Consistency Checks for Fiscal, Financial and Monetary Policy Evaluation and Design*

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(I) Introduction

In this paper I outline a simple operational approach for checking the internal consistency of budgetary, financial, and monetary policies. The intended audience is country economists, that is, macroeconomists who are working on the design, implementation, and evaluation of budgetary policy, defined to encompass public expenditure, taxation and other current revenues, and the financing (internal and external, monetary or nonmonetary) of public sector financial deficits. In addition I propose some easily implemented "ready reckoners" for assessing the sustainability of the processes governing the public debt and the external debt of a nation. I make no claim to originality. The consistency checks outlined here are derived essentially from the method outlined by Anand and van Wijnbergen (1989). This method made operational ideas that I and many others have been developing since the late 1970s (Siegel 1979; Buiter 1983a, 1983b, 1985, 1986, 1990a, 1990b; Buiter and Patel 1990; Miller 1982; Miller and Babbs 1983).

The essence of the "approach" (if that is not too grand a characterization of what is no more than a procedure for systematically filling in an accounting identity) is as follows. First, the budget identity (sources and uses of funds) is derived for the consolidated public sector and central bank. Second, all but one of the items in the budget identity are assigned numerical values. These can be initial values of stocks of assets and liabilities; estimates or guesses for real and nominal interest rates, real exchange rate depreciation rates, and real growth rates; and target values or values reflecting exogenous (for instance, external) constraints for such variables as external debt accumulation, international reserve accumulation, and the primary public sector deficit. The endogenously determined "residual" variable can be internal debt accumulation, monetary financing (which implies, via the demand function for base money, a rate of inflation), or any other item occurring in the budget identity.

The value of the residual variable implied by the calculation can be compared with any benchmark value of that variable. This benchmark may be a target value. That will be the case when monetary financing is the residual and there is a target inflation rate. Alternatively, the benchmark could be the value (implicitly) assumed in the calculation or estimation of one or more of the variables that are treated as exogenously given in the exercise; for instance, an assumption about inflation is likely to have been one of the ingredients in the calculations of the domestic real interest rates and the rate of depreciation of the real exchange rate. The benchmark could also be a maximum tolerance limit (for example, in the case of internal debt accumulation) or any other kind of prior notion as to what is a reasonable, likely, or tolerable value for the endogenously determined variable. If the benchmark value differs from the value implied by the exercise, one or more of the numerical inputs have to be altered until a consistent set of figures emerges. The behavioral economic content of the exercise is limited to the specification of the link between monetary financing and inflation and the (implicit) theorizing that generates the benchmark numbers.

All this exercise can achieve is the *internal* consistency of the numerical values assumed by a set of fiscal, financial and monetary variables. The could of course all be wrong, despite being internally consistent.

The accounting exercise often appears more transparent when it is done in continuous time. Since this note is applications-oriented, however, I have used the more cumbersome discrete time expressions. The data do come that way and large approximation errors can arise when one blithely applies the (deterministic) continuous time expressions to a sampled data system. For instance, if the inflation rate is π and the rate of growth of real income is g, the deterministic continuous time analog would erroneously give us the growth rate of nominal income as $\pi+g$, instead of the correct $\pi + g + \pi g$. With high rates of inflation the difference can be huge.

One can view the procedure as using a one-equation (or one-identity) model to determine some fiscal or financial variable that is of interest. This single function has as its arguments a myriad of variables that are properly viewed as jointly endogenous with the variable of interest. Lack of a reliable complete macroeconomic model requires that values be assigned to these arguments using any systematic or seat-of-the-pants method one can come up with. In the Anand and van Wijnbergen version, described below, the single-identity model is augmented with a small model describing the relationship between monetary financing and the rate of inflation.

Section II treats in some detail the accounts of the public sector and the central bank. Section III introduces the basic fiscal-financial-monetary consistency checks. Section IV discusses the link between seigniorage (monetary financing) and inflation. Section V contains a brief discussion of a consistency check on the external financing assumptions made as part of the basic fiscal-financial-monetary consistency checks. In Section VI the approach is reinterpreted by relating it to the evaluation of the solvency of the public sector and of the nation as a whole.

Notational Conventions.

All stocks are end-of-period; for example, B_{t-1} is the nominal stock of domestic debt held at the beginning of period t. Δ is the difference operator, with $\Delta B_t \equiv B_t - B_{t-1}$

(II) The Public Sector Financial Accounts

(a) The Non-central bank Public Sector

It is essential to consolidate all government entities: the central government, state or provincial governments, local and municipal authorities, and state enterprises (financial and nonfinancial). All intragovernment assets, liabilities, payments, and receipts are to be netted out against each other. Extra-budgetary funds must also be included, regardless of the particular accounting conventions adopted in the country. This includes social security trust funds, public bodies like the Resolution Trust Corporation in the United States, and so forth. For brevity I shall refer to the public sector exclusive of the central bank as the Treasury.

The basic budget identity of the Treasury is given in equation 1. $\tilde{D}$ is the primary (noninterest) financial deficit of the Treasury; B is the nominal stock of domestic debt held outside the central bank and i the nominal interest rate on the debt;[1/][2/] B^{cb} is central bank holdings of Treasury debt and i^{cb} its nominal interest rate; L is the nominal stock of Treasury credit to the private sector and i^{L} its nominal interest rate; B^{*} is foreign debt of the Treasury and i^{*} it nominal interest rate. E is the nominal spot exchange rate. B includes accumulated domestic, and B^{*} foreign, arrears of interest and of principal. For simplicity the arrears are assumed to carry the same interest rate as the regular debt.

$$(1) \qquad \tilde{D}_t + i_t B_{t-1} + i_t^{cb} B_{t-1}^{cb} + i_t^{*} E_t B_{t-1}^{*} - i_t^{L} L_{t-1} \equiv \Delta B_t + \Delta B_t^{cb} + E_t \Delta B_t^{*} - \Delta L_t$$

1/ For notational simplicity, debt is assumed to be the fixed nominal market value, variable interest rate variety. The framework can be extended without trouble to include variable market value debt. It is also assumed that debt denominated in domestic currency is held only be domestic residents and that debt denominated in foreign currency is held only by nonresidents. Again, this assumption can be relaxed without any problems.

2/ If there is a long-dated government debt outstanding, internal, or external, there can, of course, be variations in the market price of that debt without this reflecting any default or repudiation risk discounts. Obvious minor changes will have to be made on the right-hand side of (1) where, with B' now denoting the number of long-dated debt instruments and $P_{B'}$ their market value, $P_{B'} \Delta B'$ will enter instead of ΔB. Similarly with ω denoting the coupon payment on a long-dated debt instrument, interest payments now are $\omega B'$ rather than iB.

The primary deficit of the Treasury is defined in equation 2. C is government consumption spending, A is gross government capital formation, T is taxes and other current revenues net of current transfers and subsidies (excluding T^{CB}), T^{cb} is payments by the central bank to the Treasury, N^* is foreign aid, and S is the profits of the public enterprise sector. All these flows are measured at current prices.

$$(2) \qquad \tilde{D}_t \equiv C_t + A_t - T_t - T_t^{cb} - N_t^* E_t - S_t .$$

The left-hand side of equation 1 gives the financial deficit of the Treasury, the right-hand side its financing. Note that only changes in *financial* liabilities and assets are shown on the right-hand side of equation 1. In principle, sales and purchases of any existing assets, real and financial, by the public sector should be included on the right-hand side. Proceeds from asset sales such as privatization proceeds or revenues from sales of publicly owned land or mineral resources etc. are financing items and enter (with a positive sign), on the righthand side of equation 1. Outlays for the nationalization of private sector assets (such as the nationalization of large parts of the S&L sector in the United States) belong on the right-hand side with a negative sign.

Spending on the acquisition of newly produced capital goods is clearly not consumption spending. Yet gross public sector capital formation, A, is included in $\tilde{D}$ on the lefthand side of equation 1. In principle, one could decompose A into net capital accumulation ΔK and depreciation δK, where K is the real public sector capital stock (valued at current reproduction costs) and δ is the proportional rate of capital depreciation. Making the simplifying assumption that the nominal reproduction cost of capital follows the GDP deflator P gives:

$$A_t \equiv P_t(\Delta K_t + \delta K_{t-1})$$

Depreciation, or capital consumption, could be lumped with other public sector consumption spending, *C*, and $P\Delta K$ could be moved to the right-hand side of equation 1. The cash returns accruing to the public sector capital stock *S* could be moved out of the primary deficit and included with the other interest payments and asset income. While this is theoretically preferable, such purity is rather far away from conventional practice and shall not be pursued further here.

It is not always obvious what "the" exchange rate used in equations 1 and 2 is. Often there are multiple official exchange rates as well as a black and grey market rate. In principle, one should use, for each transaction, the rate that the government actually pays or receives. There can be many of these, for different kinds of transactions.

(b) The Central Bank

It is helpful here to start from the central bank's balance sheet. R^* is the stock of official foreign exchange reserves, *CU* the currency held by the public, *RR* the reserves held by the commercial banking sector with the central bank, and NW_t the net worth of the central bank at the beginning of period *t*+1. currency and bank reserves carry a zero nominal rate of interest. Therefore, the balance sheet of the central bank at the beginning of period t+1 is

Assets	Liabilities
B_t^{cb}	CU_t
$R_t^* E_{t+1}$	RR_t
	NW_t

The budget identity of the central bank is given by

(3) $$T_t^{cb} - i_t^{cb} B_{t-1}^{cb} - i_t^* R_{t-1}^* E_t \equiv \Delta CU_t + \Delta RR_t - \Delta B_t^{cb} - E_t \Delta R_t^*$$

From the balance sheet of the central bank it follows that

$$\Delta NW_t \equiv \Delta B_t^{cb} + E_t \Delta R_t^* + R_t^* \Delta E_{t+1} - \Delta CU_t - \Delta RR_t$$

From this equation and equation 3, it follows that

(4) $$\Delta NW_t \equiv i_t^{cb} B_{t-1}^{cb} + i_t^* R_{t-1}^* E_t - T_t^{cb} + R_t^* \Delta E_{t+1}.$$

For simplicity it is assumed that the central bank has no current expenses, such as salaries. The change in the central bank's net worth is the sum of current after-tax profits and capital gains on the existing portfolio of assets and liabilities, in this case due only to changes in the exchange rate.

(c) The Consolidated Public Sector

The accounts of the Treasury and the central bank must be consolidated. Otherwise, items can be shuffled between these two agencies far too easily. For instance, the central bank (or its subsidiaries) could take over much of the lending operations from the Treasury, creating another, potentially open-ended source of monetary financing. Henceforth the consolidated Treasury and central bank will be referred to simply as "the government" or the "public sector."

The consolidated government budget identity is given in the following equation:

(5) $$D_t + i_t B_{t-1} + i_t^* \left(B_{t-1}^* - R_{t-1}^*\right) E_t - i_t^L L_{t-1} \equiv \Delta B_t + E_t \Delta \left(B_t^* - R_t^*\right) - \Delta L_t + \Delta CU_t + \Delta RR_t$$

The public sector primary deficit D is given by

(6) $$D_t \equiv C_t + A_t - T_t - N_t^* E_t - S_t.$$

Noting that the high-powered money stock H is the sum of currency in the hands of the public and commercial bank reserves held with the central bank and that domestic credit expansion *DCE* is Treasury borrowing from the central bank, we have

(7) $H \equiv CU + RR$.

and

(8) $DCE \equiv \Delta H - E\Delta R^{*} \equiv \Delta B^{cb} + T^{cb} - i^{cb} B^{cb} - i^{*}ER^{*}$

If the Treasury taxes away all central bank profits, equation (8) simplifies to

(8') $DCE \equiv \Delta H - E\Delta R^{*} \equiv \Delta B^{cb}$

(d) A Minor Disagreement with Anand and van Wijnbergen

Anand and van Wijnbergen (1989) point out (correctly) that the central bank often lends to the commercial banking sector and to the nonbank private sector. Let L^{cb} be the nominal stock of central bank loans to the private sector and $i^{L^{cb}}$ the interest rate charged on these loans. The revised balance sheet for the central bank is now

Assets	Liabilities
B_t^{cb}	CU_t
$R_t^{*}E_{t+1}$	RR_t
L_t^{cb}	NW_{t+1}

The revised central bank budget identity becomes equation 3'

(3') $T_t^{cb} - i_t^{cb}B_{t-1}^{cb} - i_t^{*}R_{t-1}^{*}E_t - i_t^{L^{cb}}L_{t-1}^{cb} \equiv \Delta CU_t + \Delta RR_t - \Delta B_t^{cb} - E_t\Delta R_t^{*} - \Delta L_t^{cb}$

The revised expression for the change in the central bank's net worth becomes:

$$(4')\qquad \Delta NW_t \equiv i_t^{cb}B_{t-1}^{cb} + i_t^{*}R_{t-1}^{*}E_t + i_t^{L_{cb}}L_{t-1}^{cb} - T_t^{cb} + R_t^{*}\Delta E_{t+1}$$

Anand and van Wijnbergen define what they call the *adjusted* monetary base, H^a, as in (9):

$$(9)\qquad H^a \equiv H - L^{cb} \equiv CU + RR - L^{cb}$$

They then assert that this *adjusted* monetary base, the monetary base minus central bank credit to the private sector,

> "equals the central bank net non-interest-bearing liabilities to the private sector and is the appropriate concept to use for calculations of consistency of fiscal deficits with levels of inflation tax revenue" (p. 24).

I cannot follow this argument. It is the *unadjusted* monetary base, *H*, that is special for two reasons. First, its nominal rate of return is zero. Second, despite it being such a poor (rate-of-return-dominated) store of value, there may be (for a variety of reasons, including the domestic legal tender label attached to currency and the existence of legal reserve requirements) a stable demand function for this asset (or perhaps for *CU* and *RR* separately). This then makes the unadjusted monetary base a key link in the inflation mechanism.

It clearly is not correct, except in one special case, that the adjusted monetary base equals the central bank's net non-interest-bearing liabilities to the private sector. That would be the case only if the central bank's loans to the private sector were interest-free. Even if the central bank were to make such loans, it does not follow that those loans would be perfect substitutes for conventional base money (or for currency in the hands of the public and bank reserves separately). It is hard to think of conditions under which there would be a stable demand function for the adjusted monetary base.

It is much cleaner and clearer, even if central bank lending to the private sector is at a zero nominal interest rate, to continue to separate out the conventional monetary base and to allow for the financial implications of central bank loans to the private sector (whether they be interest-free loans, below-market rate loans, or loans charging the public sector's opportunity cost of funds) by entering the loans properly into the budget identities (and change-in-net-worth calculations), as shown in equations 3' and 4'. Nominal interest rates on central bank credit to the private sector that respond differently to changes in inflation or other events than do other interest rates (such as *i*) will, of course, have to be allowed for when numerical values are assigned in the consistency checks considered later.

Consolidating the Treasury budget identity and the modified central bank budget identity 3' yields

$$(5')\qquad \begin{aligned} &D_t + i_t B_{t-1} + i_t^* \left(B_{t-1}^* - R_{t-1}^*\right) E_t - i_t^L L_{t-1} - i_t^{L_{cb}} L_{t-1}^{cb} \\ &\equiv \Delta B_t + E_t \Delta \left(B_t^* - R_t^*\right) - \Delta L_t - \Delta L_t^{cb} + \Delta CU_t + \Delta RR_t \end{aligned}$$

This way of presenting the identity emphasizes the point that lending by the Treasury and central bank is simply a form of negative borrowing.

An alternative way of presenting these same data is given below. It emphasizes the view that an increase in government lending to the private sector (both by the non-central bank government sector and by the central bank) effectively amounts to a current subsidy (that is a negative component of T) and that whatever interest charges are paid are more akin to taxes or levies than to market-related interest payments. This *modified* primary deficit D' is defined as follows:

$$(10)\qquad D_t^{'} \equiv D_t + \Delta L_t + \Delta L_t^{cb} - i_t^L L_{t-1} - i_t^{L_{cb}} L_{t-1}^{cb}.$$

The public sector budget identity can now be written as

$$(5'') \quad D_t' + i_t B_{t-1} + i_t^*\left(B_{t-1}^* - R_{t-1}^*\right)E_t \equiv \Delta B_t + E_t \Delta \left(B_t^* - R_t^*\right) + \Delta CU_t + \Delta RR_t .$$

My own preferred presentation involves separating out the subsidy element in public sector lending. Public sector loans to the private sector are viewed as negative public sector debt. In the interest account, the interest payments on these loans are entered at their opportunity cost, which I take to be the government domestic borrowing rate *i*. Any excess of *i* over i^L and over $i^{L_{cb}}$ is entered explicitly as a subsidy in the *adjusted* primary deficit D^a.

$$(11) \quad D_t^a \equiv D_t + \left(i_t - i_t^l\right)L_{t-1} + \left(i_t - i_t^{l_{cb}}\right) L_{t-1}^{cb} .$$

In this case the public sector budget identity is written as

$$(5''') \quad \begin{aligned} &D_t^a + i_t\left(B_{t-1} - L_{t-1} - L_{t-1}^{cb}\right) + i_t^*\left(B_{t-1}^* - R_{t-1}^*\right)E_t \\ &\equiv \Delta\left(B_t - L_t - L_t^{cb}\right) + E_t\Delta\left(B_t^* - R_t^*\right) + \Delta CU_t + \Delta RR_t \end{aligned} .$$

(e) So-Called "Inflation-and-Real-Growth-Corrected Deficits"

It is often convenient (although of no substantive significance) to rewrite the budget identities by expressing all stocks and flows as proportions of GDP. *Y* denotes real GDP and P^* the foreign price level. Lower case letters denote the corresponding upper case letters as proportions of GDP, for example, $c_t \equiv C_t/(P_tY_t)$ *and* $b_t^* \equiv E_tB_t^*/(P_tY_t)$. Where confusion might result from this convention, I chose instead the lower-case Greek alphabet equivalent, for example, $\rho^* \equiv R^*/(PY)$ *and* $\rho\rho \equiv RR/(PY)$. π is the domestic rate of inflation; *r* is the domestic real interest rate $(1 + r \equiv (1 + i)/(1 + \pi))$; π^* is the foreign rate of inflation; r^* is the foreign real interest rate $(1+r^* \equiv (1+i^*)/(1+\pi^*))$; g is the growth rate of real GDP; γ is the percentage depreciation of the real exchange rate EP^*/P; $\sigma_t \equiv (H_t - H_{t-1})/(P_t Y_t)$ is monetary financing as a fraction GDP. Monetary financing is referred to here as *seigniorage*.

The consolidated government budget identity can then be rewritten as in equation 12.

$$(12) \quad b_t - \ell_t - \ell_t^{cb} + b_t^* - \rho_t^* \equiv \left(b_{t-1} - \ell_{t-1} - \ell_{t-1}^{cb}\right)\left(1 + r_t\right)\left(1 + g_t\right)^{-1} + \left(b_{t-1}^* - \rho_{t-1}^*\right)\left(1 + r_t^*\right)\left(1 + \gamma_t\right)\left(1 + g_t\right)^{-1} + d_t^a - \sigma_t$$

where

$$(13) \quad d_t^a \equiv d_t + \ell_{t-1}\left(i_t - i_t^L\right)(1 + g_t)^{-1} + \ell_{t-1}^{cb}\left(i_t - i_t^{L_{cb}}\right)(1 + g_t)^{-1}$$

and

$$(14) \quad d_t \equiv c_t + a_t - \tau_t - n_t^* - s_t .$$

To economize on notation, the adjusted domestic debt (as a proportion of GDP) is defined as b^a:

$$(15) \quad b^a \equiv b - \ell - \ell^{cb}$$

The public sector budget identity 12 can now be rewritten more compactly as

$$(16) \quad b_t^a + b_t^* - \rho_t^* \equiv b_{t-1}^a(1 + r_t)(1 + g_t)^{-1} + \left(b_{t-1}^* - \rho_{t-1}^*\right)(1 + r_t^*)(1 + \gamma_t)(1 + g_t)^{-1} + d_t^a - \sigma_t .$$

Equation 16 can be rewritten to yield

$$(17) \quad \Delta b_t^a + \Delta b_t^* - \Delta\rho_t^* + \sigma_t \equiv d_t^a + (1+g_t)^{-1}\left\{\left[r_t - g_t\right]b_{t-1}^a + \left[(1+r_t^*)(1+\gamma_t) - (1+g_t)\right](b_{t-1}^* - \rho_{t-1}^*)\right\} .$$

The terms on the right-hand side of equation 17 are sometimes referred to as the inflation-and-real-growth-corrected deficit (as a proportion of GDP). The interest terms added to the primary deficit evaluate interest payments not at the nominal interest rates actually paid, but at real (that is, inflation-corrected) interest rates net of the proportional rate of growth of real output. This deficit must be financed either by raising the net (internal or external) debt-GDP ratio or by printing money.

Equation 17 is still the same old public sector budget identity. All that has changed is the numéraire, which is units of real GDP in equation 17. The consistency checks can be done equally well using the accounts expressed in domestic nominal units (as in equation 5'''), in the foreign nominal units, in real terms, or in pounds avoirdupois. It is purely a matter of presentation.

Some practitioners like to express the accounts in real units because they believe asset demands, including the demand for debt, are demands for *real* stocks of assets or debt. The real required rate of return on this asset will change only if the real stock of the asset changes. Other economists prefer to express stocks and flows as fractions of GDP (that is, they use current GDP as the numéraire) because they have the notion that demand functions for real assets and debt contain a real scale variable (permanent income, wealth, or some such thing). GDP is viewed as a proxy for that scale variable. If asset demands are proportional to the scale variable at given real rates of return (something that those who like to think of balanced growth have to assume), then there will be pressure for real rates of return to change only when the ratio of the supply of an asset to GDP varies. One, of course, may hold such views on the correct behavioral specification of asset demand functions and still use Mars Bars as the numéraire for accounting purposes.

(III) A Consistency Check of Fiscal, Financial and Monetary Policies

Equation 17 is the starting point for the consistency checks. The initial values of the domestic debt-GDP ratio b^a_{t-1}, the foreign debt-GDP ratio b^*_{t-1} and the international reserve-GDP ratio ρ^*_{t-1} are known and given.

Estimates of the domestic real interest rate r_t, the foreign real interest rate r^*_t, the proportional rate of depreciation of the real exchange rate γ_t and the proportional growth rate of real GDP g_t must always be provided as inputs. The assignment of numerical values to r, r^* and γ is likely to be based on projections of nominal interest rates, i and i^*, rates of inflation π and π^*, and the nominal rate of exchange rate depreciation ϵ. This leaves the five potential policy targets: $d^a_t, \Delta b^a_t, \Delta b^*_t, \Delta\rho^*_t$ *and* σ_t, four of which can be specified independently, with the fifth determined residually from the identity 17.

With a little bit of theory and estimation, the seigniorage target, σ, can be related to the domestic rate of inflation. This shall be considered in more detail in Section IV.

The two most common ways of using identity 17 are to treat either Δb^a or σ (that is the rate of inflation) as the residual. I shall consider them in turn.

Case 1: What consequences do fiscal and inflation objectives have for domestic debt accumulation ?

Data: b^a_{t-1}, b^*_{t-1}, ρ^*_{t-1}.

Estimates or projections: r_t, r^*_t, γ_t, g_t.

Targets or constraints: d^a_t, $\Delta\rho^*_t$, *and* σ_t (through a target for π_t).

Endogenous: Δb^a_t.

Given the stocks of government liabilities and assets outstanding and the estimates of growth, real interest rates, and the rate of depreciation of the real exchange rate, one can determine the inflation-and-growth-corrected interest bill. The government's target for the primary deficit (d_t^a) then determines the inflation-and-growth-corrected deficit. A target for or constraint on the behavior of international reserves gives $\Delta\rho_t^*$. The external targets or constraints faced by the government determine the amount (if any) it can borrow abroad. This gives Δb_t^*. For instance, if the government would like to borrow but is rationed out of the international financial markets, then $\Delta B_t^* = 0$, that is, $\Delta b_t^* = -b_{t-1}^*(\pi_t + g_t)(1+\pi_t)^{-1}(1+g_t)^{-1} < 0$ *if* $b_{t-1}^* > 0$. I shall return to the external borrowing of the government in Section V.

The government's inflation target determines σ_t. Given these inputs, the behavior of the domestic debt-GDP ratio Δb_t^a follows.

The next step is to verify whether the implied behavior of the domestic debt-GDP ratio makes sense. Note that one can repeat the analysis for as many successive periods as one wishes. The first period's calculation yields b_t^a, b_t^* and ρ_t^*, which can be used as inputs in an identical set of calculations for the next period and so on.

If the implied domestic debt-GDP ratio is rising steadily, with no end in sight, the fiscal policy targets and the monetary financing or inflation targets are not consistent. (Indeed, a very steep implied rise in the internal debt-GDP ratio may lead one to question and revise the domestic real interest rate estimates that were one of the inputs in the exercise.) Something has to give. One can redo the exercise with a different primary deficit, with a different assumption about seigniorage (that is, a different assumption about inflation) or, more generally, with different assumptions about any or all of the exogenous inputs into the exercise, until something emerges that makes sense and is acceptable.

Case 2. What is the rate of inflation implied by the government's fiscal and financial strategy?

Data: b^a_{t-1}, b^*_{t-1}, ρ^*_{t-1}

Estimates or projections: r_t, r^*_t, γ_t, g_t.

Targets or constraints: d^a_t, Δb^a_t, Δb^*_t, $\Delta \rho^*_t$.

Endogenous: σ_t (that is, π_t).

In this case, the amount of real resources (as a fraction of GDP) to be extracted by running the printing presses, σ_t, is left to be residually determined from equation 17 after the economist provides b^a_{t-1}, b^*_{t-1}, ρ^*_{t-1}, r_t, r^*_t, γ_t, g_t, d^a_t, Δb^a_t, Δb^*_t *and* $\Delta \rho^*_t$. A typical benchmark for the domestic debt-GDP ratio b^a is that it should be constant. Note that if $\Delta b^a_t = 0$, nominal domestic debt grows at the rate of nominal GDP, that is, $B^a_t = (1+\pi_t)(1+g_t)B^a_{t-1}$.

In this exercise, with the deficit and the permitted net domestic and foreign debt issues given, any shortfall of sources of funds relative to uses of funds has to be made up by printing base money. Through the demand function for base money, to be discussed in Section IV, a given amount of required seigniorage implies a rate of inflation π. The purpose of the exercise is to check whether the implied rate of inflation is acceptable and, more important, whether it is the same as the assumed rate of inflation that went into the calculations that yielded inputs of domestic real interest rates and real exchange rate depreciation. Indeed, the estimate of the primary deficit, d^a, which is another input into the exercise, is bound to be a function of the rate of inflation (through the Olivera-Tanzi effect) (Tanzi 1978).

The exercises can be done, of course, with any of the non-predetermined variables in equation 17 as the residual one. It may be interesting, for example, to calculate the primary deficit, d^a , implied by the debt and reserve objectives and the inflation target (σ). If the number that comes out is too small to be feasible, at least one of the inputs will have to be revised.

(IV) From seigniorage to inflation (and back)

Formally, the consistency exercises either determine σ or use σ as an input in the endogenous determination of some other fiscal or financial variable. In practice it is often desirable (and feasible) to establish a behavioral relationship between seigniorage σ and the rate of inflation π.

Note from the definition of seigniorage that it can be written in a number of ways:

(18a) $$\sigma_t \equiv h_t - h_{t-1}(1+g_t)^{-1}(1+\pi_t)^{-1}$$

(18b) $$\sigma_t \equiv \Delta h_t - h_{t-1}(\pi_t + g_t + \pi_t g_t)(1+g_t)^{-1}(1+\pi_t)^{-1}$$

(18c) $$\sigma_t \equiv \mu_t(1+\mu_t)^{-1} h_t .$$

Here $\mu_t \equiv (H_t - H_{t-1})/H_{t-1}$, the proportional rate of growth of the nominal stock of base money in period t.

The second term on the right-hand side of equation 18a illustrates how inflation and real growth "amortize" the inherited ratio of base money to GDP. $\pi_t h_{t-1}$ is sometimes called the inflation tax. Clearly, a higher *realized* rate of inflation raises the inflation tax and permits a higher amount of seigniorage to be extracted. A higher *anticipated* rate of inflation however, in all likelihood reduces the demand for real money balances, that is, it lowers the first term on the right-hand side of 18a or 18b: the inflation tax base is eroded. To raise very large amounts of seigniorage, one would wish to have both a high realized rate of inflation and a low anticipated rate of inflation. That, of course, is not a feasible long-run proposition if the private sector is reasonably well-informed and rational.

The effects of inflation on the budget identity are not restricted to σ. I already noted the Olivera-Tanzi effect, through which inflation (even if anticipated) may increase the primary deficit by reducing the real value of tax receipts. If the private sector holds fixed-interest public debt denominated in the domestic currency, then inflation that is unanticipated at the date the debt is issued, will reduce the realized real return on these debt instruments. One can no doubt think of other channels through which anticipated and unanticipated inflation affect the entries in the budget identity, and they should be allowed for in the forecasts, estimates, and guesses that produce the numerical inputs for the consistency checks.

Consider the case where realized and anticipated inflation are the same. If the absolute value of the elasticity of demand for base money with respect to the (expected) rate of inflation is greater than unity, then higher (actual and expected) inflation will actually reduce the real value of seigniorage. This is the so-called "seigniorage Laffer curve."

Four examples of simple long-run or steady-state base money demand functions are shown below in equations 19a-d. In steady state, *h* is constant, and actual and expected rates of inflation are given by $1+\pi=(1+\mu)/(1+g)^{-1}$. If the long-run demand for base money is linear in π (19a), or if the logarithm of the demand for money is linear in π (19b), there will be a unimodal seigniorage Laffer curve, with σ rising with π for low rates of inflation and falling with π for high rates of inflation.

(19a) $h = \alpha - \beta\pi \qquad \alpha,\ \beta > 0$

(19b) $ln(h) = \alpha' - \beta'\pi \qquad \beta' > 0$

(19c) $ln(h) = \alpha'' - \beta'' ln\ (1 + \pi) \qquad \beta'' > 0$

(19d) $h^{-1}(1 + \pi)(1 + g) = \alpha''' + \beta'''\pi$
$\alpha''',\beta''' > 0$

Many other empirically plausible specifications of base money demand do not have a Laffer curve, of course. In the constant elasticity model, shown in equation 19c, σ always rises (falls) with π if the absolute value of the constant elasticity is less (greater) than unity. If the income velocity of circulation of base money $V_t \equiv \frac{P_t Y_t}{M_{t-1}} = (1+\pi_t)(1+g_t)\,h_{t-1}^{-1}$

is a positive linear function of the rate of inflation (19d), seigniorage σ increases with π if $\alpha''' + g(a''' - \beta''') > 0$, and as π goes to infinity, σ asymptotically approaches a maximum value of $(1 + g)/\beta'''$.

In equations 19a-d it is assumed that the steady state demand for real base money depends negatively on the expected rate of inflation and positively (and with unit elasticity) on real GDP. These steady state base money demand functions are unlikely to be useful for inferring the rate of inflation over any period of historical time.

In their empirical approach to estimating the demand for base money, Anand and van Wijnbergen split the demand for base money into its two components, demand for currency and demand for bank reserves. These two demand functions then are estimated with some allowance for the lags that characterize any "real time" adjustment pattern. The Anand-van Wijnbergen approach can be summarized as in equations 20 to 26. (All stocks are relative to GDP). Required reserves $\rho\rho$ are split into reserves on demand deposits $\rho\rho^{dd}$ and reserves on time deposits $\rho\rho^{td}$. The fraction of demand deposits backed by reserves is θ^{dd}, and the fraction of time deposits backed by reserves is θ^{td}. One could make the reserve ratios behavioral, but for simplicity I assume that the legal reserve requirements are strictly binding. The demands for currency cu, demand deposits *dd* and time deposits *td* are determined in a simple portfolio model. The expected rate of inflation $\hat{\pi}$ and the nominal interest rates on demand deposits (i^{dd}) and on time deposits (i^{td}) are arguments in the three asset demand functions 22, 25 and 26. The gross substitutes priors on the signs of the rate of return effects in the asset demand functions are written below the arguments in the equations. (It is a very rare event to find any significant and "correctly signed" effect of domestic interest rates on financial asset demands in developing countries with repressed financial markets.)

Other likely candidates for inclusion as arguments in the money demand functions are the expected rates of return on foreign-currency-denominated assets $\hat{e}$ *and* $i^* + \hat{e}$ where $\hat{e}$ denotes the expected proportional rate of depreciation of the nominal spot exchange rate. The former would be expected to have an effect if (limited) direct currency substitution (dollarization) is a possibility. The second would occur if domestic residents are able to shift into interest-bearing foreign-currency-denominated assets. A variable such as real financial wealth (perhaps as a proportion of GDP, ω) might also be included. The assumption of a unit elasticity of asset demands with respect to income should be tested.

(20) $h \equiv cu + \rho\rho$

(21) $\rho\rho \equiv \rho\rho^{dd} + \rho\rho^{td}$

$$(22) \quad cu = f^{c}\left(\underset{(-)}{\hat{\pi}}, \underset{(-)}{i^{dd}}, \underset{(-)}{i^{td}}; \underset{(-)}{\hat{e}}, \underset{(-)}{i^{*} + \hat{e}}, \underset{(+)}{\omega}; \underset{(+)}{Y}\right)$$

(+ dynamic specification)

$$(23) \quad \rho\rho^{dd} = \theta^{dd} dd \qquad 0 < \theta^{dd} < 1$$

$$(24) \quad \rho\rho^{td} = \theta^{td} td \qquad 0 < \theta^{td} < 1$$

$$(25) \quad dd = f^{dd}\left(\underset{(-)}{\hat{\pi}}, \underset{(+)}{i^{dd}}, \underset{(-)}{i^{td}}; \underset{(-)}{\hat{e}}, \underset{(-)}{i^{*} + \hat{e}}, \underset{(+)}{\omega}; \underset{(+)}{Y}\right)$$

(+ dynamic specification)

$$(26) \quad td = f^{td}\left(\underset{(-)}{\hat{\pi}}, \underset{(+)}{i^{dd}}, \underset{(-)}{i^{td}}; \underset{(-)}{\hat{e}}, \underset{(-)}{i^{*} + \hat{e}}, \underset{(+)}{\omega}; \underset{(?)}{Y}\right)$$

(+ dynamic specification)

Estimating the three asset demand functions enables one to obtain the demand function for base money. This must therefore include as arguments such variables as $\hat{\pi}$, i^{dd}, i^{td}, θ^{dd} *and* θ^{td} (and possibly also expected rates of return on foreign-currency-denominated assets, private financial wealth, and real income, as well as lags of the dependent and independent variables). To evaluate the implications for inflation of a given seigniorage requirement σ, these arguments will all have to be provided as inputs. What comes out of the calculations is, of course, the expected rate of inflation, $\hat{\pi}$, not the realized rate of inflation.

One way to minimize the amount of effort required is to ask what *steady state* (actual and expected) rate of inflation would permanently yield the amount of seigniorage determined by the consistency check. This might then be interpreted as the long-term inflationary consequences of pursuing a fiscal-financial strategy that implies a given need for seigniorage revenue. Unless the seigniorage required (as a fraction of GDP) is indeed constant over time (and unless all the other arguments in the base money demand function are also constant), this kind of calculation cannot be expected to yield an accurate picture of the actual inflationary consequences of the fiscal-financial mix, not even over a long period of calendar time.

The problem of determining the relationship between the amount of seigniorage to be extracted in any given period and the rate of inflation in that period can be even more awkward if the expected rate of inflation in the money demand function determining h_t is the expectation of the *future* rate of inflation π_{t+1}, if expectations are modeled as rational and if inflation is governed by a process that is not completely backward-looking.

In practice, some shortcut will have to be made. A common one is to postulate some simple adjustment process for inflation. In the example given below, the excess of the actual rate of inflation π_t over the current "core" rate of inflation $\tilde{\pi}_t$ is an increasing function of the excess of the current base money-GDP ratio h_{t-1} over the long-run desired base money-GDP ratio $\tilde{h}_t$, which can be interpreted as the long-run money demand function outlined in equations 20-26. Expectational variables affecting $\tilde{h}$ are handled in some mechanical way (through adaptive, extrapolative, or similar expectations mechanisms). Core inflation (sometimes identified with expected inflation) is some moving average of past actual inflation.

$$\pi_t = \eta(h_{t-1} - \tilde{h}_t) + \tilde{\pi}_t \qquad \eta>0$$

$$\tilde{\pi}_t = \psi\pi_{t-1} + (1-\psi)\tilde{\pi}_{t-1} \qquad 0 \leq \psi \leq 1$$

Even when less ad-hoc approaches to linking up inflation with seigniorage are adopted, it is likely that the estimated relationship will be a loose and imprecise one, not just period by period, but even over any reasonable run of calendar time.

(V) A Consistency Check of the External Financing Assumptions.

Key inputs in the fiscal consistency checks are the change in net foreign government debt Δb^* and the change in the stock of foreign exchange reserves $\Delta\rho^*$. It is helpful to check the plausibility of the values assigned to these two variables by considering the external account of the country, given in equation 27. X denotes the domestic currency value of exports, Z^* the foreign currency value of imports, F^* net foreign assets of the private sector (for simplicity they are assumed to earn the rate i^*), DFI net direct foreign investment in the country and DIV earnings remitted abroad by foreign-owned firms. For simplicity, only the government is assumed to receive foreign aid. Because B^* includes properly compounded arrears, the account given in equation (27) is complete.

$$(27) \quad X_t - E_t Z_t^* + E_t N_t^* + i_t^* E_t\left(R_{t-1}^* + F_{t-1}^* - B_{t-1}^*\right) - DIV_t \equiv E_t\left(\Delta R_t^* - \Delta B_t^* + \Delta F_t^*\right) - DFI_t$$

Equivalently, using lower-case letters again to stand for upper-case variables as a proportion of GDP,

$$(28) \quad \begin{aligned} & x_t - z_t^* + n_t^* - div_t + dfi_t \\ & + (1+g_t)^{-1}\left[(1+r_t^*)(1+\gamma_t) - (1+g_t)\right]\left(\rho_{t-1}^* + f_{t-1}^* - b_{t-1}^*\right) \\ & \equiv \Delta\left(\rho_t^* + f_t^* - b_t^*\right) \end{aligned}$$

The official international reserve and public sector foreign debt objectives used as inputs in the fiscal consistency checks have to be consistent with the projected current account surplus (the left-hand side of equation 28), the expected net foreign direct investment inflow, and the projected amount of net private investment abroad, some or all of which is sometimes dubbed "capital flight." The real GDP growth and real exchange rate depreciation assumptions used as inputs in the fiscal consistency checks have direct implications for the trade balance $x - z^*$ and should be consistent with it.

What is sketched here is a framework for extending the one-equation model to a two-equation model. In the case under consideration, that involves the introduction of more than one additional variable, so I have not progressed very far toward a complete sequential general equilibrium model. Equations 27 and 28 do, however, serve to focus attention on a key relationship involving variables that can be monitored and measured relatively easily (with the possible exception of the capital flight component of Δf^*). If it is difficult to come up with balance-of-payments projections that are consistent with the government's planned recourse to foreign financing $(\Delta b^* - \Delta \rho^*)$, then the policymakers and their advisers will have to go back to the drawing board until they match.

(VI) Solvency and Sustainability

(a) **Government Solvency and the Sustainability of the Public Debt Process**

The approach outlined in the first four sections of this paper can be related to the concept of government solvency in a way that may be illuminating. In the following notation,

(29a) $$\bar{b} \equiv b^a + b^* - \rho^*$$

(29b) $$\bar{d}_t \equiv d_t^a + \left(\frac{r_t^* + \gamma_t + r_t^* \gamma_t - r_t}{1 + g_t}\right)(b_{t-1}^* - \rho_{t-1}^*)$$

$\bar{b}$ is the nonmonetary debt of the consolidated public sector and central bank as a proportion of GDP; $\bar{d}$ is the *augmented* primary deficit of the consolidated public sector and central bank as a proportion of GDP.

Note that $r^* + \gamma + r^*\gamma - r = \frac{(1+i^*)(1+\epsilon) - (1+i)}{(1+\pi)}$. If uncovered (nominal) interest parity (UIP) holds, then $(1 + i^*)(1 + \hat{\epsilon}) = 1 + i$. I shall refer to $(1 + i^*)(1 + \epsilon)$ as the (ex-post) UIP value of the foreign rate of interest.

The public sector budget identity in equation 16 is rewritten as follows:

$$\bar{b}_{t-1} \equiv \frac{(1+g_t)}{(1+r_t)}\left[\bar{b}_t - \bar{d}_t + \sigma_t\right]. \tag{30}$$

In equation 30 a common interest rate (the domestic one) is imputed to all government debt, domestic and foreign. The augmented public sector primary deficit $\bar{d}$ allows for the fact that the government may have net foreign debt that does not equal zero $(b^* - \rho^* \neq 0)$ and that this debt may pay a foreign rate of interest that is different from the UIP rate. Take the case where $b^* - \rho^*$ is positive. Then if the domestic interest rate exceeds the UIP value of the foreign interest rate, the government saves resources by borrowing externally rather than domestically; that is, the true interest cost of the public debt is less than the interest cost with all debt costed at the domestic rate of interest. The augmented primary deficit in equation 29b includes these interest savings.

The budget identity 30 can be solved recursively forward in time. Since future interest rates, growth rates, primary deficits, and seigniorage are not known with certainty, it makes sense to write the resulting expression in terms of expected, rather than realized, values. $\mathcal{E}_t$ is the expectation operator conditional on information available at the beginning of period t. This yields

$$\bar{b}_{t-1} = \mathcal{E}_t\left\{\sum_{j=0}^{\infty} \Omega_t^j\left[-\bar{d}_{t+j} + \sigma_{t+j}\right]\right\} \tag{31}$$

$$+ \lim_{j \to \infty} \mathcal{E}_t \left\{ \Omega_t^j \, \bar{b}_{t+j} \right\}$$

$$\Omega_t^j \equiv \prod_{i=0}^{j} \frac{(1 + g_{t+i})}{(1 + r_{t+i})} \qquad j \geq 0$$

The Ω_t^j are the discount factors between periods t and t+j. The first expression on the right-hand side of equation 31 is the present discounted value of expected future (augmented) primary surpluses and expected future seigniorage. The second expression is the expectation of the present discounted value of the public debt in the infinitely distant future.

In equation 31 the stocks and flows are expressed as proportions of GDP, and the discount rates are real interest rates net of real growth rates. One can, of course, restate equation 31 in an entirely equivalent manner using stocks and flows measured in domestic currency (with domestic nominal interest rates used for discounting) or using real stocks and flows (using domestic real interest rates for discounting). With minor and obvious changes in the definition of the augmented public sector deficit, one could express 31 equivalently using stocks and flows measured in foreign currency (with foreign nominal interest rates used for discounting).

The condition for public sector solvency in a world without an obvious finite terminal date is a straightforward generalization of the solvency condition in an economy with a known finite terminal date. For the finite-horizon economy, the solvency condition is that the government cannot leave a positive stock of debt in the last period. The infinite-horizon extension of this criterion is that in the limit as the terminal date goes to infinity, the present discounted value of the public debt should not be positive. Formally, (32a) $$\lim_{j\to\infty} \mathcal{E}_t \left\{\Omega_t^j \, \bar{b}_{t+j}\right\} \leq 0.$$

Equations 31 and 32a together imply

(32b) $$\bar{b}_{t-1} \leq \mathcal{E}_t \left\{\sum_{j=0}^{\infty} \Omega_t^j \left[-\bar{d}_{t+j} + \sigma_{t+j}\right]\right\}.$$

Equation 32b states that the expected present discounted value of future (augmented) primary surpluses and seigniorage should be at least as large as the outstanding stock of public debt. For practical purposes, no generality is lost if it is assumed that 32a and 32b hold with strict equality.

The meaning of the solvency constraint is well-known: the public debt is not permitted to grow forever at a rate exceeding the rate of interest. A solvent government cannot indefinitely pay the interest it owes on its outstanding debt simply by borrowing more. At some point it will have to run primary surpluses or print money. The solvency constraint presumes a world where the real interest rate is not forever below the growth rate of capital, or, more generally, one in which the net marginal social product of capital is not forever below the growth rate of capital. In such a dynamically efficient world, Ponzi games are ruled out.

The solvency criterion is extremely weak. It can be satisfied even if the public debt-GDP ratio grows without bound, as long as the growth rate of the debt-GNP ratio is less than the excess of the interest rate over the growth rate of GNP! The criterion says that if there is a positive stock of debt outstanding, primary surpluses and/or positive seigniorage will have to occur in the future. In principle, one can make up for a thousand years of primary deficits "up front" with a subsequent ten thousand years of primary surpluses, but the time consistency, and therefore the credibility of such a strategy is obviously in doubt.

In addition, public debt-GDP ratios that rise without bound are only feasible if all of GDP and all of the interest bill on the outstanding stock of debt can be taxed away costlessly in lump-sum fashion. Since both collection costs and dead-weight losses are likely to rise with the tax burden, a finite upper bound on the public debt-GDP ratio has to be a key property of any feasible strategy. Nevertheless, it is not hard to come up with empirical examples of time series processes for the public debt that exhibit parameter constancy over extended periods of time and that would, if they were followed into the indefinite future, fail to satisfy even the very weak solvency criterion given in equations 32a and 32b (see, for example, Hamilton and Flavin [1986], Wilcox [1989], Grilli [1989], Buiter and Patel [1992], Corsetti [1991], Corsetti and Roubini [1991].

The presence of an infinite sum on the right side of equation 32b may seem to make it nonoperational: who can predict what interest rates, growth rates, primary deficits, and seigniorage will be a thousand years from now? In practice, with real interest rates well in excess of real growth rates in most developing, semi-industrialized, and former centrally planned economies, primary deficits and seigniorage in the distant future will not significantly affect the valuation of the right-hand side of 32b. Given the historical debt-GDP ratio $\bar{b}_{t-1}$ and projections of future interest rates and growth rates, one can use 32b to check whether future planned primary deficits and monetization satisfy the solvency constraint. If they do not, something will have to give. If interest rates, expected future growth, primary surpluses, or monetization cannot adjust, there will be a partial or complete default on the public debt, ensuring that, ex-post, the solvency constraint is satisfied (at the new, post default value of the public debt).

The future planned or expected seigniorage flows can, of course, be related to an inflation scenario, using the kind of models described in Section IV.

Note that, to prevent the solvency constraint from being completely toothless, the valuation of the outstanding public debt ($\bar{b}_{t-1}$) will have to be purged of any influence of the market's perception of the risk of default. (The solvency test would be rather too easily satisfied using the market's valuation of that debt if a country's external debt is valued at five cents on the dollar). In the case of very short-term debt with a fixed nominal face value or of debt with a variable interest rate, it is easy to determine what the valuation of that debt is absent default risk. In the case of long-dated fixed interest debt there may be problems, as the market value of such debt may vary (and may differ from its issue, par or redemption values) for reasons that have nothing to do with the perception by the market of default risk.

The interest rates used in checking whether the solvency constraint is satisfied (for the values of future primary surpluses and seigniorage assumed in a particular, possibly counterfactual, calculation) should be the "safe" rates exclusive of any default risk premia.

Some very easy steady-state "ready reckoners" can be obtained from the solvency constraint (or, indeed, from the budget identity in equation 30). The public debt-GDP ratio is stabilized during period t ($\bar{b}_t = \bar{b}_{t-1}$) if equation 33 holds.

(33) $$\sigma_t - \bar{d}_t = \frac{(r_t - g_t)}{1 + g_t}\,\bar{b}_{t-1}.$$

Also, for any given debt-GDP ratio $\underline{\bar{b}}$ (not necessarily the current one), and for any (gu)estimates of the long-run real interest rate $\underline{r}$ and the long-run growth rate of real GDP $\underline{g}$, one can calculate, as shown in equation 34, the permanent share of GDP that the government must appropriate, (either through an excess of current revenues over current outlays $-\underline{\bar{d}}$ or by running the printing presses $\underline{\sigma}$), in order to satisfy the solvency constraint given in equations 32a,b.

(34) $\underline{\sigma} - \underline{\bar{d}} = \frac{(\underline{r} - \underline{g})}{1+\underline{g}}\underline{\bar{b}}.$

If one has a firm long-run primary deficit target (or constraint) $\underline{\bar{d}}$, then steady-state seigniorage $\underline{\sigma}$ can be determined residually from equation 33. A model of the steady-state relationship between seigniorage and inflation can than be used to determine the long-run implications for inflation of an attempt to stabilize the public debt-GDP ratio at some particular value, for a given long-run primary surplus-GDP ratio.

Alternatively, one can fix the long-run rate of inflation, calculate the long-run seigniorage-GDP ratio this implies and so determine residually the permanent primary surplus-GDP ratio that will have to be generated to satisfy the solvency constraint for a given initial public debt-GDP ratio.

Note that the government's intertemporal budget constraint in equation 32b indeed represents a constraint on the single-period budget identities discussed in Sections II, III, and IV. The constraint is a weak one $\left(\lim_{j\to\infty} E_t \left\{\Omega_t^j \, \bar{b}_{t+j}\right\} \le 0\right)$, but one that is both well-motivated and, in principle, testable.

The final ready reckoner for the public sector is the present discounted value of the public debt itself. Useful information is contained in the behavior over time of $\Omega_t^j \, \bar{b}_{t+j}$. Looking at the behavior of the debt-GDP ratio discounted using real interest rates net of the growth rates of real GDP is, of course, equivalent to considering the behavior of the nominal market value of the public debt $\bar{B} = B - L - L^{cb} + E(B^* - R^*)$, discounted back to some initial date t, say, using the safe domestic nominal interest rate i.

(35) $\tilde{\mathfrak{Z}}_{t,t_0} \equiv \bar{B}_t \prod_{j=0}^{t-t_0-1} \left(\frac{1}{1+i_{t-j}}\right) \qquad t > t_0$

If there are many domestic nominal interest rates to choose from, the one most closely approximating the average safe cost of funds to the government should be used in equation 35.

A persistent increase in the discounted debt $\tilde{\mathfrak{Z}}_{t,t_0}$ should act as a red flag, signaling that a Ponzi game may be in progress: more is being borrowed than is required to pay the cost of servicing the existing debt. Good economic reasons may explain even a long sequence of increasing values of $\tilde{\mathfrak{Z}}_{t,t_0}$. It could, for example, reflect borrowing to finance public sector capital formation which will, directly or indirectly, result in large future government current revenue increases and primary surpluses that will be ultimately be associated with a declining discounted debt. The discounted debt process characterizing the historical sample would be a poor choice for out-of-sample forecasts in this case.

(b) National Solvency and the Substainability of the External Debt Process.

The solvency of the nation as a whole can be analyzed along the same lines as that of the government. To simplify notation, I define a number of new variables. q is the primary surplus or the noninterest current account surplus of the nation as a fraction of GDP:

$$q_t \equiv x_t - z_t^* + n_t^* \tag{36}$$

$\bar{q}$ is the augmented primary surplus or augmented national noninterest current account surplus, as a fraction of GDP:

$$\bar{q}_t \equiv q_t - div_t + dfi_t + \left[\frac{(1+r_t^*)(1+\gamma_t) - (1+r_t)}{1+g_t}\right]\left(\rho_{t-1}^* + f_{t-1}^* - b_{t-1}^*\right) \tag{37}$$

Note that for practical reasons (we do not in general have adequate data on the value of the stock of foreign direct investment or the rate of return on foreign direct investment), the augmented primary surplus defined in equation 37 includes the net flow of foreign direct investment, dfi, minus the value of the flow of earnings from foreign direct investment, div.

w denotes the value of the net external liabilities of the country, as a fraction of GDP. Note that the value of the stock of foreign direct investment in the country is not counted as a liability here. This is the counterpart to including in the definition of $\bar{q}$ given in equation 37 the net flow of foreign direct investment in the country, dfi, minus the value of foreign direct investment earnings remitted abroad, div.

(38) $$w_t \equiv b_t^* - \rho_t^* - f_t^*$$

Note that the augmented primary surplus adds to the conventionally measured primary surplus the excess of the actual earnings on net external assets over what these earnings would be if uncovered interest parity prevailed.

Equation 28 can be rewritten as

(39) $$\Delta w_t = -\bar{q}_t + \frac{(r_t - g_t)}{(1 + g_t)} w_{t-1}$$

The first ready reckoner for the national economy as a whole is the primary surplus that stabilizes the external debt ratio in period t. By setting $\Delta w_t = 0$ in equation (39) this is easily seen to be given by

(40) $$\bar{q}_t = \frac{(r_t - g_t)}{(1 + g_t)} w_{t-1}$$

Practitioners of country risk evaluation often take the external debt-to-exports ratio to be a good proxy for the relative magnitudes of a nation's external liabilities and its ability to service these liabilities. This seems unduly pessimistic and myopic. First, external debt can be serviced not just by exporting but also by import compression or import substitution. Second, actual exports need bear no relationship to the nation's potential production of exportable commodities, especially in the longer run when resources can be shifted from the production of nontraded goods to the production of traded goods. If the relative prices of traded and nontraded goods reflect the marginal social rate of transformation, the external debt-GDP ratio will be a better indicator than the external debt-exports ratio of how a nation's external liabilities stack up against its potential for servicing them.

Solvency of the nation as a whole requires that the condition given in equation 38 be satisfied. This then implies that 41 holds.

$$\lim_{j\to\infty} \mathcal{E}_t \left\{\Omega_t^j \, w_{t+j}\right\} \le 0 \tag{41}$$

$$w_{t-1} \le \mathcal{E}_t \left\{\sum_{j=0}^{\infty} \Omega_t^j \overline{q}_{t+j}\right\} \tag{42}$$

Equation 41 is the "no Ponzi game" condition that in the limit as t goes to infinity, the value of the discounted debt be nonpositive. If that is the case, the initial value of the debt will be no greater than the present discounted value of all future primary surpluses (equation 42).

This suggests that the discounted external debt be used as a red flag ready reckoner for the sustainability of the path of external indebtedness. Letting $W_t \equiv w_t P_t Y_t$ denote the value of the external financial indebtedness measured in domestic currency,

$$(43)\qquad \mathfrak{C}_{t,t_0} \equiv W_t \prod_{j=0}^{t-t_0-1} \left[\frac{1}{1+i_{t-j}}\right]$$

If the external interest rate rather than the domestic interest rate, is a better approximation of the average "safe" cost of servicing the external debt, equation 43 can be replaced by

$$(44)\qquad \mathfrak{C}^{*}_{t,t_0} \equiv W^{*}_t \prod_{j=0}^{t-t_0} \left[\frac{1}{1+i^{*}_{t-j}}\right]$$

where $W^{*} \equiv W/E$ is the value of the external debt measured in foreign currency.

Finally, let $\underline{\bar{q}}$ be defined by:

$$(45)\qquad \underline{\bar{q}} \equiv \frac{(\underline{r}-\underline{g})}{1+\underline{g}} w_{t-1}$$

$\underline{\bar{q}}$ is the constant ratio of the (augmented) primary external surplus to GDP that would achieve solvency of the nation, when the current net external debt-GDP ratio is wt-1. $\underline{\bar{q}}$ can be compared with the actual current (augmented) primary external surplus. GDP ratio, $\bar{q}_t$, and with the currently planned permanent (augmented) primary external surplus-GDP ratio $\bar{q}^{P}_t$, defined by

$$(46)\qquad \bar{q}^{P}_t \equiv \left(\frac{\underline{r}-\underline{g}}{1+\underline{g}}\right) \mathcal{E}_t \sum_{j=0}^{\infty} \Omega^{j}_t \, \bar{q}_{t+j}$$

$\bar{q} - \bar{q}^P$ measures the permanent increase in the augmented external primary surplus-GDP ratio that must be achieved to maintain external solvency.

(VII) Conclusion

The analysis in Sections II to IV can be viewed as an attempt to shortcut the awkward forward-looking evaluations inherent in analyses that take the government solvency constraint as their point of departure. The myopic methods of these earlier sections achieve this shortcut by specifying particular values for the future public debt-GDP ratio (or for its components). These values are likely not to be particularly well-motivated by considerations of optimality, desirability, or robustness.

The myopic methods of Sections II through V and the ready reckoners of Section VI may well be the best that are feasible given the constraints on data, formal modeling capacity and manpower that bedevil most attempts to do serious, yet timely, counterfactual analyses of important fiscal, financial, and monetary policy options. The user should, however, always be aware of the dangers involved in overestimating the significance of the "consistency checks," an approach that amounts to no more than a systematic "adding up check" on a small segment of a dynamic general equilibrium model. Similarly, the pitfalls involved in using a finite (and often quite short) time series to make inferences about the long-run behavior of an aggregate such as the discounted debt should be foremost in the mind of the practitioner.

Notation.

A:	public sector gross capital formation (in domestic currency).
$a \equiv A/(PY)$:	public sector gross capital formation as a proportion of GDP.
B:	short or variable interest rate public sector debt, denominated in domestic currency, held outside the central bank. For simplicity, it is assumed that this debt is not held by the rest of the world. It is measured at par or face value, and it includes accumulated arrears of interest and principal on the domestic debt, which for simplicity are assumed to carry the same interest rate i as the "regular" domestic debt.
$b \equiv B/(PY)$:	domestic non-monetary public debt (held outside the central bank) as a proportion of GDP.
$\bar{b}$:	net nonmonetary financial debt of the consolidated public sector and central bank. $\bar{b} \equiv b^a + b^* - \rho^*$
$b^a \equiv b - \ell - \ell^{cb}$:	adjusted non-monetary domestic debt as a proportion of GDP.
B^{cb}:	Central bank holdings of non-central bank government sector debt.
$b^{cb} \equiv B^{cb}/(PY)$:	non-central bank public sector debt held by the central bank as a proportion of GDP. (Central bank domestic credit to the public sector as a proportion of GDP.)
B^*:	short or variable interest public debt, denominated in foreign currency, held outside the central bank. It is measured at par or face value, and it includes accumulated arrears of principal and interest on the foreign debt, which are for simplicity assumed to carry the same interest rate i* as the "regular" foreign debt.
$b* \equiv EB^*/(PY)$:	foreign debt as a proportion of GDP.
$\tilde{\mathbf{3}}_{t,t_0}$:	net government nonmonetary debt in period t, discounted back to period t_0 .

C: public sector current expenditure (procurement and wages and salaries) in domestic currency.

$c \equiv C/(PY)$: public sector consumption spending as a proportion of GDP.

CU: nominal stock of currency held outside the central bank.

$cu \equiv CU/(PY)$: currency in the hands of the public as a proportion of GDP.

γ: proportional rate of depreciation of the real exchange rate. $1+\gamma_t \equiv (1+e_t)(1+\pi^*_t)/(1+\pi_t)$

D: primary (noninterest) financial deficit of the consolidated public sector and central bank (in domestic currency).

$d \equiv D/(PY)$: public sector primary deficit as a proportion of GDP.

$\tilde{D}$: primary (noninterest financial deficit of the non-central bank public sector (in domestic currency).

$\tilde{d} \equiv \tilde{D}/(PY)$: non-central bank public sector primary deficit as a proportion of GDP.

$\bar{d}$: augmented primary deficit of the consolidated public sector and central bank.

$$\bar{d}_t \equiv d^a_t - (r^*_t + \gamma_t + r^*_t\gamma_t - r_t)\frac{\left(b^*_{t-1} - \rho^*_{t-1}\right)}{1+g_t}$$

Da: adjusted public sector deficit.

$$D^a_t \equiv D_t + (i_t - i^l_t)L_{t-1} + (i_t - i^{l_{cb}}_t)L^{cb}_{t-1}$$

$d^a \equiv D^a/(PY)$: adjusted public sector primary deficit as a proportion of GDP.

DCE: domestic credit expansion (measured in domestic currency).

dd: demand deposits as a proportion of GDP.

DFI: direct foreign investment (measured in domestic currency).

dfi: direct foreign investment as a fraction of GDP. $dfi \equiv DFI/(PY)$.

DIV:	earnings remitted abroad by foreign-owned firms (measured in domestic currency).
div:	remittances abroad by foreign-owned companies as a fraction of GDP. $div \equiv DIV/(PY)$.
E:	spot nominal exchange rate (number of units of domestic currency per unit of foreign currency).
ϵ:	proportional rate of change of the exchange rate. $\epsilon_t \equiv (E_t - E_{t-1})/E_{t-1}$.
$\hat{\epsilon}$:	expected proportional rate of exchange rate depreciation.
F*:	private stock of foreign assets (in foreign currency).
f*:	private stock of foreign assets as a proportion of GDP.
g:	proportional growth rate of GDP. $g_t \equiv (Y_t - Y_{t-1})/Y_{t-1}$.
H:	nominal stock of high-powered, outside, or base money (in domestic currency).
$h \equiv H/(PY)$:	base money as a proportion of GDP.
$\tilde{h}$:	(long-run) desired ratio of base money to GDP.
θ^{dd}:	fractional reserve requirement for demand deposits.
θ^{td}:	fractional reserve requirement for time deposits.
i:	nominal interest rate on B.
i^{cb}:	nominal interest rate on Bcb.
i^{dd}:	nominal interest rate on demand deposits.
i^{L}:	nominal interest rate on L.
i^{Lcb}:	nominal interest rate charged on central bank lending to the private sector.
i^{td}:	nominal interest rate on time deposits.
i*:	nominal interest rate on B*.

L: stock of non-central bank public sector loans to the private sector (denominated in domestic currency).

L^{cb}: stock of central bank loans to the private sector (denominated in domestic currency).

$\ell \equiv L/(PY)$: loans by non-central bank public sector loans to the private sector as a proportion of GDP.

$\ell^{cb} \equiv L^{cb}/(PY)$: loans by the central bank to the private sector as a proportion of GDP.

μ: proportional growth rate of the stock of nominal base money. $\mu_t \equiv (H_t - H_{t-1})/H_{t-1}$.

N*: foreign aid (in foreign currency).

$n^* \equiv EN^*/(PY)$: foreign aid as a proportion of GDP.

NW: net worth of the central bank.

P: domestic GDP deflator.

P*: foreign GDP deflator.

π: proportional rate of change of P (rate of inflation). $\pi_t \equiv (P_t - P_{t-1})/P_{t-1}$.

$\hat{\pi}$: expected rate of inflation.

$\tilde{\pi}$: core rate of inflation.

π^*: foreign rate of inflation. $\pi^*_t \equiv (P^*_t - P^*_{t-1})/P^*_{t-1}$.

Q: noninterest current account surplus or primary surplus on the balance of payments. $q \equiv Q/PY$.

$\bar{q}$: augmented primary surplus on the balance of payments, as a fraction of GDP.

$$\bar{q}_t \equiv q_t - div_t + dfi_t + \left[\frac{(1+r^*_t)(1+\gamma_t) - (1+r_t)}{1+g_t}\right]\left(\rho^*_{t-1} + f^*_{t-1} - b^*_{t-1}\right)$$

R*: official gold and foreign exchange reserves held by the central bank (in foreign currency). They are assumed to earn an interest rate i*.

r: real interest rate on domestic debt. $1 + r \equiv (1+i)/(1+\pi)$

r^*: foreign real interest rate.

$$1 + r^* \equiv (1 + i^*)/(1 + \pi^*).$$

r^L: real interest rate on Treasury lending to the private sector.

$$1 + r^L \equiv (1 + i^L)(1 + \pi)^{-1}$$

$r^{L_{cb}}$ real interest rate on central bank lending to the private sector.

$$1 + r^{L_{cb}} \equiv (1 + i^{L_{cb}})(1 + \pi)^{-1}$$

$\rho^* = ER^*/(PY)$: international reserves as a proportion of GDP.

RR: nominal stock of required reserves held at the central bank by the commercial banks.

$\rho\rho \equiv RR/(PY)$: reserves held by the commercial banking system with the central bank as a proportion of GDP.

$\rho\rho^{dd}$: reserves backing demand deposits as a fraction of GDP.

$\rho\rho^{td}$: reserves backing time deposits as a fraction of GDP.

S: profits of public sector enterprises (in domestic currency).

$s \equiv S/(PY)$: profits from the public sector enterprises as a proportion of GDP.

σ: "seigniorage" as a proportion of GDP; change in the nominal stock of base money, expressed as a proportion of GDP.

$$\sigma_t \equiv (H_t - H_{t-1})/(P_t Y_t)$$

T: current revenues net of current transfers, subsidies, etc., excluding payments from the central bank, foreign aid, and profits from public sector enterprises (in domestic currency).

T^{cb}: current payments by the central bank to the Treasury (in domestic currency).

td: time deposits as a proportion of GDP.

$\tau \equiv T/(PY)$: taxes net of transfers as a proportion of GDP.

$\tau^{cb} \equiv T^{cb}/(PB)$: payments from the central bank to the Treasury as a proportion of GDP.

V: income velocity of circulation of base money. $V_t \equiv P_t Y_t / H_{t-1}$.

W: net external liabilities of the country. $W_t \equiv E_t(B^*_t - R^*_t - F^*_t)$

$w \equiv W/PY$ net external liabilities of the country as a proportion of GDP

ω: private financial wealth as a proportion of GDP.

$\mathfrak{E}_{t,t_0}$: external debt of the nation in period t, discounted back to period t_0.

X: exports of goods and services (excluding foreign factor income) in domestic currency.

x: exports as a fraction of GDP. $x \equiv X/(PY)$.

Y: real GDP.

Z*: imports of goods and services (excluding factor income paid abroad) in foreign currency.

z*: imports as a fraction of GDP. $z^* \equiv EZ^*/(PY)$.

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