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FISCAL POLICY IN OPEN, INTERDEPENDENT ECONOMIES

Willem H. Buiter

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ABSTRACT

The paper studies the effects of alternative financing policies in the open economy.

There is a non-trivial role for financial policy because of the failure of first-order debt neutrality due to uncertain private lifetimes. Both the single-country case and the interdependent two-country case are considered. Capital formation is endogenous and there are unified global financial and goods markets determining the interest rate, each country's "Tobin's q" and the terms of trade. The government's present value budget constraint or solvency constraint and the assumption that the interest rate exceeds the growth rate imply that, given spending, current tax cuts imply future tax increases. Such policies boost the outstanding stock of public debt, raise the world interest rate, crowd out capital formation at home and abroad, and lead to a loss of foreign assets. Provided a "supply-side-response-corrected" transfer criterion is satisfied, the terms of trade will improve in the short run and worsen in the long run.

Willem H. Buiter
London School of Economics
Houghton Street
London WC2A 2AE
England

Section I

Introduction

The paper studies fiscal policy in the open economy. It proceeds from the very small country, which is a price taker in the world financial market and in the markets for imports and exports, via the semi-small country, which has some market power in the market for its exportable, to the interdependent two-country case. To keep the analysis tractable, a very simple production structure is assumed: each country consumes both domestic and foreign output but is wholly specialized in the production of its exportable. So as to be able to analyse "crowding out" issues in the short run and the long run, firms in each country can engage in capital formation. Only domestic output can be transformed into domestic capital, and the investment process is subject to strictly convex internal costs of adjustment. There is no money in the model, but international portfolio lending and borrowing can occur in an integrated global financial market. There is no direct foreign investment. Rational point expectations and certainty equivalence are assumed throughout, so all stores of value are perfect substitutes in private portfolios. As cyclical, Keynesian issues are not the focus of this paper, full employment is assumed throughout.

The fiscal policy issue that is our primary concern is the choice of borrowing versus tax financing of a given programme of public spending on goods and services ("exhaustive spending"). The model can of course be used equally well for the analysis of alternative spending-cum-financing policies. So as not to get side-tracked into issues of excess burdens and deadweight losses, all taxes are assumed to be lump-sum. From the government's budget constraint, or rather from its "present value budget constraint", or solvency constraint, it then follows that our concern is with the consequences for private saving and capital formation of intertemporal redistributions of the tax burden. This means that, given the perfect financial markets that are assumed in the model, the analysis could stop right here if we specified households either as infinite-lived or as endowed with operative intertemporal gift and bequest motives (see e.g. Frenkel and Razin [1984a]). To get a non-trivial analysis of the central issue of financial policy one therefore either has to adopt the overlapping generations framework without gift and bequest motives (see e.g. Buiter [1981])or the "uncertain lifetime" approach first developed by Yaari [1965] and applied to macroeconomic issues of fiscal policy in open and closed economies by Blanchard [1983a, b]. The overlapping generations approach has the major drawback that its most popular variant, the two-period life cycle model, has a unit period of about 38 years. This makes it a suitable vehicle, at most, for the study of the Kondratieff cycle. To obtain a more interesting periodization a high price is paid in the form of higher order difference equation systems and difficult aggregation problems. The Yaari-Blanchard approach, adopted in the present paper, captures the essential notion of finite private decision horizons while preserving lower-order dynamics and easy aggregation. There is a price to be paid of course: the instantaneous probability of death is assumed to be independent of age and consequently there are no characteristic life-cycle patterns of saving and wealth.[1/]

This paper investigates the consequences of domestic and foreign governments' taxation-borrowing mixes for saving and capital formation in the two countries and for the interest rate and the real exchange rate. Qualitative, analytical methods are relied on as much as possible, but a large part of the dynamic analysis can only be performed using numerical simulation algorithms.

Section II presents the model. The very small country case is studied in Section III, followed by the semi-small country case in Section IV. The two-country model is put through its paces in Section V.

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1. After completing an earlier version of this paper I became aware of two other applications of the Yaari-Blanchard consumption model to the analysis of fiscal policy in a two-country setting. The first, by Alberto Giovannini [1984] has the same behavioural equations as the model of the present paper, except for the investment functions which are specified without internal or external costs of adjustment. Since only steady-state analysis is conducted, this is not a serious flaw. The second by Frenkel and Razin [1984b] does a dynamic analysis but has no capital formation (i.e. exogenous output) and only considers the one commodity case.

II. The Model

The paper studies the effects of fiscal policy in a dynamic, 2-country, 2-good rational expectations model. Each country is completely specialized in the production of its exportable. Fixed domestic capital formation takes the form of accumulation of domestic output only (subject to internal costs of adjustments). Each country's labour market clears and the world markets for the two traded commodities are in competitive equilibrium. There is a single, integrated, global financial market in which a bond denominated in terms of home country output is traded.

The derivation of the behavioural equations of the model is given in Appendix 1. Consumer behaviour follow's Yaari's [1965] uncertain lifetime approach, as applied to an aggregated macroeconomic model by Blanchard [1983a, b]. Investment behaviour is governed by a Tobin's "q" type relationship based on increasing and strictly convex internal costs of adjustment.

The equations governing the two-country model are :

(1) $q(s) = (\theta + \lambda)(w(s) + h(s)) \qquad \theta, \lambda > 0$

or

(1') $\dot{q}(s) = (r(s) - \theta)q(s) - (\theta + \lambda)\lambda w(s)$

(2) $\dot{w}(s) = r(s)\ w(s) + j(K(s)) - \tau(s) - q(s)$

(3) $\dot{h}(s) = \tau(s) - j(K(s)) + (r(s) + \lambda)h(s)$

or

(3') $$h(s) = \int_s^{\infty} (j(K(t)) - \tau(t)) e^{-\int_s^t (r(u) + \lambda) du} dt$$

(4) $$c_x(s) = \alpha\, q(s) \qquad 0 \leq \alpha \leq 1$$

(5) $$c_y(s) = (1-\alpha) \frac{q(s)}{\pi(s)}$$

(6) $$\dot{K}(s) = (\psi(s) - 1)\zeta^{-1} K(s) \qquad \zeta > 0$$

(7) $$\frac{\dot{\psi}(s)}{\psi(s)} = r(s) - \frac{f'(K(s)}{\psi(s)} - \frac{1}{2\psi(s)} \zeta \left(\frac{\dot{K}(s)}{K(s)}\right)^2$$

(8) $$\dot{b}^G(s) = g_x(s) + \pi(s)\, g_y(s) + r(s)\, b^G(s) - \tau(s)$$

(9) $$q^*(s) = (\theta^* + \lambda^*)(w^*(s) + h^*(s)) \qquad \theta^*, \lambda^* > 0$$

or

(9') $$\dot{q}^*(s) = \left(r(s) - \frac{\dot{\pi}(s)}{\pi(s)} - \theta^*\right) q^*(s) - (\theta^* + \lambda^*)\lambda^*\, w^*(s)$$

(10) $$\dot{w}^*(s) = \left(r(s) - \frac{\dot{\pi}(s)}{\pi(s)}\right) w^*(s) + j^*(K^*(s)) - \tau^*(s) - q^*(s)$$

(11) $$\dot{h}^*(s) = \tau^*(s) - j^*(K^*(s)) + \left(r(s) - \frac{\dot{\pi}(s)}{\pi(s)} + \lambda^*\right) h^*(s)$$

or

(11') $$h^*(s) = \int_s^{\infty} (j^*(K^*(t)) - \tau^*(t)) e^{-\int_s^t \left(r(u) - \frac{\dot{\pi}(u)}{\pi(u)} + \lambda^*\right) du} dt$$

(12) $$c_x^*(s) = \alpha^* \pi(s) q^*(s) \qquad 0 \leq \alpha^* \leq 1$$

(13) $$c_y^*(s) = (1 - \alpha^*) q^*(s)$$

(14) $$\dot{K}^*(s) = (\psi^*(s) - 1)\zeta^{*-1} K^*(s)$$

(15) $$\frac{\dot{\psi}^*(s)}{\psi^*(s)} = r(s) - \frac{\dot{\pi}(s)}{\pi(s)} - \frac{f^{*\prime}(K^*(s))}{\psi^*(s)} - \frac{\frac{1}{2}\zeta^*}{\psi^*(s)}\left(\frac{\dot{K}^*(s)}{K^*(s)}\right)^2$$

(16) $$\dot{b}^{*G}(s) = g_x^*(s) + \pi(s)\, g_y^*(s) + r(s)\, b^{*G}(s) - \pi(s)\tau^*(s)$$

(17) $$f^*(K^*(s)) = c_y(s) + g_y(s) + \dot{K}^*(s) + c_y^*(s) + g_y^*(s) + \frac{1}{2}\zeta^* \frac{(\dot{K}^*(s))^2}{K^*(s)}$$

(18) $$f(K(s)) = c_x(s) + g_x(s) + \dot{K}(s) + c_x^*(s) + g_x^*(s) + \frac{1}{2}\zeta \frac{\dot{K}(s)^2}{K(s)}$$

(19) $$F(s) = w(s) - \psi(s)\, K(s) - b^G(s)$$

These nineteen equations determine the behaviour over time of q, q^*, w, w^*, h, h^*, K, K^*, c_x, c_x^*, c_y, c_y^*, ψ, ψ^*, b^G, b^{*G}, r, π and F given the values of the fiscal instrument, τ, τ^*, g_x, g_x^*, g_y and g_y^*.

Equations (1) through (8) describe the domestic economy. Aggregate consumption, q (measured in terms of domestic output), is a constant linear function of total (human h plus non-human w) wealth (equation 1). The constant of proportionality is the sum of the pure rate of time preference, θ and the constant (i.e. age-independent) instantaneous probability of death, λ. The familiar infinite-lived consumer world with its debt-neutrality properties is the special case of our model when $\lambda = 0$. The rate of change of aggregate private non-human capital $\dot{w}$, equals private disposable income minus private consumption. Disposable income is labour income $y \equiv j(.)$, plus interest income rw minus taxes τ. Private financial wealth consists of private holdings of domestic government bonds, b^G, of claims on the rest of the world, F, and of claims on the domestic capital stock, K. All assets are perfect substitutes in private portfolios, so their expected rates of return are equalized. All bonds, whether issued by domestic or foreign private or public agents are denominated in terms of domestic output (good x) and are of the fixed market value, variable interest rate variety. r(s) is the instantaneous own rate of interest on these bonds. Labour income y is the product of the wage rate and the exogenous labour supply, which is fully employed. Choice of units sets employment equal to unity. The production function is linear homogeneous in capital, K, and labour, is strictly concave and satisfies the Inada conditions. Output (and output per worker) u is therefore given by $u = f(K)$, $f' > 0$, $f'' < 0$, $f(0) = 0$, $\lim_{K \to 0} f'(K) = +\infty$, $\lim_{K \to \infty} f'(K) = 0$. Under competitive market-clearing conditions, the wage rate (and labour income) is given by $j(K) = f(K) - Kf'(K)$ with $j' = -Kf'' > 0$. Equation (3) or (3) expresses human capital h as the present discounted value of future after-tax labour income. Note that the discount rate equals the market rate of interest, r, plus the instantaneous probability of death, λ. h(s) is the human capital of those currently alive. They do not expect to be around forever even though a population of constant size is around forever.

The instantaneous utility of current consumption function is Cobb-Douglas in the consumption of the domestic good c_x and consumption of the foreign good c_y. The constant share of consumption of domestic output in total ccnsumption spending (q) is α. The relative price of foreign goods in terms of domestic goods (i.e. the reciprocal of the terms of trade) is denoted by π.

Each country is completely specialized in the production of its own exportable. Households and governments consume both domestic and foreign goods. Capital accumulation in each country only involves that country's own output. Investment is subject to quadratic internal adjustment costs. Depreciation is ignored. Because both the production function and the cost-of-adjustment function are assumed to be linear homogeneous, the shadow price of domestic capital("Tobin's marginal q"), also equals the value of a unit of existing, installed capital in terms of current output, ψ. In equation (6) ζ is the cost-of-adjustment parameter. Equation (7) is the familiar condition that value of the marginal product of capital (corrected for adjustment costs) equals the cost of capital, i.e. the sum of the interest rate and the expected proportional rate of change of ψ. The government budget constraint is given in (8). g_x denotes government spending on domestic consumption goods and g_y government spending on imports.

Equations (9), (9'), (10), (11), (11'), (12), (13), (14) and (16) are self-explanatory foreign counterparts of domestic behavioural relationships. q^*, w^* and h^* are measured in terms of foreign output.

Equation (15) is the foreign cost-of-capital equation with the assumption of a perfectly integrated set of financial markets substituted in. The (costs-of-adjustment corrected) value of the marginal product of foreign capital (in terms of foreign output) equals the domestic interest rate minus the expected proportional rate of change of the relative price of foreign goods minus the expected proportional rate of change of ψ^*.

There are three economy-wide market equilibrium conditions: market-clearing for foreign output (17), for domestic output (17') and for financial claims. Because of Walras' Law, the financial market equilibrium condition will be dropped from explicit consideration. Equation (19) defines net domestic claims on the rest of the world F. Its rate of change equals the domestic current account surplus, i.e.

$$(19') \quad \dot{F}(s) = f(K(s)) + r(s)\, F(s) - \left(q(s) + g_x(s) + \pi(s)\, g_y(s) + \dot{K}(s) + \frac{1}{2}\zeta \frac{\dot{K}(s)^2}{K(s)}\right)$$

Note that (17) and (18) imply the following loanable funds type flow equilibrium condition:

$$(18') \quad r(s)(w(s) + \pi(s)w^*(s)) + j(K(s)) + j^*(K^*(s))\pi(s) - \tau(s) - \tau^*(s)\pi(s) - q(s) - \pi(s)\, q^*(s) = \psi(s)\dot{K}(s) + \pi(s)\psi^*(s)\dot{K}^*(s) + \dot{b}^G(s) + \dot{b}^{*G} + K(s)\dot{\psi}(s) + K^*(s)\pi(s)\dot{\psi}^*(s) + K^*(s)\psi^*(s)\dot{\pi}(s)$$

Its stock equilibrium counterpart is

$$(18') \quad w(s) + \pi(s)w^*(s) = \psi(s)K(s) + \pi(s)\psi^*(s)K^*(s) + b^G(s) + b^{*G}(s).$$

In the derivation of the optimal household consumption programme use has been made of the household "present value budget constraint" or "solvency constraint".

$$(20) \quad \int_s^\infty \bar{q}(t,v)e^{-\int_s^v (r(u)+\lambda)du} dv = \bar{w}(t,s) + \int_s^\infty (\bar{j}(t,v) - \bar{\tau}(t,v))e^{-\int_s^v (r(u)+\lambda)du} dv - \lim_{\ell\to\infty} \bar{w}(t,\ell)e^{-\int_s^\ell (r(u)+\lambda)du}$$

($\bar{q}(t,v)$ denotes consumption at time v by a household born at time t and similarly for $\bar{w}(t,v)$, $\bar{j}(t,v)$ and $\bar{\tau}(t,v)$). The conventional transversality condition is :

$$(20') \quad \lim_{\ell\to\infty} \bar{w}(t,\ell)e^{-\int_s^\ell (r(u)+\lambda)du} = 0.$$

The operational meaning of this is that the present value of future household debt must ultimately go to zero. It precludes household Ponzi games. Equally important is the public sector's "present value budget constraint" or "solvency constraint"

$$(21) \quad \int_s^\infty [g_x(t) + \pi(t)\ g_y(t)]e^{-\int_s^t r(u)du} dt = \int_s^\infty \tau(t)e^{-\int_s^t r(u)du} dt - b^G(t) + \lim_{\ell\to\infty} b^G(\ell)e^{-\int_s^\ell r(u)du}$$

The terminal condition we impose here,

$$\lim_{\ell\to\infty} b^G(\ell)e^{-\int_s^\ell r(u)du} = 0 \qquad (21')$$

rules out Ponzi games for the public sector: the present value of future spending programmes and the servicing of the existing stock of debt must be equal to the present value of future taxes. The condition that it is not feasible to service debt through further borrowing indefinitely is of course only plausible if the real interest rate exceeds the natural (long run) proportional rate of growth of real economic activity. Note that while existing households discount future taxes at a rate $r+\lambda$, the government, which knows it will tax both existing and yet-to-be born households, discounts future taxes at a rate r.

III. The very small open economy

The very small open economy treats both the interest rate and the terms of trade as parameteric. The behaviour of the household sector is given by

$$\dot{q} = (r-\theta)q - (\theta+\lambda)\lambda w$$

$$\dot{w} = rw + j(K) - \tau - q$$

The remaining dynamics can be summarized by :

$$\dot{K} = \left(\frac{\psi - 1}{\zeta}\right) K$$

$$r = \frac{f'(K)}{\psi} + \frac{\dot{\psi}}{\psi} + \frac{1}{2}\frac{\zeta}{\psi}\left(\frac{\dot{K}}{K}\right)^2$$

and

$$\dot{F} = f(K) + rF - \left(q + \dot{K} + g_x + \pi g_y + \frac{1}{2}\zeta\frac{\dot{K}^2}{k}\right)$$

Under the assumption of exogenous factor income, this model is studied in Blanchard [1983b]. Since the capital stock dynamics is a function only of the exogenous rate of interest r, we can analyse the behaviour of q, w and F in response to any shocks other than interest rate changes, while treating K as exogenous. Indeed, by considering an initial position of stationary equilibrium, with $\dot{K} = \dot{\psi} = 0$ (and $\psi = 1$), K can be treated as constant throughout. Furthermore, the (q, w) subsystem is self-contained and $\dot{F}$ is determined recursively given q and w. The state-space representation of the system (with $\dot{K} = 0$) is given in (22).

(22)
$$\begin{bmatrix} \dot{q} \\ \dot{w} \\ \dot{F} \end{bmatrix} = \begin{bmatrix} (r-\theta) & -(\theta+\lambda)\lambda & 0 \\ -1 & r & 0 \\ -1 & 0 & r \end{bmatrix} \begin{bmatrix} q \\ w \\ F \end{bmatrix} + \begin{bmatrix} 0 \\ j(K) - \tau \\ f(K) - (g_x + \pi g_y) \end{bmatrix}$$

The (q, w) subsystem has two characteristic roots, $r - (\theta + \lambda)$ and $r + \lambda$. It will be saddle-point stable if $-\lambda < r < \theta + \lambda$. q is non-predetermined and, with ψ exogenous, w is predetermined. We assume this condition to be satisfied. In addition we assume $r > 0$. The third root (the one governing F) is r.

The steady-state conditions for the private economy are, for $\lambda > 0$:

$$(23) \quad w = \frac{-(r-\theta)}{r^2-\theta(r+\lambda)-\lambda^2}(y-\tau) = \frac{-(r-\theta)}{[r-(\theta+\lambda)][r+\lambda]}(y-\tau)$$

$$(24) \quad q = \frac{-(\theta+\lambda)\lambda}{r^2-\theta(r+\lambda)-\lambda^2}(y-\tau) = \frac{-(\theta+\lambda)\lambda}{[r-(\theta+\lambda)](r+\lambda)}(y-\tau)$$

The case of the infinite-lived consumer $(\lambda = 0)$ yields no meaningful solution unless $r = \theta$. We shall not consider it any further here.

The "saddlepoint" condition implies that the denominator of (23) and (24) is negative.

Even if the private economy settles down to a stationary equilibrium, the equation of motion for the current account could, apparently, exhibit perpetual deficits or surpluses. Indeed, the root governing the predetermined variable F in (22) is $r > 0$, implying explosive behaviour. This unfortunate feature of many small open economy models with a perfect international capital market is, however, ruled out by the government's present value budget constraint (PVBC) given in (21) and the no-Ponzi game transversality condition given in (21'). We can therefore only consider budgetary and financial strategies which are consistent with bounded values of government debt. Consider any set of strategies which has the property that the stock of public sector debt converges, possibly asymptotically, to a finite stationary value. In steady-state equilibrium such strategies, are, from the government budget constraint in (8) characterized by

(25) $g_x + g_y + rb^G = \tau$

with $\dot{b}^G = 0$.

Note that with (25), a stationary equilibrium for w implies a stationary equilibrium for F. Consider alternative stationary equilibria with identical constant values for exhaustive public spending g_x and g_y but different values of taxes τ and public debt b^G. Across such steady states a lower value of taxes will be associated with higher private consumption and lower public debt. Private non-human wealth will be higher (lower) when steady-state taxes are lower, according as to the interest rate is above (below) the pure rate of time preference. All this holds for a constant level of exhaustive public spending.

$$\frac{dq}{d\tau} = \frac{(\theta+\lambda)\lambda}{r^2-\theta(r+\lambda)-\lambda^2} < 0$$

$$\frac{db^G}{d\tau} = \frac{1}{r} > 0$$

$$\frac{dw}{d\tau} = \frac{(r-\theta)}{r^2-\theta(r+\lambda)-\lambda^2} \lessgtr 0 \quad \text{if } r \gtrless \theta$$

$$\frac{dF}{d\tau} = \frac{r-\theta}{r^2-\theta(r+\lambda)-\lambda^2} - \frac{1}{r} \leq 0 \quad \text{if } r \geq \theta$$

E.g. if $r = \theta$, public debt displaces foreign assets one-for-one in private portfolios across steady states. Note that when private agents have infinite horizons ($\lambda = 0$) changes in τ and b^G have no effect (short run or long run) on consumption or net foreign assets if exhaustive public spending is constant.[2/]

Note that we cannot use (25) by itself to analyse the "real time" consequence of a change in τ with g_x and g_y constant. b^G is predetermined at a point in time. Except through default, a government cannot engineer a finite, discrete change in b^G at a point in time; real-time changes in b^G have the dimension of $\dot{b}^G$, i.e. b^G is a continuous function of time. What we must do in order to be able to use (25) to derive the "real time" long-run effects of a change in τ is to specify rules for spending, taxation and borrowing that are consistent with convergence to a steady-state equilibrium for b^G and with unchanged values of g_x and g_y across steady states.

Ideally, such rules would reflect the optimizing behaviour of governments. In this paper only ad-hoc rules that are likely to satisfy the government's solvency constraint are considered.

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2. $r = \theta$ must be assumed when $\lambda = 0$.

A fairly general stabilizing rule for taxes is given in equations (26) and (27) for the home country and the foreign country respectively:

$$\tau = \tau_1 + \mu_\tau \dot{b}^G + \nu_\tau b^G \tag{26}$$

$$\tau^* = \tau_1^* + \mu_\tau^* \frac{\dot{b}^{*G}}{\pi} + \nu_\tau^* \frac{b^{*G}}{\pi} \tag{27}$$

Thus taxes have a lump-sum component but also respond either to the deficit and/or to the size of the debt.

The behaviour of domestic and foreign public debt under these rules is given by :

$$\dot{b}^G = \left(\frac{r - \nu_\tau}{1 + \mu_\tau}\right) b^G + \frac{1}{1 + \mu_\tau} (g - \tau_1) \tag{28}$$

$$\dot{b}^{*G} = \left(\frac{r - \nu_\tau^*}{1 + \mu_\tau^*}\right) b^{*G} + \frac{1}{1 + \mu_\tau^*} \pi (g^* - \tau_1^*) \tag{29}$$

where

$$g \equiv g_x + \pi g_y \tag{30a}$$

and

$$g^* \equiv \frac{g_x^*}{\pi} + g_y^* \tag{30b}$$

Total taxes under this rule evolve according to

$$(31) \qquad \tau = \frac{1}{1+\mu_\tau}\tau_1 + \frac{\mu_\tau}{1+\mu_\tau} g + \left(\frac{r\mu_\tau + \nu_\tau}{1+\mu_\tau}\right) b^G$$

$$(32) \qquad \tau^* = \frac{1}{1+\mu^*_\tau}\tau^*_1 + \frac{\mu^*_\tau}{1+\mu^*_\tau} g^* + \left(\frac{r\mu^*_\tau + \nu^*_\tau}{1+\mu^*_\tau}\right) \frac{b^{*G}}{\pi}$$

Any rule with $\frac{r - \nu_\tau}{1 + \mu_\tau} < 0$ satisfies home country government's solvency requirement. In most of the examples considered below we assume $\nu_\tau = \nu^*_\tau = 0$ in which case the solvency condition becomes $\mu_\tau, \mu^*_\tau < -1$.

The main fiscal experiment will be a change in τ_1 or τ^*_1. With $\nu_\tau = \nu^*_\tau = 0$, this change in the lump-sum component of the tax bill also equals the steady-state change in total tax receipts.

Note, however, from (31) and (28) that a long-run, steady-state tax increase is "achieved" by a short-run tax cut which results in an initial budget deficit. Indeed, the authorities run a net cumulative budget deficit during the adjustment process towards the new long-run equilibrium. With a given spending programme on goods and services, the higher long-run taxes will be exactly sufficient to service the higher volume of public debt.

If we wish to use (22) to analyse the short-run and long-run consequences of a constant, exogenous change in taxes, the government's present value budget constraint (PVBC) can only be satisfied if we assume that public spending $(g_x + \pi g_y)$ is adjusted to maintain solvency.

By analogy with (26) and (27) we could specify potentially stabilizing public spending rules that make spending responsive to the level and/or rate of change of public debt, e.g.

$$g = g_1 + \mu_g \dot{b}^G + \nu_g b^G \tag{30}$$

$$g^* = g_1^* + \mu_g^* \frac{\dot{b}^{*G}}{\pi} + \nu_b^* \frac{b^{*G}}{\pi} \tag{31}$$

The behaviour of domestic and foreign public debt under these rules is given by

$$\dot{b}^G = \left(\frac{r + \nu_g - \nu_\tau}{1 - \mu_g + \mu_\tau} \right) b^G + \frac{1}{1 - \mu_g + \mu_\tau} (g_1 - \tau_1) \tag{32}$$

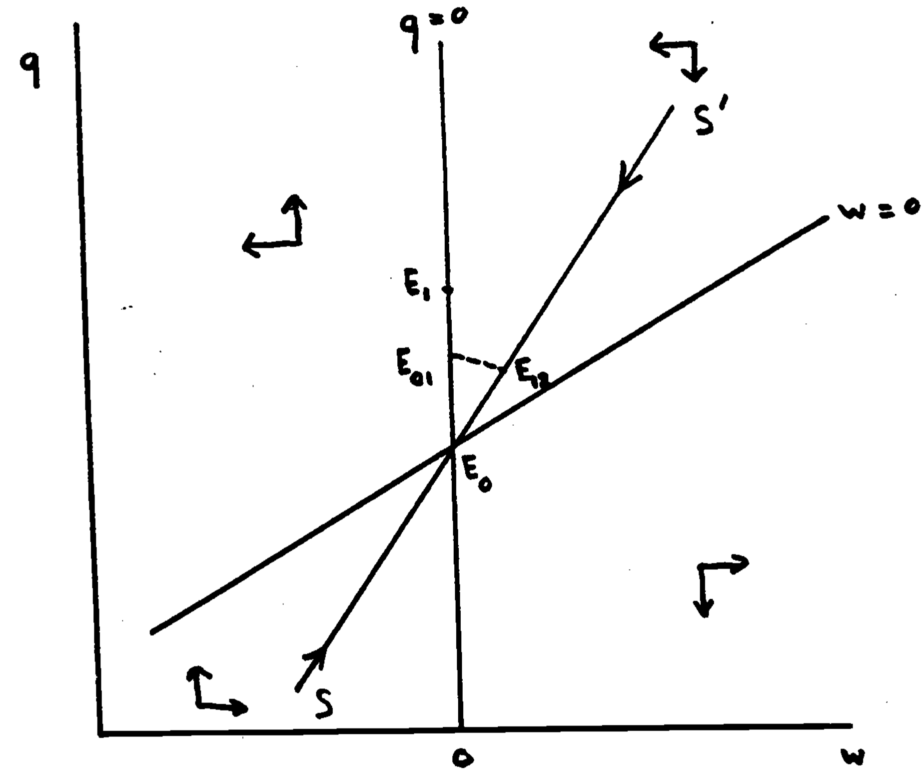
$$\dot{b}^{*G} = \frac{(r + \nu_g^* - \nu_\tau^*)}{1 - \mu_g^* + \mu_\tau^*} b^{*G} + \frac{1}{1 - \mu_g^* + \mu_\tau^*} \pi (g_1^* - \tau_1^*) \tag{33}$$

Even with exogenous taxes ($\nu_\tau = \nu_\tau^* = \mu_\tau = \mu_\tau^* = 0$) the spending rule parameters can be chosen in such a way as to make (32) and (33) convergent processes. For illustrative purposes, consider the case of exogenous taxes and $\nu_g = 0$. A convergent debt process requires $\mu_g > 1$ since

$$\dot{b}^G = \frac{r}{1 - \mu_g} b^G + \frac{1}{1 - \mu_g} (g_1 - \tau_1) \tag{34}$$

The behaviour of the (q,w) system with exogenous taxes and endogenous spending (equation (30) with $\nu_g = 0$ and equation (34)) is illustrated in Figure 1 for the case $r = \theta$. The unique convergent saddlepath SS' in q-w space is upward-sloping.

Figure 1



Starting from E_o, an unanticipated, immediate permanent tax cut moves the q, w system immediately to its new stationary equilibrium value at E_1. The unexpected announcement at t_o of an immediate temporary tax cut, to be ended at $t_1 > t_o$ causes consumption to increase immediately to some intermediate position E_{o1}, between E_o and E_1. From there the system moves gradually along a divergent trajectory drawn with reference to the E_1 equilibrium, until at t_1 it arrives at E_{12} on SS' from where it converges asymptotically to E_o. Between t_o and t_1 part of the tax cut is saved. Dissaving takes place from t_1 onwards.

Since $F \equiv W - K - b^G$ and $\dot{K} = 0$, the current account surplus is given by $\dot{F} = \dot{w} - \dot{b}^G$.

For the unexpected, immediate and permanent tax change, w is constant throughout and foreign assets are crowded out one-for-one by domestic debt. With $\mu_g > 1$, the permanent cut in taxes is accompanied by transitory spending cuts which result in a budget surplus and a current account surplus. The government obtains the means to finance the permanent tax cut by reducing its debt in the short run. As public spending converges back to its original value, the public sector PVBC exhibits matching reductions in a liability, b^G and in an asset - the present value of future taxes.

In general, with exogenous taxes and endogenous spending given by (30) with $\nu_g = 0$ (and therefore by 34)), the behaviour of q, w and F is governed by :

$$(35)\quad \begin{bmatrix} \dot{q} \\ \dot{w} \\ \dot{F} \end{bmatrix} = \begin{bmatrix} r-\theta & -(\theta+\lambda)\lambda & 0 \\ -1 & r & 0 \\ -1 & \frac{-r\mu_g}{1-\mu_g} & \frac{r}{1-\mu_g} \end{bmatrix} \begin{bmatrix} q \\ w \\ F \end{bmatrix} + \begin{bmatrix} 0 \\ j(K) - \tau_1 \\ f(K) + \frac{r\mu_g}{1-\mu_g} K - \frac{1}{1-\mu_g} g_1 + \frac{\mu_g}{1-\mu_g}\tau_1 \end{bmatrix}$$

The characteristic roots are $r - (\theta+\lambda)$, $r + \lambda$ and $\frac{r}{1-\mu_g}$. With $-\lambda < r < \theta + \lambda$ and $\mu_g > 1$ there will be two stable and one unstable characteristic roots and the system will have the desired saddlepoint configuration for two predetermined variables (w and F) and one non-predetermined variable, q.

With a temporary tax cut there is no change in the long run values of q, w, F and $\bar{b}^G$. On impact, however, since $g = \frac{1}{1-\mu_g} g_1 + \frac{\mu_g r}{1-\mu_g} b^G - \frac{\mu_g}{1-\mu_g} \tau_1$ and $\mu_g > 1$, spending is cut by more than taxes and the home country runs a current account surplus. In Figure 1, between E_{01} and E_{12} the private sector saves. There is no private investment and the public sector has a budget surplus. When the tax cut is reversed at t_1, spending is raised by more and the government runs a budget deficit. The private sector also dissaves from E_{12} to E_0 in Figure 1 and the economy as a whole has a current account deficit that vanishes asymptotically.

Consider now instead a long-run tax cut financed by short-run tax increases without any changes in the public spending programme. Since the interest rate and the terms of trade are exogenous, this policy leaves the present value of the future tax programme (discounted at r) unchanged and merely redistributes taxes over time and between generations. Under this rule the equations of motion are given by:

$$(36)\quad \begin{bmatrix} \dot{q} \\ \dot{w} \\ \dot{F} \end{bmatrix} = \begin{bmatrix} (r-\theta) & -(\theta+\lambda)\lambda & 0 \\ -1 & \frac{r}{1+\mu_\tau} & \frac{r\mu_\tau}{1+\mu_\tau} \\ -1 & 0 & r \end{bmatrix} \begin{bmatrix} q \\ w \\ F \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ j(K) + \frac{r\mu_\tau}{1+\mu_\tau} K - \frac{1}{1+\mu_\tau}\tau_1 - \frac{\mu_\tau}{1+\mu_\tau} g \\ f(K) - g \end{bmatrix}$$

The three characteristic roots of the system are $r-(\theta+\lambda)$, $r+\lambda$, and $\frac{r}{1+\mu_\tau}$. The first two are the same ones that governed the q-w subsystem in (35) when taxes were exogenous. The third comes from the government debt equation $\dot{b}^G = \frac{r}{1+\mu_\tau} b^G + \frac{1}{1+\mu_\tau}(g-\tau_1)$. With $-\lambda < r < \theta + \lambda$ and $\mu_\tau < -1$, there will be two stable roots and one unstable root and the system will again have the proper saddlepoint configuration.

What happens on impact to aggregate consumption in response to a long run tax cut (τ_1 down) financed through a short-run tax increase, depends entirely on what happens to human capital h. From equation (3') we see that, with K given, the effect on human capital depends only on the effect on the present value of future taxes discounted at $r+\lambda$. Let

$$T(s) \equiv \int_s^{\infty} \tau(t) e^{-\int_s^t (r(u)+\lambda)du} dt$$

It is easily checked that

$$\frac{dT(s)}{d\tau_1} = \frac{1}{r+\lambda} - \frac{-\ 1}{r+\lambda\frac{(1+\mu_\tau)}{\mu_\tau}}$$

This confirms the intuition that with infinite-lived households ($\lambda = 0$) and perfect capital markets, redistribution of taxes over time has no real effects. However, with finite lives ($\lambda > 0$) and $\mu_\tau < -1$, $\frac{dT(s)}{d\tau_1} < 0$: raising taxes in the long run and lowering them in the short run so as to keep their present value discounted at r constant, lowers their present value discounted at $r + \lambda$.

Thus a policy of cutting taxes in the long run and financing this by raising taxes in the short run, will lower human capital and lower private consumption in the short run, even though it will raise both in the long run.

<u>Changes in the terms of trade</u>

It is apparent from equation (22) that changes in the terms of trade, π^{-1}, have no effect, short-run or long-run, on the behaviour of aggregate private consumption q and private sector non-human wealth w. Furthermore, except insofar as public spending on goods and services $g_x + \pi g_y$ is a function of π, the current account is independent of the terms of trade in the short-run and in the long-run. This is the powerful simplifying effect of our choice of utility function. A more general analysis of the Harberger-Laursen-Metzler effect can be found in Razin and Svensson [1983] and Bean [1984].

Changes in the interest rate

In a steady state, $r = r'(K)$ or $K = k(r)$ $k' < 0$. The long-run effects of a change in the world interest rate on private consumption and non-human wealth are in general ambiguous, as shown in (37a, b), where we use the fact that $j'k' = -K$.

(37a) $$\frac{dq}{dr} = \frac{qr + (w-K)(\theta+\lambda)\lambda}{-[r^2-\theta(r+\lambda) - \lambda^2]}$$

(37b) $$\frac{dw}{dr} = \frac{(r-\theta)(w-K) + q}{-[r^2-\theta(r+\lambda)-\lambda^2]}$$

If $r = \theta$, then $\frac{dw}{dr} > 0$ but $\frac{dq}{dr}$ is still ambiguous, as it equals

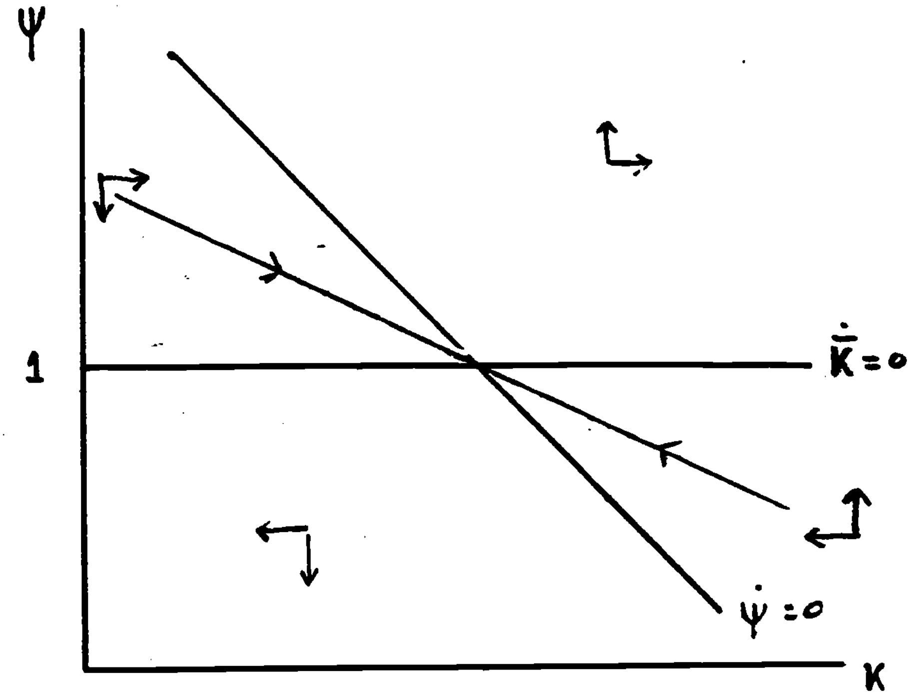
$$\frac{(\theta+\lambda)[rh - \lambda K]}{-[r^2-\theta(r+\lambda)-\lambda]} .$$

In a neighbourhood of the steady-state equilibrium $K = K_o$, $\psi = 1$, the behaviour of K and ψ can be linearly approximated by (38).

(38) $$\begin{bmatrix} \dot{K} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & \frac{K_o}{\zeta} \\ -f''(K_o) & r_o \end{bmatrix} \begin{bmatrix} K - K_o \\ \psi - 1 \end{bmatrix} + \begin{bmatrix} 0 \\ r - r_o \end{bmatrix}$$

This yields the familiar saddlepoint equilibrium shown in Figure 2.

Figure 2



The (K, ψ) dynamics is entirely self-contained, but feeds into the q, w dynamics as shown in (39) for the case where public spending adjusts to satisfy the government's PVBC.

$$(39)\quad \begin{bmatrix} \dot{K} \\ \dot{\psi} \\ \dot{q} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} 0 & \frac{K_o}{\zeta} & 0 & 0 \\ -f''(K_o) & r_o & 0 & 0 \\ 0 & 0 & -(r_o-\theta) & -(\theta+\lambda)\lambda \\ j'(K_o) & 0 & -1 & r_o \end{bmatrix} \begin{bmatrix} K-K_o \\ \psi-1 \\ q-q_o \\ w-w_o \end{bmatrix} + \begin{bmatrix} 0 \\ r-r_o \\ q_o(r-r_o) \\ w_o(r-r_o) \end{bmatrix}$$

We saw that when K is treated as constant, the (q, w) subsystem is saddlepoint-stable under mild restrictions. It will therefore have a (locally) well-behaved solution when K is governed by an exogenous process provided K does not "explode too fast" (see Buiter [1984]). Since K in turn will be (locally) well-behaved provided only that r doesn't explode too fast, the system given in

(39) will be a saddlepoint equilibrium with two stable and two unstable characteristic roots. ψ and q are non-predetermined, the boundary condition for K takes the form of an initial condition and w is subject to the simple linear restriction that at $t = t_o$, say

(42) $$w(t_o) = w(t_o^-) + (\psi(t_o) - \psi(t_o^-))K(t_o)$$

where $$x(t_o^-) = \lim_{\substack{\Delta \to 0 \\ \Delta > 0}} x(t_o - \Delta)$$

while $w(t_o^-)$ is predetermined (inherited from the past), discontinuous jumps in ψ in response to 'news' at t_o can cause discontinuous jumps in $w(t_o)$.

The four roots are $\frac{1}{2}\left(r_o \pm \sqrt{r_0^2 - \frac{4K_o}{\zeta} f''(K_o)}\right)$, $r_o - (\theta+\lambda)$ and $r_o + \lambda$.

IV The semi-small open economy

The semi-small open economy is a price taker in world financial markets but has some influence on the world price of its exportable. Price-taking behaviour in the world capital market is specified as the home country taking $r^* \equiv r - \frac{\dot{\pi}}{\pi}$ as given. Through the endogeneity of π therefore, the domestic interest rate, r, becomes endogenous, at any rate in the short run. The terms of trade are endogenized through the domestic output equilibrium condition

$$f(K) = \alpha q + g_x + \left(\frac{\psi-1}{\zeta}\right)K + \alpha^*\pi q^* + g_x^* + \frac{1}{2}(\psi-1)^2 \frac{K}{\zeta}$$

We also assume that g_x is independent of π and that $(\alpha^*\pi q^* + g_x^*)\pi^{-1}$ is independent of π, i.e. that $(\alpha^*\pi q^* + g_x^*)\pi^{-1} = M^{-1}$, say, which permits us to solve for π as

$$\text{(43)} \quad \pi = [f(K) - \alpha q - g_x + \frac{(1-\psi)K}{\zeta} - \frac{1}{2}(\psi-1)^2 \frac{K}{\zeta}]\, M$$

This has the sensible property that an increase in world demand for the home good <u>cet. par.</u> raises its relative price (lowers π). Note that this representation makes π a function of domestic demand for domestic output but not of domestic demand for foreign output.[3]

The long-run comparative statics of q, w, K and ψ are the same as those of the very small open economy, since in long-run equilibrium $r = r^* = f'(K)$ and q is independent of π. If in addition total

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3. We must of course assume that $f - \alpha q - g_x + \frac{(1-\psi)K}{\zeta} - \frac{1}{2}(\psi-1)^2 \frac{K}{\zeta} > 0$ for π to be positive.

government spending, measured in domestic output ($g \equiv g_x + \pi g_y$) is independent of π, the long-run comparative statics of b^G and F are also the same as in the very small open economy.

The long-run effects of fiscal policy changes and changes in the world rate of interest on π all follow straightforwardly from the unit elastic foreign demand for domestic output function. The steady-state version of (43), given in (44), yields the following long-run comparative static results:

(44) $$\pi = [f(K) - \alpha q - g_x]M$$

(45a) $$\frac{d\pi}{d\tau} = -M\alpha\frac{(\theta+\lambda)\lambda}{\Omega} > 0$$

(45b) $$\frac{d\pi}{dg_x} = -M < 0$$

(45c) $$\frac{d\pi}{dg_y} = 0$$

(45d) $$\frac{d\pi}{dr^*} = \frac{Mf'}{f''} + \frac{M\alpha}{\Omega}[qr + (w-K)(\theta+\lambda)\lambda] < 0$$ 4/

where

(45e) $$\Omega \equiv r^2 - \theta(r+\lambda) - \lambda^2 < 0$$

Dynamic response of the semi-small open economy

The essential dynamics of the semi-small open economy are described by the following six equations:

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4. We assume $\frac{dq}{dr} \geq 0$

$$\dot{K} = (\psi - 1)\zeta^{-1}K$$

$$\dot{\psi} = \psi r - f'(K) - \frac{1}{2}\zeta\left(\frac{\dot{K}}{K}\right)^2$$

$$\dot{q} = (r-\theta)q - (\theta+\lambda)\lambda w$$

$$\dot{w} = rw + j(K) - \tau - q$$

$$r = r^* + \frac{\dot{\pi}}{\pi}$$

$$\pi = \left(f(K) - q - g_x - \dot{K} - \frac{1}{2}\zeta\frac{\dot{K}^2}{K}\right)M$$

Having solved for the behaviour of K, ψ, q, w, r and π, we can then solve for the behaviour of public debt and net external assets from

$$\dot{b}^G = g_x + \pi g_y + rb^G - \tau$$

and

$$\dot{F} = f(K) + rF - \left(q + \dot{K} + g_x + \pi g_y + \frac{1}{2}\zeta\frac{\dot{K}^2}{K}\right)$$

<u>A long-run tax cut financed by a temporary spending cut.</u>

Consider first the consequences of a permanent cut in the exogenous level of taxes, τ, under the strongly stabilizing public spending rule given by (30) with $\nu_g = 0$, i.e.

$$g = g_1 + \mu_g\dot{b}^G \qquad \mu_g > 1$$

which implies

$$\dot{b}^G = \frac{r}{1-\mu_g}b^G + \frac{1}{1-\mu_g}(g_1 - \tau)$$

In order to simplify the analysis further, it is also assumed that all variations in public spending take the form of variations in public spending on imports, i.e. g_x is held constant throughout.

We eliminate r from the system through the real interest equalization condition and obtain $\frac{\dot{\pi}}{\pi}$ by logarithmic differentiation of the relative price equation. In a neighbourhood of the stationary equilibrium $[K_o, \psi_o, q_o, w_o]$ corresponding to $[r^*_o, \tau_o$ and $g_{x_o}]$ we can represent the behaviour of K, ψ, q and w by :

$$\text{(46a)} \quad \begin{bmatrix} \dot{K} \\ \dot{\psi} \\ \dot{q} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} 0 & \frac{K_o}{\zeta} & 0 & 0 \\ \frac{-f''\zeta(\pi+M\alpha q_o)}{\Lambda} & r^* & \frac{-\zeta M\alpha(r^*-\theta)}{\Lambda} & \frac{\zeta M\alpha(\theta+\lambda)\lambda}{\Lambda} \\ \frac{q_o M K_o f''}{\Lambda} & 0 & \frac{(\pi\zeta+MK_o)(r^*-\theta)}{\Lambda} & \frac{-(\pi\zeta+MK_o)(\theta+\lambda)\lambda}{\Lambda} \\ g'+\frac{w_o MK_o f''}{\Lambda} & 0 & -\left[1+\frac{w_o\zeta M\alpha(r^*-\theta)}{\Lambda}\right] & r^*+\frac{w_o\zeta M\alpha(\theta+\lambda)\lambda}{\Lambda} \end{bmatrix} \begin{bmatrix} (K-K_o) \\ (\psi-1) \\ (q-q_o) \\ (w-w_o) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ \frac{\pi_o\zeta}{\Lambda} & 0 & \frac{-\zeta M}{\Lambda} \\ q_o\frac{\pi_o\zeta}{\Lambda} & 0 & \frac{-q_o\zeta M}{\Lambda} \\ w_o\frac{\pi_o\zeta}{\Lambda} & -1 & \frac{-w_o\zeta M}{\Lambda} \end{bmatrix} \begin{bmatrix} r^*-r^*_o \\ (\tau-\tau_o) \\ \dot{g}_x \end{bmatrix}$$

(46b) $\Lambda = \pi_o\zeta + M\alpha q_o\zeta + MK_o$

and

(46c) $\pi - \pi_o \simeq Mf'(K_o)(K - K_o) - M\alpha(q - q_o) - MK_o\zeta^{-1}(\psi - 1) - M(g_x - g_{x_o})$

(46d) $$\frac{\dot{\pi}}{\pi} \simeq \frac{MK_o f''(K_o)}{\Lambda}(K - K_o) + \frac{M\alpha(\theta+\lambda)\lambda\zeta}{\Lambda}(w - w_o) - \frac{M\alpha\zeta(r^* - \theta)}{\Lambda}(q - q_o)$$

$$- \frac{(M\alpha q_o\zeta + MK_o)}{\Lambda}(r^* - r_o^*) - \frac{\zeta M}{\Lambda}\dot{g}_x$$

The characteristic equation of the state matrix in (46a) does not appear to factorize in any convenient manner. Experimentation with a range of plausible numerical values did, however, consistently yield the right kind of saddlepoint configuration with two stable and two unstable characteristic roots. The following results of simulations involving a world interest rate increase and a tax cut for two distinct numerical versions of the model illustrate its general properties.[5/] For the first version of the model, $\alpha = .6$, $\theta = .03$, $\zeta^{-1} = .031$[6/] and $\lambda = .03$. The production function is Cobb-Douglas with $f(K) = K^\beta$ and $\beta = .25$ in both versions of the model.

In version I, $r^* = \theta = .03$. This means that the steady-state value for w equals zero. We also assume that the initial value of taxes, τ_o, is 25 per cent of initial labour income, that g_{x_o} is fifteen per cent of initial labour income, that

5. These simulations made use of the algorithm "Saddlepoint" of Austin and Buiter [1982].

6. The value of ζ was taken from Summers [1981].

$\pi_o g_{y_o}$ is five per cent of labour income and that $M = .8968$. The initial steady state value of the relative price of foreign goods, π_o, is then equal to unity. The initial steady-state equilibrium is furthermore characterized by $K_o = 16.90$; $f(K_o) = 2.03$; $j(K_o) = 1.52$; $\psi_o = 1$, $b_o^G = 2.54$ and $F_o = -19.43$.

The characteristic roots of the linearized system (given in 46a) are all real and given by (- .02284; -.01226; .03696; .05841). Because the stable roots are so small, convergence to the steady state tends to be slow. The root governing b^G is $\frac{r^*}{1-\mu_g}$. We assume $\mu_g = 2$, so the public debt converges with a mean lag of 33.3 periods when $r^* = .03$. The consequences of a permanent tax cut financed by a temporary cut in spending are as follows. The long-run effect is to boost domestic private consumption, because human wealth increases, and to lower the relative price of foreign goods, π. K, w, ψ and r are unchanged in the long run; since $r^* = \theta$, public debt decreases and net foreign assets increase by equal amounts.

If the tax cut is not only permanent but also unanticipated, the full long-run adjustment of all endogenous variables other than b^G and F takes place immediately, i.e. π falls instantaneously to clear the market for the domestic good at the higher level of domestic consumption demand and nothing else changes.

An anticipated future permanent tax cut also causes an immediate, discrete upward move in aggregate consumption, q, at the date that the unexpected announcement of the future tax cut occurs. After the initial jump, q is still slightly below its new steady state level and continues to rise towards it. It is the effect of future tax cuts on current human capital that causes this response. The relative price of foreign output falls discretely on impact, but by less than the long-run decline: it 'undershoots' its long-run equilibrium value. Since π continues to fall after the initial drop, the interest rate declines on impact and stays below the world level throughout the adjustment process. ψ increases on impact and capital begins to accumulate. This process, however, is reversed in due course as ψ falls below unity and the capital stock returns to its initial value. At the announcement date, w increases because of the increase in ψ. Saving, however, is very negative initially, as consumption is raised before the tax cuts come through. w becomes negative, reaches a minimum at the date the tax cut comes through and then returns to zero.

With the unanticipated tax cut, b^G declines and F rises throughout the adjustment process (with $\dot{b}^G = -\dot{F}$). In the case of the anticipated future tax cut, F falls until the tax cut is actually implemented and rises thereafter. b^G rises until the tax cut occurs and declines thereafter. It is no longer the case that $\dot{F} + \dot{b}^G = 0$ at each instant.

An unexpected permanent increase in the world real interest rate r^* lowers K in the long run, raises w and q and lowers π. F increases and b^G falls. The impact effect on w, q and π with K predetermined, is the exact opposite. ψ falls discretely, reducing wealth (w becomes negative). Consumption falls because both w and h are lower. With private domestic consumption and investment demand down, but output still predetermined, π rises. In the long run, of course, π falls, so $\frac{\dot{\pi}}{\pi}$ becomes negative immediately following the initial increase in π. The positive effect of the increase in r* on r is not, however, offset completely by the (anticipated) decline in π and r rises on impact. Savings are positive and w increases steadily after the initial capital loss. Capital decumulates steadily.

The second version has $r^*_o = .05$, $\theta = .04$, $\lambda = .03$, $\zeta^{-1} = .031$, $\beta = .25$, $M = 1.3155511$, $\alpha = .6$ and as before, $\tau_o = .25\ j(K_o)$, $g_{x_o} = .15\ j(K_o)$ and $\pi_o g_{y_o} = .05\ j(K_o)$. This implies that $K_o = 8.55$, $b^G = 1.28\ (= j(K_o))$ and $F_o = -3.82$. The four characteristic roots of the K, w, q, ψ subsystem are (-.01558, -.01115, .05820 and .07853). The root governing b^G (with $\mu_g = 2$) is -.05.

A permanent tax cut raises q and w in the long run, leaves K unaffected and lowers π. F increases by more than b^G decreases. If the tax cut is unexpected, ψ increases on impact, thus raising w. Consumption jumps up discretely and continues to rise gradually thereafter. w, after the initial capital gain, rises smoothly. The capital stock increases initially, as ψ exceeds unity, but then falls back to its original value. π drops sharply on impact (because none of the public spending cuts fall on demand for domestic output) and then declines gradually to its new steady-state value, i.e. it undershoots on impact. The interest rate falls on impact and rises gradually back to its original value.

An anticipated future tax cut (of the same magnitude), leads to a smaller discrete increase in q at the announcement date. π also declines by less on impact and its subsequent rate of decline is initially smaller numerically than under the immediate tax cut, but becomes larger subsequently. ψ rises by more on impact. Dissaving takes place between the announcement date and the implementation date. Once the tax cut is in effect saving becomes positive again.

A permanent increase in r* has the long-run effect of lowering K, b^G and π and raising w, F and q. Again the impact effects on w, q and π are in the opposite direction. The analysis is qualitatively very similar to the case where $r^* = \theta$.

A long-run tax cut financed by a temporary tax increase

We now treat both $g \equiv g_x + \pi g_y$ and g_x as parametric and have taxes determined by (26) with $\nu_\tau = 0$, i.e. by

$$\tau = \tau_1 + \mu_\tau \dot{b}^G \qquad \mu_\tau < -1$$

and, therefore,

$$\dot{b}^G = \frac{r}{1+\mu_\tau} b^G + \frac{1}{1+\mu_\tau} (g - \tau_1).$$

Note that a change in τ_1 in the semi-small open economy model, unlike in the very small open economy model, does not merely involve the redistribution over time of a given present discounted value of future taxes; while the spending programme is given, the interest rate is endogenous in the semi-small open economy outside the steady state.[7/]

The values of the parameters for both numerical versions are the same as with the "spending endogenous" policy except that now $\mu_g = 0$ and $\mu_\tau = -2$. The long-run effects of changes in τ_1 (and in r^*) are the same under the "tax endogenous" policy as under the spending endogenous policy. The transitional dynamics are, however, very different.[8/]

Both in version 1 ($r^* = \theta = .03$) and in version 2 ($r^* = .05 > \theta = .04$) an unexpected, immediate reduction in τ_1 lowers consumption in the short run. The reason is that total taxes τ are actually increased initially by so much that human capital declines. ψ (and therefore w) and π increase on impact.

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7. In the two-country model interest rates are endogenous both in the short run and in the long run.

8. The two versions can be summarized as follows:

   Version 1

   $r^* = \theta = .03$; $\lambda = .03$; $\zeta^{-1} = .031$; $\beta = .25$; $M = .897$; $\alpha = .6$; $\tau_o = .380$; $g_{x_o} = .228$; $\pi_o g_{y_o} = .076$; $K_o = 16.895$; $b_o^G = 2.534$; $F_o = -19.429$; $\mu_\tau = -2$.

   Characteristic roots of K, b^G, w, ψ, q system: -.03268; -.02159; -.01216; .03662; .05746.

   Version 2

   $r_o^* = .05$; $\theta = .04$; $\lambda = .03$; $\zeta^{-1} = .031$; $\beta = .25$; $M = 1.3155511$; $\alpha = .6$; $\tau_o = .321$; $g_{x_o} = .192$; $\pi_o g_{y_o} = .064$; $K_o = 8.550$; $b_o^G = 1.282$; $F_o = -3.821$; $\mu_\tau = -2$.

   Characteristic roots of K, b^G, w, ψ, q system: -.05094; -.01563; -.01105; .05781; .07800.

After the initial capital gain, dissaving takes place. This is ultimately reversed with w going back to the original steady state value of zero in version 1 ($r^* = \theta$) and rising beyond it in version 2 ($r^* > \theta$). Capital accumulates for a while and then reverts to its original level. The initial decline in consumption is reversed as human wealth increasingly reflects the long-run tax cuts and non-human wealth recovers. Government debt is retired continuously, even after total tax receipts have become less than in the initial equilibrium. This reflects the budgetary effects of lower debt service payments. The current account is in surplus throughout.

The unexpected announcement of a future cut in τ_1 has a qualitatively smaller impact effecton ψ, w, q and π, although the direction is unchanged.

After the initial capital gain at the "announcement date", non-human wealth, w, continues to accumulate until the moment τ_1 is actually cut. The sharp increase in total taxes at the "implementation date" starts a process of dissaving which is in due course reversed again as the long-run tax cut comes through. There still is a current account surplus and a public sector budget surplus throughout, although the latter is very small until τ_1 is actually cut (and τ increased).

V The two-country model

Stationary equilibrium

In a long-run stationary equilibrium the exogenous variables are constant and all state variables have become stationary. The crucial steady-state conditions are:

(47) $\psi = \psi^* = 1$

(48) $r = f'(K) = f^{*\prime}(K^*)$

(49) $h = \dfrac{j(K) - \tau}{r + \lambda}$

(50) $h^* = \dfrac{(j^*(K^*) - \tau^*)}{r + \lambda^*}$

(51) $w = \dfrac{(\theta - r)}{[r - (\theta+\lambda)](r + \lambda)} (j(K) - \tau)$

(52) $w^* = \dfrac{(\theta^* - r)}{(r - (\theta^*+\lambda^*))(r + \lambda^*)} (j^*(K^*) - \tau^*)$

(53) $q = \dfrac{-(\theta + \lambda)\lambda}{(r - (\theta+\lambda))(r + \lambda)} (j(K) - \tau)$

(54) $q^* = \dfrac{-(\theta^* + \lambda^*)\lambda^*}{(r - (\theta^*+\lambda^*))(r + \lambda^*)} (j^*(K^*) - \tau)$

(55) $f(K) = \alpha q + g_x + \alpha^* \pi q^* + g_x^*$

(56) $f^*(K^*) = (1 - \alpha)\dfrac{q}{\pi} + g_y + (1 - \alpha^*)q^* + g_y^*$ [9]

9. Equations (55) and (56) imply that $r(w + w^*) + j(K) + \pi j^*(K^*) - \tau - \pi\tau^* - q - \pi q^* = 0$. This in turn implies that $w + \pi w^* = K + \pi K^* + b^G + b^{*G}$.

(57) $$b^G = \frac{\tau - (g_x + \pi g_y)}{r}$$ (Domestic government balanced budget condition)

(58) $$b^{*G} = \frac{\pi\left[\tau^* - \left(\frac{g_x^*}{\pi} + g_y^*\right)\right]}{r}$$ (Foreign government balanced budget condition)

(59) $$F = \frac{q + g_x + \bar{\pi} g_y - f(K)}{r}$$ (Current account balance condition)

It is informative to solve the domestic ouput market equilibrium condition (55) and the foreign output market equilibrium condition (56) for the two "fundamental"long-run endogenous variables r and π.

The analysis can be simplified somewhat by specifying public spending analogously with private consumption in the following sense:

(60a) $$g_x + \pi g_y = g$$ where g is independent of π.

(60b) $$g_x = \frac{\beta^x}{\beta^x + \beta^y} g \qquad \beta^x,\ \beta^y \geq 0$$

(60c) $$g_y = \left(\frac{\beta^y}{\beta^x + \beta^y}\right) \frac{g}{\pi}$$

(61a) $$\frac{g_x^*}{\pi} + g_y^* = g^*$$ where g^* is independent of π

(61b) $$g_x^* = \left(\frac{\beta^{*x}}{\beta^{*x} + \beta^{*y}}\right) \pi g^* \qquad \beta^{*x},\ \beta^{*y} \geq 0$$

(61c) $$g^*_y = \frac{\beta^{*y}}{\beta^{*y}+\beta^{*y}} g^*$$

Noting from (48) that $K = k(r)$, $k' = \frac{1}{f''} < 0$ and $K^* = k^*(r)$, $k^{*'} = \frac{1}{f^{*''}} < 0$ we get

(62) $$f(k(r)) = \frac{-\alpha(\theta+\lambda)\lambda}{(r-(\theta+\lambda))(r+\lambda)}(j(k(r))-\tau) + \frac{\beta^x}{\beta^x+\beta^y} g$$

$$- \frac{\alpha^*(\theta^*+\lambda^*)\lambda^*}{(r-(\theta^*+\lambda^*))(r+\lambda^*)} \pi(j^*(k^*(r)-\tau^*) + \frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}} \pi g^* \quad (yy)$$

(63) $$f^*(k^*(r)) = \frac{(1-\alpha)(\theta+\lambda)\lambda}{(r-(\theta+\lambda)(r+\lambda)} \frac{(j(k(r)))-\tau)}{\pi} + \left(\frac{\beta^y}{\beta^x+\beta^y}\right)\frac{g}{\pi}$$

$$- \frac{(1-\alpha^*)(\theta^*+\lambda^*)\lambda^*}{(r-(\theta^*+\lambda^*))(r+\lambda^*)} (j^*(k^*(r))-\tau^*) + \frac{\beta^{*y}}{\beta^{*x}+\beta^{*y}} g^* \quad (y^*y^*)$$

The linearized yy locus is given by:

(64) $$\left[rk' + \frac{\alpha[qr+(\theta+\lambda)\lambda(w-K)]}{\Omega} + \alpha^*\pi \frac{[q^*r+(\theta^*+\lambda^*)\lambda^* (w^*- K^*)]}{\Omega^*}\right] dr$$

$$- \left[\alpha^*q^* + \frac{\beta^{*y}}{\beta^{*y}+\beta^{*y}} g^*\right] d\pi = \frac{\alpha(\theta+\lambda)\lambda}{\Omega} d\tau + \frac{\alpha^*(\theta^*+\lambda^*)\lambda^*\pi}{\Omega^*} d\tau^*$$

$$+ \frac{\beta^x}{\beta^x+\beta^y} dg + \pi \frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}} dg^* \quad (yy)$$

The linearized y^*y^* locus is given by :

$$(65) \quad \left\{ rk^{*\prime} + \frac{(1-\alpha)}{\Omega}\,\frac{[qr + (\theta+\lambda)\lambda(w-K)]}{\pi} + \frac{(1-\alpha^*)}{\Omega^*}\left[q^*r + (\theta^*+\lambda^*)\lambda^*(w^*-K^*)\right]\right\} dr$$

$$+ \left\{ (1-\alpha)\frac{q}{\pi^2} + \left(\frac{\beta^y}{\beta^x+\beta^y}\right)\frac{g}{\pi^2}\right\} d\pi = \frac{(1-\alpha)(\theta+\lambda)\lambda}{\Omega\pi}\, d\tau + \frac{(1-\alpha^*)(\theta^*+\lambda^*)\lambda^*}{\Omega^*}\, d\tau^*$$

$$+ \left(\frac{\beta^y}{\beta^x+\beta^y}\right)\frac{1}{\pi}\, dg + \frac{\beta^{*y}}{\beta^{*x}+\beta^{*y}}\, dg^* \qquad (y^*y^*)$$

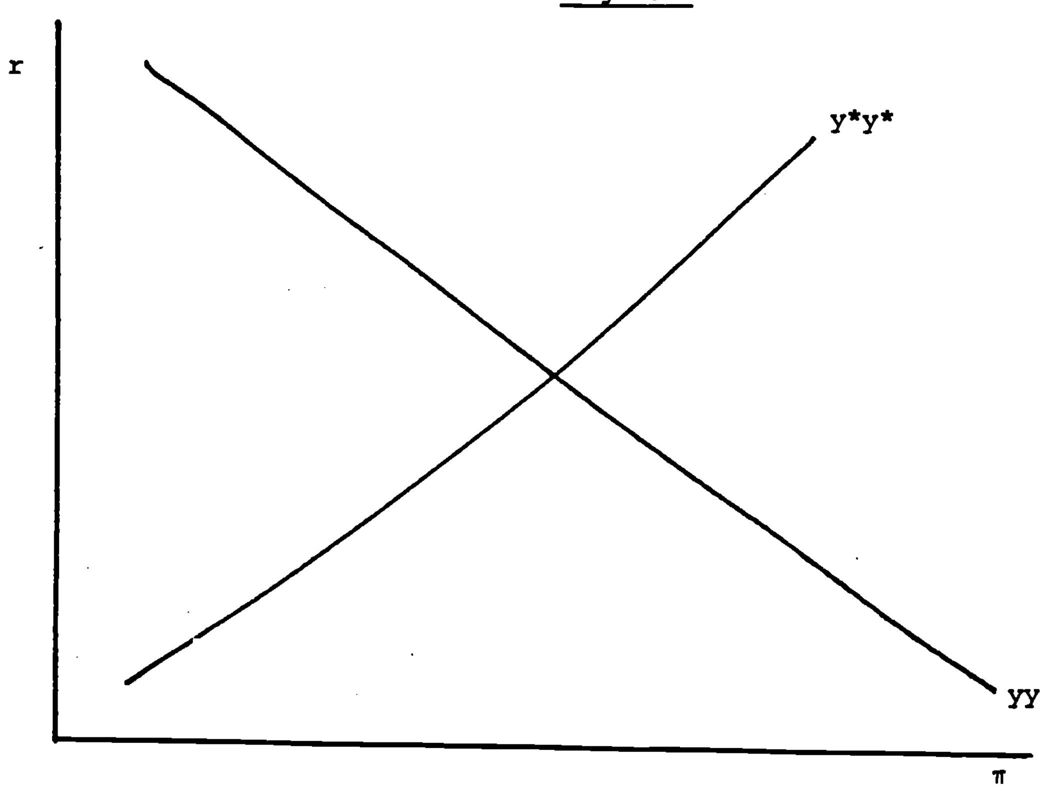
where

$$(66a) \quad \Omega = r^2 - \theta(r+\lambda) - \lambda^2 < 0$$

$$(66b) \quad \Omega^* = r^2 - \theta^*(r+\lambda^*) - \lambda^{*2} < 0$$

Under very mild restrictions, the yy locus is downward-sloping and the y*y* locus upward-sloping as drawn in Figure 3. Note that in this case, if a stationary equilibrium exists, it will be unique.

Figure 3



The long-run comparative static effects of changes in public spending and taxation, at home and abroad, on r and π are summarized in equation (67a, i).

$$\text{(67a)}\qquad \frac{dr}{d\tau} = \frac{\left[\alpha(1-\alpha)q + \alpha^*(1-\alpha)\pi q^* + \alpha\frac{\beta^y}{\beta^x+\beta^y}g + \frac{(1-\alpha)\beta^{*x}}{\beta^{*x}+\beta^{*y}}\pi g^*\right]\frac{(\theta+\lambda)\lambda}{\Omega\pi^2}}{\Delta} > 0$$

$$\text{(67b)}\qquad \frac{d\pi}{d\tau} = \frac{\frac{(\theta+\lambda)\lambda}{\Omega\Omega^*}\left[q^*r + (\theta^*+\lambda^*)\lambda^*(w^* - K^*)\right](\alpha^*-\alpha) + \frac{r(\theta+\lambda)\lambda}{\Omega}\left[\frac{(1-\alpha)k'}{\pi} - \alpha k^{*\prime}\right]}{\Delta}$$

$$\text{(67c)}\qquad \frac{dr}{d\tau^*} = \frac{\left[\alpha^*(1-\alpha)q + \alpha^*(1-\alpha^*)\pi q^* + \alpha^*\frac{\beta^y}{\beta^x+\beta^y}g + (1-\alpha^*)\frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}}\pi g^*\right]\frac{(\theta^*+\lambda^*)\lambda^*}{\Omega^*\pi}}{\Delta} > 0$$

$$\text{(67d)}\qquad \frac{d\pi}{d\tau^*} = \frac{\frac{(\theta^*+\lambda^*)\lambda^*}{\Omega\Omega^*}\left[qr + (\theta+\lambda)\lambda(w-K)\right](\alpha-\alpha^*) + \frac{r(\theta^*+\lambda^*)\lambda^*}{\Omega^*}\left[(1-\alpha^*)k' - \alpha^*k^{*\prime}\pi\right]}{\Delta}$$

$$\text{(67e)}\qquad \frac{dr}{dg} = \frac{\frac{\beta^x}{\beta^x+\beta^y}\left[\frac{(1-\alpha)q}{\pi^2} + \left(\frac{\beta^y}{\beta^x+\beta^y}\right)\frac{g}{\pi^2}\right] + \left(\frac{\beta^y}{\beta^x+\beta^y}\right)\frac{1}{\pi}\left[\alpha^*q^* + \frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}}g^*\right]}{\Delta} < 0$$

$$\text{(67f)}\qquad \frac{d\pi}{dg} = \frac{\left[\frac{qr+(\theta+\lambda)\lambda(w-K)}{\Omega\pi}\right]\left[\alpha - \left(\frac{\beta^x}{\beta^x+\beta^y}\right)\right] + \left[\frac{q^*r+(\theta^*+\lambda^*)\lambda^*(w^*-K^*)}{\Omega^*}\right]\left[\alpha^* - \left(\frac{\beta^x}{\beta^x+\beta^y}\right)\right]}{\Delta}$$

$$+ \frac{r\left[\frac{k'}{\pi}\left(1 - \left(\frac{\beta^x}{\beta^x+\beta^y}\right)\right) - k^{*\prime}\left(\frac{\beta^x}{\beta^x+\beta^y}\right)\right]}{\Delta}$$

$$\text{(67g)}\qquad \frac{dr}{dg^*} = \frac{\frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}}\left[\frac{(1-\alpha)q}{\pi} + \frac{\beta^y}{\beta^x+\beta^y}\frac{g}{\pi}\right] + \frac{\beta^{*y}}{\beta^{*x}+\beta^{*y}}\left[\alpha^*q^* + \frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}}g^*\right]}{\Delta} < 0$$

$$\text{(67h)}\quad \frac{d\pi}{dg^*} = \frac{\left[\frac{qr+(\theta+\lambda)\lambda(w-K)}{\Omega}\right]\left[\alpha-\left(\frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}}\right)\right]+\pi\left[\frac{q^*r+(\theta^*+\lambda^*)\lambda^*(w^*-K^*)}{\Omega^*}\right]\left[\alpha^*-\left(\frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}}\right)\right]}{\Delta}$$

$$+\frac{r\left[k'\left(1-\frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}}\right)-k^{*\prime}\pi\frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}}\right]}{\Delta}$$

$$\text{(67i)}\quad \Delta \equiv \left\{rk'+\alpha\frac{[qr+(\theta+\lambda)\lambda(w-K)]}{\Omega}+\alpha^*\pi\frac{[q^*r+(\theta^*+\lambda^*)\lambda^*(w^*-K^*)]}{\Omega^*}\right\}\left[\frac{(1-\alpha)}{\pi^2}q+\left(\frac{\beta^y}{\beta^x+\beta^y}\right)\frac{g}{\pi^2}\right]$$

$$+\left\{rk^{*\prime}+\frac{(1-\alpha)[qr+(\theta+\lambda)\lambda(w-K)]}{\pi\Omega}\right.$$

$$\left.+\frac{(1-\alpha^*)}{\Omega^*}\left(q^*r+(\theta^*+\lambda^*)\lambda^*(w^*-K^*)\right)\right\}\left[\frac{\alpha^*q^*}{\pi}+\frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}}g^*\right]$$

$$< 0$$

The effect of lower taxes and higher public spending (domestic or foreign) is to lower the long-run world interest rate. This is only a paradox until one remembers that lower taxes or higher spending are, across steady states, associated with lower public debt (see (57) and (58)). Of course the process of adjustment towards such a lower debt steady state will involve transitorily higher taxes and/or lower public spending to achieve the surpluses necessary for retiring the debt. The association of higher spending with lower interest rates is present regardless of the composition of the spending increase between domestic and foreign output.

The effect of tax cuts on the terms of trade in the long run can be explained in terms of a familiar transfer problem criterion plus a correction for supply adjustments. E.g. from (67b), ignoring supply-side adjustments ($k' = k^{*\prime} = 0$), a domestic tax cut will raise the relative price of domestic output $\left(\frac{d\pi}{d\tau} > 0\right)$ if $\alpha > \alpha^*$, i.e. if the marginal (and average) propensity to consume domestic output is greater at home than abroad. Lower taxes also are associated with lower interest rates, higher capital-labour ratios and higher output. If there is no bias in domestic private consumption towards home goods ($\alpha = \frac{1}{2}$) and if technologies are similar in the sense that $k' = k^{*\prime}$, the output adjustment term vanishes. If there is a bias towards home goods ($\alpha > \frac{1}{2}$) and if k' and $k^{*\prime}$ are similar, then the supply effect reinforces the transfer effect and $\frac{d\pi}{d\tau} > 0$ <u>a-fortiori</u>. By exactly analogous reasoning, given supply, $\frac{d\pi}{d\tau^*} > 0$ if $\alpha^* > \alpha$ or $1-\alpha^* < 1-\alpha$. A cut in foreign taxes will raise the relative price of foreign goods if foreigners allocate a larger fraction of their total consumption spending to foreign goods than do domestic residents. Again the same kind of supply side effect that was discussed for a domestic tax cut must be allowed for. We shall not consider it any further here or below when the effects of spending increases on π are discussed.

An increase in domestic public spending raises the relative price of domestic output $\left(\frac{d\pi}{dg} < 0\right)$ if the domestic public sector's marginal propensity to spend on domestic output $\left(\frac{\beta^y}{\beta^x+\beta^y}\right)$ exceeds a weighted average of the domestic and foreign private marginal propensities to spend on domestic output, i.e. if (ignoring supply effects)

$$\frac{\beta^x}{\beta^x+\beta^y} > \frac{[qr + (\theta+\lambda)\lambda(w-K)]\Omega^*}{[qr + (\theta+\lambda)\lambda(w-K)] + \pi[q^*r + (\theta^*+\lambda^*)\lambda^*(w^*-K^*)]\Omega\Omega^{*-1}}\alpha$$

$$+ \frac{+\pi[q^*r + (\theta^*+\lambda^*)\lambda^*(w^*-K^*)]}{\Omega^*\Omega^{-1}[qr + (\theta+\lambda)\lambda(w-K)] + \pi[q^*r + (\theta^*+\lambda^*)\lambda^*(w^*-K^*)]}\alpha^*$$

Similarly, an increase in foreign public spending will raise the relative price of foreign output $\left(\frac{d\pi}{dg^*} > 0\right)$ if the foreign public sector's marginal propensity to spend on foreign output $\left(1 - \frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}}\right)$ exceeds a weighted average of domestic foreign private marginal propensities to spend on foreign output $1-\alpha$ and $1-\alpha^*$. The exact condition can be obtained from (67h).

In terms of Figure 3 , a domestic tax increase or spending cut shifts yy up and to the right while y*y* shifts up and to the left. A foreign tax increase or spending cut shifts yy and y*y* in the same directions.

Having derived the steady-state effects on π and r, the remaining long-run comparative statics is straightforward. Any policy change that raises r lowers the capital stock at home and abroad. Lower long-run domestic taxes are associated with a lower long-run stock of domestic public debt, with a lower global interest rate and a higher capital stock in both countries. Domestic human capital is higher and consumption is almost certain to increase. Foreign human capital (measured in foreign output) is also increased because of the lower r and higher K*; the lower interest rate is likely to reduce foreign non-human wealth and foreign consumption in the long run.

With τ^* and g^* given, a lower world interest rate still requires, at a given value of π, a larger long-run stock of foreign public debt (assuming $b^{*G} > 0$ initially). This could be reversed by a decline in π.

Finally, there is the effect of domestic and foreign fiscal policy on the long run value of F.

From equation (59) it is easily checked that, in response to a change in some exogenous variable z, F changes as follows :

$$(68) \quad \frac{dF}{dz} = -\frac{1}{r}\left[F + \frac{rq + (\theta+\lambda)\lambda(w-K)}{\Omega} + f'k'\right]\frac{dr}{dz} + \frac{1}{r}\frac{(\theta+\lambda)\lambda}{\Omega}\frac{d\tau}{dz} + \frac{1}{r}\frac{dg}{dz}$$

At a given interest rate, a higher steady state level of public spending must be associated with a larger stock of claims on the rest of the model, in order to finance the increase in the excess of domestic absorption over domestic income. Since increased public spending is associated in the long run with a lower interest rate, and thus a larger domestic capital stock and domestic output level, this indirect effect will tend to lower F. The reduction in private consumption likely to be associated with a lower value of r will also tend to lower F. Finally, a lower value of r will <u>cet. par.</u> worsen (improve) the current account through the debt service (foreign investment income) component rF if the country is a net creditor with $F > 0$ (a net debtor with $F < 0$). Thus cet. par. a positive (negative) value of F will tend to make $\frac{dF}{dg}$ positive (negative) through the net foreign interest component. With minor and obvious changes, the argument about the effect of an increase in g on F also applies to a cut in τ. It is also easily seen that an increase in g^* will tend to have the opposite effect on F of an increase in g and that a cut in τ^* will tend to have the opposite effect on F of a cut in τ. It seems plausible that the "direct" effect of an increase in g or a cut in τ of raising F in the long run will outweigh the indirect effects through production, private consumption and net foreign investment income. Our numerical examples do indeed all have this property, although it is not implied by all parameter values consistent with saddlepoint stability.

Dynamic adjustment

The linearized structural form of the two-country model is given in Appendix 2. The production functions in the two countries are assumed to be Cobb-Douglas with competitive capital shares β and β^* respectively. To calculate the initial stationary equilibrium we must assign values to β, β^*, ζ, ζ^*, θ, θ^*, λ, λ^*, α, α^*, $\frac{\beta^x}{\beta^x+\beta^y}$, $\frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}}$, g, g^*, τ_1 and τ_1^*. A full characterization of the dynamic behaviour also requires values for μ_τ and μ_τ^*. There are eight linearly independent state variables in the linearized state-space representation of the model. A convenient choice of state variables is b^G, b^{*G}, K, K*, w, h, h* and ψ. For a (locally) unique convergent saddlepoint equilibrium solution to exist we therefore require five stable and three unstable characteristic roots in the state matrix.

<u>The one-commodity case</u>

First consider the special case of the model where the relative price of foreign goods, π, is constant throughout in response to tax changes. This requires not only that the private domestic propensity to spend on domestic output equals the foreign private propensity to spend on domestic output ($\alpha = \alpha^*$) but also that the supply responses to the interest rate changes induced by the fiscal policy have no further effect on π (see equations (67b) and (67d)). This extension of the familiar transfer criterion for a change in the terms of trade is of course unnecessary when output is exogenous ($k' = k^{*'} = 0$).

The numerical details of the first simulation are given in Table 1. The two countries are identical (with $\alpha = \alpha^* = .5$) and the initial long-run equilibrium is one with F = 0.

The policy experiment is an increase in τ_1. Because of our tax function this policy experiment amounts to a short-run tax cut followed by (and indeed necessitating) a long-run tax increase through its effect on the stock of outstanding public debt.

Table 1 - One-good economy; zero external debt of home country

Parameter values

$\beta = \beta^* = .25; \quad \zeta^{-1} = \zeta^{*-1} = .031; \quad \lambda = \lambda^* = .03; \quad \theta = \theta^* = .035;$

$\alpha = \alpha^* = .5; \quad \frac{\beta^x}{\beta^x+\beta^y} = \frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}} = .5\,; \quad \tau_{1,0} = \tau^*_{1,0} = .3206205;$

$g_o = g^*_o = .1469511; \quad \mu_\tau = \mu^*_\tau = -2.$

Key initial equilibrium values

$r_o = .05; \quad \pi_o = 1; \quad F_o = 0.$

Characteristic roots

$-.05326; \quad -.05; \quad -.01543; \quad -.01539; \quad -.015; \quad .06539; \quad .07628; \quad .08.$

Figure 4 shows the dynamic response of the key variables in the two countries.

The long-run response to the increase in τ_1 is a larger stock of domestic public debt, a small reduction in the foreign stock of public debt; equal reductions in the domestic and foreign capital stocks; a small increase in domestic private non-human wealth; a larger increase in foreign non-human wealth; a large reduction in domestic human capital and a smaller reduction in foreign human capital. The interest rate goes up, domestic consumption falls and foreign consumption rises. The net foreign asset position of the home country becomes negative as domestic government debt crowds out domestic net foreign assets as well as domestic and foreign real capital. π is, of course, unaffected in the long run as in the short run. The system exhibits (local) saddlepoint stability.

The dynamic response to an unexpected, immediate (at $t = 0$) and permanent increase in τ_1 as follows.

In the short run, taxes are cut by about the same amount they will be raised in the long run. The tax cut is reversed gradually and becomes an increase from period 14 on. The stock markets (ψ and ψ^*) fall on impact in both countries, reflecting higher anticipated future interest rates. The behaviour of ψ and ψ^* and of K and K^* is identical. The interest rate rises only gradually. Domestic consumption increases on impact, reaches a peak soon after and then begins a steady decline. Foreign consumption drops on impact, reaches a trough soon afterwards

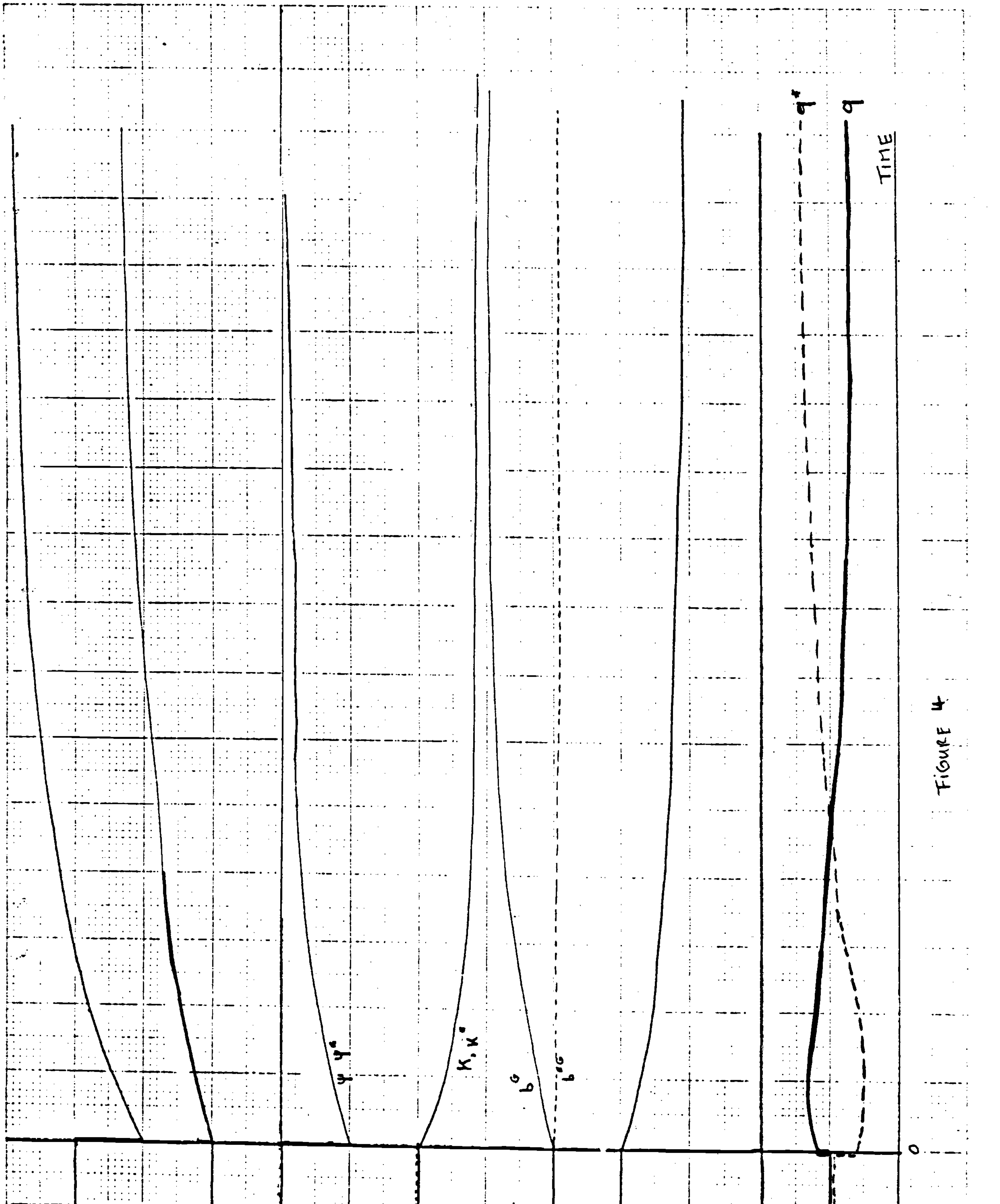


FIGURE 4

and increases steadily thereafter to its higher new long-run level.

If the foreign country were to raise τ_1^* at the same time and by the same amount as the home country raises τ_1, the results are the following. In the long run, all "country-specific" endogenous variables change by the same amount in each country. $\bar{F}$ of course remains unchanged. b^G and b^{*G} increase by the same amount and the change in $b^G + b^{*G}$ is exactly twice that when only τ_1 was increased. K and K* fall by twice as much while r increases twice as much. Consumption declines in both countries, as does human capital. The current account remains in balance throughout the adjustment process. In fact, all domestic and foreign variables (b^G and b^{*G}, K and K*, w and w*, h and h*, ψ and ψ^*, q and q*, τ and τ^*) move in the same way. The long-run tax increase is preceded, in both countries by an initial cut in total taxes which boosts consumption and creates budget deficits which only vanish asymptotically. The joint move towards fiscal expansion in the short run creates a steeper decline in the two countries' stock markets.

The only way for the foreign country to avoid the higher world interest rate resulting from the short-run expansionary fiscal action in the home country, is for the former to engage in short-run contractionary fiscal action. Consider e.g. a policy response by the foreign country which consists of a reduction in τ_1^* (and therefore a short run increase in τ^*) equal in magnitude to the increase in τ_1. The result is no change in r, ψ, ψ^*, K and K* in the short run or in the long run. All other country-specific endogenous variables change by opposite amounts in the long-run and during the adjustment process. The home country runs budget deficits and current account deficits throughout while the foreign country runs budget surpluses and current account surpluses. In the long run w and h are down (and w* and h* are up by equal amounts). Consumption falls at home in the long run and declines abroad. The short term response is in the opposite direction.

These results are not affected qualitatively by changing parameter values in such a way that the initial stationary equilibrium is one in which the home country is a net creditor ($F_o > 0$) or a net debtor ($F_o < 0$). E.g. consider $g_o = .1939$ and $g^*_o = .10$ while keeping the other parameter values the same as those given in Table 1. This "shifting" of public spending towards the home country lowers the initial long-run value of b^G_o to 2.534, raises that of b^{*G}_o to 4.412 but leaves r_o and π_o unchanged. The home country becomes a net creditor with $F_o = .939$. The characteristic roots are virtually the same as in the case where $F_o = 0$ (specifically, the saddlepoint equilibrium configuration with 5 stable and 3 unstable roots persists) and even quantitatively the short-run and long-run response of the system is not much affected. Shifting public spending the other way, towards the foreign country, with $g_o = .10$, $g^*_o = .1939$ and $b^G_o = 4.412$, $b^{*G}_o = 2.534$ and $F_o = -.9390213$ again does not yield a picture that is significantly different from that shown in Figure 4.

The two-commodity case

We now introduce a bias in private spending towards a country's own good, i.e. $\alpha > \alpha^*$. The first numerical example, specified fully in Table 2a, again has $F_o = 0$. The adjustment process following an unanticipated permanent increase in τ_1 is shown in Figure 5. The long-run effect of an increase in τ_1 on most endogenous variables is qualitatively the same as in the one-good case. One exception is that b^{*G} shows a small increase rather than a small decline. The reason for this, as can be seen from equation (58), is the long-run increase in π. The long-run reduction in home (consumption) demand worsens the home country's terms of trade since the share of domestic consumption spending falling on domestic output is higher than the share of foreign consumption spending allocated to domestic output ($\alpha > \alpha^*$).

Because π now varies over time, there is a certain amount of "decoupling" between domestic and foreign capital formation during the adjustment process. The short-run domestic tax cut outweighs the long-run tax increase, so h and q increase on impact. With q^* falling, the terms of trade improve in the short run. After the initial discontinuous drop in π, however, π rises smoothly throughout the adjustment process. This rise in π is anticipated. Since $r^* = r - \frac{\dot{\pi}}{\pi}$ is the interest rate governing ψ^*, the foreign stock market falls by less initially than the domestic one and domestic capital decumulates more swiftly than foreign capital.

A simultaneous, equal increase in τ_1 and τ_1^* has exactly the same effect as it has in the one-good version of the model since π remains constant throughout. While this stabilizes the real exchange rate relative to a unilateral increase in τ_1, it reinforces the effect on the interest rate.

A policy of reducing τ_1^* by the same amount as the increase in τ_1 does

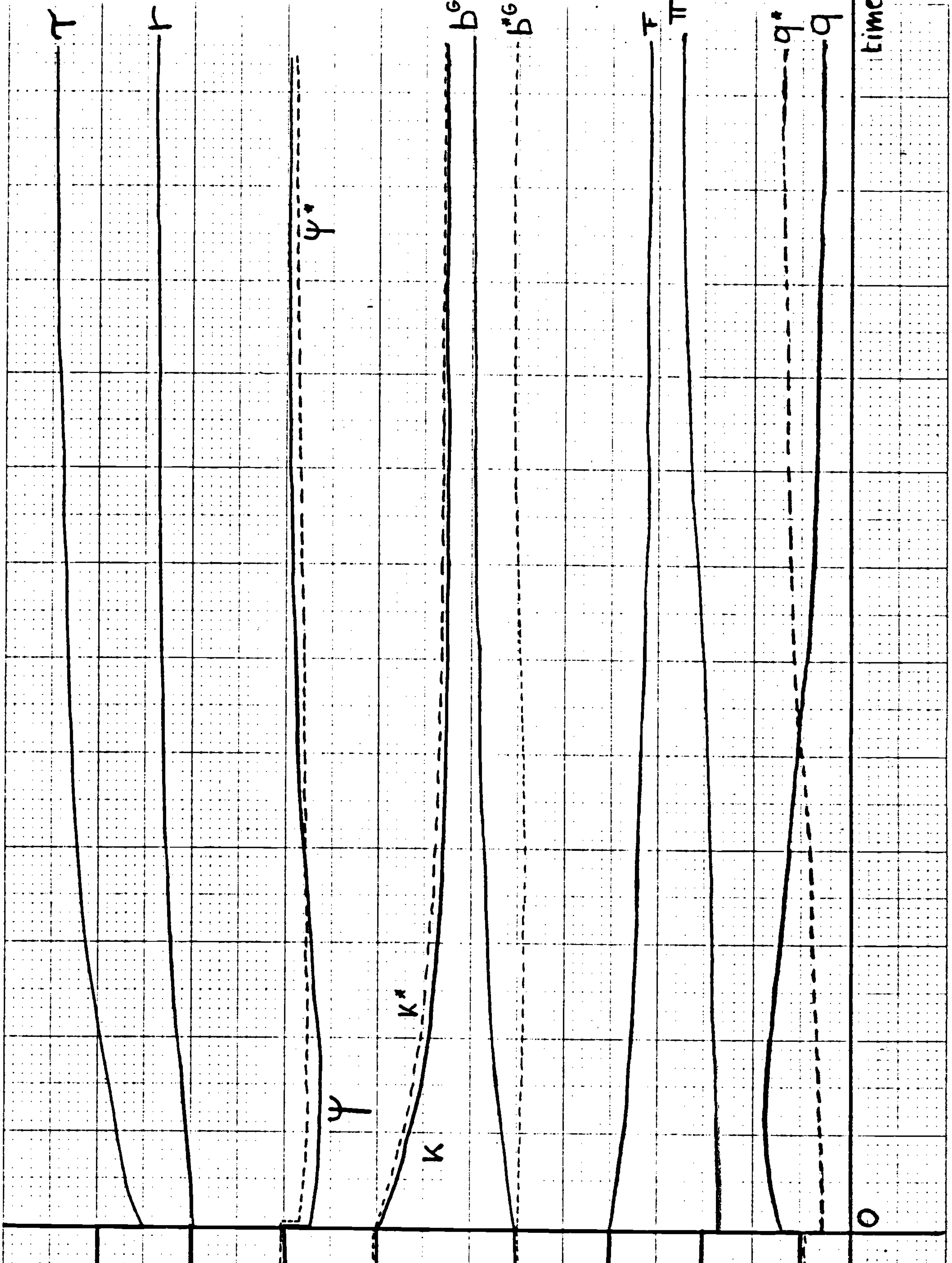


FIGURE 5

indeed stabilize the interest rate (r) but reinforces the swings in π which falls by more in the short run and rises by more in the long run. To prevent both π and r from changing, two fiscal instruments (e.g. τ^* and g^*) must be used.

Even when the terms of trade are endogenous, the sign of the initial external net worth position of the two countries does not appear to have crucial implications for the qualitative stability or saddlepoint properties of the model. Table 2 shows how the desired saddlepoint configuration is present when F_o is negative (Table 2b) and when F_o is positive (Table 2c). Qualitatively, the long-run and short-run responses of the endogenous variables in the net external creditor and the net external debtor cases are similar to each other and to the zero net external debt case.

Table 2 : Two-good economy

a) Home country has zero net external debt ($F_o = 0$).

Parameter values:

$\beta = \beta^* = .25$; $\zeta^{-1} = \zeta^{*-1} = .031$; $\lambda = \lambda^* = .03$; $\theta = \theta^* = .035$;

$\alpha = 1 - \alpha^* = .7$; $\dfrac{\beta^x}{\beta^x+\beta^y} = 1 - \dfrac{\beta^{*x}}{\beta^{*x}+\beta^{*y}} = .7$; $\tau_{1,0} = \tau^*_{1,0} = .3206205$;

$g_o = g_o^* = .1469511$; $\mu_\tau = \mu_\tau^* = -2$

Key initial equilibrium values

$r_o = .05$; $\pi_o = 1$; $F_o = 0$.

Characteristic roots

$-.05326$; $-.04861$; $-.01545$; $-.01505$; $-.01068$; $.06101$;

$.07628$; $.07834$.

b) Home country is external debtor ($F_o < 0$)

Parameter values:

$\beta = \beta^* = .25$; $\zeta^{-1} = \zeta^{*-1} = .031$; $\lambda = \lambda^* = .03$; $\theta = .04$;

$\theta^* = .035$; $\alpha = 1-\alpha^* = .6$; $\dfrac{\beta^x}{\beta^x+\beta^y} = .75$; $\dfrac{\beta^{*x}}{\beta^{*x}+\beta^{*y}} = .3624$;

$\tau_{1,0} = \tau^*_{1,0} = .3206205$, $g_o = .2885585$; $g_o^* = .3059254$;

$\mu_\tau = \mu_\tau^* = -2$.

Key initial equilibrium values

$r_o = .05;\quad \pi_o = 1;\quad F_o = -3.1795.$

Characteristic roots

$-.05059;\quad -.04992;\quad -.01605;\quad -.01533;\quad -.01470;\quad .06334;$

$.07763;\quad .07937.$

c) Home country is external creditor ($F_o > 0$)

Parameter values

$\beta = \beta^* = .25;\quad \zeta^{-1} = \zeta^{*-1} = .031;\quad \lambda = \lambda^* = .03;\quad \theta = .035;$

$\theta^* = .04;\quad \alpha = 1-\alpha^* = .6;\quad \dfrac{\beta^x}{\beta^x+\beta^y} = .6375548;\quad 1 - \dfrac{\beta^{*x}}{\beta^{*x}+\beta^{*y}} = .75;$

$\tau_{1,0} = \tau^*_{1,0} = .3206205;\quad g_o = .3059254;\quad g^*_o = .2885585;\quad \mu_\tau = \mu^*_\tau = -2.$

Key initial equilibrium values:

$r_o = .05;\quad \pi_o = 1;\quad F_o = 3.1795.$

Characteristic roots:

$-.05039;\quad -.04981;\quad -.02130;\quad -.01562;\quad -.01520;\quad .06320;$

$.07766;\quad .07942.$

Conclusion

The purpose of this paper has been to study certain aspects of public debt and deficits in the open economy using a model in which private sector behavioural relationships have been derived explicitly from optimizing behaviour. The public sector's present value budget constraint or solvency constraint, together with the assumption that the real interest rate exceeds the rate of growth, was shown to tie together current tax cuts and future tax increases.[11/] In a two-country setting such a policy would raise the interest rate in an integrated global capital market, crowd out private capital at home and abroad and worsen a country's external net worth position. If in addition private spending shows a preference, at the margin, for domestic output over foreign output, then the policy would improve the terms of trade in the short run but cause them to worsen in the long run.

The analysis brings out the central role of the interest rate in transmitting disturbances between countries when capital markets are highly integrated. While there always exist paths (or contingent rules) for the domestic fiscal policy instrument that can neutralize any incipient shocks to the path of interest rates originating from abroad, such "stabilizing" policy actions inevitably involve costs. Even if lump-sum taxes are used, intertemporal (and therefore intergenerational) redistribution of the tax burden is inevitable. If lump-sum taxes are not available, dead-weight losses and excess burdens will be imposed. Varying the public spending programme involves distortions in the intertemporal allocation of public consumption. If public sector

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11. This is the same point as was emphasized in the context of a monetary economy by Sargent and Wallace [1981].

capital formation (not considered in this paper) is varied yet other costs are incurred. Taxing international capital flows may be an interesting second-best policy.

The finite private decision horizon (or the excess of the effective private discount rate over the government's discount rate) permits a non-trivial analysis of one of the central current issues of financial policy: the consequences for private saving and capital formation of varying the time pattern of taxation and borrowing.

Several possible extensions of the model came to mind. The first is to add money to the asset menu. To do this properly would be a major task, but the ad-hoc inclusion of domestic money as an argument in the direct utility function may be a useful first step. If non-interest-bearing government fiat money is added to the instantaneous utility function in logarithmic form ($+ \gamma \ln m$, where m is the nominal money stock deflated by the Cobb-Douglas price index, p, appropriate to the utility function) the extension is trivial.

Money demand is given by $m = \frac{\gamma}{r + \frac{\dot{p}}{p}} q$ and is unit elastic with respect to the nominal interest rate $r + \frac{\dot{p}}{p}$. If all non-money assets are index-linked, money is a veil. Super-neutrality prevails in the short run as in the long run and in response to any kind of monetary shock. Real interest rates are unaffected by monetary policy. The real seigniorage the authorities can extract through monetary expansion is independent of the rate of growth of nominal money and of the rate of inflation. The terms of trade are independent of the behaviour of the nominal exchange rate.

If nominally denominated interest-bearing public debt exists in addition to money, unanticipated monetary policy changes which cause discontinuous jumps in the general price level can inflict capital losses or gains on the holders of these nominal assets. A non-unitary interest elasticity of demand for real money balances permits the consideration of seigniorage issues.

A second important issue is the de facto non-existence of lump-sum taxation. Barro [1979] analyzed the problem of the optimal inter-temporal pattern of distortionary taxation and (under very strong conditions) derived a version of the "uniform tax rate over time" result for an economy in which "first-order debt neutrality" held. It would be rather more relevant to study this problem in a world which does not have this strong first order debt neutrality property, such as the Yaari-Blanchard model or the overlapping generations model without operative intergenerational gifts and bequests.

A further desirable extension would be to relax the unattractive, highly restrictive perfect capital market assumption which permits private agents, once allowance is made for their finite expected lifetimes, to borrow on the same terms as the government.

Fourth, labour market disequilibrium could be added as a feature to the model. The simplest approach simply posits different combinations and degrees of nominal and real wage rigidity. The obvious starting point here is the work of Sachs [1983].

Finally, the model is inhabited by well-informed, rationally anticipating and optimizing private agents and rather mechanically acting governments. Clearly, government behaviour should be endogenized in a more satisfactory manner. The interaction between the two national governments could be strategic in nature. Recent developments in differential game theory and its applications to economics hold considerable promise (see e.g. Miller and Salmon [1983]).

The Yaari-Blanchard model, as developed in this paper would seem to be a flexible vehicle for the analysis of a wide range of interesting issues in international economics.

APPENDIX 1

Private sector decision rules

Private consumption behaviour and asset demand

The essential features of the model of consumer behaviour are taken from Yaari [1965], as presented in Blanchard [1983a, b]. Time is continuous. At each instant a new age cohort, composed of many agents, is born. The size of each cohort is normalized to λ, $0 \leq \lambda < 1$. During their lifetime each agent faces a common, constant instantaneous probability of death λ. All surviving agents therefore have a life expectancy of λ^{-1}. λ is also taken to be the proportion of agents in each cohort which die at each instant. The size of the surviving cohort at time t which was born at time t_o is therefore $\lambda e^{\lambda(t-t_o)}$. Total population at any time t is constant and given by $\lambda \int_{-\infty}^{t} e^{-\lambda(t-s)} ds = 1$.

All surviving agents have the same labour income. Private agents can save or dissave by buying or selling bonds and domestic capital (which are perfect substitutes) or by buying or selling annuities in a perfect insurance market. There is no direct foreign investment. Bonds are short and have a fixed value in terms of good x, the domestically produced good. The instantaneous interest rate is $r(t)$. Since there is no bequest motive and negative bequests are not permitted, agents will contract to have their entire non-human wealth returned to the life insurance company in the event of their death. The life insurance industry is competitive and subject to free entry. Thus if an agent's non-human wealth is $\bar{w}$ they will receive $\lambda\bar{w}$ at each instant they are alive and pay $\bar{w}$ to the insurance company the day they die.

Each agent born at time t has the utility function (1) which he maximizes at each instant s subject to the budget constraint (2).

$$\text{(1a)}\quad \max E_s \int_s^{\infty} [\ln \bar{c}(t,v) + \beta^x \ln g_X(t,v) + \beta^y \ln g_Y(t,v)] e^{\theta(v-s)} dv$$

$$\beta^x, \beta^y, \gamma, \theta > 0$$

$$\text{(1b)}\quad \bar{c}(t,v) = \bar{c}_X^{\alpha}\, \bar{c}_Y^{(1-\alpha)}$$

$$0 < \alpha < 1$$

$$\text{(2)}\quad \frac{d}{ds}\bar{w}(t,s) = (r(s) + \lambda)\bar{w}(t,s) + j(t,s) - \bar{\tau}(t,s) - \bar{c}_X(t,s) - \pi(s)\bar{c}_Y(t,s)$$

E_s is the expectation operator conditional on information up to time s; $\bar{c}_X$ is private consumption of domestic output; $\bar{c}_Y$ is private consumption of foreign output. The government provides domestic (g_X) and foreign output (g_Y) as public goods. For any variable $\bar{m}$, say, m denotes the economy-wide aggregate. θ is the pure rate of time preference, $\bar{w}$ non-human wealth measured in units of good x, $\bar{j}$ labour income, $\bar{\tau}$ taxes net of transfers and π the relative price of foreign output (competitiveness or the reciprocal of the terms of trade).

Expectations are rational and single-valued, i.e. held with complete subjective certainty. Using certainty equivalence, optimizing (1a) is therefore equivalent to optimizing (3).

$$(3) \qquad \max \int_s^\infty [\ln \bar{c}(t,v) + \beta^x \ln g_x(t,v) + \beta^y \ln g_y(t,v)] e^{-(\theta+\lambda)(v-s)} dv$$

Note that the private sector wealth constraint or present value budget constraint (PVBC) corresponding to (2) is

$$(4) \qquad \int_s^\infty [\bar{c}_x(t,v) + \pi(v)\bar{c}_y(t,v)] e^{-\int_s^v (r(u)+\lambda)du} dv = \bar{w}(t,s)$$

$$+ \int_s^\infty (\bar{j}(t,v) - \bar{\tau}(t,v)) e^{-\int_s^v (r(u)+\lambda)du} dv - \lim_{\ell\to\infty} \bar{w}(t,\ell) e^{-\int_s^\ell (r(u)+\lambda)du}$$

The conventional transversality condition $\lim_{\ell\to\infty} \bar{w}(t,\ell) e^{-\int_s^\ell (r(u)+\lambda)du} = 0$ gives the familiar "lifetime" household budget constraint. The total value, in terms of domestic output, of current private consumption spending $\bar{q}$ is defined by

$$(5) \qquad \bar{q}(t,s) \equiv \bar{c}_x(t,s) + \pi(s)\bar{c}_y(t,s)$$

From the first-order conditions for an optimum we find that

$$(6) \qquad \bar{q}(t,s) = (\theta+\lambda)[\bar{w}(t,s) + \bar{h}(t,s)]$$

where human capital, $\bar{h}(t,s)$ is defined by :

(7) $$\bar{h}(t,s) \equiv \int_s^\infty [\bar{j}(t,v) - \bar{\tau}(t,v)] e^{-\int_s^v (r(u)+\lambda)du} dv$$

(8a) $$\bar{c}_x(t,s) = \alpha \bar{q}(t,s)$$

(8b) $$\bar{c}_y(t,s) = (1-\alpha) \frac{\bar{q}(t,s)}{\pi(s)}$$

As in Blanchard's model, optimal private consumption spending is governed by

(9) $$\frac{d}{ds}\bar{q}(t,s) = (r(s) - \theta)\bar{q}(t,s)$$

Also, along the optimal trajectory,

(10) $$\frac{d}{ds}\bar{w}(t,s) = (r(s)+\lambda)\,\bar{w}(t,s) + \bar{j}(t,s) - \bar{\tau}(t,s) - \bar{q}(t,s)$$

$$= (r(s)-\theta)\,\bar{w}(t,s) + \bar{j}(t,s) - \bar{\tau}(t,s) - (\theta+\lambda)\,\bar{h}(t,s)$$

For any individual household variable $\bar{m}(t,s)$ we define the corresponding aggregate $m(s)$ by

$$m(s) = \lambda \int_{-\infty}^{s} \bar{m}(t,s) e^{\lambda(t-s)} dt$$

If labour income and taxes are the same for all agents alive, regardless of age, then

$\bar{j}(t,s) = \lambda j(s)$

and

$\bar{\tau}(t,s) = \lambda \tau(s)$

It then follows that

(11) $q(s) = (\theta + \lambda)(w(s) + h(s))$

or

(11') $\dot{q}(s) = (r - \theta)\, q(s) - (\theta + \lambda)\, \lambda\, w(s)$

(12a) $c_x(s) = \alpha\, q(s)$

(12b) $c_y(s) = (1-\alpha) \frac{q(s)}{\pi(s)}$

(13) $\dot{w}(s) = r(s)\, w(s) + j(s) - \tau(s) - q(s)$

(14) $h(s) = \int_s^{\infty} (j(t) - \tau(t)) e^{-\int_s^t (r(u)+\lambda) du} dt$

or

(14') $\dot{h}(s) = \tau(s) - j(s) + (r(s) + \lambda)\, h(s)$

Foreign consumption behaviour is determined analogously. Note that all bonds are denominated in terms of home country output. w^*, q^* and h^* are measured in terms of foreign country output.

Production and private investment

We consider a competitive economy with continuous full employment. The production function has constant returns to labour and capital and satisfies the Inada conditions. Domestic output y is therefore given by

$$(15)\quad y = f(K) \qquad f' > 0; \quad f'' < 0; \quad f(0) = 0; \quad \lim_{K\to 0} f' = \infty$$

$$\lim_{K\to\infty} f' = 0$$

It follows that labour income, j is given by

$$j = f(K) - Kf'(K)$$

or

$$j = j(K) \qquad g' > 0$$

Private capital formation involves the transformation of domestic output into capital and is subject to quadratic internal costs of adjustment. The firm's objective functional is

$$\max_{\{L(s),\dot{K}(s)\}} \int_t^{\infty} \left[L(s) f\left(\frac{K(s)}{L(s)}\right) - w(s)L(s) - \dot{K}(s) - \frac{1}{2}\zeta\frac{\dot{K}(s)^2}{K(s)}\right] e^{-\int_t^s r(u)du} ds.$$

where L(s) is the firm's employment of labour and $\zeta > 0$.

Since the production function and the cost-of-adjustment function are linear homogeneous Tobin's marginal q equals his average q or ψ and we can write the investment function as :

$$\dot{K}(s) = \frac{(\psi(s) - 1)}{\zeta} K(s) \tag{16}$$

$$r = \frac{f'(K)}{\psi} + \frac{\dot{\psi}}{\psi} + \frac{1}{2}\frac{\zeta}{\psi}\left(\frac{\dot{K}}{K}\right)^2 \tag{17}$$

Again, the foreign investment decision rule can be derived analogously.

APPENDIX 2

The linear approximation to the two-country model

The general non-linear model is given by equations (1) – (17).

(1) $$\dot{b}^G = g - \tau + rb^G$$

(2) $$\dot{b}^{*G} = \pi(g^* - \tau^*) + rb^{*G}$$

(3) $$\dot{K} = \frac{(\psi - 1)}{\zeta} K$$

(4) $$\dot{K}^* = \frac{(\psi^* - 1)}{\zeta^*} K^*$$

(5) $$\dot{w} = rw + j(K) - \tau - q$$

(6) $$\dot{h} = \tau - j(K) + (r + \lambda)h$$

(7) $$\dot{h}^* = \tau^* - j^*(K^*) + (r - \frac{\dot{\pi}}{\pi} + \lambda^*)h^*$$

(8) $$\dot{\psi} = r\psi - f'(K) - \frac{1}{2}\zeta\left(\frac{\dot{K}}{K}\right)^2$$

(9) $$q = (\theta + \lambda)(w + h)$$

(10) $$q^* = (\theta^* + \lambda^*)(w^* + h^*)$$

(11) $$\tau = \tau_1 + \mu_\tau \dot{b}^G$$

(12) $$\tau^* = \tau_1^* + \mu_\tau^* \frac{\dot{b}^{*G}}{\pi}$$

(13) $$f(K) = \alpha q + \alpha^* \pi q^* + \frac{\beta^x}{\beta^x + \beta^y} g + \frac{\beta^{*x}}{\beta^{*x} + \beta^{*y}} \pi g^* + \dot{K} + \frac{1}{2}\zeta \frac{\dot{K}^2}{K}$$

(14) $$f^*(K^*) = (1-\alpha)\frac{g}{\pi} + (1-\alpha^*)q^* + \left(\frac{\beta^y}{\beta^x+\beta^y}\right)\frac{g}{\pi} + \frac{\beta^{*y}}{\beta^{*x}+\beta^{*y}} g^* + \dot{K}^* + \frac{1}{2}\zeta^* \frac{\dot{K}^{*2}}{K^*}$$

(15) $$r = \frac{\dot{\psi}^*}{\psi} + \frac{\dot{\pi}}{\pi} + \frac{f^{*\prime}(K^*)}{\psi^*} + \frac{1}{2}\frac{\zeta^*}{\psi^*}\left(\frac{\dot{K}^*}{K^*}\right)^2$$

(16) $$F = w - \psi K - b^G$$

(17) $$w + \pi w^* = \psi K + \pi\psi^* K^* + b^G + b^{*G}$$

A convenient representation of the linearized model involves 8 state variables ($x \equiv \{b^G, b^{*G}, K, K^*, w, h, h^*, \psi\}'$), eleven output variables or short-run endogenous variables ($y \equiv \{q, q^*, r, \tau, \tau^*, F, \pi, \psi^*, \dot{\pi}, \dot{\psi}^*, w^*\}'$) and six exogenous or forcing variables ($z \equiv \{\tau_1, \tau_1^*, g, g^*, \dot{g}, \dot{g}^*\}'$). The inclusion of both π and ψ^* and their rates of change as output variables is merely a device to let the computer do more of the work.

The boundary conditions for four of the state variables are given by the assignment of given initial values:

(18a) $$b^G(0) = b_o^G$$

(18b) $$b^{*G}(0) = b_o^{*G}$$

(18c) $$K(0) = K_o$$

(18d) $$K^*(0) = K_o^*$$

The initial value of w is given by the linear restriction:

(18e) $$w(0) = w(0^-) + (\psi(0) - \psi(0^-))K_o = w(0^-) + (\psi(0) - 1)K_o.$$ 10/

10. We make use of the assumption that $\psi(0^-) = \psi^*(0^-) = 1$.

The remaining boundary conditions (for h, h* and ψ) take the form of the restriction that the system has to lie on the stable manifold. Provided the state matrix of the linearized system has three unstable roots, this suffices to ensure a unique (convergent and continuous except at those moments when "news" arrives) solution.

The linearized system is represented in (19) and (20).

$$
\begin{bmatrix}
\frac{-r_o}{1+\mu_\tau} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-r_o}{1+\mu_\tau^*} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-K_o}{\zeta} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -j'(K_o) & 0 & -r_o & 0 & 0 & 0 \\
0 & 0 & j'(K_o) & 0 & 0 & -(r_o+\lambda) & 0 & 0 \\
0 & 0 & 0 & j^{*\prime}(K_o^*) & 0 & 0 & -(r_o+\lambda^*) & 0 \\
0 & 0 & f''(K_o) & 0 & 0 & 0 & 0 & -r_o
\end{bmatrix}
\begin{bmatrix}
b^G - b^G \\
b^{*G} - b_o^{*G} \\
K - K_o \\
K^* - K_o^* \\
w - w_o \\
h - h_o \\
h^* - h_o^* \\
\psi - 1
\end{bmatrix}
$$

$$
+ \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{b}^G \\
\dot{b}^{*G} \\
\dot{K} \\
\dot{K}^* \\
\dot{w} \\
\dot{h} \\
\dot{h}^* \\
\dot{\psi}
\end{bmatrix}
$$

$$
+\begin{bmatrix}
0 & 0 & \frac{-b_o^G}{1+\mu_\tau} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{-b_o^{*G}}{1+\mu_\tau^*} & 0 & 0 & \frac{(\tau_{1,0}^*-g_0^*)}{1+\mu_\tau^*} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-K_o^*}{\zeta^*} & 0 & 0 & 0 \\
1 & 0 & -w_o & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -h_o & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -h_o^* & 0 & -1 & 0 & \frac{h_o^*}{\pi_o} & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
q - q_o \\
q^* - q_o^* \\
r - r_o \\
\tau - \tau_o \\
\tau^* - \tau^* \\
\pi - \pi \\
\dot{\pi} \\
\psi^* - 1 \\
\dot{\psi}^* \\
F - F_o \\
w^* - w_o^*
\end{bmatrix}
\tag{19}
$$

$$
+\begin{bmatrix}
\frac{1}{1+\mu_\tau} & 0 & \frac{-1}{1+\mu_\tau} & 0 & 0 & 0 & 0 \\
0 & \frac{\pi_o}{1+\mu_\tau^*} & 0 & \frac{-\pi_o}{1+\mu_\tau^*} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\tau_1 - \tau_{1,0} \\
\tau_1^* - \tau_{1,0}^* \\
g - g_o \\
g^* - g_o^* \\
\dot{g} \\
\dot{g}^*
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
$$

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & -(\theta+\lambda) & -(\theta+\lambda) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -(\theta^*+\lambda^*) & 0 \\
0 & 0 & 0 & -f^{*''}(K_o^*) & 0 & 0 & 0 & 0 \\
\frac{-\mu_\tau r_o}{1+\mu_\tau} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-\mu_\tau^* r_o}{\pi_o(1+\mu_\tau^*)} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & f^{*'}(K_o^*) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{(1-\alpha)}{\pi_o}(\theta+\lambda)\lambda & 0 & 0 & 0 \\
0 & 0 & f'(K_o) & 0 & 0 & 0 & 0 & -K_o\zeta^{-1} \\
0 & 0 & 0 & 0 & \alpha(\theta+\lambda)\lambda & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & -1 & 0 & 0 & K_o \\
-1 & -1 & -1 & -\pi_o & 1 & 0 & 0 & -K_o
\end{bmatrix}
\begin{bmatrix}
b^G - b_o^G \\
b^{*G} - b_o^{*G} \\
K - K_o \\
K^* - K_o^* \\
w - w_o \\
h - h_o \\
h^* - h_o^* \\
\psi - 1
\end{bmatrix}
$$

$$
+
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & f^{*'}(K_o^*) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & f'(K_o) & 0 & 0 & 0 & 0 & -K_o\zeta^{-1} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
b^{*G} \\
\dot{b}^{*G} \\
\dot{K} \\
\dot{K}^* \\
\dot{w} \\
\dot{h} \\
h^* \\
\dot{\psi}
\end{bmatrix}
$$

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(\theta^*+\lambda^*) \\
0 & 0 & 1 & 0 & 0 & 0 & -\frac{1}{\pi_o} & f^{*\prime}(K_o^*) & -1 & 0 & 0 \\
0 & 0 & \frac{-\mu_\tau b_o^G}{1+\mu_\tau} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{-\mu_\tau^* b_o^{*G}}{\pi_o(1+\mu_\tau^*)} & 0 & 1 & \frac{\mu_\tau^* r_o b_o^{*G}}{(1+\mu_\tau^*)\pi_o^2} & 0 & 0 & 0 & 0 & 0 \\
\frac{\alpha-1}{\pi_o} & \alpha^*-1 & 0 & 0 & 0 & \left[(1-\alpha)q_o+\frac{\beta^y}{\beta^x+\beta^y}g_o\right]\pi_o^{-2} & 0 & \frac{-K_o^*}{\zeta^*} & 0 & 0 & 0 \\
\left(\frac{\alpha-1}{\pi_o}\right)(r_o-\theta) & (\alpha^*-1)(r_o-\theta^*) & \left(\frac{\alpha-1}{\pi_o}\right)q_o+(\alpha^*-1)q_o^* & 0 & 0 & 0 & \frac{(1-\alpha^*)}{\pi_o}q_o^*+\left[(1-\alpha)q_o+\frac{\beta^y}{\beta^x+\beta^y}g_o\right]\pi_o^{-2} & 0 & \frac{-K_o^*}{\zeta^*} & 0 & (1-\alpha^*)(\theta^*+\lambda^*)\lambda^* \\
-\alpha & -\alpha^*\pi & 0 & 0 & 0 & -\left[\alpha^*q_o^*+\frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}}g_o^*\right] & 0 & 0 & 0 & 0 & 0 \\
-\alpha(r_o-\theta) & -\alpha^*\pi_o(r_o-\theta^*) & -(\alpha q_o+\alpha^*\pi_o q_o^*) & 0 & 0 & 0 & -\frac{\beta^{*x}}{\beta^{*x}+\beta^{*y}}g_o^* & 0 & 0 & 0 & \alpha^*\pi_o(\theta^*+\lambda^*)\lambda^* \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & w_o^*-K_o^* & 0 & -\pi_o K_o^* & 0 & 0 & \pi_o
\end{bmatrix}
\begin{bmatrix}
q-q_o \\ q^*-q_o^* \\ r-r_o \\ \tau-\tau_o \\ \tau^*-\tau_o^* \\ \pi-\pi_o \\ \dot{\pi} \\ \psi^*-1 \\ \dot{\psi}^* \\ F-F_o \\ w^*-w_o^*
\end{bmatrix}
$$

$$
+\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-1}{1+\mu_\tau} & 0 & \frac{-\mu_\tau}{1+\mu_\tau} & 0 & 0 & 0 \\
0 & \frac{-1}{1+\mu_\tau^*} & 0 & \frac{-\mu_\tau^*}{1+\mu_\tau} & 0 & 0 \\
0 & 0 & -\left(\frac{\beta^y}{\beta^x+\beta^y}\right)\frac{1}{\pi_o} & \frac{-\beta^{*y}}{\beta^{*x}+\beta^{*y}} & 0 & 0 \\
0 & 0 & 0 & 0 & -\left(\frac{\beta^y}{\beta^x+\beta^y}\right)\frac{1}{\pi_o} & \frac{-\beta^{*y}}{\beta^{*x}+\beta^{*y}} \\
0 & 0 & \frac{-\beta^x}{\beta^x+\beta^y} & \frac{-\beta^{*x}}{\beta^{*x}+\beta^{*y}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{-\beta^x}{\beta^x+\beta^y} & \frac{-\beta^{*x}}{\beta^{*x}+\beta^{*y}} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\tau_1-\tau_{1,0} \\
\tau_1^*-\tau_{1,0}^* \\
g-g_o \\
g^*-g_o^* \\
\dot{g} \\
\dot{g}^*
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
$$

Schematically, they can be written as :

$$(19') \quad E_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + E_2 \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + E_3 y + E_4 z = 0$$

$$(20') \quad E_5 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + E_6 \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + E_7 y + E_8 z$$

E_7 and $E_2 - E_3 E_7^{-1} E_6$ are assumed to be of full rank. x_1 contains the predetermined state variables and x_2 the non-predetermined ones. Equations (19') and (20') are reduced to state-space form.

$$(21a) \quad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \bar{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \bar{B} z$$

$$(21b) \quad y = \bar{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \bar{D} z$$

Provided (21a) has as many stable characteristic roots as predetermined variables (5 in our case) and as many unstable characteristic roots as non-predetermined variables (3 in our case) a unique convergent solution exists. This result is of course strictly local in our model.

The algorithm "Saddlepoint" of Austin and Buiter [1982] only permits boundary conditions for the predetermined variables of the form $x_1'(0) = x_{1,0}'$.

$$F_1 x_1''(0) + F_2 x_1'(0) + F_3 x_2(0) = f \tag{22}$$

Here $x_1 \equiv \begin{bmatrix} x_1' \\ x_1'' \end{bmatrix}$ and x_1' contains the predetermined variables for which initial values are assigned (b^G, b^{*G}, K and K* in our model) while x_1'' contains the predetermined variables for which the boundary conditions take the form of linear restrictions at the initial date (w in our model).

In terms of the notation of equation (22), boundary condition (18e) can be represented as in equation (23).

$$[\,1\,][\,w - w_o\,] + [\,0 \quad 0 \quad 0 \quad 0\,] \begin{bmatrix} b^G - b^G \\ b^{*G} - b_o^{*G} \\ K - K_o \\ K^* - K_o \end{bmatrix} \tag{23}$$

$$+ [\,0 \quad 0 \quad -K_o\,] \begin{bmatrix} h - h_o \\ h^* - h_o^* \\ \psi - 1 \end{bmatrix} = [\,0\,]$$

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