricardian equivalence. Without loss of generality, we therefore assume that lump-sum taxes are continuously adjusted to keep the nominal stock of public debt (monetary and non-monetary) constant, $\dot{A}(t) = 0,\ t \geq 0$, that is,

$$\begin{aligned} \tau &= g + ia + (i_M - i)m \\ &= g + i\frac{\overline{A}(0)}{p} + (i_M - i)m \end{aligned} \qquad \textbf{(16)}$$

Monetary policy

The monetary authorities peg the nominal interest rate on currency exogenously:

$$i_M = \overline{i}_M$$

We assume in what follows that the other monetary instrument is the short nominal interest rate on bonds, rather than the level or the growth rate of the nominal money stock. There are two reasons for this. First, it simplifies the exposition. Second, it is how monetary policy is actually conducted in developed market economies.

The monetary authorities are assumed to follow a simplified Taylor rule for the short nominal interest rate on non-monetary financial claims, as long as this does not put the short nominal bond rate below the interest rate on currency. A standard Taylor rule for the short nominal bond rate which restricts the short nominal bond rate not to be below the short nominal rate on currency, would be

$$\begin{aligned} i &= \overline{i} + \gamma\pi + \varepsilon y \quad \text{if} \quad \overline{i} + \gamma\pi + \varepsilon y \geq i_M \\ &= i_M \qquad\qquad\quad \text{if} \quad \overline{i} + \gamma\pi + \varepsilon y < i_M \end{aligned}$$

For our purposes, all that matters is the responsiveness of the short bond rate to the inflation rate. We therefore omit feedback from the level of real GDP (or from the output gap) in what follows. The short nominal interest rate rule then simplifies to

$$\begin{aligned} i &= \overline{i} + \gamma\pi \quad \text{if} \quad \overline{i} + \gamma\pi \geq i_M \\ &= i_M \qquad\ \ \text{if} \quad \overline{i} + \gamma\pi < i_M \end{aligned} \qquad \textbf{(17)}$$

The Taylor rule is sometimes justified as a simple, ad-hoc rule consistent with inflation targeting. If the target rate of inflation is constant at π^*, and equal to the steady-state rate of inflation achieved under the rule, the intercept in the Taylor rule, $\bar{i}$, can be given the following interpretation

$$\bar{i} = \delta + (1-\gamma)\pi^* \qquad \textbf{(18)}$$

This allows us to write the Taylor rule as

$$i = \delta + \pi^* + \gamma(\pi - \pi^*) \qquad \textbf{(19)}$$

or

$$r = \delta + (\gamma - 1)(\pi - \pi^*) \qquad \textbf{(20)}$$

For reasons of space, only a 'Keynesian' variant of the model, characterised by nominal price rigidities, is considered here.[8] In this Keynesian variant, output is demand-determined, the price level, *p,* and the rate of inflation, π, are assumed to be predetermined, and the rate of inflation adjusts to the gap between actual and capacity output through the simplest kind of accelerationist Phillips curve.

$$c + g = y \qquad \textbf{(21)}$$

$$\dot{\pi} = \beta(y - \bar{y})$$
$$\beta > 0 \qquad \textbf{(22)}$$

For simplicity, we assume capacity output to be exogenous and constant.

The behaviour of the economy can be summarised in two first-order differential equations in the non-predetermined state variable c and the predetermined state variable π. The equation governing the behaviour of private consumption growth switches, however,

[8] See Buiter and Panigirtzoglou [1999] for a longer version of the paper which includes an analysis of the liquidity trap with flexible prices.

when the floor on the short nominal interest rate becomes binding (when the economy is at the floor (i_M) for the short nominal interest rate).

$$\dot{\pi} = \beta(c + g - \bar{y}) \qquad \textbf{(23)}$$

$$\begin{aligned} \dot{c} &= [\bar{i} + (\gamma - 1)\pi - \delta]c \quad \text{if } \bar{i} + \gamma\pi \geq i_M \\ &= [i_M - \pi - \delta]c \quad \text{if } \bar{i} + \gamma\pi < i_M \end{aligned} \qquad \textbf{(24)}$$

When the short nominal interest rate floor is not a binding constraint (we shall refer to this as the 'normal' case), saddlepoint stability for the dynamic system requires $\gamma > 1$. A higher rate of inflation leads, through the policy reaction function, to a larger increase in the short nominal bond rate so as to raise the short real rate. As shown in Figure 1a, the $\dot{c} = 0$ locus in the *normal region* (denoted $(\dot{c} = 0)_N$), is vertical in a phase diagram with π on the horizontal axis and c on the vertical axis, at $\pi = \dfrac{\bar{i} - \delta}{1 - \gamma} = \pi^*$. [9]

Figure 1a,b here

When the short nominal interest rate on bonds is at its floor (henceforth in the *'floor region'*), the $\dot{c} = 0$ locus (denoted $(\dot{c} = 0)_F$) is vertical at $\pi = i_M - \delta$. We first consider the case where $i_M - \delta < 0$ and $\pi^* \geq 0$. The first of these assumptions is satisfied if the monetary authorities follow the current institutional practice of not paying interest on cash ($i_M = 0$). The second assumption too is descriptively realistic. With these assumptions the locus $(\dot{c} = 0)_F$ is to the left of $(\dot{c} = 0)_N$. This is the case considered in Figure 1a.

As long as the rate of inflation exceeds $\dfrac{i_M - \bar{i}}{\gamma}$, the short nominal bond rate exceeds the nominal interest rate on currency, and the economy is in the normal region. For inflation rates at or below $\dfrac{i_M - \bar{i}}{\gamma}$, the economy is in the floor region. The switch from the normal to the floor region occurs at $\pi = \frac{i_M - \bar{i}}{\gamma} = \frac{i_M - \delta}{\gamma} + \left(\frac{\gamma - 1}{\gamma}\right)\pi^*$. We shall refer to the boundary of the normal and the floor regions as the *NF* locus in Figure 1a,b,c. When $i_M - \delta < 0$ and $\pi^* \geq 0$, the switching value of π lies between the two $\dot{c} = 0$ loci. This assumption is reflected in Figure 1a. The *NF* locus could either be to the left or to the right of the *c* axis.

There are two steady states - the normal steady state and the liquidity trap steady state - for the nominal bond rate and the rate of inflation. The normal steady state values are:

$$\bar{c} = \bar{y} - g$$

$$\bar{r} = \delta$$

$$\bar{\pi}^N = \frac{\delta - \bar{i}}{\gamma - 1} = \pi^* \qquad \text{(Normal case)}$$

or

$$\bar{\pi}^L = \bar{i}_M - \delta \qquad \text{(Liquidity trap)}$$

$$\bar{i}^N = \frac{\gamma\delta - \bar{i}}{\gamma - 1} > \pi^* \qquad \text{(Normal case)}$$

or

$$\bar{i}^L = \bar{i}_M \qquad \text{(Liquidity trap)}$$

Note that steady state household utility is higher in the liquidity trap than in the normal case. Consumption is the same in both cases and in the liquidity trap steady state households are satiated with real money balances. The government's target rate of inflation, π^*, implicit in the Taylor rule, cannot (unless $\pi^* = \bar{i}_M - \delta$, a case considered below) be rationalised as the steady state rate of inflation that maximises steady-state household utility.

The linear approximation of the normal dynamics at the normal steady state ($c = \bar{c} = \bar{y} - g$ and $\pi = \bar{\pi} = \frac{\delta - \bar{i}}{\gamma - 1} = \pi^*$) is

[9] Here and in what follows we ignore the *c = 0* segment of the c isocline.

$$\begin{bmatrix} \dot{c} \\ \dot{\pi} \end{bmatrix} \approx \begin{bmatrix} 0 & (\gamma-1)(\overline{y}-g) \\ \beta & 0 \end{bmatrix} \begin{bmatrix} c-\overline{c} \\ \pi-\overline{\pi} \end{bmatrix}$$

The determinant of the state matrix is $(1-\gamma)(\overline{y}-g)\beta < 0$ if $\gamma > 1$. The two characteristic roots are $\pm\sqrt{\beta(\gamma-1)(\overline{y}-g)}$. Since $\gamma > 1$, the equilibrium configuration in the neighbourhood of the normal steady state (Ω^N) is a saddlepoint.

The linear approximation of the floor dynamics at the liquidity trap steady state Ω^L (with $c = \overline{c} = \overline{y} - g$ and $\pi = \overline{\pi} = i_M - \delta$) is

$$\begin{bmatrix} \dot{c} \\ \dot{\pi} \end{bmatrix} \approx \begin{bmatrix} 0 & g-\overline{y} \\ \beta & 0 \end{bmatrix} \begin{bmatrix} c-\overline{c} \\ \pi-\overline{\pi} \end{bmatrix}$$

The determinant of the state matrix is $(\overline{y}-g)\beta > 0$. The two characteristic roots are $\pm\sqrt{\beta(g-\overline{y})}$. The linearised dynamic system has two complex conjugate roots with zero real parts or pure imaginary roots. The equilibrium configuration near the liquidity trap steady state (Ω^L in Figure 1a) is neutral and cyclical.

It is also possible to characterise the global dynamics of the model.

From (23) and the normal version of (24) it follows that the slope of the integral curves in $c-\pi$ space is given by

$$\frac{dc}{d\pi} = \frac{[\overline{i} - \delta + (\gamma-1)\pi]c}{\beta(c+g-\overline{y})}$$

This can be rewritten as

$$\beta\left(1+\frac{g-\overline{y}}{c}\right)dc = [\overline{i} - \delta + (\gamma-1)\pi]d\pi$$

As this is separable in c and π, it can be integrated to yield

$$\beta[c+(g-\overline{y})\ln c] = (\overline{i}-\delta)\pi + \frac{(\gamma-1)}{2}\pi^2 + k$$

where *k* is an arbitrary constant.

Provided $(\bar{i}-\delta)^2+2(1-\gamma)\left(k-\beta[c+(g-\bar{y})\ln c]\right)\geq 0$, the integral curves in the normal case (c > 0, $\pi > \frac{i_M-\bar{i}}{\gamma}$) are given by:

$$\pi = \frac{\bar{i}-\delta \pm \sqrt{(\bar{i}-\delta)^2+2(1-\gamma)\left(k-\beta[c+(g-\bar{y})\ln c]\right)}}{1-\gamma}$$

The integral curves for the liquidity trap case (c > 0, $\pi \leq \frac{i_M-\bar{i}}{\gamma}$) are given by

$$\pi = i_M-\delta \pm \sqrt{(i_M-\delta)^2+2\left(k-\beta[c+(g-\bar{y})\ln c]\right)}$$

The liquidity trap configuration is a *center*.[10] Some neighbourhood of this steady state is completely filled by closed integral curves, each containing the steady state in its interior.

The left-hand panel of Figure 1a (to the left of the *NF* locus) shows the behaviour of the system when the dynamics are government by the floor region, the right-hand panel of Figure 1a (to the right of the *NF* curve) shows the behaviour of the system when the dynamics are governed by the normal region. On the boundary of the two regions (when $\pi = \frac{i_M-\bar{i}}{\gamma}$) and at a given level of consumption, the slope of the integral curve in the normal case, $\left.\frac{dc}{d\pi}\right|^N$ is the same as the slope of the integral curve in the liquidity trap case $\left.\frac{dc}{d\pi}\right|^L$.[11] This means that the centre orbits of the liquidity trap region and the saddlepoint solution trajectories of the normal region merge smoothly into each other at the boundary between the two regions.

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[10] Anne Sibert provided the mathematical solution for the behaviour of the system in the liquidity trap region.

[11] It is easily checked that $\left.\frac{dc}{d\pi}\right|^N_{\pi=\frac{i_M-\bar{i}}{\gamma}} = \left.\frac{dc}{d\pi}\right|^L_{\pi=\frac{i_M-\bar{i}}{\gamma}} = \frac{\left[\left(\frac{\gamma-1}{\gamma}\right)i_M+\frac{1}{\gamma}\bar{i}-\delta\right]c}{\beta(c+g-\bar{y})}$

Figure 1a shows the 'merged', global solution trajectories spanning the two regions. The stable branch SS' and the unstable branch UU' through the normal steady state merge on the boundary *NF* into an orbit drawn with reference to the liquidity trap steady state. The lowest inflation rate achieved on this orbit, $\underline{\pi}$, is the lowest starting value for the inflation rate for which well-behaved solutions are defined. Any path starting below $\underline{\pi}$ will eventually lead to an explosive solution, with inflation and consumption rising without bound.[12]

To understand the possible multiplicity of non-explosive solutions that may occur in this model, two properties of admissible solutions deserve emphasising. First, explosively divergent solutions are ruled out, if non-explosive solutions exist. Second, the inflation rate is a predetermined state variable while consumption is non-predetermined. This means that discontinuous changes in the rate of inflation are never allowed and that discontinuous changes in the level of private consumption are permitted only at instants that news arrives. In what follows, news arrives only once, at the initial date.

For all initial inflation rates below $\underline{\pi}$, there only exist explosive solutions. EE' in Figure 1a is one such explosive solution. For all initial rates of inflation less than $\frac{i_M - \bar{i}}{\gamma}$ (to the left of the *NF* locus) but above $\underline{\pi}$, there exists a continuum of solution trajectories that always stay completely within the floor region. *LL'* is one such solution. Nominal interest rates at all maturities will be zero.[13] For any initial rate of inflation below the normal steady state rate of inflation (π^*) but above $\frac{i_M - \bar{i}}{\gamma}$, there will be a continuum of possible solution orbits, all of which are at partly in the floor region. The instantaneous short nominal rate will

[12] The accelerationist Phillips curve does not bound actual output y, which is demand-determined and can, taken literally, exceed capacity output, $\bar{y}$, without bound. A richer model would rule out such explosive real output dynamics.

be zero on that part of the solution curve *LL'* that lies to the left of $\frac{i_M - \bar{i}}{\gamma}$, but there will be longer maturity nominal interest rates that are positive. When the solution trajectory is to the right of $\frac{i_M - \bar{i}}{\gamma}$, even the instantaneous nominal interest rate will be positive.

Figure 1a shows that for any initial rate of inflation below the target level (the normal steady state level π^*) but above $\underline{\pi}$, there also exists a unique orbit (and two values of *c*) that will take the system to the normal steady state. That is the solution trajectory given to the right of the *NF* locus (and to the left of π^*) by the stable branch *SS'* and the unstable branch *UU'* drawn with reference to the normal steady state Ω^N, and to the left of the *NF* locus by that closed orbit, drawn with reference to the liquidity trap steady state, Ω^L, that has tangencies to *SS'* and *UU'* on the *NF* locus at *T* and *T'* respectively. Thus, even if we (rather arbitrarily) restrict admissible solutions to those that converge to the normal steady state, there will be, for any initial rate of inflation below π^* and above $\underline{\pi}$, two initial values of consumption that are consistent with this requirement. In addition, also exists a continuum of solution orbits like *LL'* that cycle, either partly in the normal region and partly in the floor region (like *LL'*) or completely in the floor region. These orbits never reach the normal steady state. When the initial rate of inflation equals $i_M - \delta$, the continuum of solutions for consumption, ranging between ε^H and ε^L includes the liquidity trap steady state, Ω^L.

For any initial inflation rate above the normal steady state rate of inflation, π^*, there is a unique non-explosive solution trajectory. That solution puts consumption on the stable branch through the normal steady state, SS'. There is no non-explosive solution trajectory that moves the system from an initial rate of inflation above π^* into the liquidity trap region.

[13] We assume for concreteness that $i_M = 0$ here.

It is interesting to investigate what happens when the target rate of inflation implicit in the Taylor rule, π^*, equals $i_M - \delta$, the steady state inflation rule in the liquidity trap case. When the target rate of inflation equals Friedman's optimum rate of inflation, the configuration shown in Figure 1b occurs. The normal steady state, with its local saddlepoint configuration and the liquidity trap steady state with its local centre configuration coincide. Indeed, $\bar{\pi}^N = \pi^* = \bar{\pi}^L = i_M - \delta = \underline{\pi}$ in this case. Any solution starting from an inflation rate above π^* now converges along the stable branch SS' towards the unique steady state Ω^{NL}. Any solution starting from an inflation rate below π^* (and therefore also below $\underline{\pi}$) now diverges explosively.

It is not sensible to have parameter configurations where the target rate of inflation implicit in the Taylor rule is below the steady state inflation rate that supports Friedman's optimum quantity of money. Assume the contrary, i.e. that $\pi^* < i_M - \delta$. The inflation rate defining the boundary between the normal and the floor regions, π^{NF}, say, is given by:

$\pi^{NF} = \dfrac{i_M - \delta + (\gamma - 1)\pi^*}{\gamma}$. With $\gamma > 1$ it follows that $\bar{\pi}^N = \pi^* < \pi^{NF} < \bar{\pi}^L = i_M - \delta$. The steady state for the Taylor rule would lie outside the range of inflation rates for which the Taylor rule is defined. In what follows, we only consider parameter configurations supporting the solution trajectories shown in Figure 1a.

Demand shocks and the liquidity trap

We want to consider shocks that can cause the liquidity trap to be sprung, that is, shocks for which the constraint $i \geq i_M$ can become binding. We consider an economy that is initially in the normal steady state, at Ω_1^N in Figure 2, and is hit by an unexpected demand shock or supply shock that lowers current aggregate demand below current capacity output.

For concreteness we will consider the unanticipated announcement, at $t = t_0$ of an immediate and temporary reduction in public spending, *g*, which is reversed again at $t = t_1 > t_0$.

The consumption function for our model is

$$c(t) = \frac{\delta}{1+\eta}\left[\frac{M(t)+B(t)}{P(t)} + \int_t^{\infty} e^{-\int_t^v [i(u)-\pi(u)]du}[y(v)-\tau(v)]dv\right] \quad \textbf{(25)}$$

With the logarithmic utility function, the intertemporal substitution elasticity is unity. The marginal propensity to spend out of comprehensive wealth is $\frac{\delta}{1+\eta}$, which is independent of current and anticipated future real interest rates. Comprehensive wealth is the sum of financial wealth $\frac{M(t)+B(t)}{P(t)}$ and human wealth $\int_t^{\infty} e^{-\int_t^v [i(u)-\pi(u)]du}[y(v)-\tau(v)]dv$. Real interest rates affect current consumption only because they discount future real after-tax endowments. Monetary policy affects consumption to the extent that changes in current and anticipated future short nominal rates can affect current and anticipated future real discount factors, $\int_t^{\infty} e^{-\int_t^v [i(u)-\pi(u)]du}$, at any horizon $v - t \geq 0$.

An unanticipated immediate and temporary cut in public spending is contractionary in the short run because, although forward-looking Ricardian households realise that lower public spending means a correspondingly lower present discounted value of future taxes, the effect of the temporary public spending cut on household permanent (after-tax) income, and therefore on household consumption, is smaller in magnitude than the spending cut. As long as the public spending cut is in effect therefore (between t_0 and t_1), aggregate demand, $c + g$, will fall, for a given path of current and expected future real interest rates. Once the public spending cut is reversed (after t_1), aggregate demand will rise again. Aggregate demand (for a given path of current and expected future real interest rates) will be larger than it would have been absent the temporary spending cut.

As we shall see, following the contractionary fiscal shock, inflation will be lower along any of the equilibrium solution paths. Because of the Taylor-style interest rate reaction function, which has the short nominal interest rate adjusting more than one-for-one with the inflation rate, the profile of expected future short real rates is actually lower with the public spending cut than without. Future after-tax endowments are therefore discounted at a lower rate, but this is not enough to negate the net negative effect on aggregate demand of the public spending cuts.[14] Figure 2 represents the behaviour of the system following the public spending shock.

Figure 2 here

Assume the system starts, before the news arrives, at the normal steady state equilibrium Ω_1^N, with government spending expected to be constant. An unanticipated, immediate, permanent cut in public spending (the case there $t_1 \rightarrow \infty$) will result in an immediate transition to the new steady state at Ω_2^N. In the new steady state, the rate of inflation, and all real and nominal interest rates are the same as before. The level of private consumption rises by the same amount as the cut in the level of public consumption. Any initial jump in private consumption above the level corresponding to Ω_2^N would lead to explosively divergent behaviour and so would any initial jump to a level below Ω_2^N.

When the cut increase in public consumption is not permanent, the transition is as follows. Assume that at the announcement date, t_0, there is unexpected news of an immediate temporary cut in public spending, which is reversed again at $t_1 > t_0$. There is a unique

[14] If instead of the logarithmic instantaneous utility function we had adopted the constant elasticity of marginal utility function with an intertemporal substitution elasticity larger than 1, the negative effect on consumption would have been reinforced.

solution that will cause the system to return to the initial, normal steady state, Ω_1^N. This solution involves an immediate discrete jump increase in private consumption to Ω_{12}^N, situated vertically above Ω_1^N and below Ω_2^N. Note that the rate of inflation, π, is predetermined. From Ω_{12}^N, the system travels along the unstable solution trajectory, drawn with reference to the steady state Ω_2^N, that will cause it to arrive at Ω_{13}^N on the unique stable branch through Ω_1^N, at t_1, the moment the public spending cut is reversed. From then on the system converges to Ω_1^N along the stable branch through Ω_1^N, labelled $S_1S_1^{'}$. From t_0 till t_1 there is excess capacity and inflation is falling. From t_1 on inflation is rising and there is excess demand.

In addition to this unique solution that converges to the initial normal steady state Ω_1^N, there is a continuum of solutions that puts the system, at t_1, on a closed orbit that will lie partly in the normal and partly in the floor regions. One such solution is shown in Figure 2. At the initial date, t_0, there is a jump in the level of private consumption to a level below Ω_{12}^N, say Ω_{12}^L. From Ω_{12}^L the system travels along a divergent trajectory, drawn with reference to Ω_2^N, that will put it on the orbit *LL'* at t_1. Note that this solution trajectory intersects $S_1S_1^{'}$, the stable branch through Ω_1^N, before it reaches the orbit *LL'* at $t = t_1$ at the point Ω_{13}^L. There exists a continuum of possible initial jumps in private consumption, between Ω_{12}^N and Ω_1^N that will put the economy one of a continuum of closed orbits around the liquidity trap steady state.

Monetary policy actions to avoid or escape a liquidity trap in our model can take various forms. First, a one-off increase in the inflation target, π^*, which under our Taylor rule amounts to a reduction in $\bar{i}$, the intercept term in the Taylor rule. Second, a one-off reduction in the nominal interest rate on money, i_M. Third, a change in the responsiveness of the nominal interest rate to the rate of inflation, that is, a change in γ. Finally, the adoption of a *rule* for the nominal interest rate on money that ensures that it will always be below the nominal interest rate on non-monetary assets. Only the last of these measures turns out to eliminate the liquidity trap problem.

An increase in the target rate of inflation shifts the normal steady state horizontally to the right, one-for-one. The *NF* locus, marking the boundary between the normal and the floor regions also shifts to the right, but less than one-for-one. When the actual inflation rate exceeds the target inflation rate, there is only one non-explosively divergent solution trajectory. This is the trajectory that converges to the normal steady state. For any inflation rate below the target inflation rate (and above $\underline{\pi}$), there still exists a solution trajectory that converges to the normal steady state, but there also exists a continuum of solutions that take the form of closed orbits around the liquidity trap steady state. These orbits are partly in the floor region. For any initial inflation rate below $\underline{\pi}$, only explosively divergent solutions exist. Raising the target rate of inflation therefore reduces the range of initial inflation rates for which there are non-explosively divergent solutions that do not converge to the normal steady state.

Lowering the nominal interest rate on money leaves the normal steady state (and the dynamics in the normal region) unchanged. It shifts the liquidity trap steady state horizontally to the left, one-for-one, and it also shifts the *NF* locus to the left, but less than one-for-one.

Raising γ, the responsiveness of the nominal interest rate to the rate of inflation, while leaving the target rate of inflation unchanged (that is, varying $\bar{i}$ to leave $\pi^* = \dfrac{\delta - \bar{i}}{\gamma - 1}$

constant) also does not qualitatively affect the behaviour of the system.[15] To ensure that, provided the system starts off in the normal region ($i > i_M$), it cannot end up in the floor region, the Taylor rule must be modified.

A simple modification (or amplification) of the Taylor rule that avoids the liquidity trap is as follows. The exogenous own nominal interest rate assumption for money is replaced by the following simple rule:

$$\begin{aligned} &i_M = i - \alpha \\ &\alpha > 0 \end{aligned} \qquad \textbf{(26)}$$

The Taylor rule for the short nominal interest rate on non-monetary financial instruments continues to be given, as before, by equation (17), that is:

$$\begin{aligned} &i = \bar{i} + \gamma\pi \\ &\gamma > 1 \end{aligned} \qquad \textbf{(27)}$$

The rest of the model is as before. Note, however, that there is now no restriction on the domain of the nominal interest rate function. Equation (26) ensures that the constraint that the short nominal interest rate on non-monetary instruments cannot fall below the short nominal interest rate on money never becomes binding. Specifically, since the own rate of interest on money moves up and down one-for-one with the short nominal interest rate on non-monetary instruments, there is no (zero or other) lower bound to the level of the short nominal interest rate. The 'floor region' and the liquidity trap have been abolished at a stroke, by assuming that the monetary authorities follow a rule for the own nominal interest rate on money ensures that the nominal interest rate on non-monetary securities is always above the own nominal interest rate on money.

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15 Provided $\delta + \pi^* - i_M > 0$ it will shift the *NF* locus to the right towards the $\pi = 0$ locus.

Only the simplest kind of rule, maintaining a constant wedge between the two interest rates is considered here.[16] This apparently minor change in specification implies that there now is just the normal region, with its saddlepoint configuration, and that only the normal steady state exists.

The rule for the two short nominal interest rates given in equations (26) and (27) may require the payment of non-zero (positive or negative) interest rates on money. In Section IV we consider what the practical obstacles to paying negative interest on money may be.

(III) Can the zero nominal interest rate floor become binding in the UK?

Most estimates of the current level of the long real interest rate in the UK put it somewhere between 2.0 and 3.0 percent per annum. Figures 3 and 4 show the recent behaviour of medium-term and long-term real rates of interest on index-linked government securities.[17]

Figure 3 here

Figure 4 here

With an inflation target of 2.5 percent per annum (as in the UK), the long-run nominal interest rate (ignoring term- and risk premia) would be between 4.5 and 5.5 percent per annum. In steady state, the short-term nominal interest rate would also be between 4.5 and 5.5 percent per annum. We can regard this as the 'normal' level of the short nominal interest rate. If one believed that there were contingencies (such as a dramatic, spontaneous collapse

[16] Note that, because the opportunity cost of holding money, $i - i_M$, is positive and constant, the ratio of real money balances to consumption will also be constant in this model.

[17] Jenny Salvage prepared Figures 3 through 7

of aggregate demand) under which a cut in short rates of more than 4.5 to 5.5 percent would be in order, the monetary authority would be at risk of hitting the zero interest floor.

Historically, in the UK, there have been occasions when Bank Rate has swung by more than 4.5 or 5.5 percentage points. On 15 November 1979, the Bank's Minimum Lending Rate hit 17.00 percent. On March 11, 1981, it stood at 12.00 percent. On October 6, 1989, the Bank's Minimum Band 1 Dealing Rate stood at 14.88 percent. On 8 February 1994, it was down to 5.13 percent. Clearly, very large swings in Bank Rate, in excess of the 4.5 or 5.5 percent 'safety margin' associated with a 2.5 percent inflation target and a 2.0 to 3.0 percent long real interest rate, have occurred in the past.

The emphasis should, however, be on 'in the past'. These very large cuts in Bank Rate invariably took place from a very high level of rates associated with prior macroeconomic mismanagement, generally an inflationary surge that threatened to get out of control (or had indeed done so) or the desperate defence of an overvalued exchange rate peg. Figure 5 shows the behaviour of Bank Rate (or its successor rates), the inflation rate and the sterling-US$ exchange rate for the UK in the post-World War II period.

Figure 5 here

Neither situation applies today. Nor should it apply again if the political commitment to low and stable inflation and its institutional expression in an operationally independent central bank remain intact (see also Johnson, Small and Tryon [1999] for a US perspective on this and related issues).

The longer-term historical record can also be viewed as encouraging. The UK got through the period 1800-1914 without ever landing itself in a liquidity trap. As Figure 6 shows, the average rate of inflation over this 115-year period was slightly negative and the variability of the inflation rate was high. Figure 7 shows that Bank Rate did not fall below 2 per cent throughout 115 years preceding World-War I.[18]

Figure 6 here

Figure 7 here

There is a marked positive association, over time and across countries, between the level of the inflation rate and its variability (see Okun [1971, 1975], Taylor [1981], Ball and Cecchetti [1990]). If such a relationship were to be found also between the level of short nominal rates and their variability or volatility, it would further reduce the likelihood of ending up in a liquidity trap in an environment with sustained low inflation and therefore, on average, with low nominal interest rates.

As will become apparent, the available statistical evidence on the association between the level and volatility of short-term nominal interest rates is mixed, and neither weakens nor reinforces our prior belief, that it is hard to conceive of situations in which the zero nominal interest rate floor would become a binding constraint on monetary policy in the UK, with the current symmetric annual inflation target of 2.5 percent.

In principle, this hypothesis can be tested by estimating a dynamic stochastic process for the short nominal interest rate, using either time series or Markov chain models. By making distributional assumptions about the disturbances in this process, it would be possible to calculate the odds on the short nominal rate falling below zero, either conditionally, that is, given the starting values of the process, or unconditionally. We do indeed attempt this, but our efforts must be accompanied by a clear health warning.

There is an obvious, and in our view virtually insurmountable, problem with any assessment, based on historical data, of the odds that the non-negativity constraint on short

[18] The temporary collapse in the external value of the U.S. dollar starting in 1861 reflects the exceptional circumstances of the American Civil War and its aftermath, the Greenback

nominal rates will become binding. During the sample, markets undoubtedly were operating under the assumption that short nominal rates could never fall below zero. In the UK over the past 200 years, the annual Bank Rate series indeed never fell below 2 percent. With the support of the empirical distribution of nominal short rates truncated from below at zero, the historical interest rate record is unlikely to be informative about the odds on the economy getting into a liquidity trap in the future, since this would require a structural break in the interest rate process, about which the sample is uninformative. If we were to assume (counterfactually, as can be seen from Figures 9, 10 and 11) that the distribution of Bank Rate or of the error term in the Bank Rate equation is normal, there will always be a positive probability that Bank rate will go negative. If we assume instead that the distributions in question are, say, lognormal, the probability of breaching the zero floor (even asymptotically) is a-priori constrained to be zero. We try to circumvent this by calculating the asymptotic confidence bands for Bank Rate reported below from the empirical distribution of the sample residuals. Since the empirical distribution of the residuals obviously has finite support, this procedure will, if anything, underestimate the likelihood of the economy ending up in a liquidity trap.

Even ignoring the unavoidable small-sample problems, this procedure is vulnerable to the following criticism. What we are interested in is the probability that the short nominal interest rate would have had to be negative in order to avoid the economy getting into a very undesirable equilibrium.[19] If the economy *had* been in a liquidity trap in the sample, the data would reflect the liquidity trap configuration of the economy, including the endogenous responses of real activity and inflation that supported the liquidity trap floor as an equilibrium. Information on the 'deep' structural parameters of the model (the invariant

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period.

parameters governing money demand and its determinants) is necessary to recover the 'first passage' probabilities into the liquidity trap region of the economy, and these cannot be recovered from the a-theoretical, reduced-form time series processes we estimate. Finally, our statistical tests only concern the likelihood of the short nominal interest rate hitting the zero bound. As was pointed out in the Introduction, this is only a necessary condition for the economy being in a liquidity trap.

What do the data tell us about the statistical association between the level and volatility of the short nominal interest rate? The very high-frequency association between short nominal sterling rates and a measure of volatility derived from short sterling futures over the period 1987-1999 is shown in Figure 8.[20] The association between the level of short sterling and its volatility is, if anything, weakly negative.

Figure 8 here

The slightly lower frequency time-series evidence on the association between the level of short nominal interest rate and a statistical measure of its variability using weekly data is also mixed. Figure 9 shows the time series record for the UK for the period 1975-1999.

Figure 9 here

For the whole period 1975-1999, volatility and level of the three month interbank rate are positively contemporaneously correlated, but for the post-inflation targeting period 1993-1999, the correlation is slightly negative. The statistical model that generated the conditional variance measure used in Figure 9 can be found in the Appendix. We also provide an estimate of the steady state (long-run) value of the three month interbank rate implied by the

[19] In principle, there could be a positive lower bound on the nominal interest rate, even with the own rate on currency at zero.

[20] Backing a volatility estimate out of futures prices is attractive, because it avoids the need to construct statistical estimates of volatility from the time series data on interest rates.

statistical model, together with 95% steady state confidence bands for three month interbank rate.

We also investigated the statistical properties of Bank Rate at significantly lower frequencies, using a 200 year time series of annual observations. Our time series model (described in the Appendix) implies a strong positive correlation (0.81) between the level of Base Rate and its contemporaneous conditional variance. Figure 10 plots the level of Base Rate and our estimates of its conditional variance. We also provide an estimate of the steady state (long-run) value of Base rate implied by the statistical model, together with 95% steady state confidence bands for Base Rate.

Figure 10 here

The confidence bands were calculated using the distribution of the estimated sample residuals. Not surprisingly, the distribution of sample residuals is distinctly non-normal. The same holds for Base rate itself. Figure 11 shows the frequency distribution of Base Rate and Figures 12 and 13 those of the estimated interest rate residuals. The sample distribution of Bank Rate is significantly skewed to the right. Its empirical distribution is truncated from below at 2.0 percent. The distribution of the sample residuals from the two main interest rate models is rather more symmetric.

Figure 11 here

Figure 12 here

Figure 13 here

Krugman [1998d] has suggested that deflation (negative inflation) makes a liquidity trap more likely. This is indeed an implication of just about any model of liquidity traps, including the model we developed in Section II of this paper. We therefore estimate a simple time series process for annual RPI inflation over the 200 year period, and for its conditional variance. The results are reported in Figure 14, together with its estimated steady state value and steady state 95% confidence intervals. The statistical inflation model is described in the Appendix. Surprisingly, the contemporaneous correlation between inflation and its conditional variance turns out to be negative.

Figure 14 here

The steady state confidence intervals for the annual rate of RPI inflation show that there is quite a large probability of deflation. Before one gets too worried about this, three points should be kept in mind. First, the relationship between interest rates and expected inflation depends on the behaviour of the inflation risk premium. Second, the UK experienced negative trend inflation and short bouts of sharp deflation in the 19th century, without landing itself in a liquidity trap. Third, the monetary policy target in the UK is, since June 1997, a symmetric inflation target. Deviations of inflation below the 2.5 percent target are to be avoided as much as deviations above that target. The risk of sharp deflation is therefore diminished. The new monetary regime has been in operation for too short a period, however, for this to show up as a structural break in the inflation time series process.

McKinnon and Ohno [1999] have argued that, at any rate in the Japanese case, a large expected appreciation of the currency could create a liquidity trap. We investigated the likelihood of a sharp appreciation of sterling by using almost 200 years of £/$ exchange rate data to estimate a simple stochastic process for the proportional rate of depreciation of the exchange rate and its conditional variance. The results are reported in Figure 15, together with the expected long-run sterling depreciation rate and the 95% asymptotic confidence intervals. The statistical model underlying these calculations is described in the Appendix. The contemporaneous correlation between exchange rate depreciation and its conditional variance is low but negative.

Figure 15 here

It suggests that, based on this particular statistical model, there is quite a significant probability of a sizeable appreciation of sterling. Again, the caveat about the dangers of ignoring risk premia applies. It is surprising that our simple statistical model appears to handle such episodes as the American Civil War, two World Wars and the Great Depression of the Thirties quite well.

Finally, Figure 16 reports cross-sectional evidence on the relationship between the level of short nominal rates and their volatility based on a sample of 59 countries between 1989 and 1998. The source of the data is IFS.[21] The correlation between the two variables is very high at 0.89, suggesting that across countries high short-term nominal rates are accompanied by high unconditional variances.

Figure 16 here

Fuhrer and Madigan [1997] and Orphanides and Wieland [1998] use stochastic simulations of a small structural rational expectations model calibrated for the US, to investigate the consequences of the zero bound on nominal interest rates. They found that if the economy is subject to stochastic shocks similar in magnitude to those experienced by the US over the 1980s and 1990s, the consequences of the zero bound were negligible for target inflation rates as low as 2 percent. With target inflation between zero and one percent, there was a quantitatively significant deterioration in economic performance. Applying a structural non-linear VAR approach, Iwata and Wu [2001] conclude, using Japanese data from the 1990s, that the zero nominal interest rate round can be a serious constraint on monetary policy in Japan.

On balance, the data fail to offer convincing support either for or against the contention that a regime of low short nominal interest rates is likely to be a regime of stable

[21] We would like to thank Nick Hanchard for preparing this Figure.

short nominal interest rates. This is therefore not unambiguously good news, nor unambiguously bad news for a policy maker targeting low inflation, although the current UK target would seem to provide quite handsome room for monetary manoeuvre. Two hundred years of UK monetary history also favour the contention that zero bounds on short nominal interest rates are unlikely to become a policy concern.

(IV) Paying interest on currency to avoid a liquidity trap

In the model of Section II, neither the use of fiscal policy nor an increase in the target rate of inflation can guarantee that the economy will not end up at the (zero) nominal interest rate floor. When the nominal interest rate on non-monetary instruments is governed by a Taylor rule, the only way to ensure that the nominal interest rate floor cannot become a binding constraint in policy is to adopt a rule for the own nominal interest rate on currency that keeps the nominal rate on currency always below the nominal interest rate on non-monetary instruments. In general, this rule will require the payment of interest on currency. While technically and administratively awkward, the payment of interest, positive or negative, on currency is in principle feasible.

That nominal interest rate floor at zero is not a technological, immovable barrier. It is the result of a policy choice - the decision by governments or central banks to set the administered nominal interest rate on coin and currency at zero, rather than at some other (negative) level. Coin and currency are government *bearer* bonds[22]. A bearer bond is a debt

[22] Bearer securities are securities for which ownership is established by possession, without any need for registering title. Thus, a bearer bond is a bond with no owner information attached to it. The legal presumption is that the bearer is the owner. If the issuer of the bond is credit-worthy, they are almost as liquid and transferable as cash. Cash (coin and currency) is a special case of a zero interest (or zero-coupon) bearer bond issued by the state (generally through the central bank). Currency can be viewed as a zero coupon bearer consol or bearer perpetuity, since it can be interpreted as having an infinite maturity. It may actually be more

security in paper form whose ownership is transferred by delivery rather than by written notice and amendment to the register of ownership. We shall refer to all securities that are not bearer bonds as *registered* securities. Bearer bonds are negotiable, just as e.g. money market instruments such as Treasury Bills, bank certificates of deposit, and bills of exchange are negotiable.[23] Coin and currency therefore are bearer bonds. They are obligations of the government, made payable not to a named individual or other legal entity, but to whoever happens to present it for payment - the bearer. Coin and currency have three further

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informative to view currency as a zero coupon *finite* maturity bearer bond, which is issued and redeemed at par, with redemption taking the form of the one-for-one exchange of old currency for new currency which is indistinguishable from the old currency (see Buiter and Panigirtzoglou [1999, Appendix 1]).

The vast majority of 'international bonds', historically called 'eurobonds' are bearer. Bearer bonds can take two main forms. First, the traditional 'definitive' style, where the bonds literally are individual pieces of security-printed paper in denominations of, say, $10,000, which individual holders bring in to paying agents so as to receive payment of interest and principals. Second, 'global' bonds, which are technically bearer instruments but consist of a single piece of paper representing the entire issue (and so worth hundreds of millions or even billions of dollars). In practice, the terms of the global bond say that only Euroclear (the settlement system based in Brussels) or Cedelbank (the settlement system based in Luxembourg) are entitled to the proceeds of the global bond, and that Euroclear and Cedelbank will in turn divide the proceeds up amongst the end-investors whose details are stored in their electronic records. Thus the global bond is not an instrument which in practice can be passed from one owner to another, even though it is technically 'bearer'. Effectively the bonds are dematerialised.

Bearer bonds are legal and quite common in the UK. While the bearer debenture went out of use, replaced by the non-negotiable debenture or debenture stock, transferable (in the same way as common stocks) by entry in the company's register, a number of new negotiable investment securities have evolved. They include the modern bearer bond, the negotiable certificate of deposit, and the floating rate note. A limited number of gilts have also been issued with a bearer option.

Before July 1983, municipal securities in the U.S. were issued for the most part in certificate form with coupons attached. Some of these so-called old-style bearer bonds are still available in the marketplace. The issuer has no record of who owns these bonds. The owner clips the coupons and collects the interest from the issuer's paying agent. Transferring the bonds requires physical delivery and payment. Bearer bonds issued by municipal authorities were made illegal in the U.S. in 1982.

[23] A financial instrument is negotiable if it is transferable from one person to another by being delivered with or without endorsement so that the title passes to the transferee. Key elements of negotiability include the following: (1) transfer by physical delivery; (2) transfer is such as to confer upon its holder unchallengeable title and (3) a negotiable instrument benefits from a number of evidential and procedural advantages in the event of a court action.

distinguishing properties: they are government bearer bonds with infinite maturities (perpetuities or consols); their coupon payments (which define the own (or nominal) rate of interest on coin and currency) are zero, and they are legal tender (they cannot be refused in final settlement of any obligation).

There are two reasons why interest is not paid on currency. [24] The first and currently less important one has to do with the attractions of seigniorage (issuing non-interest-bearing monetary liabilities) as a source of government revenue in a historical environment of positive short nominal rates on non-monetary government debt.[25]

The second, and more important, reason why no interest is paid on coin and currency, are the practical, administrative difficulties of paying a negative interest rate on bearer bonds. Significant 'shoe leather' costs are involved both for the state and for private agents.

There is no practical or administrative barrier to paying negative nominal interest rates (market-determined or administered) on registered securities, including balances held in registered accounts, such as bank accounts.[26] The reason is that, for registered securities, the identities of both the issuer and the holder (the debtor and the creditor) are known or easily established. This makes it easy to verify whether interest due has been paid and received. Thus the non-bearer bond part of the monetary base, that is, banks' balances with the central bank, could earn a negative nominal interest rate without any technical problems. Positive

[24] From here on, 'currency' will be taken to include both coin and currency. There obviously are more severe technical problems with attaching coupons or stamps to coin than to currency notes.

[25] Of course, issuing negative interest-bearing monetary liabilities would be even more attractive, from a seigniorage point of view.

[26] The only exception is that it would not be possible to have a consol or perpetuity with a negative nominal interest rate. Assume the constant nominal coupon payment of the consol is positive. If the infinite sequence of short nominal rates is negative, the value of the consol would be unbounded positive. A negative coupon would yield an unbounded negative value for the consol.

interest payments or negative interest payments just involve simple book-keeping transactions, debit or credit, between known parties.

There are technical, administrative problem with paying negative interest on the bearer bond part of the central bank's monetary liabilities, coin and currency. While the identity of the issuer (the Central Bank) is easily verified, the identity of the holder is not. There is no obligation to register title to currency in order to establish ownership. Possession effectively provides complete title. This creates problems for paying any non-zero interest rate, because it is difficult to verify whether a particular note or coin has already been credited or debited with interest.

The problem of verifying whether interest due on bearer bonds has been paid is present even when the interest rate is positive. However, the problem of getting the anonymous holder of currency to come forward to claim his positive coupon receipt from the government is less acute than the problem of getting the anonymous holder to come forward to make a payment to the government.[27] In both cases, however, each individual currency claim has to be marked clearly as 'current', that is, as having paid or received all interest that is due. Without this, positive interest-bearing currency could be presented repeatedly for the payment of interest. Historically, the problem of paying positive interest on bearer bonds was solved by attaching coupons or stamps to the title certificate of the bearer bond. When claiming his periodic coupon payment, the appropriate coupon was physically removed ('clipped') from the title certificate and retained by the issuer.

Without further amendment, the 'coupon clipping' or stamping route would not work for bearer bonds with negative coupons. The enforcement problems involved in getting the unregistered, anonymous holders of the negative coupon bearer bonds to come forward to pay

[27]This is akin to the problem of compelling payment of taxes when the tax base cannot be verified.

the issuer would be insurmountable. The only practical way around this problem, is to make the bearer bond subject to an expiration date and a conversion procedure. In the case of currency, this could be achieved by periodically attaching coupons or stamps to currency, without which the currency would cease to be ‘current’.

For currency to cease to be ‘current’, it is not enough for the monetary authority to declare that after a certain date ‘old’ currency shall cease to be legal tender. Being legal tender certainly enhances the attractiveness of currency as a store of value, medium of exchange and means of payment, but these advantages need not be enough to induce holders of 'old' currency, which is about to lose its legal tender status, to come forward and exchange it, at a price, for 'new' currency which does have continuing legal tender status. What serves as medium of exchange and means of payment is socially determined. Being legal tender is but one among many considerations that induce people to use certain classes of object as means of payment and medium of exchange. For currency to cease to be current, the bearer has to be subject to a serious penalty, such as confiscation, if the appropriate coupon or stamp has not been attached. In other words, there have to be periodic 'monetary reforms'.

There is a long tradition on the cranky fringes of the economics profession of proposals for taxing money or taxing liquidity. Many of these proposals were part of wide-ranging, and generally hare-brained, schemes for curing the world's economic and social ills. The mechanics of taxing currency are straightforward main-stream economics, however.

The best-known proponent of taxing currency was probably Silvio Gesell (1862-1930), a German/Argentinean businessman and economist admired by Keynes, who wrote of him *"I believe that the future will learn more from the spirit of Gesell than from that of Marx"* (Keynes [1936, p. 355]). Gesell wanted to stimulate the circulation of money by getting the state to issue money that, like capital assets, depreciated in value.[28] Rather than relying on inflation to reduce the attractiveness of holding money, Gesell proposed "Stamp Scrip" - dated bills that would lose a certain percentage of value each year unless new stamps were put on them (Gesell [1949]). Irving Fisher [1933] for a while supported the issuance of stamp scrip and wrote a sympathetic account of it. Stamp Scrip was actually issued briefly during the Great Depression of the Thirties in parts of the Canadian province of Alberta by the Social Credit provincial government of the day. [29]. The Canadian federal government and the courts blocked the key measures, and in the end the provincial government refused to accept its own scrip in payment.[30] Similar local currency experiments were tried in Wörgl, Austria during the 1930s.

Thus, for negative interest on bearer bonds such as currency to be enforceable, the bearer bond has to expire after a certain date, unless it is converted into new currency. The desired interest rate on currency would be determined by the terms on which the old currency could be exchanged with the central bank for new currency. Taxing currency (or paying

[28] Gesell's motivation was not, as far as we can determine, the avoidance of or escape from liquidity traps. His aim was to eliminate the interest component of costs and prices completely from the economic system, not just in the extreme circumstances of the liquidity trap, but as a permanent feature. Our reading of his works suggest that he was a bit vague about the distinction between real and nominal interest rates. The formal model analysed in Section II of this paper has the property that the monetary authorities cannot influence the long-run real interest rate.

[29] In August 1935 the first social credit government was elected in the Canadian province of Alberta. While its ideology owed more to the writings of two other great economic cranks, Alfred Richard Orage [1917] and Major Clifford Hugh Douglas [1919] (and to the personal involvement of the latter as economic adviser to the provincial government), the Alberta Prosperity Certificates introduced in 1936 by Premier William Aberhart, were pure Gesell. Similar in appearance to a dollar bill, the certificates required a weekly endorsement of a 2c stamp, amounting to a 104 percent annual capital levy (see Hutchinson and Burkitt [1997] and Mallory [1954]).

[30] It also had failed to convince the Federal government in Ottawa to match its negative interest rates. Since Federal currency was at least as useful as a means of payment, this would require to scrip to trade at a discount with respect to the Federal currency and to appreciate vis-à-vis the federal currency at a rate that compensated for the interest differential between Federal and provincial currency.

negative interest on currency) through expiration of old currency and conversion into new currency can be visualised as follows. After the expiration date, t_1 , the issuer (the central bank) or its agents can confiscate the old currency without compensation.[31] Provided the forces of the law are strong enough, this could induce holders of the old currency to convert it, at a price, on or before the expiration date, rather than continue to use it in transactions or as a store of value after the expiration date and risk having it confiscated.

At fixed intervals of length Δt (Gesell periods, say) whose duration could, for convenience, be set at a year (or several years, in order to reduce conversion costs), and on a specific day, (Gesell day), old currency would legally revert to the issuer (the central bank). After Gesell day, the old currency has no value (because of the credible threat of confiscation) and will not be used in transactions or as a store of value. On Gesell day, *1* £ worth of new currency would be issued in exchange for $e^{-i_M \Delta t}$ £s worth of old currency, where i_M would be the policy-determined (instantaneous) nominal interest rate on currency.[32] For simplicity, we assume i_M to be constant, although it could be time-varying. The nominal rate of interest on currency would be administratively determined, that is, set by the central bank. Earlier exchanges of old for new money might be allowed at the rate of *1 £* worth of the new currency for $e^{-i_M \Delta t} e^{-\int_{t_e}^{t_1} i(s)ds}$ £s worth of the old currency, where t_1 is the date of the next Gesell day, $t_e \leq t_1$ is the time before the next Gesell day on which the old currency is exchanged for the new, and i is the instantaneous nominal interest rate on the government's non-monetary liabilities. For currency to remain rate-of-return-dominated as a store of value, it is necessary

[31]Less drastic penalties might work also. For instance, old money found in circulation after its 'expiry' date would be forcibly converted into new money at the rate offered on the conversion date, but subject to an additional penalty. The confiscation scenario makes the key point very clearly, however.

[32] $e^{-i_M \Delta t} - 1$ would be the effective (Gesell) period tax rate on currency. The instantaneous tax rate would be $-i_M$.

that $i_M < i$. Both rates could be negative, and may have to be, if zero bounds are to be ruled out. Coin and currency would effectively become time-limited, finite maturity financial claims.

New currency could, in principle, be used in transactions before midnight on the Gesell day before they are formally introduced. The relative value of the old currency in terms of the new currency would change at an instantaneous rate i_M, to ensure that, at the moment the old currency expires and the new currency comes in officially, there is no discrete jump in the value of old money in terms of new money, or of goods and services in terms of money.[33] It follows that, during the period of coexistence of old and new money, the rate of inflation of the prices of goods and services would be higher in terms of old money than in terms of new money, with the excess of the old money inflation rate over the new money inflation rate equal to $-i_M$.

Our scheme for removing the zero nominal interest rate floor by taxing currency only applies to government bearer bonds with an administratively determined nominal rate of return, that is to coin and currency. Commercial banks' balances with the central bank are not bearer bonds, but registered securities, in the terminology of this paper. The nominal interest rate on these balances is determined administratively, but paying negative interest on them is as simple as paying positive interest. Bank deposits, which are private registered securities in our terminology, would not need to be taxed. If, when currency is taxed, the equilibrium nominal market yield on deposits, and on any other private registered securities, is negative, banks will pay a negative interest rate on deposits, without any need for taxing deposits. The same applies to private electronic or e-money, including 'money on a chip', internet accounts etc.

Clearly there are costs associated with Gesell money, even if one can come up with a slightly higher-tech (and tamper-proof) alternative to physically stamping currency. These shoe leather costs have to be set against the benefits of removing the zero floor on the nominal interest rate.

There are costs (and benefits) other than shoe-leather costs associated with taxing currency. Taxing currency would be regressive, since only the relatively poor hold a significant fraction of their wealth in currency. Taxing currency would also, however, constitute a tax on the grey, black and outright criminal economies, which are heavily cash-based. In the case of the US dollar, with most US currency held abroad (one assumes by non-US residents), it would represent a means of increasing external seigniorage.[34]

(V) Conclusion

The credible targeting of a low rate of inflation should result, on average, in low nominal interest rates. The administratively determined zero nominal interest rate on currency sets a floor under the nominal interest rate on non-monetary financial claims. An important policy issue then is the following: how likely is it that the economy ends up, as a result of shocks or endogenous fluctuations, in a situation where the zero short nominal interest floor becomes a binding constraint? If low average nominal interest rates also tend to be stable rates, the risk of ending up in a liquidity trap need not be enhanced much by targeting a low rate of inflation. The empirical evidence on the relationship between the level and volatility of short nominal rates is, however, mixed. The cross-sectional evidence supports a strong positive correlation. The time-series evidence for the UK is ambiguous.

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[33] This is just like the ex-dividend price of a share of common stock being equal, on the day the dividend is paid, to the dividend-inclusive price of the stock minus the dividend. In our example, the dividend would be negative

To avoid getting into a liquidity trap, or to get out of one once an economy has landed itself in it, there are just two policy options. The first is to wait for some positive shock to the excess demand for goods and services, brought about through expansionary fiscal measures or through exogenous shocks to private domestic demand or, in an open economy, to world demand. The second option is to lower the zero nominal interest rate floor on currency by taxing currency. If a rule were followed that kept the nominal interest rate on currency systematically below the nominal interest rate on non-monetary instruments, the economy could never end up in a liquidity trap. Such a rule would require the authorities to be able to pay interest, negative or positive, on currency, that is, to turn currency into 'Gesell money'.

The transactions and administrative costs associated with what amounts to periodic currency reforms would be non-trivial. Such currency conversion costs could be reduced by lengthening the interval between conversions, but they would remain significant. These 'shoe-leather costs' of taxing currency have to be set against the potential benefits of avoiding a liquidity trap. It may take quite a lot of shoe leather to fill an output gap.

[34] Unless drug dealers switch elastically to non-stamped currency.

Appendix: A Time-Series Investigation of the Association Between the Level and Volatility of the Short Nominal Interest Rate, the Inflation Rate and the Exchange Rate Depreciation Rate.

(1) UK 3-month interbank rate 1975-1999.

Let i denote the UK 3-month interbank rate. The time series model estimated for Figure 1 was

$$\begin{aligned}
& i_t - i_{t-1} = a + b i_{t-1} + \varepsilon_t, \\
& \varepsilon_t = \rho \varepsilon_{t-1} + u_t + \vartheta u_{t-1}, \\
& E(\varepsilon_t \mid \Psi_{t-1}) = 0, E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2, \\
& \sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 + \delta i_{t-1}
\end{aligned} \qquad \textbf{(A1)}$$

This time series model includes *AR(1)* and *MA(1)* terms in the conditional mean equation to account for the autocorrelation of standardised residuals. It includes *GARCH(1,1)* terms in the conditional variance equation to account for the autocorrelation of squared standardised residuals, as well as, a linear function of the lagged interest rate level to capture the potential dependence of the conditional variance on the lagged interest rate level.

Weekly data of the UK 3-month interbank rate from Jan 75 to April 99 were used for the model estimation. The estimation results using maximum likelihood were:

Table A.1

	Coefficient	Std.Error	Prob
a	0.047	0.030	0.119
b	−0.0062	0.0038	0.103
ρ	0.86	0.06	0.000
ϑ	−0.78	0.08	0.000
ω	−0.00042	0.00036	0.242
β	0.975	0.007	0.000
γ	0.022	0.008	0.005
δ	0.000069	0.000059	0.243

R^2 (adjusted) = 0.016

Standard errors are estimated using Quasi Maximum Likelihood. The estimates of the parameters are consistent even if the conditional normality assumption is violated. They can, however, be inefficient.

The coefficient δ that determines the dependence of the conditional variance on the lagged interest rate level is insignificant. Alternative models of the conditional variance that include higher order powers or higher order lags of the lagged interest rate level were also estimated.[35] The coefficients of the higher order powers or higher order lags of the interest rate level were also insignificant. Therefore, the dependence of the conditional variance on past interest rate levels is weak.

However, the ex-post contemporaneous relationship between the interest rate level i_t and the conditional variance σ_t^2 is strong. In fact the two variables have a correlation of 0.66. This is mainly because of the large information shocks in 1970's and 1980's when short interest rates were high. After the introduction of inflation targeting in UK (in late 92) the

[35] This includes a version replacing the conditional variance equation in (A2.1) by $\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \delta r_{t-1}^2$, a specification suggested by Chan, Karolyi, Longstaff and Sanders [1992].

relationship between the interest rate level i_t and the conditional variance σ_t^2 is weaker. In fact they are slightly negatively correlated, with a correlation coefficient of -0.26.

The steady state forecast of the level of the 3-month interbank rate is 7.546. The 95% confidence intervals are 4.02 and 14.41. The confidence intervals were constructed by assuming that the standardised steady state forecast follows the in-sample distribution of the standardised residuals which, of course, has finite support. In particular the assumed skewness and kurtosis were 0.846 and 10.49 respectively.

Using $\ln(1+i_t)$ rather than i_t as the specification of the interest rate variable in the regressions did not result in significantly different results.

(2) UK base rate 1800-1998:

Let *i* denote the UK base rate. The time series model estimated was an EGARCH model of the form:

$$
\begin{aligned}
& i_t - i_{t-1} = a + b i_{t-1} + \varepsilon_t, \\
& \varepsilon_t = \rho \varepsilon_{t-10} + u_t + \vartheta u_{t-2}, \\
& E(\varepsilon_t \mid \Psi_{t-1}) = 0,\ E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2, \\
& \log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \alpha \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}
\end{aligned}
\qquad \textbf{(A2)}
$$

The coefficients α and γ capture the potentially asymmetric impact of last periods shocks on conditional variance.

The time series model also includes AR(10) and MA(2) terms in the conditional mean equation to account for the autocorrelation of standardised residuals.

Annual data for the UK base rate from 1800 to 1998 were used for estimation. The estimation results using maximum likelihood were:

Table A2

	Coefficient	Std.Error	Prob
a	0.34	0.11	0.002
b	−0.071	0.023	0.002
ρ	0.12	0.07	0.099
ϑ	−0.28	0.08	0.000
ω	−0.064	0.086	0.455
γ	0.29	0.07	0.000
β	0.91	0.02	0.000
α	0.078	0.102	0.446

$$R^2 \text{ (adjusted)} = 0.120$$

Standard errors are estimated using Quasi Maximum Likelihood. The estimates of the parameters are asymptotically consistent even if the conditional normality assumption is violated. They can be inefficient, however, in this case.

The EGARCH model was chosen over alternative GARCH models, because its long run unconditional variance was non-explosive[36]. The coefficient γ is significant[37], but the effect is the opposite of the "leverage[38]" effect, that is, a negative shock has a negative impact on the conditional variance.

The steady state forecast of the base rate is 4.88. The 95% confidence intervals are 1.02 and 10.1. The confidence intervals were constructed by assuming that the standardised steady state forecast follows the in-sample distribution of the standardised residuals. In particular the assumed skewness and kurtosis were 0.49 and 4.197 respectively.

(3) UK annual inflation (RPI annual % changes) 1800-1998:

[36] An asymmetric component ARCH model had also a non-explosive unconditional variance, but the convergence was much slower than EGARCH model.
[37] at 95% confidence level.

Let p denote the RPI and $\pi_t = \frac{p_t - p_{t-1}}{p_{t-1}}$ its annual proportional rate of change. The time series model estimated was

$$\begin{aligned} &\pi_t - \pi_{t-1} = a + b\pi_{t-1} + \varepsilon_t, \\ &\varepsilon_t = \rho_1 \varepsilon_{t-5} + \rho_2 \varepsilon_{t-8}, \\ &E(\varepsilon_t \mid \Psi_{t-1}) = 0,\ E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2, \\ &\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \gamma\varepsilon_{t-1}^2 \end{aligned} \qquad \textbf{(A3)}$$

This time series model includes AR(5) and AR(8) terms in the conditional mean equation to account for the autocorrelation of standardised residuals. It includes GARCH(1,1) terms in the conditional variance equation to account for the autocorrelation of squared standardised residuals.

Annual data of the UK RPI annual proportional changes from 1800 to 1998 were used for the model estimation. The estimation results using maximum likelihood were:

Table A3

	Coefficient	Std.Error	Prob
a	0.013	0.006	0.031
b	–0.46	0.085	0.000
ρ_1	0.15	0.08	0.053
ρ_2	0.24	0.08	0.001
ω	0.00009	0.000103	0.380
β	0.81	0.12	0.000
γ	0.16	0.104	0.114

R^2 (adjusted) = 0.294

Standard errors are estimated using Quasi Maximum Likelihood. The estimates of the parameters are asymptotically consistent even if the conditional normality assumption is violated. They can be inefficient, however, in this case.

[38] The "leverage" effect is the negative correlation between current returns and future volatility, found mainly in stock returns data.

Alternative models of the conditional variance that include inflation rate dependence were also estimated. The coefficient of the inflation rate dependence was insignificant.

The steady state forecast of the inflation rate is 2.7%. The 95% confidence intervals are -10.7% and 21%. The confidence intervals were constructed by assuming that the standardised steady state forecast follows the in-sample distribution of the standardised residuals. In particular the assumed skewness and kurtosis were 0.434 and 5.286 respectively.

(4) £/$ Annual changes 1800-1998:

Let s denote the annual spot exchange rate and $e_t = \frac{s_t - s_{t-1}}{s_{t-1}}$ its proportional rate of change. The time series model estimated was

$$\begin{aligned} & e_t - e_{t-1} = a + br_{t-1} + \varepsilon_t, \\ & \varepsilon_t = \rho\varepsilon_{t-2} + u_t + \vartheta u_{t-3}, \\ & E(\varepsilon_t \mid \Psi_{t-1}) = 0,\ E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2, \\ & \sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \delta e_{t-1}^2 \end{aligned} \qquad \textbf{(A4)}$$

This time series model includes AR(2) and MA(3) terms in the conditional mean equation to account for the autocorrelation of standardised residuals. It includes GARCH(1,1) terms in the conditional variance equation to account for the autocorrelation of squared standardised residuals, as well as a linear term in the square of the growth rate of the exchange rate to capture the potential dependence of the conditional variance on the proportional rate of change of the exchange rate. Annual data on annual percentage changes of the £/$ exchange rate from 1800 to 1998 were used for the model estimation. The estimation results using maximum likelihood were:

Table A4

	Coefficient	Std.Error	Prob
a	−0.0059	0.0016	0.000
b	−0.75	0.09	0.000
ρ	−0.18	0.05	0.000
ϑ	−0.25	0.08	0.001
ω	0.00101	0.00027	0.000
β	0.10	0.06	0.113
γ	−0.13	0.06	0.034
δ	0.89	0.32	0.006

R^2 (adjusted) = 0.422

The coefficient δ, which measures the dependence of the conditional variance on the squared proportional exchange rate change, is highly significant. The steady state forecast of the £/$ annual percentage change is -0.8%. The 95% confidence intervals are -10.6% and 4.2%. The confidence intervals were constructed by assuming that the standardised steady state forecast follows the in-sample distribution of the standardised residuals. In particular the assumed skewness and kurtosis were 0.864 and 7.57 respectively.

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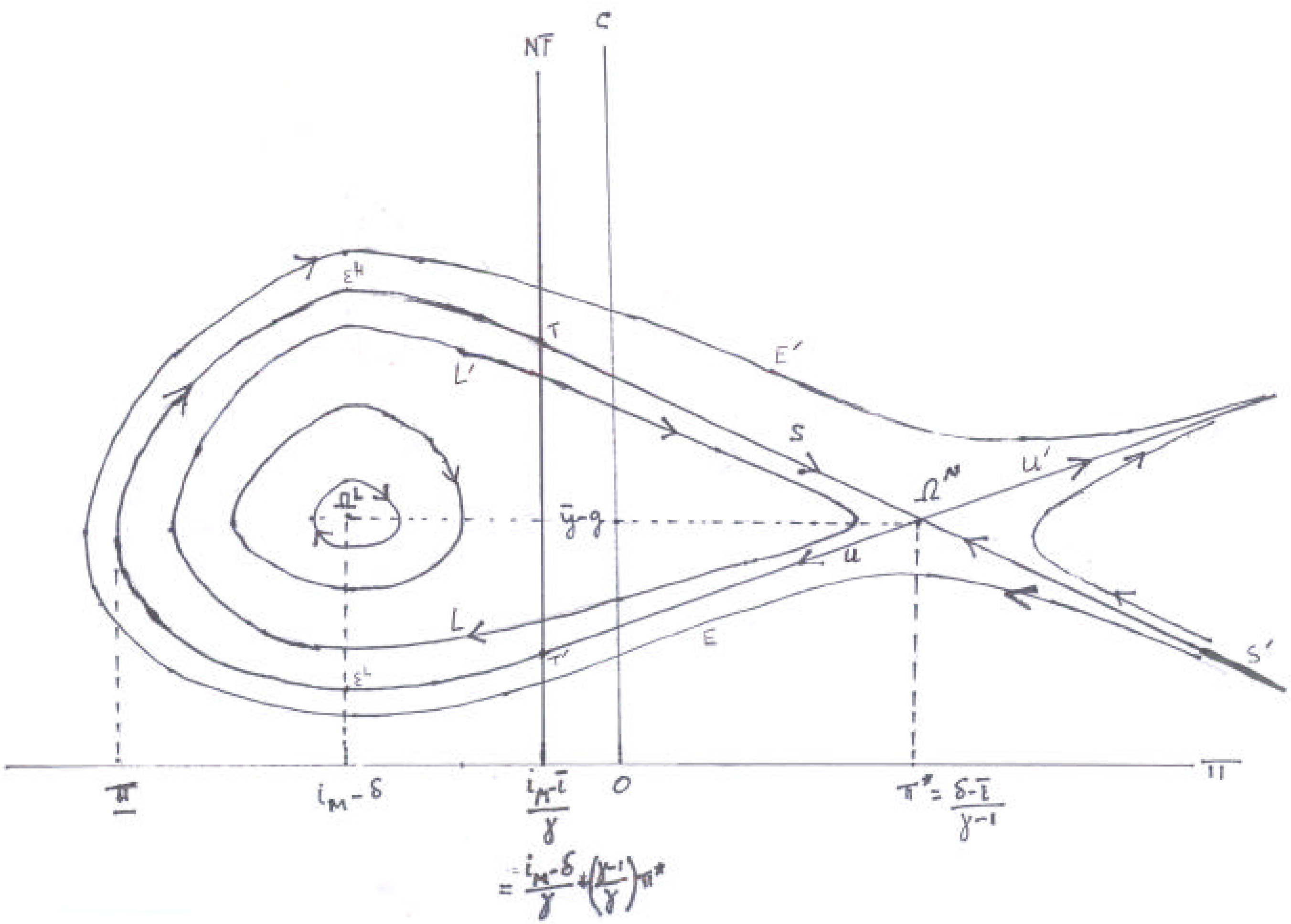
FIGURE 1a



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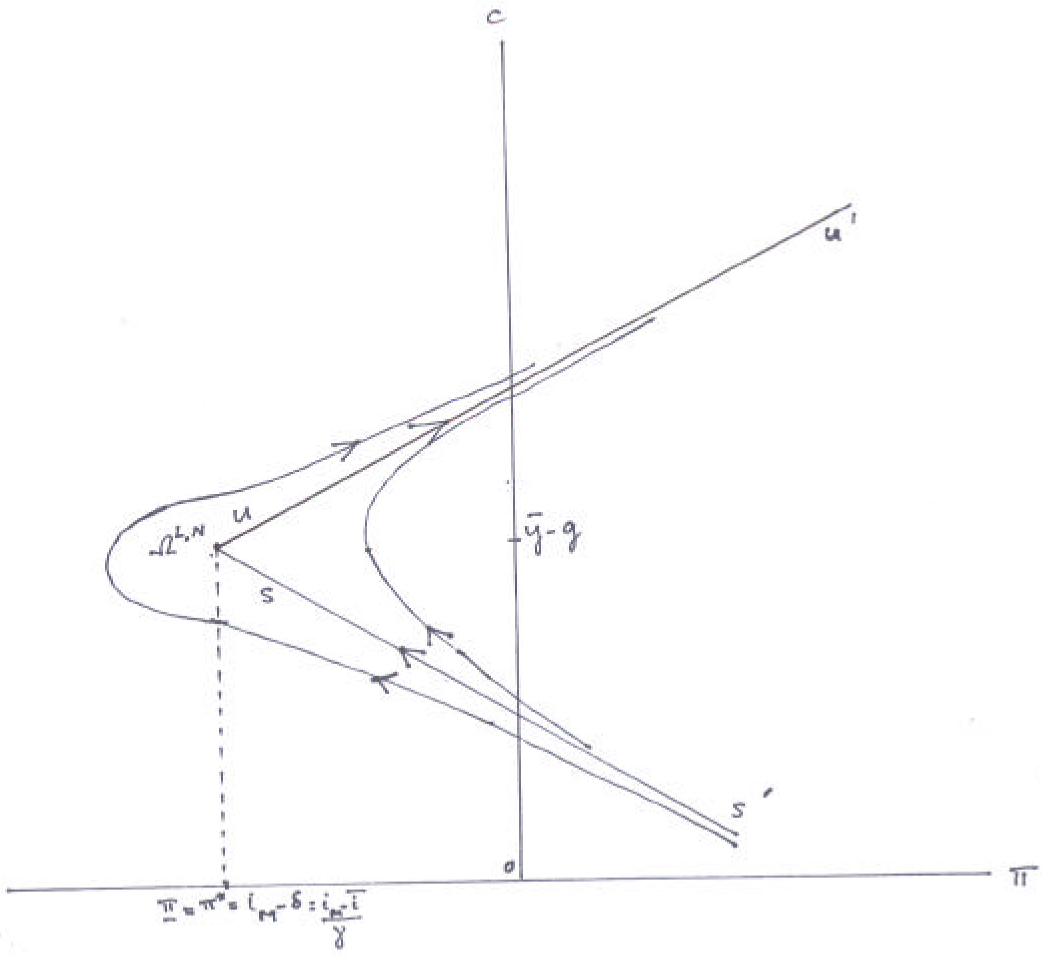
ε^H
ε^L
$\bar{y}-g$
$\underline{\pi}$
$i_M-\delta$
$\frac{i_M-\bar{\imath}}{\gamma}$
$=\frac{i_M-\delta}{\gamma}+\left(\frac{\gamma-1}{\gamma}\right)\pi^*$
0
$\pi^*=\frac{\delta-\bar{\imath}}{\gamma-1}$
π

FIGURE 1b



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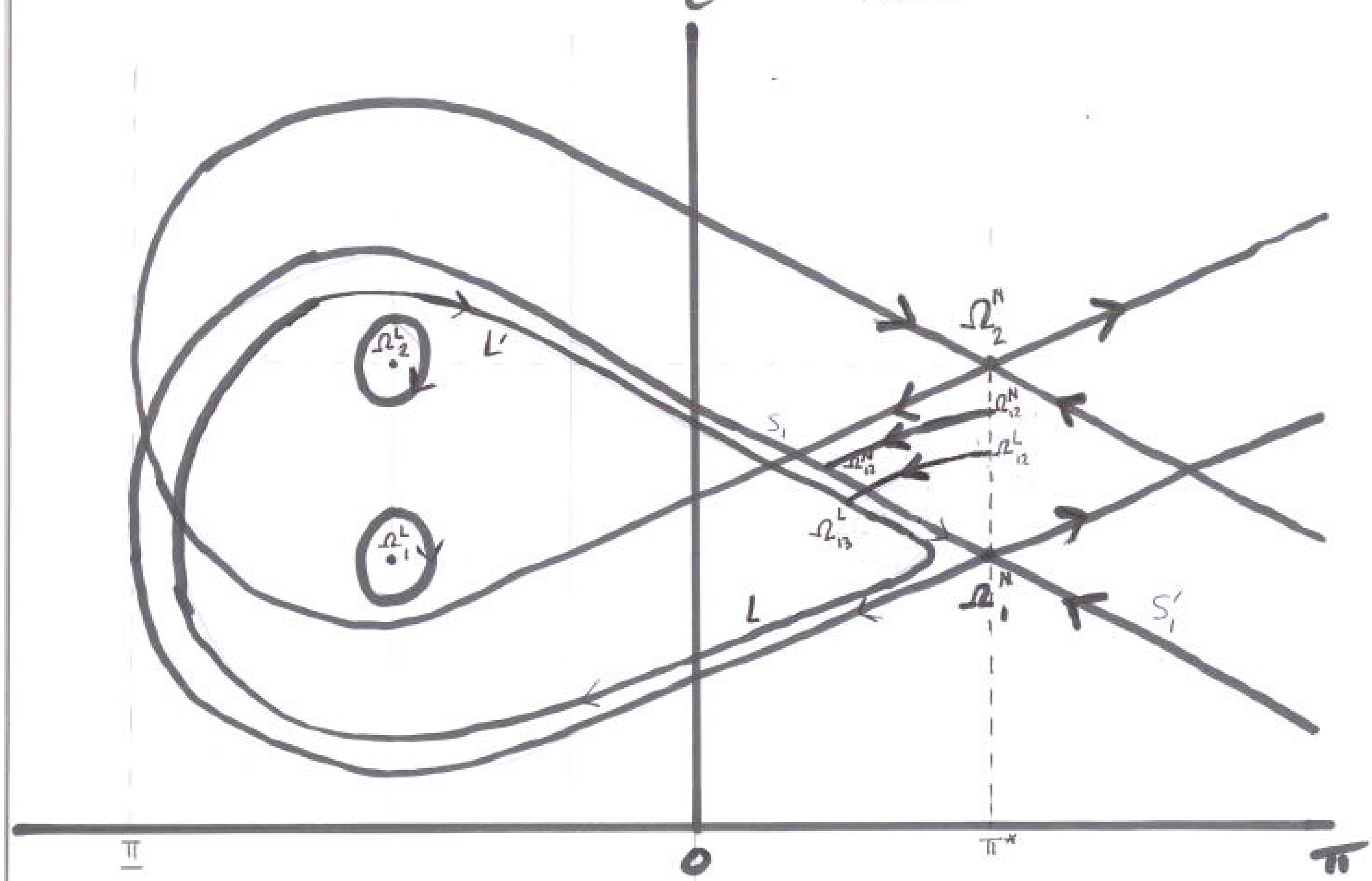
$\Omega^{L,N}$
$\bar{y}-g$
s
s'
0
π
$\underline{\pi} = \pi^* = i_M - \delta = \frac{i_M - \bar{i}}{\gamma}$

FIGURE 2



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Ω^L_{13}
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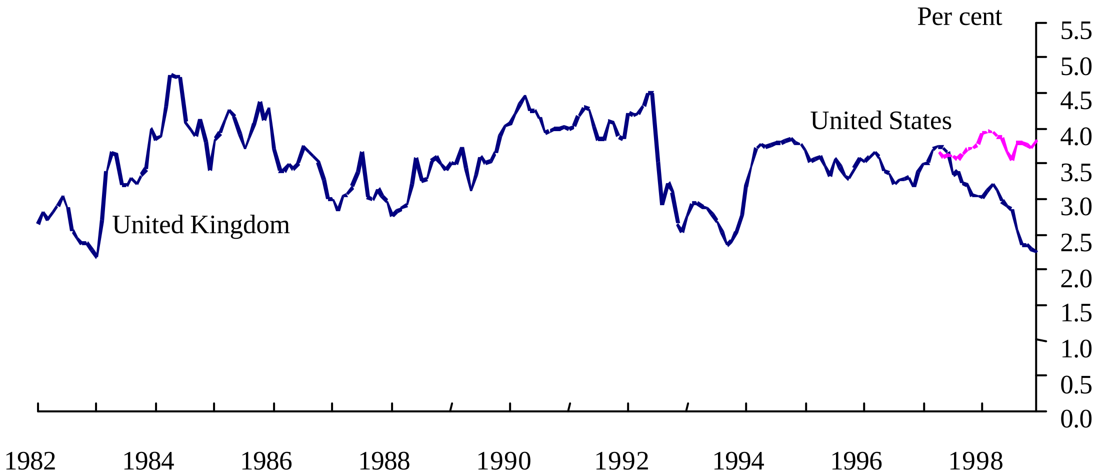
π^*
π

Figure 3

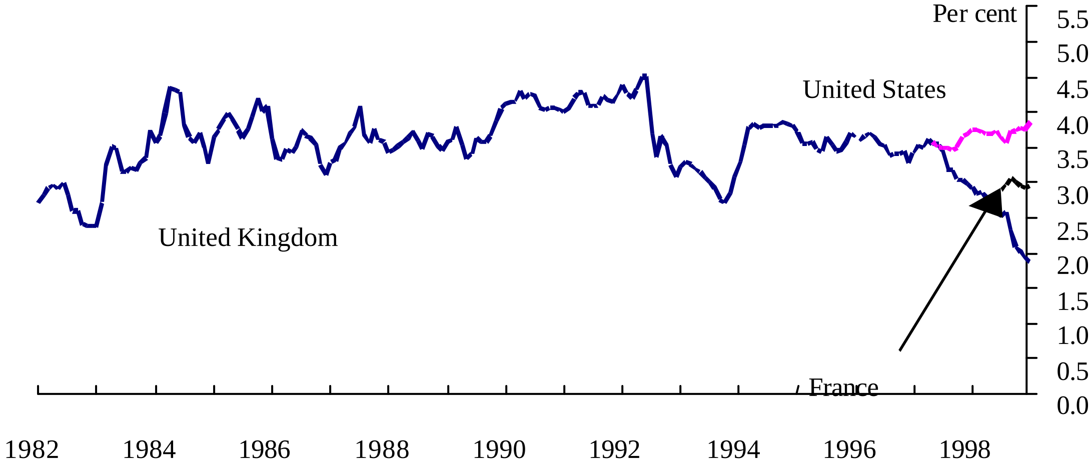
Five-year real interest rates in the United Kingdom and United States [(a)]



(a) US bonds mature in 2002.

Figure 4

Long term year real interest rates in the United Kingdom, United States and France [(a)]



(a) UK bonds mature in 2009, US bonds in 2008, French bonds
in 2009.

Figure 5

Bank Rate, inflation and £/$ exchange rate, 1945 to date

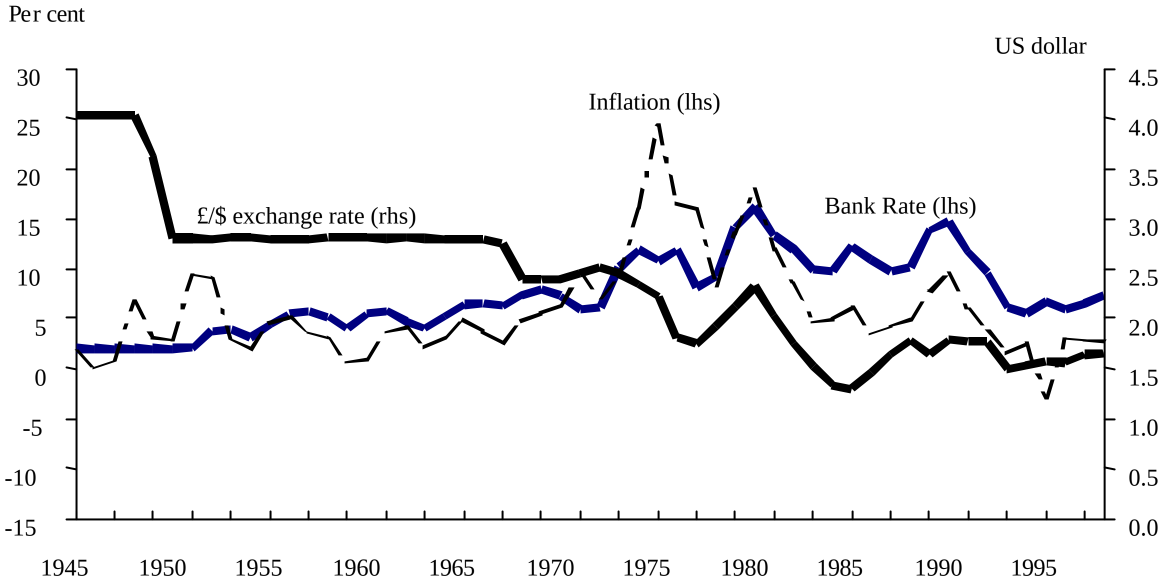


Figure 6

Price level and inflation in the United Kingdom, 1800-1914

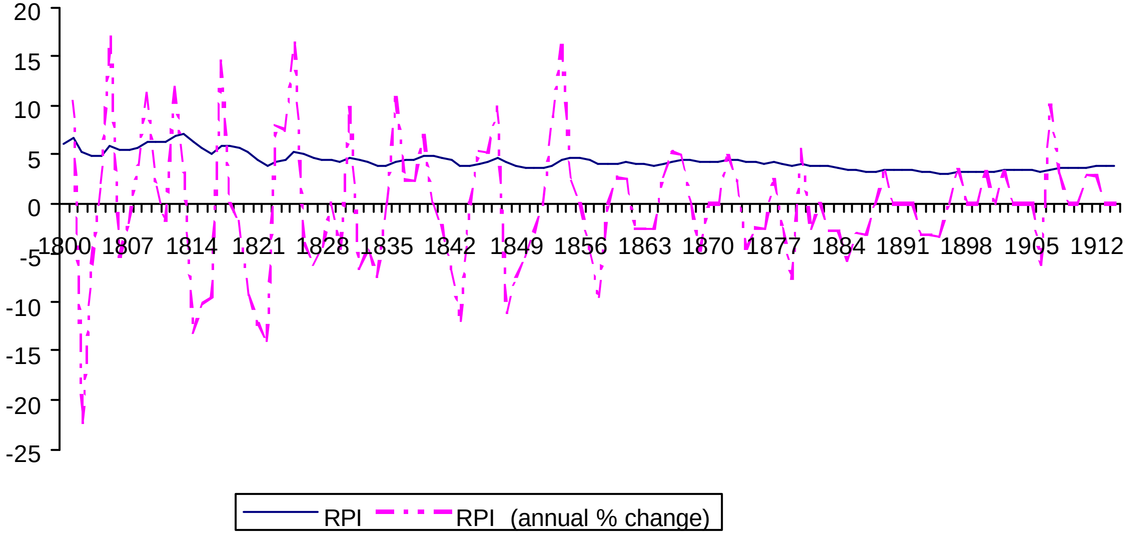


Figure 7

Bank Rate, inflation and £/$ exchange rate, 1817-1914

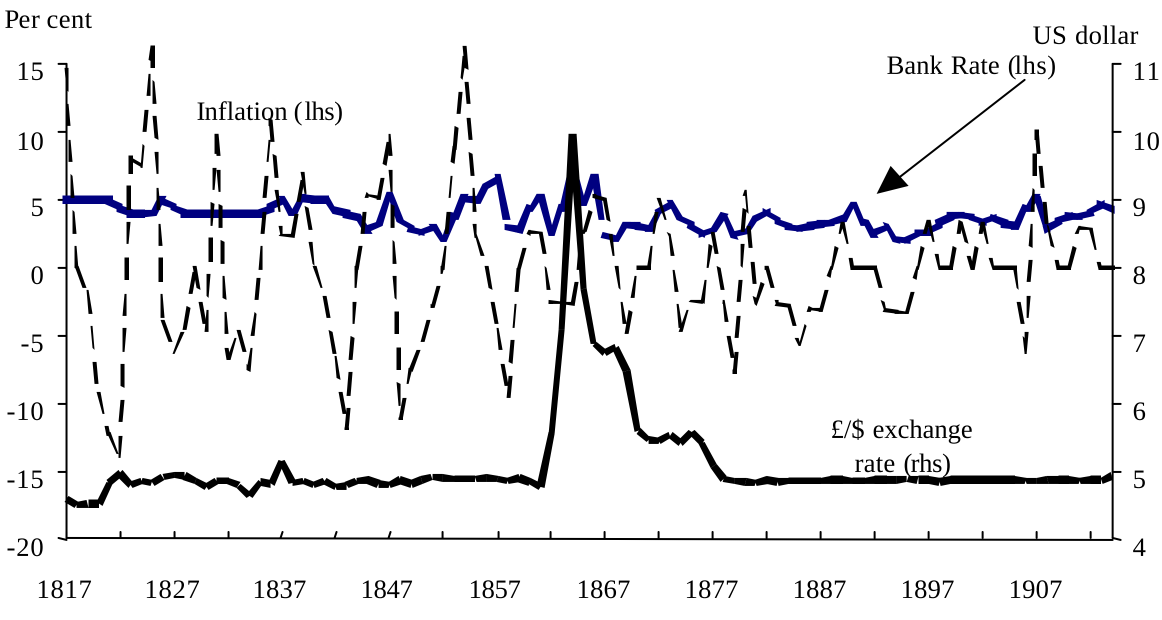
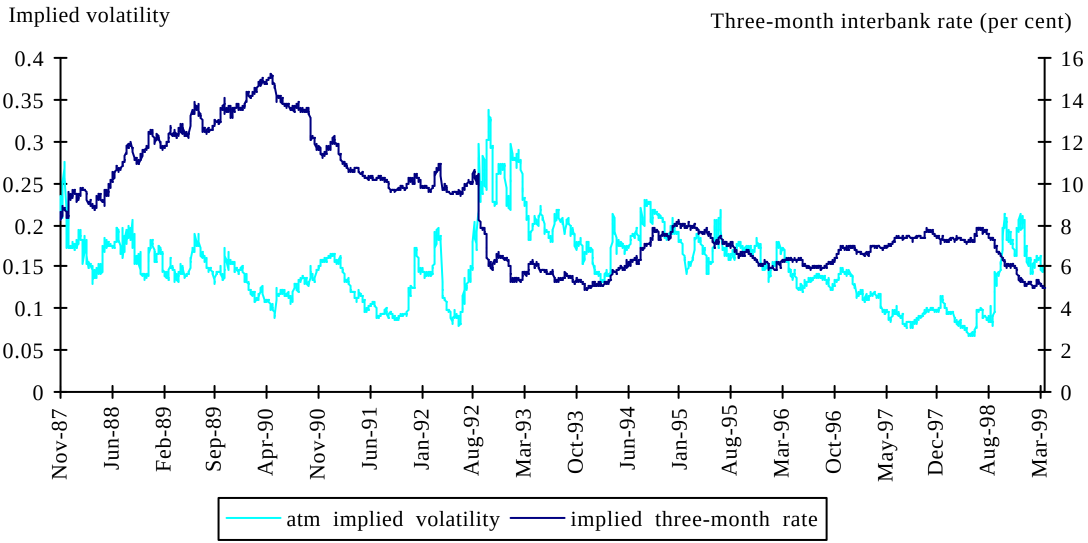


Figure 8

Short-sterling futures options (LIFFE) - constant horizon of six months

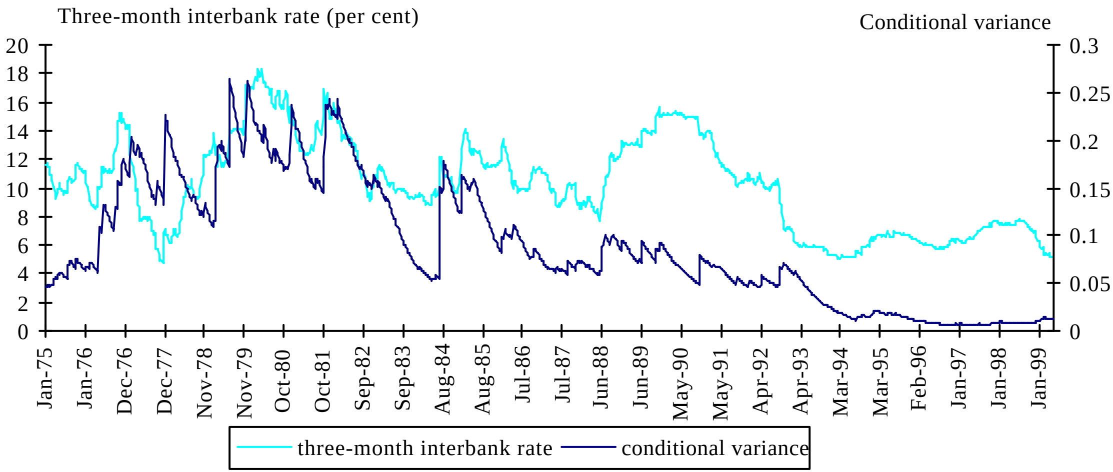


correlation between 1987-99: -0.26, 95% confidence intervals: -0.22 -0.30

correlation between 1993-99: -0.41, 95% confidence intervals: -0.35 -0.46

Figure 9

Time series model of three-month interbank rate

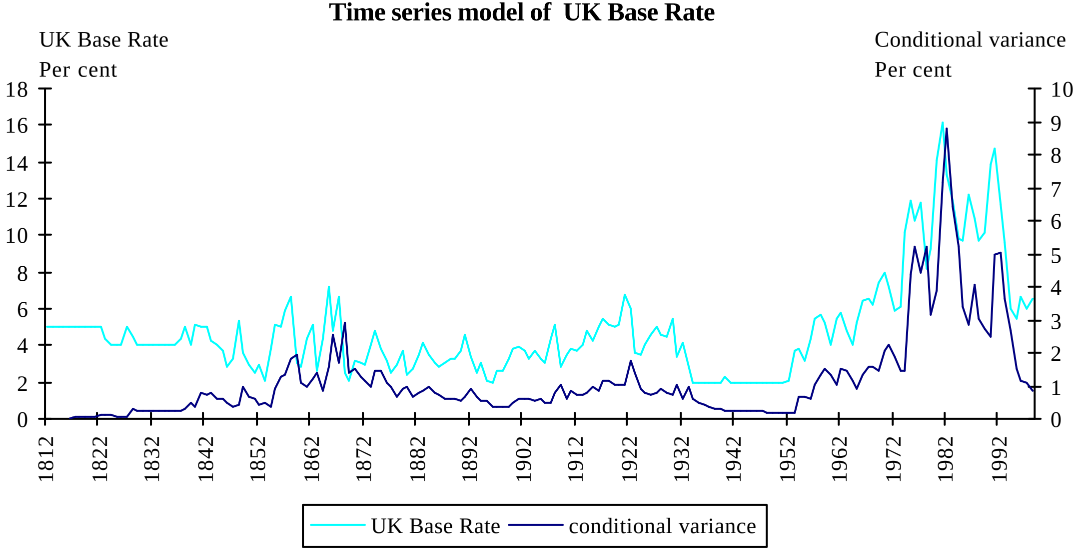


correlation between 1975-99: 0.66, 95% confidence intervals: 0.72 0.59

correlation between 1993-99: -0.26, 95% confidence intervals: -0.13 -0.38

steady-state three-month interbank rate: 7.55, 95% confidence intervals: 4.02 14.41

Figure 10



steady-state Base Rate: 4.88%, 95% confidence intervals: 1% 10%

correlation: 0.81, 95% confidence intervals: 0.66 0.97

Figure 11

UK Base Rate 1800-1999

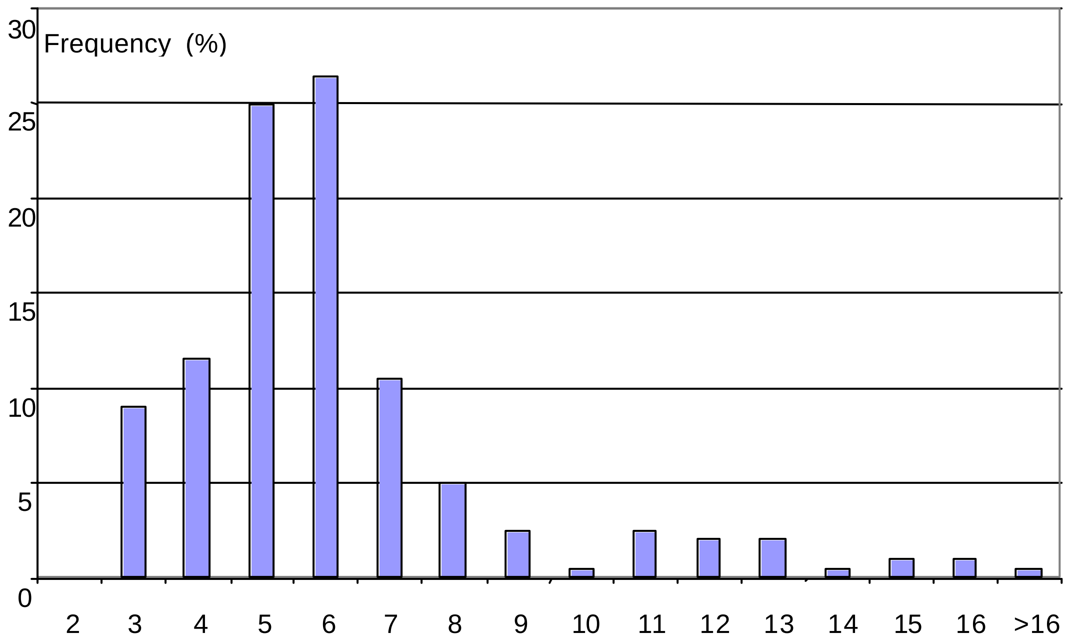
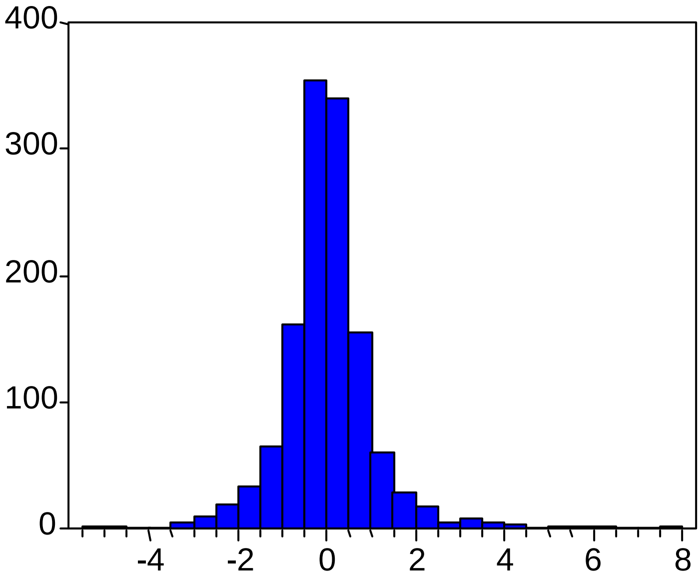


Figure 12

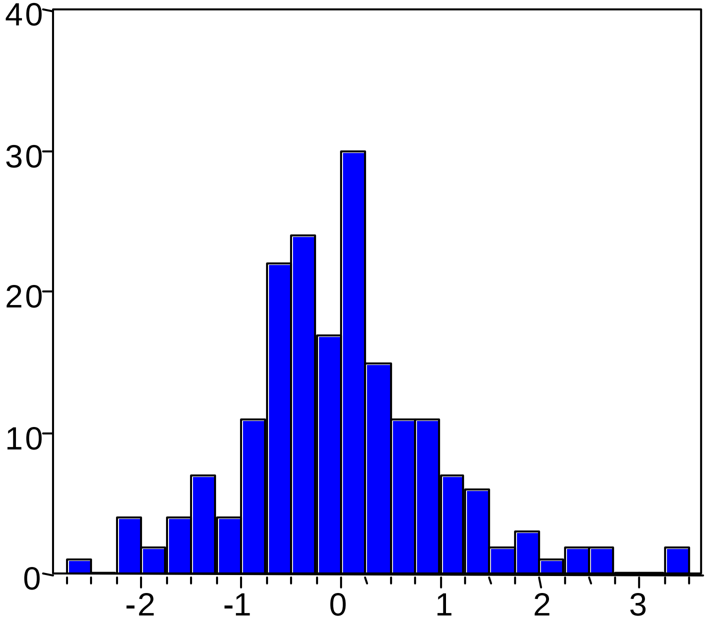
Histogram of standardised residuals: UK three-month interbank rate 1975-99



Observations 1268	
Mean	0.022144
Median	-0.009907
Maximum	7.955724
Minimum	-5.209383
Std. Dev.	1.005875
Skewness	0.846027
Kurtosis	10.49388
Jarque-Bera	3118.290
Probability	0.000000

Figure 13

Histogram of standardised residuals: UK Base Rate 1800-1998

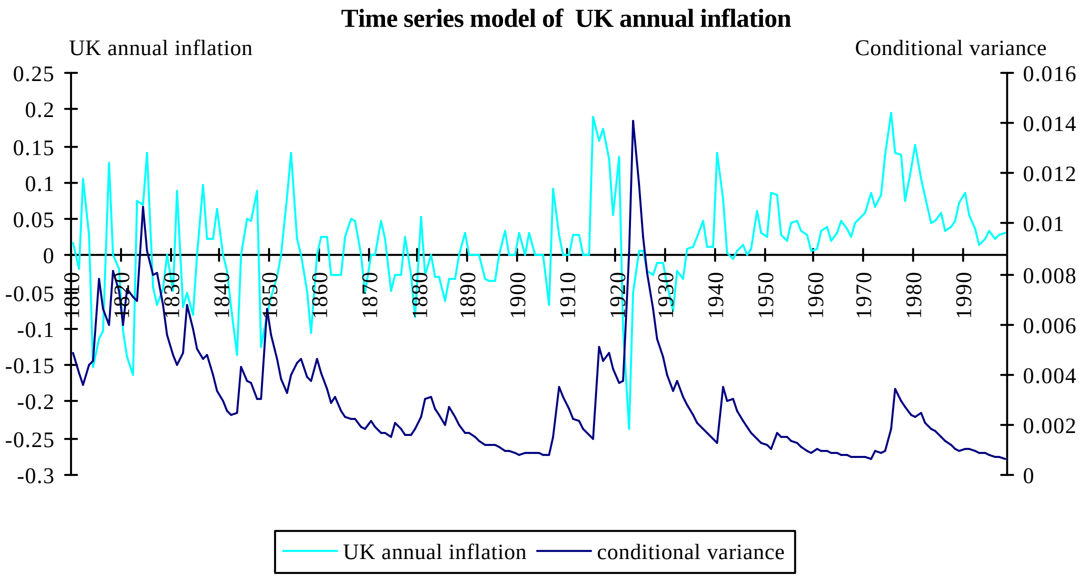


Sample 1811-1998

Observations 188

Mean	-0.008235
Median	-0.056227
Maximum	3.380759
Minimum	-2.702496
Std. Dev.	0.991252
Skewness	0.496179
Kurtosis	4.196770
Jarque-Bera	18.93341
Probability	0.000077

Figure 14

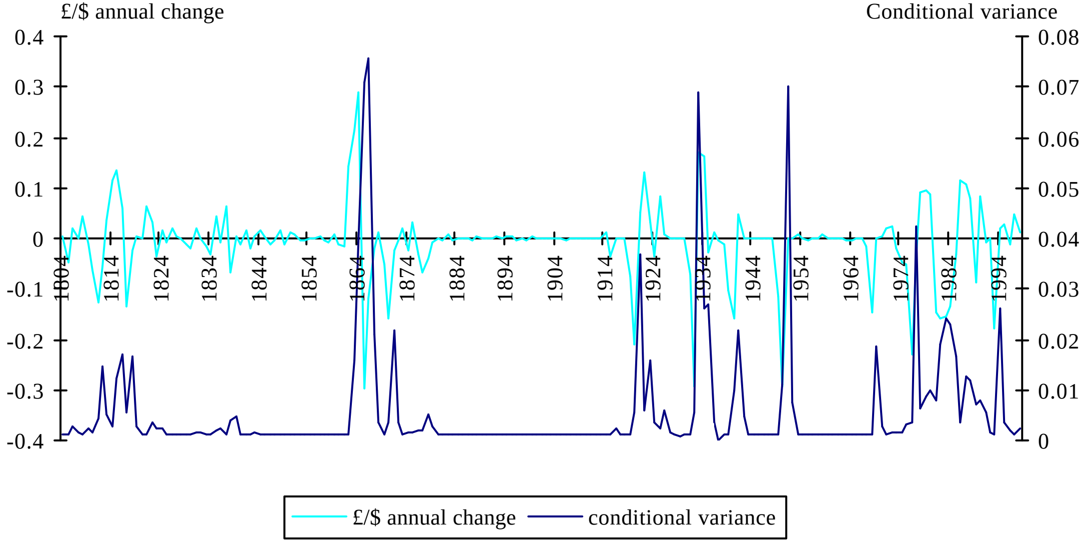


steady-state inflation: 2.7%, 95% confidence intervals: -10.7% 21.0%

correlation: -0.27, 95% confidence intervals: -0.43 -0.12

Figure 15

Time series model of £/$ annual change

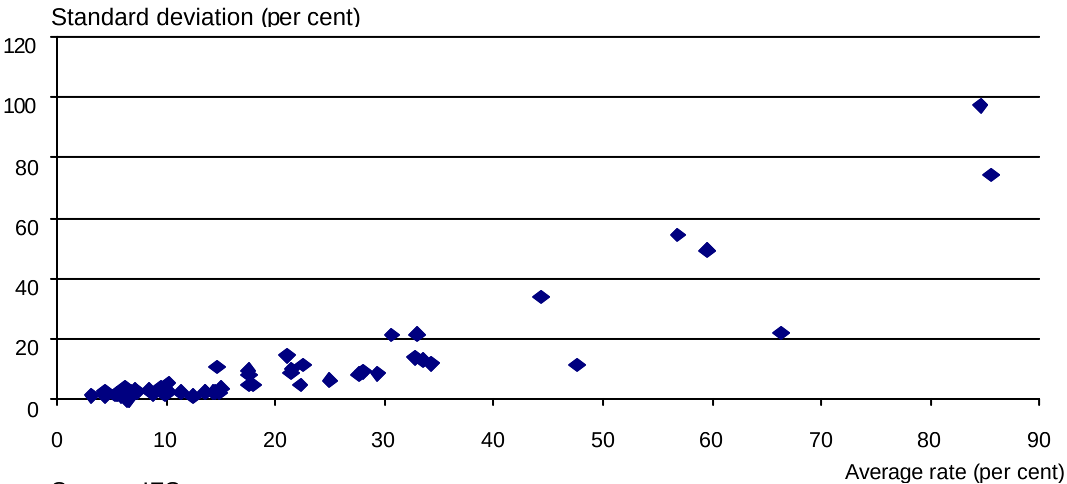


steady-state £/$ annual change: -0.8%, 95% confidence intervals: -10.6% 4.2%

correlation: -0.03, 95% confidence intervals: -0.19 0.13

Figure 16

Scatter plot of the standard deviation and average level of treasury bill rates in 59 countries, January 1988 - January 1999



Source: IFS

correlation 0.89