The Fiscal Theory of the Price Level: A Critique*

Willem H. Buiter**
Chief Economist and Special Counsellor to the President,
European Bank for Reconstruction and Development, CEPR and NBER

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Abstract

This paper argues that the 'fiscal theory of the price level' (*FTPL*) has feet of clay.

The source of the problem is a fundamental *economic* misspecification. The FTPL confuses two key building blocks of a model of a market economy: budget constraints, which must be satisfied identically, and market clearing or equilibrium conditions. The FTPL asssumes that the government's intertemporal budget constraint needs to be satisfied only *in equilibrium*.

This economic misspecification has far-reaching implications for the mathematical properties of the equilibria supported by models that impose the structure of the FTPL. It produces a rash of contradictions and anomalies.

JEL classifications: E31, E41, E51, E52, E62, E63.

Key words: Fiscal theory of the price level; Ricardian fiscal-financial monetary programmes; government budget constraint; price level indeterminacy.

Author: Willem H. Buiter, European Bank for Reconstruction and Development, One Exchange Square, London EC2A 2JN. UK
Tel. #44-20-73386805
Fax. #44-20-73386110/6111
E-mail: buiterw@ebrd.com
Web page: http://www.nber.org/wbuiter/index.htm

1 Introduction

The thesis of this paper is that the 'fiscal theory of the price level' (FTPL) is fatally flawed.[1] An economic misspecification is the origin of the problem. The FTPL confuses two key building blocks of models of a market economy: budget constraints and equilibrium conditions. Specifically, it denies that the government's intertemporal budget constraint must hold as an identity, that is, for all admissible values of the variables entering the budget constraint. Instead it requires it to be satisfied only *in equilibrium*.

Property rights, contract enforcement, budget constraints and voluntary exchange are defining features of transactions among private agents in a market economy. Certain transactions between the government and private agents allow for a form of legal 'involuntary exchange'. Unrequited transfers between the private and public sectors, that is, transfers without value-equivalent or utility-equivalent *quid-pro-quo*, are allowed. This reflects the government's ability to tax, the expression of its monopoly of the legitimate use of force - the power to prescribe and proscribe behaviour. This same power is reflected in the government's ability to declare certain of its liabilities to be legal tender.

For a given structure of property rights and contract enforcement, budget constraints define bounds on the uses and sources of funds, and therefore on decision rules, that must be satisfied by economic agents. Budget constraints apply equally to

[1] The seminal contributors, that is, the original exponents of the FTPL, are Begg, and Haque. (1984), followed by Auernheimer. and Contreras. (1990,. 1991, 1995). Later contributors tend not to credit (or debit) these earlier authors appropriately. The recent revival includes Leeper (1991), Leeper and Sims. (1994), Woodford (1994, 1995, 1996, 1998a,b), Sims (1994, 1997), Cochrane (1996, 1999a,b,c), Dupor (1997), Loyo (1997a,b), Luttmer (1997), Olivei (1997); critical evaluations include Canzoneri, *et. al*. (1998a,b), Buiter (1998, 1999), McCallum (1998), Clements *et. al*. (1998) and Janssen *et. al*. (1999).

private agents (households and firms[2]) and to the government. They apply to agents who are (and/or perceive themselves to be) small according to some appropriate metric. Examples are competitive, price-taking households and firms, or households and firms that take tax rates and public spending plans to be exogenous.

Budget constraints also apply to agents who are (and perceive themselves to be) large, as in the case of monopolistic or monopsonistic firms and of a government which recognises the impact of its past, current and future actions on equilibrium prices and quantities. They apply to optimising agents, to satisficing agents, and to agents who follow ad-hoc decision rules. They apply to non-monetary economies and to economies with inside or outside, commodity or fiat money. They apply to economies in which the supply of fiat money is a government monopoly, as is the case in most of the FTPL literature, including this paper.

Specifying the appropriate budget constraints is not always straightforward. Concepts like default, insolvency and bankruptcy are often difficult to formalise in models with incomplete markets. Even in deterministic models with complete markets but incomplete market participation, such as infinite-horizon models of market economies with overlapping generations, the government's solvency constraint, usually a 'no-Ponzi-finance' condition on the terminal indebtedness of economic agents, is awkward to rationalise in terms of generally acceptable primitive assumptions.[3] The model used in this paper is deterministic and has complete markets. The current version of the paper is restricted to the finite-horizon case for which the specification of the appropriate intertemporal budget constraint is unambiguous and straightforward. All key results can be shown to carry over,

[2] In the complete markets models under certainty considered in this paper, the budget constraint of the firm is that (the present discounted value of current and future) profits be non-negative.

however, to the infinite-horizon case, with the standard infinite-horizon solvency constraint (see Buiter (1998, 2000)).

A *fiscal-financial-monetary programme or FFMP* is a complete set of rules specifying public spending, taxes, transfers, money issuance (seigniorage) and bond issuance in each period.[4]

The FTPL is based on the distinction between *Ricardian* and *non-Ricardian* FFMPs. In what follows, the government is to be interpreted as the consolidated general government and central bank. The government spends on goods and services, makes transfer payments and raises taxes, borrows and issues monetary liabilities. When it cannot meet its contractual debt obligations (interest payments and repayment of principal) exactly, it either defaults or has to dispose of its 'supersolvency' surpluses.

The government faces an intertemporal budget constraint or solvency constraint. A *Ricardian* FFMP requires that the government's solvency constraint hold for all admissible sequences of the variables entering into the government's intertemporal budget constraint. That is, the government's intertemporal budget constraint holds *identically*, not just *in equilibrium*. With a Ricardian rule, either the government restricts itself to FFMPs that permit it to always meet its contractual debt obligations exactly, or the government will fail to meet its contractual debt obligations. In the latter case, the government's intertemporal budget constraint co-determines the *public debt revaluation factor* (which can be a default discount factor or a 'supersolvency premium') on the public debt. In other words, the government's intertemporal budget constraint becomes a pricing kernel for the public debt,

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[3] See Buiter. and Kletzer. (1998).
[4] and, for stochastic models, in each state of nature.

determining the *effective* value of the public debt and overriding its *notional* or *contractual* value. Default or 'supersolvency' are not permitted.

A *non-Ricardian* FFMP requires the government's solvency constraint to hold only in equilibrium. It also requires the government to meet its contractual debt obligations exactly.

A Ricardian FFMP does not, generically, allow a configuration in which the government is able to meet exactly all its contractual debt obligations, while at the same time fixing exogenously complete sequences for real public spending, real taxes net of transfers and real seigniorage. At least one element in one of these spending, tax or seigniorage sequences must be endogenously (or residually) determined to ensure that the government's solvency constraint is *always* satisfied and all contractual debt obligations are honoured. When the government has to meet its outstanding debt obligations exactly, a Ricardian FFMP has to have one degree of freedom left in it.

A non-Ricardian FFMP does permit the government to fix exogenously complete sequences of real public spending, real net taxes and real seigniorage while requiring at the same time that it meet its outstanding contractual obligations exactly. The government's intertemporal budget constraint has to hold *in equilibrium only*. Without this relaxation of the conditions under which the government's budget constraint must hold, a non-Ricardian FFMP is over-determined. It is clear that, in general, the government will not be able to meet its outstanding contractual debt obligations exactly, for all admissible value of the variables entering the government's intertemporal budget constraint, including, specifically, the initial value of the general price level. However, if, the government has a positive initial stock of nominally denominated contractual debt outstanding, and if the present discounted value of its

future primary surpluses plus seigniorage also happens to be positive, there exists a unique value of the initial general price level that will equate the real value of the outstanding stock of contractual debt obligations to the present discounted value of future primary surpluses plus seigniorage. Since non-Ricardian FFMPs only require the government's intertemporal budget constraint to hold *in equilibrium*, the FTPL takes the unique initial price level that satisfies the governments intertemporal budget constraint and allows the government to meet its contractual debt obligations exactly, to be the first element of its equilibrium sequence for the general price level. The same argument applies to later values of the general price level.

The general price level under the (overdetermined) non-Ricardian FFMP therefore plays the role that the endogenous public debt revaluation factor plays under an overdetermined Ricardian FFMP. This paper shows that the attempt to let the general price level mimic the role of the public debt revaluation factor leads to contradictions and anomalies.

2 The model

I use a simple dynamic competitive equilibrium model with a representative private agent and a government sector, consisting of a consolidated fiscal authority and central bank. There is no uncertainty and markets are complete. Time, indexed by t, is measured in discrete intervals of equal length, normalised to unity. There is a finite number of periods, N, indexed by t, $1 \leq t \leq N$. Initial contractual asset stocks are predetermined.

2.1 Household behaviour

Households are price takers in all markets in which they transact. They receive an exogenous perishable endowment, $y_t > 0$, each period, consume $c_t \geq 0$ and pay net real lump-sum taxes t_t. They have access to three stores of value: non-interest-bearing fiat money (a liability of the government); a nominal one-period bond with a *notional* or contractual money price $P_t^B \geq 0$ in period *t*, which entitles the buyer to a single contractual nominal coupon payment worth $\Gamma > 0$ units of money in period *t+1*; and a real or index-linked one-period bond with a *notional* or contractual money price $P_t^b \geq 0$ in period *t*, which entitles the buyer to a single contractual coupon payment worth $g > 0$ units of real output in period *t+1*. The quantities of money, nominal bonds and real bonds outstanding at the end of period *t* (and the beginning of period *t+1*) are denoted M_t, B_t and b_t, respectively. The money price of output in period *t* is P_t. The government is assumed to have a monopoly of the issuance of base money, so $M_t \geq 0,\ 0 \leq t \leq N$.

Let $i_{t,t+1}$ the one-period risk-free nominal interest rate in period *t* and $r_{t,t+1}$ the one-period risk-free real interest rate in period *t*. By arbitrage it follows that

$$1 + i_{t,t+1} = \frac{\Gamma}{P_t^B} = \frac{P_{t+1} g}{P_t^b} = (1 + r_{t,t+1}) \frac{P_{t+1}}{P_t} \qquad (1)$$

Notional or contractual bond prices are the prices that prevail if the contractual payments (Γ or g) are certain to be made exactly. The effective bond prices are the prices that actually prevail, if the government does not meet its contractual obligations exactly.

When the government does not meet its contractual obligations exactly, its debt is be valued at effective prices, $\tilde{P}_t^B$ and $\tilde{P}_t^b$ respectively. Assume that all debt has equal seniority, that is, any resources available for debt service are pro-rated equally over all outstanding contractual debt. Let $D_{t,t+1}$ denote the fraction of the contractual payments due in period $t+1$ that is actually paid. That is, the actual payments in period $t+1$ on the two debt-instruments issued in period t are

$$\tilde{\Gamma}_{t+1} = D_{t,t+1}\Gamma \tag{2}$$

$$\tilde{g}_{t+1} = D_{t,t+1}g \tag{3}$$

It follows immediately that

$$\tilde{P}_t^B = D_{t,t+1}P_t^B \tag{4}$$

$$\tilde{P}_t^b = D_{t,t+1}P_t^b, \tag{5}$$

I shall refer to $D_{t,t+1}$ as the *public debt revaluation factor* for the notional value of the public debt outstanding at the end of period t. When $0 \le D_{t,t+1} < 1$, the debt revaluation factor can be interpreted as a default discount factor.

Note that

$$1+i_{t.t+1} = \frac{\tilde{\Gamma}_{t+1}}{\tilde{P}_t^B} = \frac{P_{t+1}\tilde{g}_{t+1}}{\tilde{P}_t^b} = \left(1+r_{t,t+1}\right)\frac{P_{t+1}}{P_t} \tag{6}$$

In principle, households or firms can default as well as the government. However, throughout the FTPL literature, households and firms have been assumed to satisfy their budget constraints identically. The single-period household budget identity is, for $1 \le t \le N$

$$M_t - M_{t-1} + \tilde{P}_t^B B_t - D_{t-1,t}\Gamma B_{t-1} + \tilde{P}_t^b b_t - D_{t-1,t}g\, P_t b_{t-1} = P_t(y_t - t_t - c_t) \tag{7}$$

The solvency constraint of the household is that at the end of period N, the household cannot have positive debt,

$$\tilde{P}_N^B B_N + \tilde{P}_N^b b_N \geq 0 \tag{8}$$

I only consider equilibria in which money is weakly dominated as a store of value, that is, equilibria supporting a non-negative nominal interest rate sequence. The motive for holding money is that end-of-period real money balances are an argument in the direct felicity function. To keep the analysis as transparent as possible, the period felicity function is assumed to be iso-elastic and money is assumed to enter it in an additively separable manner. All key propositions in this paper would go through for more general functional forms and for most alternative ways of introducing money into the model including 'money in the shopping function' and 'money in the production function'. For the strict Clower (1967) cash-in-advance models, there exists no finite-horizon equilibrium with a positive price of money unless one introduces another 'closure rule' to ensure that money is accepted in exchange for goods and services in the last period of the model.

The representative competitive consumer maximises the utility functional given in equation (9) defined over non-negative sequences of consumption and end-of-period real money balances subject to (7) and (8) and given the initial contractual asset stocks. It takes the tax sequence as given.

$$u_t = \sum_{j=0}^{N-t} \left[\frac{1}{1-h} c_{t+j}^{\;1-h} + f \frac{1}{1-h} \left(\frac{M_{t+j}}{P_{t+j}} \right)^{1-h} \right] \left(\frac{1}{1+d} \right)^j$$

$$c_{t+j}, M_{t+j} \geq 0; \; h, \; f, \; d > 0 \tag{9}$$

Since utility is increasing in consumption and real balances, (8) will hold with equality.

The contractual values of the initial financial asset stocks are predetermined, that is,

$$
\begin{aligned}
B_0 &= \bar{B}_0 \\
b_0 &= \bar{b}_0 \\
M_0 &= \bar{M}_0 > 0
\end{aligned}
\tag{10}
$$

Let R_{t-1t+j} be the nominal discount factor between periods *t-1* and *t+j*, that is,

$$
\begin{aligned}
R_{t-1t+j} &\equiv \prod_{k=0}^{j} \frac{1}{1+i_{t-1+k,\,t+k}} \qquad \text{for } j \geq 0 \\
&\equiv 1 \qquad\qquad\qquad\qquad \text{for } j = -1
\end{aligned}
\tag{11}
$$

Solving the household budget identity (7) forward recursively, using (2)-(6) yields

$$
\begin{aligned}
D_{t-1,t}\left(\Gamma B_{t-1} + P_t g\, b_{t-1}\right) \geq & \sum_{j=0}^{N-t} R_{t,t+j}\left(P_{t+j}(c_{t+j} + t_{t+j} - y_{t+j}) + (M_{t+j} - M_{t+j-1})\right) \\
& + R_{t,N}(\tilde{P}_N^B B_N + \tilde{P}_N^b b_N)
\end{aligned}
\tag{12}
$$

Specifically, in the initial period, *t = 1*, we have, imposing the household solvency constraint (8),

$$
D_{0,1}\left(\Gamma B_0 + P_1 g\, b_0\right) \geq \sum_{j=0}^{N-1} R_{1,1+j}\left(P_{1+j}(c_{1+j} + t_{1+j} - y_{1+j}) + (M_{1+j} - M_{1+j-1})\right) \tag{13}
$$

The household optimal consumption programme is characterised by

$$
\left(\frac{c_{t+1}}{c_t}\right)^h = (1 + r_{t,t+1})(1+d)^{-1} \qquad 1 \leq t \leq N-1 \tag{14}
$$

$$
\frac{M_t}{P_t} = c_t\left[f\left(\frac{1+i_{t,t+1}}{i_{t,t+1}}\right)\right]^{\frac{1}{h}} \qquad 1 \leq t \leq N-1 \tag{15}
$$

$$
\frac{M_N}{P_N} = c_N f^{\frac{1}{h}} \tag{16}
$$

Equation (15) is the familiar optimality condition relating the optimal money stock in period *t* to optimal consumption in every period but the last. The money-in-the-direct-utility-function approach views money as a consumer durable yielding a flow of unspecified liquidity services each period. In the last period, *N*, money only

has value because of the liquidity services it yields that period. Effectively, real money balances in period N become a perishable commodity, as shown in equation (16), which does not involve any intertemporal relative price.

2.2 The government

The government sector is made up of the consolidated fiscal and monetary authorities. Its decision rules are exogenously given, subject only its solvency constraint or intertemporal budget constraint. The government's single-period budget identity for $1 \le t \le N$ is given in (17), its solvency constraint in (18),

$$M_t - M_{t-1} + \tilde{P}_t^B B_t - D_{t-1,t}\Gamma B_{t-1} + \tilde{P}_t^b b_t - P_t D_{t-1,t} g\, b_{t-1} \equiv P_t(g_t - t_t) \qquad (17)$$

$$\tilde{P}_N^B B_N + \tilde{P}_N^b b_N \le 0 \qquad (18)$$

The government's single-period budget identity and solvency constraint imply that, for $1 \le t \le N$

$$D_{t-1,t}\left(\Gamma B_{t-1} + P_t g\, b_{t-1}\right) \le \sum_{j=0}^{N-t} R_{t,t+j}\left(P_{t+j}(t_{t+j} - g_{t+j}) + (M_{t+j} - M_{t+j-1})\right) \qquad (19)$$

Specifically, in the initial period,

$$D_{0,1}\left(\Gamma B_0 + P_1 g\, b_0\right) \le \sum_{j=0}^{N-t} R_{1,1+j}\left(P_{1+j}(t_{1+j} - g_{1+j}) + (M_{1+j} - M_{1+j-1})\right) \qquad (20)$$

For simplicity, assume (20) holds with equality (as it will in equilibrium because of the household's intertemporal budget constraint (13)). Should one impose the constraint that $0 \le D_{0,1} \le 1$? This rules out both $D_{0,1} < 0$ (notional debtors can be effective creditors and vice versa) and $D_{0,1} > 1$ (the default discount factor can be a

'super-solvency premium'). Consider first the case for ruling out $D_{0,1} < 0$ *a-priori*. In that case equation (20) applies only if

$$\begin{aligned} &\operatorname{sgn}\left\{\Gamma B_0 + P_1 g\, b_0\right\} \\ &= \operatorname{sgn}\left\{\sum_{j=0}^{N-1} R_{1,1+j}\left(P_{1+j}\left(t_{1+j} - g_{1+j}\right) + \left(M_{1+j} - M_j\right)\right)\right\} \end{aligned} \tag{21}$$

Consider the case where (21) is violated. For instance, let the private sector hold a positive contractual stock of public debt in period *1*, although the government's present discounted value of future primary surpluses plus seigniorage is negative. If one did insist on imposing (21) and required $D_{0,1} \geq 0$, it would follow that there exists no feasible FFMP and therefore no equilibrium.

Against this, consider a government that is credibly committed to the spending, tax and monetary issuance sequences on the RHS of (20), which incorporates the solvency constraint. If the RHS of (20) were to be negative, while the outstanding value of the contractual debt at the beginning of period *1* is positive, this government would have no option but to impose an immediate capital levy on the private sector in period *1*, large enough to create a stock of public sector credit (negative public debt) equal in value to the present discounted value of the excess of current and future public spending over taxes plus seigniorage. Thus a positive notional or contractual value of the public debt would have to be transformed or revalued, in period *1*, into a negative effective value of the public debt. A negative value of $D_{0,1}$, the initial government debt revaluation factor, would, on this interpretation, make sense. It is the direct implication of a natural minimal consistent planning requirement on the government's FFMP. I will adopt this second approach

and admit negative values of $D_{0,1}$, although the case for the inadmissibility of the FFMP does not depend on it.

The constraint $D_{0,1} \leq 1$ says that public debt can trade at a *discount* on its notional or contractual value, but not at a *premium*. If $D_{0,1} > 1$ is ruled out, government bond-holders do not get more than the government is contractually obliged to pay them, if the present discounted value of future primary surpluses and seigniorage exceeds the contractual value of the public debt. This means that equation (20) should be replaced by (22).

$$D_{0,1} = \min\left\{ 1\,,\ \frac{\sum_{j=0}^{N-1} R_{1,1+j}\left(P_{1+j}\,(t_{1+j} - g_{1+j}) + (M_{1+j} - M_j)\right)}{\Gamma B_0 + P_1 g\, b_0} \right\} \qquad (22)$$

If one chose to impose (22), one would need a theory for determining how any surplus of the present discounted value of future primary surpluses and seigniorage over the contractual value of the outstanding stocks of public debt is disposed of. For simplicity, I will not restrict the magnitude or sign of $D_{0,1}$. A constraint similar to (21) will be relevant when the FTPL is considered below.

The fiscal-financial-monetary programme. I will consider two monetary 'regimes', an exogenous nominal money rule and an exogenous nominal interest rate rule.

The *exogenous nominal money rule* specifies an exogenous positive sequence for the nominal money stock,

$$\begin{array}{l} \left\{ M_t = \bar{M}_t > 0; 0 \leq t \leq N \right\} \\ \left\{ \dfrac{\bar{M}_{t+1}}{\bar{M}_t} \geq \dfrac{1}{1+d} \right\} \end{array} \quad . \qquad (23)$$

The second inequality in (23) ensures non-negative equilibrium nominal interest rates in our model. The nominal interest rate is endogenous under this rule.

The *exogenous nominal interest rate rule* specifies an exogenous non-negative sequence for the nominal interest rate,

$$\left\{ i_{t-1,t} = \bar{i}_{t-1,t} \geq 0; 1 \leq t \leq N-1 \right\} \tag{24}$$

The nominal money stock is endogenous under this exogenous nominal interest rate rule. Note that, since the nominal interest rate is a real variable – it represents the real pecuniary opportunity cost of holding money balances, an economy with an exogenous nominal interest rate rule lacks a policy-provided nominal anchor.

The real government spending sequence is exogenous and constant.

$$\begin{aligned} & g_t = \bar{g}_t = \bar{g} \qquad\qquad 1 \leq t \leq N \\ & 0 \leq \bar{g} < y_t \end{aligned} \tag{25}$$

Ricardian fiscal-financial-monetary programmes. There are two kinds of Ricardian FFMPs, those which require outstanding contractual debt obligations to be met exactly and those that permit the public debt revaluation factor to be different from unity.

Definition: A Ricardian FFMP with contract fulfilment

A Ricardian FFMP <u>*with contract fulfilment*</u> *is a set of sequences for real public spending,* $\{g_t, t = 1,...,N\}$*, net real taxes* $\{t_t, t = 1,...,N\}$ *and either a sequence of nominal money stocks,* $\{M_t, t = 0,1,...,N\}$ *or a sequence of nominal interest rates,* $\{i_{t,t+1}, t = 0,1,...,N\}$ *which identically satisfy the government's*

intertemporal budget constraint (20) and ensure that all outstanding contractual debt obligations are met exactly, that is, $D_{0,1} \equiv 1$.

Given the nominal money stock sequence or given the nominal interest rate sequence, and given the (constant) real public spending sequence, at least one element in the sequence of taxes must become endogenous. Since the model with its representative agent and lump-sum taxes exhibits debt neutrality or Ricardian equivalence, any rule for taxes that permits the government's intertemporal budget constraint (20) to be satisfied is appropriate (and equivalent) for our purposes. For concreteness, I shall assume that taxes are set to achieve a zero nominal non-monetary debt from the end of period *1* on, that is,

$$\begin{aligned} t_1 &= \bar{g} - \frac{M_1 - M_0}{P_1} + D_{0,1}\left(\frac{\Gamma B_0}{P_1} + g b_0\right) \\ t_t &= \bar{g} - \frac{M_t - M_{t-1}}{P_t} \qquad 2 \le t \le N \end{aligned} \tag{26}$$

Equations (23), (25), (26) and $D_{0,1} \equiv 1$ define our Ricardian FFMP with contract fulfilment and an exogenous nominal money rule. Equations (24), (25), (26) and $D_{0,1} \equiv 1$ define our Ricardian FFMP with contract fulfilment and an exogenous nominal interest rate rule.

Many other Ricardian FFMPs with contract fulfilment are possible, including programmes based on ad-hoc feedback rules or optimising rules for the government's instruments. An important example of a Ricardian FFMP with contract fulfilment is the 'switching' rule studied by Sargent. and Wallace. (1981) in their *Unpleasant Monetarist Arithmetic* paper. This rule specifies constant sequences for real public spending and real taxes net of transfers. There are two policy regimes. In regime 1, for $1 \le t < t_1$, the authorities fix the growth rate of the nominal money stock

exogenously at $\overline{m}$. Government borrowing is the residual. There is only index-linked or real government debt ($B_t \equiv 0,\ 1 \le t \le N$). Regime 2 starts when, at $t = t_1 \ge 1$, the government stabilises the real stock of public debt, that is, it borrows or lends just enough to keep the real stock of public debt constant until the one-but last period, *N-1*.[5] In the last period, *N*, the government cannot leave any positive debt.[6] In regime 2, the nominal money stock becomes endogenous and adjusts to satisfy the government's single-period budget identity and intertemporal budget constraint. Sargent and Wallace assume that the (endogenous) growth rate of the nominal money stock for $t \ge t_1$ is constant. The Unpleasant Monetarist Arithmetic FFMP is summarised below:

$$
\begin{aligned}
& g_t = \overline{g} && \\
& t_t = \overline{t} && 1 \le t \le N \\
& M_{t+1} = (1+\overline{m})M_t && 0 \le t \le t_1 - 1 \\
& \frac{M_{t+1}}{M_t} = \frac{M_t}{M_{t-1}} && t_1 \le t \le N-1 \\
& B_t = 0 && 0 \le t \le N \\
& b_{t+1} = b_t && t_1 \le t \le N-2 \\
& b_N \le 0 &&
\end{aligned}
\tag{27}
$$

In the Unpleasant Monetarist Arithmetic model, the government meets its contractual debt obligations exactly, $D_{0,1} \equiv 1$. The Sargent and Wallace FFMP

[5] In the Sargent and Wallace model, the real *per capita* stock of public debt is stabilised.

therefore represents a Ricardian FFMP with contract fulfilment. In regime 1, there is an exogenous nominal money stock phase, followed by, in regime 2, an endogenous nominal money stock. The endogeneity of monetary growth in regime 2 provides the degree of freedom in the FFMP to ensure that the intertemporal budget constraint is always satisfied. The Sargent-Wallace FFMP respects the proper roles of budget constraints and equilibrium conditions and is *not* an example of the FTPL confusion at work.

Definition: A Ricardian FFMP without contract fulfilment

A Ricardian <u>*FFMP without contract fulfilment*</u> *is an* <u>*overdetermined*</u> *FFMP which identically satisfies the government's intertemporal budget constraint (20), but for which outstanding contractual debt obligations do not have to be met exactly.* $D_{0,1}$ *is therefore determined endogenously by the requirement that the government's intertemporal budget constraint be satisfied identically.*

There are many Ricardian FFMPs without contract fulfilment. I will use a very simple rule for real public spending, real net taxes and real seigniorage proposed by Woodford (1995) and also used by Cochrane (1999a).

Woodford proposes the following tax rule:

$$t_t = \overline{s}_t - \frac{M_t - M_{t-1}}{P_t} \qquad 1 \le t \le N \qquad (28)$$

where $\{\overline{s}_t\}$, $1 \le t \le N$ is an exogenously given real sequence of taxes plus seigniorage.

[6] The Sargent Wallace model is a 2-period OLG model with an infinite number of generations. In that model, the stock of real per capita public debt is kept constant at its $t = t_1$ value forever after.

Equations (23), (25), and (28) define the Ricardian FFMP without contract fulfilment and an exogenous nominal money rule. $D_{0,1}$ is endogenous. Equations (24), (25), and (28) define the Ricardian FFMP without contract fulfilment and an exogenous nominal interest rate rule. Again $D_{0,1}$ is endogenous.

It is clear from equation (28) that an exogenous nominal money stock sequence is consistent with the Ricardian fiscal rule without contract fulfilment. With $\{\overline{g}_t, 1 \le t \le N\}$ and $\{\overline{s}_t, 1 \le t \le N\}$ given, the sequence of lump-sum taxes $\{t_t, 1 \le t \le N\}$ can adjust passively to accommodate any exogenous nominal money stock sequence $\{\overline{M}_t, 0 \le t \le N\}$.

Non-Ricardian fiscal-financial-monetary programmes

Definition: A Non-Ricardian FFMP

A Non-Ricardian FFMP is an <u>overdetermined</u> FFMP which satisfies the government's intertemporal budget constraint (20) <u>in equilibrium only</u>, but for which outstanding contractual debt obligations must be met exactly, that is, $D_{0,1} \equiv 1$. A *non-Ricardian FFMP with an exogenous nominal money rule* will be defined by an exogenous sequence of real public spending, g_t, an exogenous sequence of real net taxes plus real seigniorage, $\overline{s}_t$, and an exogenous strictly positive sequence of nominal money stocks, that is, by equations (23), (25), (28), and $D_{0,1} \equiv 1$.

A *non-Ricardian FFMP with an exogenous nominal interest rate rule* will be defined by an exogenous sequence of real public spending, g_t, an exogenous sequence of real net taxes plus seigniorage, $\overline{s}_t$ and an exogenous non-negative sequence of nominal interest rates, that is, by equations (24), (25), (28) and $D_{0,1} \equiv 1$.

2.3 Market clearing

The goods market clears each period, that is,

$$y_t = c_t + g_t \qquad 1 \le t \le N \qquad (29)$$

For simplicity, I assume in what follows that the real fundamentals are constant, that is,

$$\begin{aligned} y_t &= \bar{y} \\ g_t &= \bar{g} \qquad 1 \le t \le N \end{aligned}$$

Only non-negative equilibrium price sequences are permissible.

2.4 Equilibrium under the Ricardian FFMP with contract fulfilment and an exogenous nominal money rule

The equilibrium is characterised by equation (23) and (30) to (36). Note that $D_{0,1} \equiv 1$, since contract fulfilment is imposed.

$$c_t = c = \bar{y} - \bar{g} \qquad 1 \le t \le N \qquad (30)$$

$$r_{t,t+1} = d \qquad 1 \le t \le N-1 \qquad (31)$$

$$1 + i_{t,t+1} = (1+d)\frac{P_{t+1}}{P_t} \qquad 1 \le t \le N-1 \qquad (32)$$

$$\frac{M_t}{P_t} = (\bar{y} - \bar{g})\left[\frac{f(1+d)\frac{P_{t+1}}{P_t}}{(1+d)\frac{P_{t+1}}{P_t} - 1}\right]^{\frac{1}{h}} \qquad 1 \le t \le N-1 \qquad (33)$$

$$\frac{M_N}{P_N} = (\bar{y} - \bar{g}) f^{\frac{1}{h}} \qquad (34)$$

$$\frac{\Gamma B_0}{P_1} + g\, b_0 \equiv \sum_{j=0}^{N-1}\left(\frac{1}{1+d}\right)^j \left[t_{1+j} + \left(\frac{M_{1+j} - M_j}{P_{1+j}}\right) - \bar{g}\right] \qquad (35)$$

$$t_1 = \bar{g} - \frac{M_1 - M_0}{P_1} + \frac{\Gamma B_0}{P_1} + g\, b_0$$

$$t_t = \bar{g} - \frac{M_t - M_{t-1}}{P_t} \qquad 1 < t \le N \tag{36}$$

$$\begin{aligned} M_0 &= \bar{M}_0 > 0 \\ B_0 &= \bar{B}_0 \\ b_0 &= \bar{b}_0 \end{aligned} \tag{37}$$

This economy has multiple equilibria for the general price level sequence.[7] One equilibrium has an infinite price level (a zero price of money) in each period. A second has an infinite price level only in the last period, *N*. Since money is worthless in period *N*, the demand for money in period *N-1* takes the same form as the demand for money in the terminal period, given in equation (34). One can work backwards from this to an initial value for price level and the real money stock. Indeed, for every period *t*, $N \ge t > 1$, there exists an equilibrium in which money is valueless for all periods *s*, $N \ge s \ge t$, but valued up to that time.[8]

There is also a unique equilibrium in which money has positive value in each period. The monetary equilibrium conditions (33, 34) provide *N* equations that uniquely determine the *N* (finite) equilibrium prices P_t , $t = 1, \ldots, N$. Equation (34) determines P_N as a function of the nominal stock of money in the last period, $\bar{M}_N$. The remaining *N-1* monetary equilibrium conditions given by (33) determine P_{N-1} down to P_1, given the solution for the price level in period *N* and the exogenous

[7] There are therefore also multiple equilibria for the real money stock sequence and, if non-zero contractual nominal debt is outstanding, for the real debt sequence.

[8] I am indebted to Chris Sims for pointing out, in private correspondence, the existence of more than one equilibrium in which money is valueless in some period(s). Note that this does not require nominal bonds be absent. Any change in the real value of the nominal stock of bonds associated with a change in the general price level will, because of our assumption of a Ricardian FFMP with contract fulfilment, be offset by a matching change in real lump-sum taxes, according to (36).

values of the nominal money stocks in periods *1* to *N-1*. The tax rule given in (36) then determines the *N* values of the lump-sum tax sequence. Given that tax rule, the government's solvency constraint holds identically.

Another way of putting this is that the government's solvency constraint (and the assumed exogeneity of the real public spending sequence and the nominal money stock sequence) implies that the tax sequence become endogenous, if all contractual debt obligations are to be met exactly.

The equilibrium real and nominal interest rate sequences and the equilibrium consumption sequence are always uniquely determined.

Under this Ricardian FFMP with contract fulfilment and an exogenous nominal money rule, money is *conditionally neutral* (see Buiter (1998)). Holding constant the initial stock of nominal non-monetary debt, B_0, equal proportional changes in the sequence of nominal money stocks (including the initial nominal stock of money), $\{\bar{M}_t\}, 0 \le t \le N$, and in the sequences of all endogenous nominal prices $\{P_t, P_t^B, P_t^b\}, 1 \le t \le N$ leave the real equilibrium unchanged. If the initial stock of non-monetary nominal debt is non-zero, the sequence of (endogenous) real lump-sum taxes will change (according to (36)), because the real value of the initial stock of nominal non-monetary government debt, $\frac{B_0}{P_1}$, changes when the initial price level changes.

Under the Ricardian FFMP with contract fulfilment and an exogenous nominal money rule, money and the initial stock of nominal non-monetary debt are *jointly unconditionally neutral* (see Buiter (1998)). Equal proportional changes in the sequence of nominal money stocks (including the initial nominal stock of money), $\{\bar{M}_t\}, 0 \le t \le N$, in the initial stock of nominal non-monetary debt, B_0, and in the

sequences of all endogenous nominal prices $\{P_t, P_t^B, P_t^b\}, 1 \le t \le N$ leave the real equilibrium unchanged. The (endogenous) sequence of real lump-sum taxes will not need to change (again according to (36)).

I summarise this as Proposition 1.

Proposition 1.

Under the Ricardian FFMP with contract fulfilment and an exogenous nominal money rule, money is neutral in equilibria in which money has value in each period.

2.5 Equilibrium under the Ricardian FFMP with contract fulfilment and an exogenous nominal interest rate rule

With an exogenous non-negative nominal interest rate sequence (and endogenous nominal money stocks), the key monetary equilibrium conditions under a Ricardian FFMP with contract fulfilment can be rewritten as follows:

$$\frac{M_t}{P_t} = (\bar{y} - \bar{g})\left[\frac{f(1+\bar{i}_{t,t+1})}{\bar{i}_{t,t+1}}\right]^{\frac{1}{h}} \qquad 1 \le t \le N-1 \qquad (38)$$

$$\frac{M_N}{P_N} = (\bar{y} - \bar{g}) f^{\frac{1}{h}} \qquad (39)$$

The monetary equilibrium conditions (38,39) provide N equations that uniquely determine the N equilibrium real money stocks, M_t/P_t , $t = 1, \ldots, N$. The endogenous equilibrium nominal money stock sequence $\{M_t\}$, 1 £ t £ N and the equilibrium price sequence $\{P_t\}$, 1 £ t £ N are indeterminate. The tax rule given in (36) then determines the N values of the lump-sum tax sequence. If the initial stock of nominal non-monetary debt, B_0, is non-zero, the first term in the equilibrium real

tax sequence, t_1, which depends on $\frac{B_0}{P_1}$, is also indeterminate. However, it continues to be the case that, given that tax rule in (36), the government's intertemporal budget constraint holds identically. Whatever the general price level turns happens to be, the period *1* lump-sum tax will assume the value required to satisfy the first equation in (36). The equilibrium real interest rate sequence, the equilibrium inflation rate sequence and the equilibrium consumption sequence are also uniquely determined.

Price level indeterminacy under a Ricardian nominal interest rate rule is a familiar result. It is not paradoxical or surprising, let alone anomalous. In a frictionless economy, with flexible (that is, non-predetermined), market-clearing nominal prices, an exogenous nominal interest rate sequence does not provide a nominal anchor for the system. The reason is that, despite its name, the short *nominal* interest rate is a *real* variable, the real pecuniary opportunity cost of holding money balances.

I summarise this as Proposition 2.

Proposition 2.

Under the Ricardian FFMP with contract fulfilment and an exogenous nominal interest rate rule, all nominal equilibrium values are indeterminate. The real equilibrium is uniquely determined.

2.6 Equilibrium under the Ricardian FFMP without contract fulfilment and an exogenous nominal money rule

Define the *effective* real value of the initial net public debt, $\tilde{L}_0$, as follows:

$$\tilde{L}_0 \equiv D_{0,1}\left(\frac{\Gamma \bar{B}_0}{P_1} + g\,\bar{b}_0\right) \tag{40}$$

Let $L_0 \equiv \frac{\Gamma B_0}{P_1} + g\, b_0$ be the contractual or notional value of the government's initial contractual debt obligations, so $\tilde{L}_0 = D_{0,1} L_0$. Substitute the rule given by (28), that real tax revenue plus the real value of seigniorage is exogenously given, into the government's intertemporal budget constraint (20), but without imposing the constraint that all contractual debt obligations are met exactly. The key equilibrium conditions determining $D_{0,1}$ and the equilibrium price sequence can then be represented by (41), and (33,34).

$$\tilde{L}_0 \equiv D_{0,1} L_0 \equiv D_{0,1}\left(\frac{\Gamma \bar{B}_0}{P_1} + g\, \bar{b}_0\right) \equiv \sum_{j=0}^{N-1}\left(\frac{1}{1+d}\right)^j \left[\bar{s}_{1+j} - \bar{g}\right] \tag{41}$$

The remaining equilibrium conditions are given by (30), (31), (32) and (37).

For reasons of space I will concentrate exclusively on the unique equilibrium in which money has positive value in each period.

The right-hand side of (41) is exogenous. Everything on the left-hand-side of (41), except for P_1 and $D_{0,1}$, is exogenous or predetermined. Assume again that the exogenous and strictly positive nominal money stock sequence satisfies $\frac{\bar{M}_{t+1}}{\bar{M}_t} \geq \frac{1}{1+d}$.

The monetary equilibrium conditions (33,34) still provide N equations that uniquely determine the N equilibrium prices P_t , $t = 1, \ldots, N$.

Given the value of the initial price level, determined by the monetary equilibrium conditions and the exogenous nominal money stock sequence, the government's intertemporal budget constraint determines the government debt revaluation factor, $D_{0,1}$. Except for a set of parameter configurations of measure zero, this endogenous value of $D_{0,1}$ will be different from *1*.

For a Ricardian FFMP without contract fulfilment and with an exogenous nominal money rule always to support an equilibrium, it must be possible to turn positive (negative) contractual net debt into negative (positive) effective net debt ($D_{0,1} < 0$), and to permit contractual debt not only to be effectively discounted ($D_{0,1} < 1$) but also to be effectively priced at a premium ($D_{0,1} > 1$).

If one does not, on a-priori grounds, accept values of $D_{0,1}$ that are greater than *1* or negative, one would have to conclude that no equilibrium exists under a Ricardian FFMP without contract fulfilment and with an exogenous nominal money rule, if equation (41) were to yield a value for $D_{0,1}$ that was negative or greater than *1*. My interpretation of the government's intertemporal budget constraint as a consistency requirement imposed on all FFMPs permits values for $D_{0,1}$ that are negative or greater than *1*. The critique of the FTPL does not depend on whether one excepts Ricardian equilibria with $D_{0,1} < 0$ or $D_{0,1} > 1$, or whether one interprets $D_{0,1} < 0$ or $D_{0,1} > 1$ as implying that no Ricardian equilibrium exists.

The government's intertemporal budget constraint can therefore be viewed as an effective public debt pricing kernel, that is, an equation determining the effective real value of the net public debt or the public debt revaluation factor.[9] The present discounted value of future primary surpluses and seigniorage equals ('determines', if the real seigniorage and real primary surplus sequences are taken as given) the effective real value of the initial net government debt. If the notional or contractual value of the initial debt differs from its effective value, the government solvency 'overwrites' the contractual value.

[9] In general, the default discount factor is determined simultaneously with all other endogenous variables by the complete set of equilibrium conditions.

I summarise this discussion as Proposition 3.

Proposition 3.

Under a Ricardian FFMP without contract fulfilment and with an exogenous nominal money rule, the government's intertemporal budget constraint and the overdetermined FFMP determine the effective real value of the net public debt. This will in general be different from the notional or contractual value of the government's outstanding debt obligations.

The remaining properties of the equilibrium are familiar:

Proposition 4.

Under the Ricardian FFMP without contract fulfilment and with an exogenous nominal money rule, money is neutral in equilibria in which it has value in each period.

2.7 Equilibrium under the Ricardian FFMP without contract fulfilment and an exogenous nominal interest rate rule?

As under the Ricardian FFMP with contract fulfilment and an exogenous non-negative nominal interest rate rule (and endogenous nominal money stocks), the monetary equilibrium conditions (38,39) determine the real money stock sequence under the Ricardian FFMP without contract fulfilment and with an exogenous nominal interest rate rule. The remaining equilibrium conditions are (28), (30), (31), (32), (37) and (41).

The government's intertemporal budget constraint (41), with its overdetermined FFMP, now determines the effective real value of the initial net

public debt, $\tilde{L}_0$. The endogenous nominal money stock sequence and the price level sequence are indeterminate. Note that, if the initial stock of contractual nominal debt, B_0, is non-zero, both the initial price level, P_1, and the government debt revaluation factor, $D_{0,1}$ are indeterminate. The effective real value of the initial net public debt, $\tilde{L}_0$, however, is well-determined. All other real equilibrium values, including the inflation rate, are well-determined.

2.8 Equilibrium under the non-Ricardian FFMP with an exogenous nominal money rule

The equilibrium conditions are the same as for the Ricardian FFMP without contract fulfilment and with an exogenous nominal money rule, except for the imposition of the additional constraint that contractual government debt obligations are met exactly, that is, $D_{0,1} \equiv 1$. The equilibrium price sequence is determined by the government's intertemporal budget constraint with $D_{0,1} \equiv 1$, reproduced below as (42), and the monetary equilibrium conditions, (33,34) with the exogenous nominal money stock sequence imposed.

$$\frac{\Gamma \overline{B}_0}{P_1} + g\,\overline{b}_0 \equiv \sum_{j=0}^{N-1} \left(\frac{1}{1+d} \right)^j \left[\overline{s}_{1+j} - \overline{g} \right] \tag{42}$$

The remaining equilibrium conditions are given by (28), (30), (31), (32) and (37).

Restricting consideration again to equilibria with a positive value for money in each period, it is clear that the system (42) and (33,34) is overdetermined. The initial price level is determined twice, once from the monetary equilibrium conditions and once from the government's intertemporal budget constraint. Except through a fluke,

these two values of the initial price level will not be the same. This should not be surprising. The equilibrium under the Ricardian FFMP with an exogenous nominal money rule (with or without contract fulfilment) was exactly determined. The non-Ricardian FFMP with an exogenous nominal money rule has a further restriction imposed on it.

Proposition 5.

Under the non-Ricardian FFMP with an exogenous nominal money rule, the price level is overdetermined.

2.9 Equilibrium under the non-Ricardian FFMP with an exogenous nominal interest rate rule: could this be the FTPL?

Under a non-Ricardian FFMP with an exogenous nominal interest rate rule, the equilibrium conditions are the same as under the Ricardian FFMP without contract fulfilment and with an exogenous nominal interest rate rule, except for the addition of the constraint $D_{0,1} \equiv 0$. Outstanding contractual debt obligations have to be met exactly, despite the overdetermined FFMP.

It may seem that the price level indeterminacy characteristic of the Ricardian FFMPs with a fixed nominal interest rate rule can now be resolved. The monetary equilibrium conditions (33,34) determine the equilibrium real money balances for each period. The government's intertemporal budget constraint (42) (which has $D_{0,1} \equiv 1$ imposed) determines the initial price level, P_1, and equation (43) permits all subsequent price levels to be determined.

$$1+\bar{i}_{t,t+1} = (1+d)\frac{P_{t+1}}{P_t} \qquad 1 \le t \le N-1 \qquad (43)$$

The FTPL, with its overdetermined non-Ricardian FFMP, lets the initial price level do the work done by the government debt revaluation factor in the overdetermined Ricardian FFMP. The general price level revalues the outstanding stock of contractual nominal government debt to make it consistent with the overdetermined real spending, tax and seigniorage sequences. The effective real value of the initial public debt adjusts to satisfy the government's intertemporal budget constraint in equilibrium, *and* remains equal to the notional or contractual real value of the initial public debt. Could this be the FTPL?

Three questions arise. First, when is this fiscal theory of the price level mathematically consistent? Second, what else does the FTPL imply, and do these other implications make sense? Third, how robust is the FPTL?

3 Implications of the fiscal theory of the trice level: contradictions and anomalies

3.1 An arbitrarily restricted domain of existence

The general price level, P_1, cannot be negative. A necessary condition for the government's intertemporal budget constraint under the non-Ricardian FFMP to support an equilibrium is therefore that condition (44) be satisfied. Note that (44) is similar to condition (21), which ensures a non-negative value of the public debt revaluation factor.[10] It is the same as (21) if all government debt is nominally denominated.

$$\operatorname{sgn}\left\{\overline{B}_0\right\} = \operatorname{sgn}\left\{\sum_{j=0}^{N-1}\left(\frac{1}{1+d}\right)^j\left[\overline{s}_{1+j} - \overline{g}\right] - g\,\overline{b}_0\right\} \qquad (44)$$

[10] Note that (21) is not necessary for the validity of the Ricardian approach. It is irrelevant for the Ricardian approach with contract fulfilment. It is not necessary for the Ricardian approach without contract fulfilment, as it could be argued quite

Condition (44) says that the initial stock of non-monetary nominal public debt must be positive (negative) if the excess of the present discounted value of future real primary government surpluses plus future real seigniorage revenues over the value of the initial stock of index-linked government debt is positive (negative).

Everything on either side of equation (44) is exogenous or predetermined. There is no reason why arbitrary configurations of $B_0, b_0, d, \overline{g}$ and $\{\overline{s}_t\}, 1 \le t \le N$ would always satisfy (44), although they may do so. If there is only index-linked public debt, there can be no FTPL. If, in an open economy extension of this model, all public debt is foreign-currency-denominated, there likewise is no FTPL. Arbitrary restrictions on the predetermined and exogenous variables in the government solvency constraint are required to support a non-negative equilibrium price level sequence.

3.2 The FTPL and the price of phlogiston: the price of money in an economy without money

Taken at face value, the FTPL can determine the price of money (the reciprocal of P) when (44) is satisfied, even in a world in which there is no demand for money, and even in a world where money does not exist as a physical object or intangible fiduciary financial claim. In our model this will be the case when $f = 0$. One interpretation of this world is one in which non-interest-bearing government fiat money, or cash, exists, but has become redundant as a medium of exchange and means of payment, and is dominated as a store of value by money-denominated securities with a positive nominal interest rate. Another interpretation is that of a world in which money does not exist, except as a name.

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persuasively that, when the government's intertemporal budget constraint implies $D_{0,1} < 0$ or $D_{0,1} > 1$, there exists no Ricaridian equilibrium.

There are interesting and important issues that arise when an economy gradually demonetises over time, say in response to technological and regulatory developments that permit households to economise on money to an ever-increasing degree and that may, ultimately, make the government's transactions medium completely redundant. The FTPL sheds no light on these issues. It does, however, permit the price of money to be determined in a world in which money only exists as a name, that is, as a numéraire, unit of account and invoicing unit. In this world, money does not exist as a physical object or as a financial claim. It can be thought of as an imaginary substance, like *phlogiston*, the imaginary element formerly believed to cause combustion. Private or public agents can issue securities denominated in terms of this pure numéraire.

According to the FTPL, under a non-Ricardian FFMP with an exogenous nominal interest rate rule, the price of phlogiston can be determined uniquely from the government's intertemporal budget constraint, if two conditions are satisfied. First, an interest-bearing financial claim denominated in terms of phlogiston is issued by the government.[11] Second, the constraint in (44) is satisfied.

Under these conditions, the government's intertemporal budget constraint (42) alone can, under the non-Ricardian exogenous nominal interest rate rule, determine the initial general price level in terms of phlogiston, P_1. From (43) all future price levels can then be determined. Equations (38,39) play no longer any role, with $f = 0$. A theory capable of pricing phlogiston, something that does not exist, except as a name, is an intellectual bridge too far.

The anomaly is eliminated if we replace the non-Ricardian intertemporal budget constraint (42) by the Ricardian intertemporal budget constraint, with or

without contract fulfilment. In either case, only real quantities of money, debt or taxes are determined. The nominal equilibrium sequences are indeterminate.

From the Ricardian perspective, the only thing determined by the government's intertemporal budget constraint when the FFMP is overdetermined, as it is in (41), is the effective real value of the net public debt in the initial period, $\tilde{L}_0$. It really does not matter what the contractual debt is denominated in, be it 'money', commodities or phlogiston. The government's intertemporal budget constraint determines the effective real value of the initial net public debt regardless of the denomination of the contractual debt obligations.

3.3 The FTPL and the HTPL

One could apply the logic of the FTPL to the household sector and view the household's intertemporal budget constraint is viewed as a condition that need only be satisfied not only in equilibrium. Following the logic of the FTPL, one could then one can overdetermine the household's optimal consumption and portfolio allocation programme and fix c_1 at some arbitrary positive level. The household solvency constraint then determines the period *1* price level. This gives us the 'household intertemporal budget constraint theory of the price level' or HTPL.

3.4 The FTPL when the price-level is predetermined

Consider again the case where $f > 0$, for which, when the price level is non-predetermined, an equilibrium exists with a positive price of money in each period. If the price level is predetermined, that is, the price level in period *t* depends on the price

[11] The payment(s) that define this security cannot, of course, be made in phlogiston, that is, by the transfer of units of phlogiston.

level in one or more periods before *t*, the FTPL leads to an overdetermined price level even when the authorities adopt an exogenous nominal interest rate rule. In Buiter (1999), I consider a simple 'Keynesian' example of such an economy, in which output is demand-determined and the price level is predetermined and updated through a simple accelerationist Phillips curve.[12] With the price level in period *1* predetermined, it cannot do the job of mimicking a revaluation factor on the public debt in the government's intertemporal budget constraint. Under Ricardian FFMPs, the model continues to be exactly determined.

Whether the government's intertemporal budget constraint is to be treated as an equilibrium condition or an identity should not depend on whether the price level is non-predetermined or predetermined, just as it should not depend on whether the nominal money stock sequence is endogenous or exogenous.

The infinite-horizon version of the model is considered in a longer version of this paper (Buiter (2000)). All results, inconsistencies and anomalies of the finite-horizon case carry over to the infinite-horizon case with one exception. Proposition 5, that under a non-Ricardian FFMP with an exogenous rule for the nominal money stock, the general price level is *overdetermined*, now only applies when the velocity of circulation of money does not depend on the nominal interest rate and, through that, on expected future price levels. When the demand for money responds to the opportunity cost of holding money, the price level is not overdetermined when the nominal money stock is exogenous, but other anomalies, such as explosive or implosive behaviour of the price level occur (see also Buiter (1998)).

[12] In the example the first difference of the price level (or the rate of inflation) is also predetermined.

5 Conclusion

The Ricardian approach to the government budget constraint recognises two cases that are well-posed. If the outstanding contractual debt obligations of the government have to be met exactly, the fiscal-financial-monetary programme has to leave one degree of freedom. If no degrees of freedom are left, the government will not, in general be able to meet its outstanding contractual debt obligations exactly. Default, partial or complete, or 'supersolvency' results in this case.

The non-Ricardian approach, which purports to support the fiscal theory of the price level, admits a third case. Here the outstanding contractual debt obligations of the government have to be met exactly, but nevertheless the fiscal-financial-monetary programme has no degrees of freedom. From the Ricardian perspective, this third case is ill-posed and represents an overdetermined fiscal-financial-monetary programme. In the fiscal theory of the price level, the general price level is required to mimic the role paid in well-posed Ricardian models by the default discount (or premium) factor on the public debt. Generically, the general price level cannot play that role. Once the possibility of explicit default is properly allowed for, non-Ricardian regimes become Ricardian regimes and the fiscal theory of the price level vanishes.

The fiscal theory of the price level confuses the roles of budget constraints and equilibrium conditions in models of a market economy. The resulting 'equilibria' are either inconsistent – they are overdetermined or imply a negative price level – or vitiated by anomalies, including the ability to price money in a world without money.

The Ricardian view of budget constraints as identities avoids all inconsistencies and anomalies. Unlike the fiscal theory of the price level, which can only hold when the government sets nominal interest rates, the Ricardian approach

remains valid regardless of whether the government controls the nominal interest rate or the nominal money stock. Unlike the fiscal theory of the price level, the Ricardian approach is valid regardless of the initial configuration of the government's contractual debt obligations. Unlike the fiscal theory of the price level, the Ricardian approach remains valid when the price level is predetermined. And unlike the fiscal theory of the price level, the Ricardian approach applies equally in a world without money as in a monetary economy.

The fiscal theory of the price level rests on a fundamental confusion between equilibrium conditions and budget constraints. It therefore does not constitute a valid starting point for further research in monetary economics.

References

Auernheimer, L. and Contreras, B. (1990), 'Control of the interest rate with a government budget constraint: price level determinacy, and other results', mimeo, Texas A&M University, February.

Auernheimer, L. and Contreras, B. (1991), 'A Nominal Interest Rate Rule: the Case of the Open Economy', mimeo, Texas A&M University, July, revised March 1997.

Auernheimer, L. and Contreras, B. (1995), 'Control de la tasa de interes con restriccion presupuestaria: Determinacion de los precios y otros resultados', *El Trimestere Economico*, 62(3), July-September, pp. 381-96.

Begg, D. K. H. and Haque, B. (1984), 'A Nominal Interest Rate Rule and Price Level Indeterminacy Reconsidered', *Greek Economic Review*, 6(1), pp. 31-46.

Buiter, W. H. (1987), 'A fiscal theory of hyperdeflations? Some surprising monetarist arithmetic', *Oxford Economic Papers*, 39, pp. 111-18.

Buiter, W. H. (1998), 'The Young Person's Guide to Neutrality, Price Level Indeterminacy, Interest Rate Pegs and Fiscal Theories of the Price Level', National Bureau of Economic Research Working Paper No. 6396, February.

Buiter, W. H. (1999), 'The Fallacy of the Fiscal Theory of the Price Level', National Bureau of Economic Research Working Paper W7302, August.

Buiter, W. H. and Kletzer, K. M. (1998), 'Uses and Limitations of Public Debt', in Steven Brakman, Hans van Ees and Simon K. Kuipers eds. *Market Behaviour and Macroeconomic Modelling*, MacMillan Press Ltd, London, 1998, pp. 275-307.

Canzoneri, M. B., Cumby, R. E. and Diba, B. T. (1998a), Does Fiscal or Monetary Policy Provide a Nominal Anchor: Evidence from the OECD Countries', Mimeo, Georgetown University.

Canzoneri, M. B., Cumby, R. E. and Diba, B. T. (1998b), 'Is the Price Level Determined by the Needs of Fiscal Solvency?', CEPR Discussion Paper No. 1772, January.

Clements, M., Herrendorf, B. and Valentinyi, A. (1998), 'Price Level Determination and Default on Long-term Government Debt', University of Warwick Mimeo.

Clower, R. W. (1967), 'A reconsideration of the microfoundations of monetary theory.', *Western Economic Journal*, 6, December, pp. 1-8.

Cochrane, J. H. (1996), 'Maturity Matters: Long-Term Debt in the Fiscal Theory of the Price Level,' unpublished, University of Chicago, December.

Cochrane, J. H. (1999a), 'A Frictionless View of U.S. Inflation', *NBER Macroeconomics Annual, 1998*, Ben. S. Bernanke and Julio J. Rotemberg eds., the MIT Press, Cambridge, Massachusetts, pp. 323-384.

Cochrane, J. H. (1999b), 'Long-term debt and optimal policy in the fiscal theory of the price level' mimeo, University of Chicago, June 16.

Cochrane, J. H. (1999c), 'Money as a Stock: Price Level Determination with no Money Demand', Mimeo, August 1999.

Concise Oxford Dictionary (1995), Ninth Edition, Della Thompson ed., Clarendon Press, Oxford.

Dupor, B. (1997), Exchange Rates and the Fiscal Theory of the Price Level', unpublished, University of Pennsylvania, October, forthcoming, *Journal of Monetary Economics*.

Helpman, E. (1981), 'An Exploration of the Theory of Exchange Rate Regimes', *Journal of Political Economy*, 89, October, pp. 865-890.

Janssen, N., Nolan C. and Thomas, R. , ' Money, Debt and Prices in the UK, 1705-1996'. Mimeo, Bank of England.

Leeper, E. M. (1991), 'Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies', *Journal of Monetary Economics*, 27, pp. 129-147.

Leeper, E. M. and Sims, C. A. (1994), 'Towards a Modern Macroeconomic Model Usable for Policy Analysis', *NBER Macroeconomics Annual 1994*, pp. 81-118.

Loyo, E. (1997a), 'Going International With the Fiscal Theory of the Price Level', Unpublished, Princeton University, November.

Loyo, E. (1997b), 'The Wealth Effects of Monetary Policy and Brazilian Inflation', unpublished, Princeton University, November.

Lucas, R. E. (1982), 'Interest Rates and Currency Prices in a Two-Country World', *Journal of Monetary Economics*, 22, July, pp. 335-60.

Luttmer, E. G.J. (1997), 'Notes on Price Level Determinacy', mimeo, London School of Economics, October.

McCallum, B. T. (1998), 'Indeterminacy, Bubbles and the Fiscal Theory of Price Level Determination', NBER Working Paper No. 6456, March.

Obstfeld, M. and Rogoff, K. (1983), 'Speculative hyperinflations in maximizing models: Can we rule them out?', *Journal of Political Economy*, 91, August, pp. 675-87.

Obstfeld, M. and Rogoff, K. (1986), 'Ruling out divergent speculative bubbles', *Journal of Monetary Economics*, 17, May, pp. 346-62.

Obstfeld, M. and Rogoff, K. (1996), *Foundations of International Macroeconomics*, MIT Press, Cambridge, Massachusetts.

Olivei, G. (1997), 'Government Spending Shocks and the Real Interest Rate under Alternative Fiscal Regimes', Mimeo, Princeton University.

Sargent, T. J. and Wallace, N. (1981), 'Some Unpleasant Monetarist Arithmetic', *Quarterly Review, Federal Reserve Bank of Minneapolis*, Fall.

Sims, C. A. (1994), 'A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy', *Economic Theory*, 4, pp. 381-399.

Sims, C. A. (1997), Fiscal Foundations of Price Stability in Open Economies', unpublished, Yale University, September.

Tanzi, V. (1978), 'Inflation, real tax revenue and the case for inflationary finance: theory with an application to Argentina', *IMF Staff Papers*, 25, September.

Woodford, M. (1994), 'Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy', *Economic Theory*, 4, pp. 345-380.

Woodford, M. (1995), 'Price-level Determinacy Without Control of a Monetary Aggregate', *Carnegie-Rochester Conference Series on Public Policy* 43, pp. 1-46.

Woodford, M. (1996), 'Control of the Public Debt: A Requirement for Price Stability?', NBER Working Paper No. 5684, July.

Woodford, M. (1998a), 'Doing without money: controlling inflation in a post-monetary World', *Review of Economic Dynamics*, 1, 173-219.

Woodford, M. (1998b), 'Public Debt and the Price Level', Mimeo, Princeton University, May.