Journal of International Economics 23 (1987) 221–239. North-Holland

BORROWING TO DEFEND THE EXCHANGE RATE AND THE TIMING AND MAGNITUDE OF SPECULATIVE ATTACKS

Willem H. BUITER*

Yale University, New Haven, CT 06520, U.S.A.
NBER and CEPR

Received March 1986, revised version received January 1987

The paper analyses the implications for the timing and magnitude of a speculative attack on a fixed exchange rate of government borrowing to increase the stock of international reserves. It is shown that such borrowing reduces the likelihood of an early collapse and increases the likelihood of an eventual collapse. The magnitude of the speculative attack also increases.

1. Introduction

The purpose of this paper is to extend the recent literature on collapsing managed exchange rate regimes by allowing explicitly for the government budget constraint and especially for the interest cost of servicing the public debt. The seminal paper of Salant and Henderson (1978) analysed the implications of government attempts to peg the price of gold or to defend a price ceiling by managing a stockpile; it demonstrated the inevitability of an eventual speculative attack and of the collapse of such schemes. Krugman (1979) provided the first application of this analytical approach to the macroeconomic problem of defending a fixed exchange rate parity. Since then there have been many other, mainly theoretical, studies of the viability of a variety of managed exchange rate regimes and of the nature and likelihood of speculative attacks. [See, for example, Flood and Garber (1983, 1984a, 1984b), Obstfeld (1984, 1986a, 1986b), Connolly and Taylor (1984) and Dornbusch (1984)]. Related relevant theoretical work on speculative attacks, runs, etc. can be found in Salant (1983) which analyses price stabilization schemes and Diamond and Dybvig (1983) which deals with bank runs. Empirical work in this tradition is still relatively scarce, but notable examples are Cumby and van Wijnbergen (1983), Grilli (1984, 1986b), Collins (1984) and Garber and Grilli (1986).

*I would like to thank Sweder van Wijnbergen for many useful discussions on the subject of fiscal policy and the viability of managed exchange rate regimes. An anonymous referee made many helpful suggestions.

The existing literature almost invariably treats domestic credit expansion (dce) as the exogenous government control and fundamental forcing variable driving the stock of reserves and/or the exchange rate. [One notable exception is Cumby and van Wijnbergen (1983).] This paper 'goes behind' dce and considers separately the primary (non-interest) government deficit, the interest cost of servicing the public debt and the net issues of interest-bearing government debt. By doing this, the otherwise implicit fiscal and financial aspects of decisions to alter the level and/or the rate of growth of the stock of domestic credit are brought out explicitly. While this is a worthwhile exercise in its own right, it is especially useful when, as in Garber and Grilli (1986) and Grilli (1986b), the consequences for the likelihood and timing of a speculative attack on the exchange rate of a government decision to borrow (internationally) to augment its stock of reserves is analysed.

In sections 2 and 3 I analyse the consequences of a decision by the government to replenish its stock of non-interest-bearing foreign exchange reserves through a once-off, stock-shift open market sale of interest-bearing bonds. Without a fundamental fiscal correction (i.e. a decision to cut the primary deficit by an amount equal to the increase in the interest cost of servicing the debt), the consequences of this decision to borrow for the timing and magnitude of the speculative attack that causes the collapse of the fixed exchange rate regime are as follows. In the deterministic, continuous time model of section 2, the timing of the speculative attack is brought forward if the borrowing takes place sufficiently long before the date at which the collapse would have occurred absent the borrowing. It is delayed if the borrowing takes place sufficiently close to the collapse date without borrowing. The magnitude of the attack (measured by the size of the final stock-shift reduction in the stock of reserves to their critical level) always increases as a result of the borrowing. In the stochastic discrete time model of section 3, the probability of collapse k periods or less after the borrowing falls for small k and increases for large k. As in the deterministic case this reflects the fact that, because of the interest cost of servicing the newly incurred debt, an act of borrowing reduces the *level* of the stock of domestic credit (increases the *level* of the stock of reserves) but also increases the rate of domestic credit expansion (increases the rate at which the stock of reserves declines). Finally, in an interesting special case the expected length of the time interval until the collapse is, under mild conditions, increased by an act of borrowing.

2. Government borrowing and the timing and magnitude of speculative attacks; the continuous time deterministic case

The simple continuous time deterministic small country open economy model is given in eqs. (1)–(4):

$$\frac{M}{SP^*} = l(i, y), \quad l_i < 0;\ l_y > 0, \tag{1}$$

$$P = SP^*, \tag{2}$$

$$i(t) = i(t^*) + \frac{E_t \dot{S}(t)}{S(t)}, \quad i^* > 0, \tag{3}$$

$$\dot{M} + S\dot{B}^* - S\dot{R}^* \equiv \Delta + i^*SB^*. \tag{4}$$

M is the nominal stock of non-interest-bearing domestic high-powered money, i the domestic nominal interest rate, y domestic real output, P the domestic price level, P^* the foreign price level, S the spot nominal exchange rate, i^* the foreign nominal interest rate, B^* the stock of government debt, R^* the stock of official foreign exchange reserves and Δ the nominal primary, i.e. net of interest on the public debt, deficit. Foreign exchange reserves are assumed to be non-interest-bearing, as was the case with gold under the gold standard.

P^*, i^* and y are exogenous. There is a single traded good and strict purchasing power parity (PPP) holds [eq. (2)]. There is a perfect international financial market with risk-neutral speculators. The domestic nominal interest rate is therefore given by the uncovered interest parity condition (3). E_t denotes the expectation operator conditional on information available at time t. (4) denotes the open economy consolidated government sector budget identity. All government borrowing is assumed to be denominated in foreign currency. This is for convenience only. The paper is not concerned with the use of (unexpected) devaluation as a means of reducing the real value of the authorities' interest-bearing debt. For that issue the currency denomination of the debt is of course crucial. Issues of political or sovereign risk are also ignored. Δ is the current value of public sector 'exhaustive' spending on goods and services minus taxes net of transfers, excluding interest on the debt, i^*SB^*.

Using the monetary authority's balance sheet identity we can reinterpret (4) as the familiar condition that the government deficit $\Delta + i^*SB^*$ is financed by borrowing $S\dot{B}^*$ or domestic credit expansion $\dot{D}$, i.e.

$$\dot{D} + S\dot{B}^* \equiv \Delta + i^*SB^*. \tag{4'}$$

A crucial assumption is that the interest rate paid on government debt, $i^* > 0$, exceeds the interest rate on international reserves, r^*. Given that assumption, the further assumption made in this paper that the interest rate on reserves equals zero, only serves to simplify the algebra. If the two interest rates were equal, there would, even in an only minimally rational world, never be any foreign exchange crises (or liquidity crises) which were not also government solvency crises. The solvency constraint of the government is given in eq. (5). It is obtained by integrating the government budget identity [including interest on reserves, given in (4″)] forward in time and imposing a 'no Ponzi game' transversality condition:[1]

$$\dot{M} + S\dot{B}^* - S\dot{R}^* \equiv \Delta + i^*SB^* - r^*SR^*. \tag{4''}$$

$$S(t)[B^*(t) - R^*(t)] \leqq \int_t^{\infty} \mathrm{E}_t\left\{\exp\left(-\int_t^{v} i^*(u)\,\mathrm{d}u\right) \times [-\Delta(v) + \dot{M}(v) - (i^*(v) - r^*(v))S(v)R^*(v)]\right\}\mathrm{d}v. \tag{5}$$

Eq. (5) states that the value of the government's net non-monetary debt should not exceed the present value of future expected primary surpluses $-\Delta(v)$, plus the present value of future expected seigniorage revenue $\dot{M}(v)$ minus the present value of the expected future cost of holding reserves $(i^*(v) - r^*(v))S(v)R^*(v)$. Clearly, if $i^* = r^*$, the opportunity cost of holding reserves is zero. A successful open market sale by the government [equal increases in $B^*(t)$ and $R^*(t)$] which raises the future expected path of the stock of reserves [$\mathrm{E}_t\delta R^*(v) \geqq 0$ for all $v \geqq t$][2] will not, in that case, affect the solvency of the government; the r.h.s. of (5) is unaffected by it. If, however, $i^* > r^*$, then borrowing to replenish the stock of reserves will worsen the government's solvency position. If (5) holds with strict equality, then either future primary surpluses must be raised, or future seigniorage revenue must be boosted in order to avoid insolvency. With $i^* = r^*$, any sensible government would raise both B^* and R^* to arbitrarily high levels, thus eliminating the risk of running out of reserves, without this in any way affecting their solvency. $i^* > r^*$ is therefore a necessary condition for there to be a reserve problem separate from a solvency problem. With $i^* = r^*$, debt repudiation or default would accompany any foreign exchange crisis. The assumption that $i^* > r^*$ is historically appropriate for the gold standard regime analysed by Garber and Grilli (1985) and Grilli (1986b). In a sterling–dollar world, with a fixed dollar price of gold and a fixed sterling price of gold, $r^* = 0$ while sterling and

[1] This condition is

$$\lim_{v \to \infty} \exp\left(-\int_t^{v} i^*(u)\,\mathrm{d}u\right) S(v)[B^*(v) - R^*(v)] = 0.$$

See, for example, Buiter (1985).

[2] Note $S(v) = \bar{S}$ for $v \leqq \bar{t}$, where $\bar{t}$ is the date of collapse and $R^*(v) = 0$, $v \geqq \bar{t}$. $i^*(v)$ and $r^*(v)$, $v < \bar{t}$ are unaffected by the open market sale. A successful open market sale raises $\bar{t}$.

dollar short nominal interest rates were positive. Where modern financial developments have greatly reduced or even eliminated the financial opportunity cost of holding reserves, the analysis of this paper has to be qualified, since in the limit as r^* goes to i^* a foreign exchange crisis is merely one manifestation of a solvency crisis.

Initially the economy is on a fixed exchange rate with $S=\bar{S}$. When the stock of reserves falls below a threshold level $\bar{R}^*$ the authorities stop defending the exchange rate and a free float of indefinite duration ensues. Other scenarios have been analysed [e.g. by Obstfeld (1986a)] but for our purposes the simplest case suffices. The threshold level is set equal to zero. Following the now familiar argument that efficient financial markets rule out anticipated future discrete or discontinuous changes in the level of the exchange rate, we can calculate the date $t=\bar{t}$ at which the fixed exchange rate regime collapses and a free float begins, from the boundary condition that at $\bar{t}$ the exchange rate that would prevail if the exchange rate collapsed and floated at $\bar{t}$, $\tilde{S}(\bar{t})$, equals the fixed exchange rate $\bar{S}$. The example is chosen such that reserves are lost continuously (the 'structurally weak' currency case) and a collapse in finite time is certain. Grilli (1986a) analyses a more symmetric small country model for which in addition to the lower bound on reserves (which when crossed compels a float or a devaluation) there also is an upper bound (which when crossed compels a float or a revaluation). In a two-country model the same results could be obtained with each national authority establishing a lower bound for its stock of reserves.

In the example, when the fixed exchange rate regime collapses, the expected proportional rate of exchange rate depreciation becomes positive. The domestic nominal interest rate therefore increases discretely at $\bar{t}$ and there is a discrete ['stock-shift'] reduction in the demand for real money balances. With $S(\bar{t})$ given at $\bar{S}$ (because of the required continuity of the exchange rate) the reduction in the real money stock is brought about by a reduction in the nominal money stock. The stock-shift reduction in the money stock $\delta M(\bar{t})$ is brought about by a speculative attack at $\bar{t}$ in which the stock of reserves undergoes a stock-shift reduction to its critical level. The total stock of domestic claims on the rest of the world cannot, of course, change at a point in time (barring repudiation). The stock-shift loss of reserves by the authorities is therefore matched by domestic private agents acquiring interest-bearing claims on the rest of the world.

The literature has traditionally treated domestic credit expansion $\dot{D}$ as the fundamental (exogenous) forcing variable. While there is nothing logically wrong with such a specification, it suppresses and therefore tends to obscure the *fiscal* and debt management aspects of exchange rate regime viability and breakdown. The importance of this issue is greatest when the policy event under consideration is a major financial operation of the government. An example of such an event is contained in two stimulating papers by Garber and Grilli (1986) and Grilli (1986b) on the Belmont–Morgan Syndicate bond issue and its role in defending the gold standard in 1895. Briefly their argument is that when the U.S. Treasury borrowed abroad through the Syndicate to replenish its stock of gold reserves, this act of borrowing increased the viability of the gold standard in the sense that it reduced uniformly the probability of collapse after the loan was secured, compared to what would have been the case without the loan.

The analysis of this section and of the next demonstrates that the Garber–Grilli argument implicitly assumes that the act of borrowing was accompanied by a *fiscal correction*, specifically by a reduction in the primary deficit which kept dce after the borrowing on the path it would have been on without the borrowing. Without such a fiscal correction the need to service the additional debt requires either increased domestic credit expansion or further borrowing. If there exists an upper bound on the stock of interest-bearing debt a government is able or willing to countenance, increased borrowing now means increased domestic credit expansion later and therefore increased loss of reserves and a greater probability of collapse later. This argument is of course simply an open economy extension of Sargent and Wallace's 'Unpleasant Monetarist Arithmetic'.[3]

For simplicity I consider the case in which the authorities engage in one act of borrowing, at $t=t_0$. At that date there is a stock-shift, once-off open market sale of bonds. Given perfect capital mobility, it is immaterial whether we visualize the government as borrowing at home or borrowing abroad. The net effect at t_0 is a stock-shift reduction in the stock of domestic credit and an equal increase in the stock of reserves and in the stock of interest-bearing public debt.

After the open market sale, no further borrowing occurs. This may well be an acceptable stylized representation of the situation in the late 1890s when government debt issues were very much the exception to the rule. It may also describe the situation of a number of developing and semi-industrialized economies that are faced with an external credit constraint and are given the option of a once-off relaxation of that constraint. Because of the higher debt service component in the public sector deficit, the rate of domestic credit expansion will be higher after t_0, than it would have been without the borrowing, if the primary deficit path is unaffected by the borrowing. The authorities effectively purchase a once-off reduction in the stock of domestic credit at t_0 for a permanently higher rate of domestic credit expansion after t_0. Equivalently, they obtain a once-off increase in the stock of reserves at t_0 for a higher rate of reduction in the stock of reserves after t_0 and until the fixed exchange regime collapses at $t=\bar{t}$.[4]

[3] I assume throughout that the long-run real interest rate exceeds the long-run growth rate of the real tax base, to rule out feasible Ponzi games.

[4] $\bar{t}$ itself, as shown below, will be affected by the open market sale.

In the continuous time, deterministic model of this section the effect of the open market sale on the timing of the collapse is ambiguous. If the borrowing occurs just before the exchange rate regime would have collapsed absent the borrowing, the collapse is postponed ($\bar{t}$ increases). If the borrowing occurs long enough before the exchange rate regime would have collapsed in the absence of borrowing, the collapse is brought forward ($\bar{t}$ declines). In the latter case the higher rate of decline of reserves dominates the once-off increase in the level of the stock of reserves; in the former case the opposite holds.

In the discrete time stochastic model of section 3, the probability at t_0 of collapse in or before period t_0+i, $i=1,2,\ldots$, falls as a result of borrowing at t_0 for small i, but rises for large i. The intuition again is strong, a stochastic version of 'live now and pay later'. For an act of borrowing to lower uniformly the probability of collapse it must be accompanied by a fiscal correction, i.e. by a reduction in the primary deficit.

As long as the fixed exchange rate regime survives, the behaviour of the stock of reserves, except at those instants that the monetary authorities engage in open market operations, is given by

$$\bar{S}\dot{R}^* = -(\Delta + i^*\bar{S}B^*) + \gamma + \bar{S}\dot{B}^*, \tag{6a}$$

$$\gamma \equiv pl_i\dot{i}^* + pl_y\dot{y} + \frac{\dot{p}^*}{p^*}M, \text{ the determinants of the growth in demand for money balances.}[5] \tag{6b}$$

In order to assure that the exchange rate regime is headed for collapse it is assumed that $\Delta + i^*\bar{S}B^* - \gamma > 0$. Since only one stock-shift open market sale of bonds at t_0 is considered, $\dot{B}^* = 0$ for all $t > t_0$. Domestic credit expansion therefore exceeds the growth of money demand and a collapse is certain. What remains to be determined is the timing and the magnitude of the speculative attack that forces the abandonment of the fixed exchange rate regime and the way in which both timing and magnitude are affected by borrowing without a fundamental fiscal correction.

We first calculate the shadow floating exchange rate at time t, $\tilde{S}(t)$, i.e. what the exchange rate would be at t if it floated freely for all future time with R^* at its critical value zero, i.e. with $M = D$.

A linear approximation of the model gives us the equation of motion for

[5]From eq. (4), $\bar{S}\dot{R} = -(\Delta + i^*\bar{S}B^*) + \bar{S}\dot{B}^* + \dot{M}$. Since money demand equals money supply, we have, differentiating the money demand function in eq. (1),

$$\dot{M} = Sp^*l_i\frac{\mathrm{d}}{\mathrm{d}t}i + Sp^*l_y\dot{y} + \left[\frac{\dot{S}}{S} + \frac{\dot{p}^*}{p^*}\right]M.$$

As long as the fixed exchange rate regime survives, $\dot{S}/S = 0$ and $i = i^*$. Since $p = Sp^*$, substitution of the expression for $\dot{M}$ into the balance-of-payments identity yields eqs. (6a) and (6b).

the expected exchange rate:

$$\mathrm{E}_t\dot{\tilde{S}}(t)=\alpha_s\tilde{S}(t)-\alpha_M D(t)+z(t) \tag{7}$$

where

$$\alpha_s=-\left[l_i^{-1}\frac{D}{P}\right]_0>0;\qquad \alpha_M=-\left[\frac{l_i^{-1}}{P^*}\right]_0>0,$$

and

$$z(t)=-\left[\left[l_i^{-1}\frac{D}{P}\frac{\tilde{S}}{P^*}\right]_0 P^*(t)+[\tilde{S}l_i^{-1}l_y]_0 y(t)+[\tilde{S}]_0 i^*(t)\right].$$

The solution for the exchange rate is given by[6]

$$\tilde{S}(t)=\int_t^{\infty}\mathrm{e}^{\alpha_s(t-u)}\,\mathrm{E}_t[\alpha_M D(u)-z(u)]\,\mathrm{d}u. \tag{8}$$

The shadow floating rate is the 'present discounted value' of future expected fundamentals. The fundamentals are future expected money stocks (or stocks of domestic credit) and the future expected determinants of money demand $z(\cdot)$, i.e. the future foreign price level, the future foreign interest rate and future real output.

The fixed exchange rate regime will collapse and a free float will commence at $t=\bar{t}$, if and only if

$$\tilde{S}(\bar{t})=\bar{S}. \tag{9}$$

Let the foreign interest rate i^* and the primary government deficit Δ be constant over time, i.e.

$$D(t)=D(t_0)+[\Delta+i^*\bar{S}B^*(t_0)](t-t_0).^7 \tag{10}$$

At t_0 there is a stock-shift open market purchase and thereafter B^* is kept constant.

From (8), (9) and (10) we find that

$$\bar{S}=\frac{\alpha_M}{\alpha_S}\left[D(t_0)+[\Delta+i^*\bar{S}B^*(t_0)][\bar{t}-t_0]+\frac{\Delta+i^*\bar{S}B^*(t_0)}{\alpha_S}\right]+Z(\bar{t}), \tag{11}$$

[6] We choose the unique continuously convergent solution.

[7] I ignore as of second-order magnitude the effect of changes in the exchange rate on debt service. Since S is endogenous from t_0 on, $D(t)$ is, strictly speaking, given by

$$D(t)=D(t_0)+(\Delta+i^*\bar{S}B^*(t_0))(t-t_0)+i^*B^*(t_0)\int_{\bar{t}}^{t}(S(v)-\bar{S})\,\mathrm{d}v.$$

where

$$Z(\bar{t}) = -\int_{\bar{t}}^{\infty} \mathrm{E}_t z(u) \mathrm{e}^{-\alpha_s(u-\bar{t})} \mathrm{d}u.$$

The interval between the open market purchase and the date of collapse is therefore given by

$$\bar{t} - t_0 = \frac{\bar{S} - \frac{\alpha_M}{\alpha_S} D(t_0) - Z(\bar{t})}{[\Delta + i^* \bar{S} B^*(t_0)] \frac{\alpha_m}{\alpha_s}} - \frac{1}{\alpha_s}. \tag{12}$$

The open market sale reduces $D(t_0)$ by the same amount it increases $\bar{S}B^*$ and $\bar{S}R^*$. The effect of the borrowing on the timing of the collapse is therefore given by[8]

$$\left. \frac{\partial(\bar{t} - t_0)}{\bar{S}\, \partial B^*(t_0)} \right|_{\mathrm{OM}} = \frac{1}{\Delta + i^* \bar{S} B^*(t_0)} - \frac{\left[\bar{S} - \frac{a_M}{\alpha_s} D(t_0) - Z(\bar{t}) \right] i^*}{[\Delta + i^* \bar{S} B^*(t_0)]^2 \frac{\alpha_M}{\alpha_S}}$$

$$= \frac{1}{\Delta + i^* \bar{S} B^*(t_0)} \left[\frac{\alpha_s - \alpha_s i^* (\bar{t} - t_0) - i^*}{\alpha_s} \right]. \tag{13}$$

The first term on the right-hand side of (13) measures the beneficial effect of having a lower stock of domestic credit and acquiring a larger stock of reserves at t_0. It obviously postpones the day of collapse. The second term represents the effect of the increased rate of domestic credit expansion after t_0 due to the increased debt service. It brings forward the day of collapse.

Assuming, as we do, that $\Delta + i^* \bar{S} B^* > 0$ and $i^* > 0$, the condition for $\bar{t} - t_0$ to increase or decrease with an open market sale at t_0 can be rewritten as

$$\left. \frac{\partial(\bar{t} - t_0)}{\bar{S}\, \partial B^*(t_0)} \right|_{\mathrm{OM}} \gtrless 0 \Leftrightarrow \frac{1}{i^*} - \frac{1}{\alpha_S} \gtrless \bar{t} - t_0. \tag{14}$$

[8] $\left. \frac{\partial(\bar{t} - t_0)}{\bar{S}\, \partial B^*(t_0)} \right|_{\mathrm{OM}}$

denotes

$$\left. \frac{\partial(\bar{t} - t_0)}{\bar{S}\, \partial B^*(t_0)} \right|_{\bar{S}\,\mathrm{d}B^*(t_0) = -\mathrm{d}D(t_0)}.$$

Thus, the mathematics bears out the intuition that if reserves are replenished by an open market sale just before the regime would have collapsed, the date of collapse is postponed. If the replenishment takes place far enough in advance of the collapse date without replenishment, the date of collapse is actually brought forward.

Fig. 1 illustrates the time paths of the stock of reserves with and without borrowing at t_0. Clearly, the open market sale tilts the reserve path: it shifts it up vertically by $\delta R^*(t_0)$ at t_0 and increases the steepness of the negatively sloped path after t_0. $\bar{t}$ is the date of collapse without borrowing at t_0 and $R^*(\bar{t}_-)$ the stock of reserves the instant before the speculative attack at $\bar{t}$.

Fig. 1(a) represents the case where $\bar{t}-t_0$ increases as a result of govern-

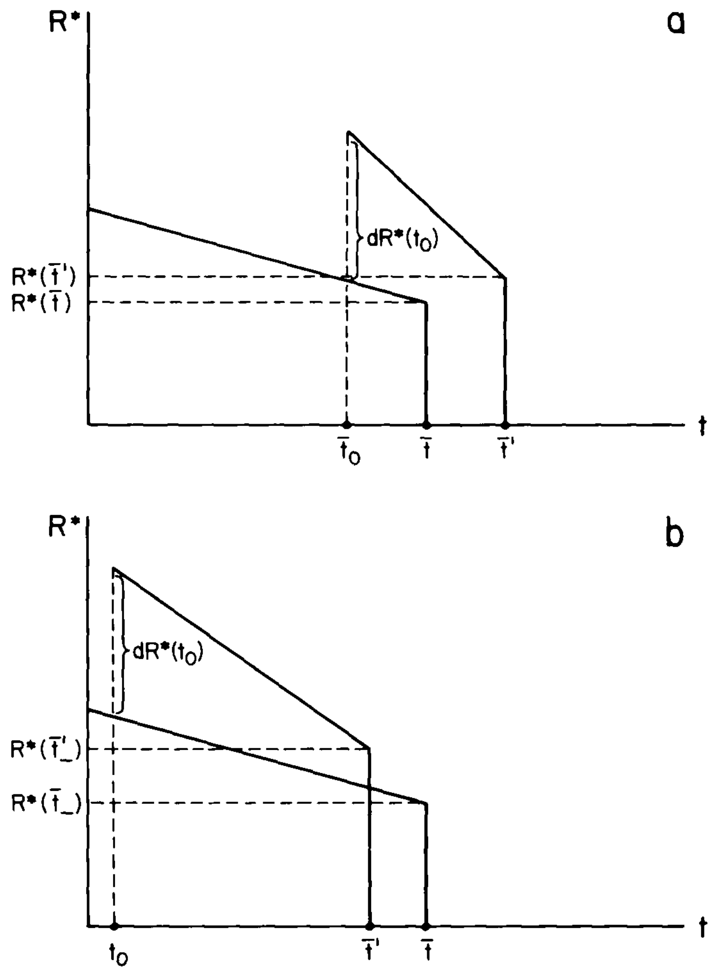

Fig. 1. Borrowing and the timing and magnitude of the collapse of a fixed exchange rate regime.

ment borrowing at t_0 and fig. 1(b) the case where the attack is brought forward.

To complete the picture we must determine what happens to the stock of reserves at $\bar{t}_-$, just before the collapse, as a result of the open market sale. The answer is known once we known what happens to the nominal interest rate at $\bar{t}$. At $\bar{t}_-$, $i(\bar{t}_-)=i^*$, with or without borrowing. At $\bar{t}$, $i(\bar{t})=i^*+\mathrm{E}_{\bar{t}}[\dot{\tilde{S}}(t)/\tilde{S}(\bar{t})]$. If the proportional rate of exchange rate depreciation $\dot{\tilde{S}}/\tilde{S}$ is higher (lower) at $\bar{t}$ with borrowing at t_0 than without it, then the stock of reserves at $\bar{t}_-$ (and the collapse to zero of the stock of reserves at $\bar{t}$) will be larger (smaller) with borrowing than without it. It is easily checked that

$$\left.\frac{\partial\dot{\tilde{S}}(\bar{t})}{\bar{S}\,\partial B^*(t_0)}\right|_{\mathrm{OM}}=\frac{\alpha_M i^*}{\alpha_S}>0. \tag{15}$$

Since $\tilde{S}(\bar{t})=\bar{S}$, the proportional rate of exchange rate depreciation at the moment of collapse is raised by borrowing at t_0, regardless of whether the collapse is postponed or brought forward. Borrowing at t_0 therefore increases the magnitude of the speculative attack as measured by the stock-shift loss of reserves at $\bar{t}$.

For a country headed towards an unwanted devaluation, the analysis has the following implications. First, do not borrow too soon, for without any reduction in the primary deficit, too early borrowing may precipitate the collapse. Second, 'last minute' borrowing to defend the exchange rate is useful if the time it buys is used to implement a superior fundamental fiscal correction, i.e. a more effective package of measures for reducing the primary deficit or to invest in a future reputation for fiscal restraint through a current demonstration of fiscal restraint.

3. Government borrowing and the timing of a speculative attack; the discrete time stochastic case

The discrete time analogue of the model of eqs. (1)–(4) is given by

$$\frac{M(t)}{S(t)P^*(t)}=l[i(t),y(t)], \tag{16}$$

$$P(t)=S(t)P^*(t), \tag{17}$$

$$1+i(t)=\frac{1+i^*(t)}{S(t)}\mathrm{E}_t S(t+1), \tag{18}$$

$$M(t+1)-M(t)+S(t)[B^*(t+1)-B^*(t)]-S(t)[R^*(t+1)-R^*(t)]$$
$$\equiv \Delta(t)+i^*(t)S(t)B^*(t). \tag{19}$$

As before, t_0 is the period during which an open market purchase is performed and $\tilde{S}(t)$ is the shadow floating exchange rate at time t.

We first define and derive the probability, at t_0, that the fixed exchange rate regime collapses in period $t > t_0$ (neither earlier nor later) and the probability, at t_0, that it collapses no later than period $t > t_0$. The probability of collapse in period t is denoted $\pi(t, t_0)$ and the probability of collapse in or before period t is denoted $\Pi(t, t_0)$. Formally:

$$\pi(t, t_0) \equiv \mathrm{Prob}_{t_0}[\tilde{S}(t) \geqq \bar{S} \text{ and } \tilde{S}(t-i) < \bar{S};\ 1 \leqq i < t - t_0] \tag{20}$$

and

$$\Pi(t, t_0) \equiv \sum_{j=t_0+1}^{t} \pi(j, t_0) = \mathrm{Prob}_{t_0}(\tilde{S}(t) \geqq \bar{S}).^{9} \tag{21}$$

Having calculated $\Pi(t, t_0)$ we can investigate how it is influenced by an open market sale. Intuition suggests that the magnitude and even the *sign* of the effect are likely to be different for small values of $t - t_0$ than for large values: an open market sale should lower the probability of an early collapse but raise the probability of a collapse over a longer time horizon. The formal model confirms this intuition.

$\pi(t, t_0)$ is the probability density function of 'collapse intervals', $t - t_0$. For the stochastic process considered in this paper, there is an interesting special case for which it is simple to derive the expectation of this distribution, i.e. the mean duration of the interval until the collapse. An open market sale is then shown, under weak conditions, to raise this mean duration.

Linearizing the model of eqs. (16)–(19) we obtain the following first-order equation of motion for the shadow exchange rate:

$$\tilde{S}(t) = a_s \mathrm{E}_t \tilde{S}(t+1) + a_m D(t) + v(t), \quad 0 < a_s < 1,\ a_m > 0,^{10} \tag{22}$$

[9] The second equality in (21) only holds because of the monotonicity of $\tilde{S}$, i.e. because the shadow exchange rate in our model always depreciates.

Take, for example, the two-period horizon, i.e. the probability that the fixed exchange rate regime collapses in no more than two periods:

$$\begin{aligned}\Pi(t_0+2, t_0) &\equiv \pi(t_0+1, t_0) + \pi(t_0+2, t_0)\\ &= \mathrm{Prob}_{t_0}[\tilde{S}(t_0+1) > \bar{S}] + \mathrm{Prob}_{t_0}[\tilde{S}(t_0+2) > \bar{S}]\\ &\quad - \mathrm{Prob}_{t_0}[\tilde{S}(t_0+1) > \bar{S} \text{ and } \tilde{S}(t_0+2) > \bar{S}].\end{aligned}$$

But since $\tilde{S}(t)$ is strictly increasing in t [see eq. (26) below]

$$\mathrm{Prob}_{t_0}[\tilde{S}(t_0+1) > \bar{S} \text{ and } \tilde{S}(t_0+2) > \bar{S}] = \mathrm{Prob}_{t_0}[\tilde{S}(t_0+1) > \bar{S}],$$

$$\begin{aligned}[\mathrm{Prob}_{t_0}[\tilde{S}(t_0+2) > \bar{S} \text{ and } \tilde{S}(t_0+1) > \bar{S}] &= \mathrm{Prob}_{t_0}[\tilde{S}(t_0+2) > \bar{S} \mid \tilde{S}(t_0+1) > \bar{S}] \cdot \mathrm{Prob}[\tilde{S}(t_0+1) > \bar{S}]\\ &= \mathrm{Prob}_{t_0}[\tilde{S}(t_0+1) > \bar{S}]].\end{aligned}$$

Therefore

$$\Pi(t_0+2, t_0) = \mathrm{Prob}_{t_0}[\tilde{S}(t_0+2) > \bar{S}].$$

[10] $$a_s = \left[\frac{(1+i^*)p}{(1+i)p - l_i^{-1}M}\right]_0, \qquad a_m = \left[\frac{Sl_i^{-1}}{(1+i)p - l_i^{-1}M}\right]_0.$$

where $v(t)$ represents the influence of output, the foreign price level and the foreign interest rate on money demand.[11]

Choosing the fundamental, convergent forward-looking solution for $\tilde{S}(t)$ using (22) we find:

$$\tilde{S}(t)=a_m D(t)+v(t)+\sum_{k=1}^{\infty} a_s^k \mathrm{E}_t[a_m D(t+k)+v(t+k)]. \tag{23}$$

The stock of domestic credit evolves according to (24):

$$D(t+k)=D(t_0)+(t+k-t_0)i^*\bar{S}B^*(t_0)+\sum_{j=t_0}^{t+k-1} \Delta(j). \tag{24}$$

Following Flood and Garber (1984) we assume that $\Delta(j)$ is governed by the following stochastic process:

$$\Delta(j)=\Delta+\varepsilon(j), \tag{25a}$$

$$\varepsilon(j)=-\lambda^{-1}+\eta(j), \quad \lambda>0. \tag{25b}$$

The $\eta(j)$ are i.i.d. random variables with an exponential density f given by

$$f(\eta(j))=\begin{cases}\lambda \mathrm{e}^{-\lambda\eta(j)}, & \eta(j)>0,\\ 0, & \eta(j)\leqq 0,\end{cases} \tag{25c}$$

$$\Delta+i^*\bar{S}B^*(t_0)\geqq\lambda^{-1}. \tag{25d}$$

This specification ensures that domestic credit expansion, although stochastic, is always positive. Note that since arbitrarily large realizations of η can occur, there is always a non-zero probability that any stock of reserves, however large, will be exhausted by a speculative attack the next period. As pointed out by Flood and Garber (1984b), the currency will therefore always stand at a forward discount while the fixed exchange rate regime survives.

We now substitute the stochastic process governing the stock of domestic credit [eqs. (24), (25a), (25b)] into the shadow exchange rate process (23).

Assuming for simplicity that $v(t)=\mathrm{E}_t v(t+k)=0$ for all t and for all $k>1$,

[11]
$$v(t)=\left[\frac{Sl_i^{-1}M}{p^*[(1+i)p-l_i^{-1}M]}\right]_0 p^*(t)+\left[\frac{pSl_i^{-1}l_y}{(1+i)p-l_i^{-1}M}\right]_0 y(t)+\left[\frac{p\mathrm{E}_tS(t+1)}{1+i^*}\right]_0 i^*(t).$$

eq. (23) becomes:

$$\tilde{S}(t)=\frac{a_m}{1-a_s}\left[D(t_0)+(t-t_0)(i^*\bar{S}B^*(t_0)+\Delta-\lambda^{-1})+\sum_{j=t_0}^{t-1}\eta(j)\right]$$

$$+\frac{a_m a_s}{(1-a_s)^2}[\Delta+i^*\bar{S}B^*(t_0)]. \tag{26}$$

In order to derive an expression for $\Pi(t,t_0)$, the probability at t_0, of a collapse occurring no later than period t, we define:

$$\bar{\eta}(t,t_0)=\sum_{j=t_0}^{t-1}\eta(j) \tag{27a}$$

and

$$K(t,t_0)=\left[\frac{1-a_s}{a_m}\right]\bar{S}-\frac{a_s}{1-a_s}[\Delta+i^*\bar{S}B^*(t_0)]$$

$$-[D(t_0)+(t-t_0)(\Delta+i^*\bar{S}B^*(t_0)-\lambda^{-1})]. \tag{27b}$$

From (26) it then follows that

$$\Pi(t,t_0)=\mathrm{Prob}_{t_0}(\tilde{S}(t)\geqq\bar{S})=\mathrm{Prob}(\bar{\eta}(t,t_0)\geqq K(t,t_0)). \tag{28}$$

Since the $\eta(j)$, $j=t_0,\ldots,t-1$, are independent random variables each having an exponential distribution with parameter λ, it follows that $\bar{\eta}(t,t_0)$ has the gamma density $\Gamma(t-t_0,\lambda)$, where $t-t_0\geqq 1$ and

$$\Gamma(t-t_0,\lambda)=\begin{cases}\dfrac{\lambda^{t-t_0}}{(t-t_0-1)!}\bar{\eta}(t,t_0)^{t-t_0-1}\,\mathrm{e}^{-\lambda\bar{\eta}(t,t_0)}, & \bar{\eta}(t,t_0)>0,\\ 0, & \bar{\eta}(t,t_0)\leqq 0.\end{cases} \tag{29}$$

Therefore $\Pi(t,t_0)$ is given by the area under this gamma density function above $K(t,t_0)$:

$$\Pi(t,t_0)=\begin{cases}\displaystyle\int_{K(t,t_0)}^{\infty}\frac{\lambda^{t-t_0}}{(t-t_0-1)!}\bar{\eta}(t,t_0)^{t-t_0-1}\,\mathrm{e}^{-\lambda\bar{\eta}(t,t_0)}\,\mathrm{d}\bar{\eta}(t,t_0), & K(t,t_0)>0,\\ 1, & K(t,t_0)\leqq 0.\end{cases}$$

Using elementary properties of gamma functions, this simplifies to

$$\Pi(t,t_0)=\begin{cases}\displaystyle\sum_{j=0}^{t-t_0-1}\frac{(\lambda K(t,t_0))^j}{j!}\,\mathrm{e}^{-\lambda K(t,t_0)}, & K(t,t_0)>0,\\ 1, & K(t,t_0)\leqq 0.\end{cases} \tag{30}$$

Flood and Garber (1984) studied the probability of a collapse occurring the next period, $\Pi(t_0+1,t_0)$ which, from (30), is given by

$$\Pi(t_0+1,t_0)=\begin{cases} e^{-\lambda K(t_0+1,t_0)}, & K(t_0+1,t_0)>0, \\ 1, & K(t_0+1,t_0)\leqq 0. \end{cases}$$

As expected, the probability of a collapse occurring no later than period t increases with t, i.e. $\Pi(t+1,t_0)>\Pi(t,t_0)$.[12]

We now consider how, for $K(t,t_0)>0$, the probability of a collapse occurring during an interval of $t-t_0$ periods after an open market sale is affected by this financial operation.

First note that

$$\frac{1}{\bar{S}}\frac{\partial\Pi(t,t_0)}{\partial B^*(t_0)}\bigg|_{\mathrm{OM}}=\frac{1}{\bar{S}}\frac{\partial\Pi(t,t_0)}{\partial K(t,t_0)}\cdot\frac{\partial K(t,t_0)}{\partial B(t,t_0)}\bigg|_{\mathrm{OM}}. \tag{31}$$

From eq. (30) it follows that

$$\frac{\partial\Pi(t,t_0)}{\partial K(t,t_0)}=\sum_{j=0}^{t-t_0-1}\frac{[\lambda K(t,t_0)]^j}{j!}e^{-\lambda K(t,t_0)}\left[\frac{j}{K(t,t_0)}-\lambda\right]$$

$$=\frac{-\lambda(\lambda K(t,t_0))^{t-t_0-1}}{(t-t_0-1)!}e^{-\lambda K(t,t_0)}<0. \tag{32}$$

Also, from eq. (27b),

$$\frac{1}{\bar{S}}\frac{\partial K(t,t_0)}{\partial B^*(t_0)}\bigg|_{\mathrm{OM}}=1-i^*\left[t-t_0+\frac{a_s}{1-a_s}\right]. \tag{33}$$

From eqs. (31), (32) and (33) it then follows that

$$\frac{\partial\Pi(t,t_0)}{\bar{S}\,\partial B^*(t_0)}\bigg|_{\mathrm{OM}}\gtrless 0\Leftrightarrow t-t_0\gtrless\frac{1}{i^*}-\frac{a_s}{1-a_s}. \tag{34}$$

Note that since $a_s\approx 1/(1+\alpha_s)$, this discrete time stochastic result can be

[12]First, holding the K's constant, increasing t by one period adds a positive term to the sum in (30). Second, from (27b), $K(t,t_0)$ decreases or stays constant (according as to whether $\Delta+i^*\bar{S}B^*(t_0)-\lambda^{-1}\geqq 0$) as t increases. As shown in eq. (32),

$$\frac{\partial\Pi(t,t_0)}{\partial K(t,t_0)}=\frac{-\lambda(\lambda K(t,t_0))^{t-t_0-1}}{(t-t_0-1)!}e^{-\lambda K(t,t_0)}<0.$$

Any decline in $K(t,t_0)$ as t increases therefore also tends to raise Π.

seen to correspond closely to the continuous time deterministic result reported in eq. (14).

Eq. (34) says that for any values of the foreign interest rate i^* and the sensitivity of the current shadow exchange rate to the expectation of next period's shadow exchange rate a_s (a decreasing function of $-l_i^{-1}$, the interest sensitivity of money demand) there exists a critical interval $\overline{t-t_0}$ defined by

$$\overline{t-t_0}=\frac{1}{i^*}-\frac{a_s}{i-a_s}.$$ [13]

For intervals shorter than $\overline{t-t_0}$ the probability of a collapse is lowered by an open market sale at t_0, while for intervals longer than $\overline{t-t_0}$ it is raised.

The probability of an immediate collapse after an open market sale at t_0 is reduced (raised) if i^* is less than (exceeds) $1-a_s$, since, from eq. (34),

$$\left.\frac{\partial \Pi(t_0+1,t_0)}{\bar{S}\,\partial B^*(t_0)}\right|_{\mathrm{OM}} \gtrless 0 \Leftrightarrow i^* \gtrless 1-a_s.$$

From footnote 10,

$$a_s=\left[\frac{1+i^*}{1+i-l_i^{-1}M/P}\right]_0.$$

$-l_i^{-1}(M/P)$ is the reciprocal of the interest semi-elasticity of demand for high-powered money. Values around 2.0 have been found for this semi-elasticity in the United Kingdom. With $i_0=i_0^*=0.10$, $a_s=0.6875$ and it seems likely that an open market sale will lower the probability of an immediate collapse.

Also, from (34) the duration of the interval, after an open market sale, for which the probability of a collapse is reduced, is inversely related to the interest rate.

Next we derive an expression for the *expected duration* of the interval between the open market sale and the collapse of the fixed exchange rate regime, and analyse how it is affected by the sale. The expected waiting time until the collapse is denoted $\mathrm{E}_{t_0}(t-t_0)$.

$\eta(t,t_0)$, the probability of a collapse occurring in period t (neither earlier nor later) is given by $\pi(t,t_0)=\Pi(t,t_0)-\Pi(t-1,t_0)$. Therefore,

$$\mathrm{E}_{t_0}(t-t_0)=\sum_{t=t_0+1}^{\infty}(t-t_0)[\Pi(t,t_0)-\Pi(t-1,t_0)]. \tag{35}$$

[13] $\overline{t-t_0}$ should be a positive integer. For simplicity I ignore this constraint.

Remembering that $\Pi(t,t_0)\equiv \mathrm{Prob}_{t_0}(\bar{\eta}(t,t_0)\geqq K(t,t_0))$ and using eq. (30) it follows that

$$\mathrm{E}_{t_0}(t-t_0)=\sum_{t_0+1}^{\infty}(t-t_0)\left[\sum_{k=0}^{t-t_0-1}\frac{(\lambda K(t,t_0))^k}{k!}\mathrm{e}^{-\lambda K(t,t_0)}\right.$$
$$\left.-\sum_{k=0}^{t-t_0-2}\frac{(\lambda K(t-1,t_0))^k}{k!}\mathrm{e}^{-\lambda K(t-1,t_0)}\right].$$

Now consider the special case where the deterministic component of dce equals zero, i.e. $\Delta+i^*\bar{S}B^*(t_0)-\lambda^{-1}=0$. From (27b) this implies that $K(t,t_0)=K(t-1,t_0)=K(t_0,t_0)$. In this case

$$\mathrm{E}_{t_0}(t-t_0)=\sum_{t=t_0+1}^{\infty}(t-t_0)\frac{(\lambda K(t_0,t_0))^{t-t_0-1}}{(t-t_0-1)!}\mathrm{e}^{-\lambda K(t_0,t_0)}.$$

Now

$$\frac{(\lambda K(t_0,t_0))^{t-t_0-1}}{(t-t_0-1)!}\mathrm{e}^{-\lambda K(t_0,t_0)}$$

is a Poisson density with parameter $\lambda K(t_0,t_0)$. Its mean is $\lambda K(t_0,t_0)$. Therefore

$$\mathrm{E}_{t_0}(t-t_0)=\lambda K(t_0,t_0)+1. \tag{36}$$

From (33) it follows that

$$\left.\frac{\partial \mathrm{E}_{t_0}(t-t_0)}{\bar{S}\,\partial B^*(t_0)}\right|_{\mathrm{OM}}=\lambda\left[1-i^*\frac{a_s}{1-a_s}\right]. \tag{37}$$

When the deterministic component of the dce process equals zero, an open market sale at t_0 increases the expected length of the interval until the collapse of the exchange rate regime i.f.f. $i^*<(1-a_s)/a_s$. With $a_s=0.6875$, as was assumed earlier for illustrative purposes, this condition is likely to be satisfied.

4. Conclusion

The main results of the paper are summarized in the Introduction. I conclude by restating the main policy conclusion and suggesting possible extensions of the approach adopted in this paper.

The policy message of the paper is a familiar but important one.

Borrowing additional reserves to defend the exchange rate is useful to the extent that 'buying time' is useful. It might, for example, be useful if the quality of the fiscal package necessary to create a viable exchange rate regime is improved by a delay in its implementation. Borrowing without a fiscal correction lowers the likelihood of an early collapse and increases the expected duration of the interval until the collapse. As it raises the likelihood of a later collapse, viability (in the sense of long-run assured survival of the managed exchange rate regime) can be achieved only by lowering the government's need for seigniorage revenue. This requires a fundamental fiscal correction, i.e. a lowering of the primary deficit.

Desirable extensions of the analytical approach of this paper include a more flexible specification of fiscal and financial policy and a more satisfactory modelling of liquidity constraints, leading to foreign exchange crises and the collapse of managed exchange rate regimes, and solvency constraints, causing not only foreign exchange crises but also default on or repudiation of public sector debt.[14] An international cash-in-advance model may be a suitable vehicle for analysing both issues.

[14]For an interesting discussion of some of these issues, see Obstfeld (1984b).

References

Buiter, W.H., 1985, A guide to public sector debt and deficits, Economic Policy 1, no. 1, Nov., 14–79.

Collins, S.M., 1984, Exchange rate expectations and interest parity during credibility crises. The French Franc, March 1983, Harvard University, HIER Discussion Paper Series No. 10B0, Aug.

Connolly, M.B. and D. Taylor, 1984, The exact timing of the collapse of an exchange rate regime and its impact on the relative price of traded goods, Journal of Money Credit and Banking 16, no. 2, May, 194–207.

Cumby, R.E. and S. van Wijnbergen, 1983, Fiscal policy and speculative runs on the central bank under a crawling peg exchange rate regime: Argentina, 1979–1981. Mimeo (N.Y.U. Graduate School of Business Administration).

Diamond, P.W. and P.H. Dybvig, 1983, Bank runs, deposit insurance and liquidity, Journal of Political Economy 91, no. 3, June, 401–419.

Dornbusch, R., 1984, Collapsing exchange rate regimes, Mimeo.

Flood, R.P. and P.M. Garber, 1983, A model of stochastic process switching, Econometrica 51, May, 537–551.

Flood, R.P. and P.M. Garber, 1984a, Gold monetization and gold discipline, Journal of Political Economy 92, Feb., 90–107.

Flood, R.P. and P.M. Garber, 1984b, Collapsing exchange rate regimes: Some linear examples, Journal of International Economics 17, 1–13.

Garber, P. and V. Grilli, 1986, The Belmont–Morgan Syndicate as an optimal investment banking contract, European Economic Review 30, no. 3, June, 649–677.

Grilli, V., 1984, Speculative attacks and devaluations in the EMS, Mimeo.

Grilli, V., 1986a, Buying and selling attacks on fixed exchange rate systems, Journal of International Economics 20, 143–156.

Grilli, V., 1986b, Investment banking contracts in a speculative attack environment: Evidence from the 1890's, unpublished.

Krugman, Paul, 1979, A model of balance of payments crises, Journal of Money Credit and Banking 11, no. 3, Aug., 311–325.
Obstfeld, M., 1984, Balance of payments crises and devaluation, Journal of Money Credit and Banking 16, no. 2, May, 208–217.
Obstfeld, M., 1986a, Speculative attack and the external constraint in a maximizing model of the balance of payments, Canadian Journal of Economics 19, no. 1, Feb., 1–22.
Obstfeld, M., 1986b, Rational and self-fulfilling balance of payments crises, American Economic Review 76, no. 1, Mar., 72–81.
Salant, S.W., 1983, The vulnerability of price stabilization schemes to speculative attack, Journal of Political Economy 91, Feb., 1–38.
Salant, S.W. and D.W. Henderson, 1978, Market anticipations of government policies and the price of gold, Journal of Political Economy 86, Aug., 627–648.
Sargent, T.J. and N. Wallace, 1984, Some unpleasant monetarist arithmetic, in: B. Griffith and G.E. Wood, eds., Monetarism in the United Kingdom (Macmillan, London) 15–41.