

SHORT-RUN AND LONG-RUN DISEQUILIBRIUM  
IN DYNAMIC MACROMODELS

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### Abstract

Two kinds of disequilibria can be distinguished in dynamic macromodels: disequilibrium of the short-run endogenous variables relative to the short-run equilibrium (e.g. an IS/LM disequilibrium) and disequilibrium relative to a long-run steady state equilibrium with short-run equilibrium prevailing at each point in time.

This paper demonstrates that if short-run equilibrium is not established instantaneously, separating the short-run and long-run adjustment processes as is often done in the literature can be seriously misleading. Both have to be modeled simultaneously and their full interaction has to be allowed for.

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Introduction

Two kinds of disequilibria can be distinguished in dynamic macromodels: disequilibrium of the short-run endogenous variables relative to the temporary (or momentary) equilibrium and non-steady state behaviour or disequilibrium relative to a long-run steady state equilibrium. I shall refer to a disequilibrium of the first kind as a "short-run disequilibrium" or "market disequilibrium," and to a disequilibrium of the second kind as a "long-run disequilibrium". Short-run disequilibrium always implies long-run disequilibrium but temporary equilibrium in each successive period (at each point in time in the continuous case) is quite consistent with non-steady state behaviour. Negishi<sup>1</sup> makes the distinction very succinctly:

"Models of trade cycles and economic growth generate time paths of outputs, capital stock and prices, which are of a dynamic disequilibrium type, in which the supply of and demand for each commodity are assumed to be continuously equal in every market. This abstraction from the market-clearing process, which may be considered as a shorter-run phenomenon than the one under consideration, may be justified if the former is rapidly damped and can be supposed to have worked out its effects."<sup>2</sup>

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\* This paper is based on parts of my Ph.D. Dissertation, "Temporary Equilibrium and Long-Run Equilibrium," Yale 1975. I have had the benefit of the advice of James Tobin, Gary Smith and Katsuhito Iwai.

"...it is also possible to disregard the slow long-run changes and concentrate on the shorter-run process. For example, in a short-run Keynesian model of income determination it is often assumed that the stock of capital is fixed"<sup>3</sup> and "...the stability analyses a competitive economy are concerned with the behavior of the short-run market-clearing adjustment process towards temporary equilibrium within a 'week' in the sense of Hicks, i.e., it concerns the dynamics of the market-clearing process."<sup>4</sup>

In this paper I investigate whether it is legitimate to keep these two kinds of adjustment processes separate. Can one analyze price (or quantity) adjustment to market disequilibrium on the assumption that asset stocks and expectations remain constant and analyze stock adjustment processes and the formation of new expectations on the assumption that temporary equilibrium prevails all the time, when in reality these two kinds of adjustment processes occur simultaneously? The essence of the answer is that when adjustment to short-run disequilibrium takes place in real time, it constitutes a serious misspecification of the dynamic behaviour of the model not to allow for both kinds of adjustment processes simultaneously with explicit recognition of their interdependence.

#### Short-Run and Long-Run Disequilibrium in the IS/ LM Model

Another way of phrasing the question raised in the introduction is: will the behaviour of an economic system when adjustment to short-run disequilibrium is instantaneous (i.e. when temporary equilibrium prevails all the time) be a good "parable" for the behaviour of the system when adjustment to short-run disequilibrium is not instantaneous but still

very rapid relative to the adjustment of asset stocks and expectations?

To make the discussion more concrete, the simple IS/LM model used by Tobin and Buiter<sup>5</sup> will be used as an example. Since it is only used for illustrative purposes, I shall not spend a great deal of time motivating or justifying its specification.<sup>6</sup>

Notation: Y - real output and factor income  
 G - real government spending on goods and services  
 R - interest rate on bonds  
 $\theta$  - marginal income tax rate on factor income and government interest payments to the private sector  
 $\hat{\mu} = \mu(1-\theta)$  - private savers' target wealth-factor income ratio  
 M - nominal quantity of outside money  
 D - nominal quantity of bonds [bonds are fixed nominal market value, variable interest rate government obligations].  
 K - stock of real reproducible capital  
 W - private non-human wealth.

The investment and saving functions are of the stock adjustment type.

The investment function is

$$I = I(H(R)Y-K) \quad I' > 0, \quad H' < 0; \quad I(0) = 0 .$$

H(R) is the technologically determined optimal capital-output ratio.

The saving function is

$$S = S(\hat{\mu}Y-W) \quad S' > 0 \quad S(0) = 0$$

$$W = M + D + H(R)Y .$$

It is assumed that the firms' target capital stock  $H(R)Y$  equals the market value of the stock of capital  $qK$  where  $q$  denotes the valuation ratio, the ratio of the market value of a unit of existing capital to the value of a unit of capital at current reproduction costs.<sup>7</sup>

The demand for money is specified as

$$L(R(1-\theta), \frac{Y}{W})W \quad L_1 < 0; \quad L_2 > 0; \quad \frac{L_2 Y}{W} < L \quad .$$

The fraction of wealth held in the form of real money balances is inversely related to the after-tax rate of return differential between bonds and money and positively to the income-wealth ratio. The short-run income elasticity of demand for money balances is less than unity.

The short-run IS/LM equilibrium is given by:

$$(1) \quad I(H(R)Y-K) + G + (1-\theta)RD - \theta Y = S(\hat{\mu} Y - M - D - H(R)Y) \quad (IS)$$

$$(2) \quad L(R(1-\theta), \frac{Y}{M+D+H(R)Y}) (M+D+H(R)Y) = M \quad (LM)$$

Textbook short-run disequilibrium dynamics consists in making the rate of change of real output an increasing function of the excess of planned investment and government spending over planned saving and taxation and the rate of change of the interest rate an increasing function of the excess demand for money balances.<sup>8</sup> In its simplest form this can be represented by:

$$\dot{Y} = \gamma_{11} [I(H(R)Y-K) + G + (1-\theta)RD - \theta Y - S(\hat{\mu} Y - M - D - H(R)Y)]$$

$$\dot{R} = \gamma_{22} [L(R(1-\theta), \frac{Y}{M+D+H(R)Y}) (M+D+H(R)Y) - M]$$

$$\gamma_{11}, \gamma_{22} > 0$$

$\gamma_{ii}$  are the speed of adjustment coefficients.

Linearizing at the short-run equilibrium  $(Y^*, R^*; \bar{M}, \bar{D}, \bar{K})$  we get

$$(3) \begin{bmatrix} \dot{Y} \\ \dot{R} \end{bmatrix} = \begin{bmatrix} \gamma_{11} & 0 \\ 0 & \gamma_{22} \end{bmatrix} \begin{bmatrix} (I'+S')H(R) - \theta - S'\hat{\mu} & (I'+S')H'(R)Y + (1-\theta)D \\ L_2 + (L - \frac{L_2 Y}{W})H(R) & WL_1(1-\theta) + (L - \frac{L_2 Y}{W})H'(R)Y \end{bmatrix} \begin{bmatrix} Y-Y^* \\ R-R^* \end{bmatrix}$$

or  $\dot{x} = \Gamma a(x-x^*)$  where  $x$  is the vector of state variables,  $\Gamma$  is the diagonal matrix of speed of adjustment coefficients and  $a$  is the coefficient matrix.

I assume  $(I' + S')H'(R)Y + (1-\theta)D < 0$  (An increase in the rate of interest reduces effective demand. It will reduce investment  $(I'H'(R)Y)$  and it will increase saving by reducing wealth  $(S'H'(R)Y)$  but it will also increase government spending on debt service  $(1-\theta)D$ . The first two effects are assumed to dominate the third. I also assume

$$(I' + S')H(R) - \theta - S'\hat{\mu} < 0 .$$

This expression is minus the reciprocal of the simple Keynesian multiplier. Given these a priori restrictions (3) will satisfy the (2) necessary and sufficient stability conditions:

$$(4a) \quad \gamma_{11} [(I'+S') H(R) - \theta - S'\hat{\mu}] + \gamma_{22} [WL_1(1-\theta) + (L - \frac{L_2Y}{W}) H'(R)Y] < 0$$

$$(4b) \quad [(I'+S') H(R) - \theta - S'\hat{\mu}] [WL_1(1-\theta) + (L - \frac{L_2Y}{W}) H'(R)Y] \\ - [L_2 + (L - \frac{L_2Y}{W})H(R)] [(I'+S')H'(R)Y + (1-\theta)D] > 0$$

Furthermore, since the matrix  $a$  is Q-stable (qualitatively stable) the system will be stable for any positive speeds of adjustment  $\gamma_{ii}$ .<sup>9</sup>

The interpretation of the stability conditions in terms of the slopes of the IS curve and the LM curve, illustrated in Figure 1 is familiar and needs no repeat.

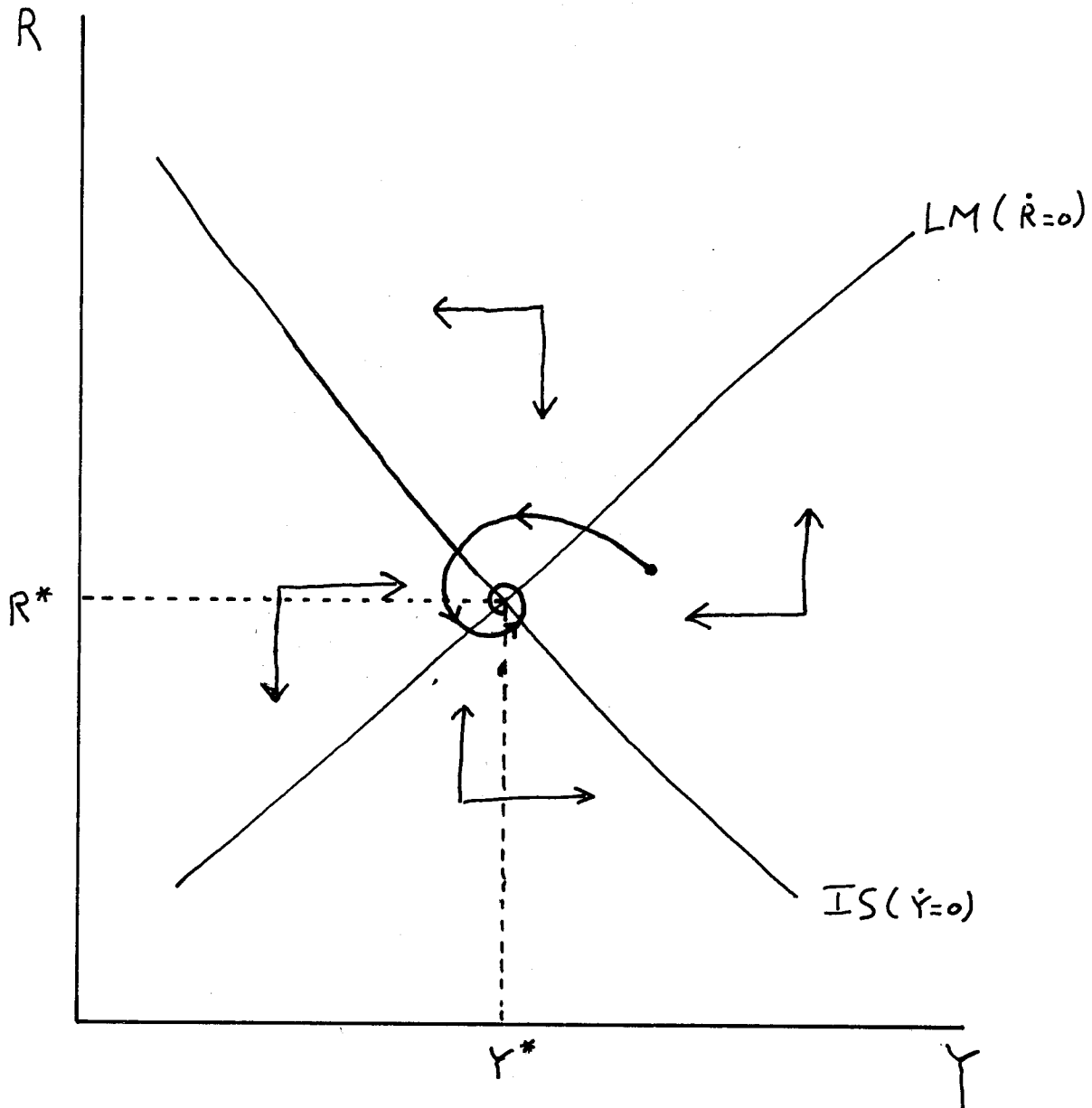
$$\left. \frac{dR}{dY} \right|_{IS} = \frac{- [(I'+S') H(R) - \theta - S'\hat{\mu}]}{(S'+I') H'(R)Y + (1-\theta)D} < 0$$

$$\left. \frac{dR}{dY} \right|_{LM} = \frac{- [L_2 + (L - \frac{L_2Y}{W})H(R)]}{WL_1(1-\theta) + (L - \frac{L_2Y}{W})H'(R)Y} > 0$$

Two points have to be made about this result. First, there is no way of knowing, without detailed empirical information, how long the adjustment



FIGURE 1



process will take. If the system is non-linear the convergence time may be either finite or infinite. If the system is linear, convergence will only be asymptotic as  $t \rightarrow \infty$ , no matter how large we choose the  $\gamma_{ii}$  to be. Second, built into the model are mechanisms changing the positions of the LM and IS curves while this short-run adjustment process takes place. In the model we are considering investment occurs, changing the stock of capital, and the government budget may be in surplus or deficit causing the stocks of government bonds and/or money to change. (In what follows I shall assume for simplicity that all deficits are financed by selling bonds.) It is reasonable to assume that current investment will not increase productive capacity till some time in the future because of the many lags between current investment spending and the entering into operation of the new capital goods as a part of the economy's productive capacity. While these capital goods that are "in the pipeline" do not constitute part of the economy's stock of finished capital goods in the sense that they cannot be used as an input into the productive process, they are part of "goods in process" and therefore belong to private sector net worth. Unfinished goods are part of private wealth if -- or to the extent that -- they are expected to become finished goods at some point in the future or if they have some alternative use as unfinished goods. While current investment may therefore affect productive capacity with a lag, it will affect net worth immediately, and this should be recognized in the adjustment process.

Second, if the short-run adjustment process is not instantaneous but takes a finite amount of time (be it very brief) short-run and long-run

adjustment processes will inevitably overlap. Assume that investment undertaken today will mature (result in additions to the stock of finished capital goods) after some fixed time interval of length  $z > 0$ . Assume also that the length of the period it takes for short-run disequilibrium to work itself out is  $z' < z$  if the parameters of the temporary equilibrium (the stocks in our model) remain constant. Let  $t$  denote time. Then  $I(t) = \dot{K}(t+z)$ . During the time interval  $(t, t+z')$  when the short-run adjustment mechanism is supposed to operate in response to some perturbation of market equilibrium at  $t$ , all investment undertaken in the past between  $t-z$  and  $t-(z-z')$  will mature, contradicting the assumption of constancy of the stock of capital during the short-run disequilibrium adjustment process. Rapid adjustment to short-run disequilibrium relative to stock adjustment is not sufficient for it to be possible to separate the two kinds of adjustment processes. Instantaneous adjustment is required for this separation to be valid.

In addition the assumption of sizeable gestation lags, while suitable for certain kinds of fixed investment, is not applicable to the deficit financing process of the government, which results in immediate changes in asset stocks and private sector net worth. Also, in the model under consideration, one of the short-run adjustment processes is the multiplier process. While the adjustment of interest rates to excess demand in highly organized financial markets may be very rapid, the adjustment of real output to changes in effective excess demand is a time-consuming process even in a situation of general unemployment of resources. The speed of adjustment of financial asset stocks through government deficits or surpluses may well be greater than that of the multiplier process.

Short-Run Equilibrium and Long-Run Disequilibrium

Assuming for simplicity that investment spending is instantaneously translated into changes in the stock of capital and that government deficits result instantaneously in changes in the stock of bonds we can write the long-run model -- if the short-run adjustment process is instantaneous -- as

$$I(H(R)Y-K) + G + (1-\theta)RD - \theta Y - S(\hat{\mu}Y - M - D - H(R)Y) = 0$$

$$L(R(1-\theta), \frac{Y}{M+D+H(R)Y})(M+D+H(R)Y) = M$$

$$\dot{K} = I(H(R)Y-K)$$

$$\dot{D} = G + (1-\theta)RD - \theta Y$$

Let  $B(K, D; \bar{M})$  and  $F(K, D; \bar{M})$  be the implicit IS/LM solutions for  $Y$  and  $R$  respectively. Substituting this into the two dynamic equations we get:

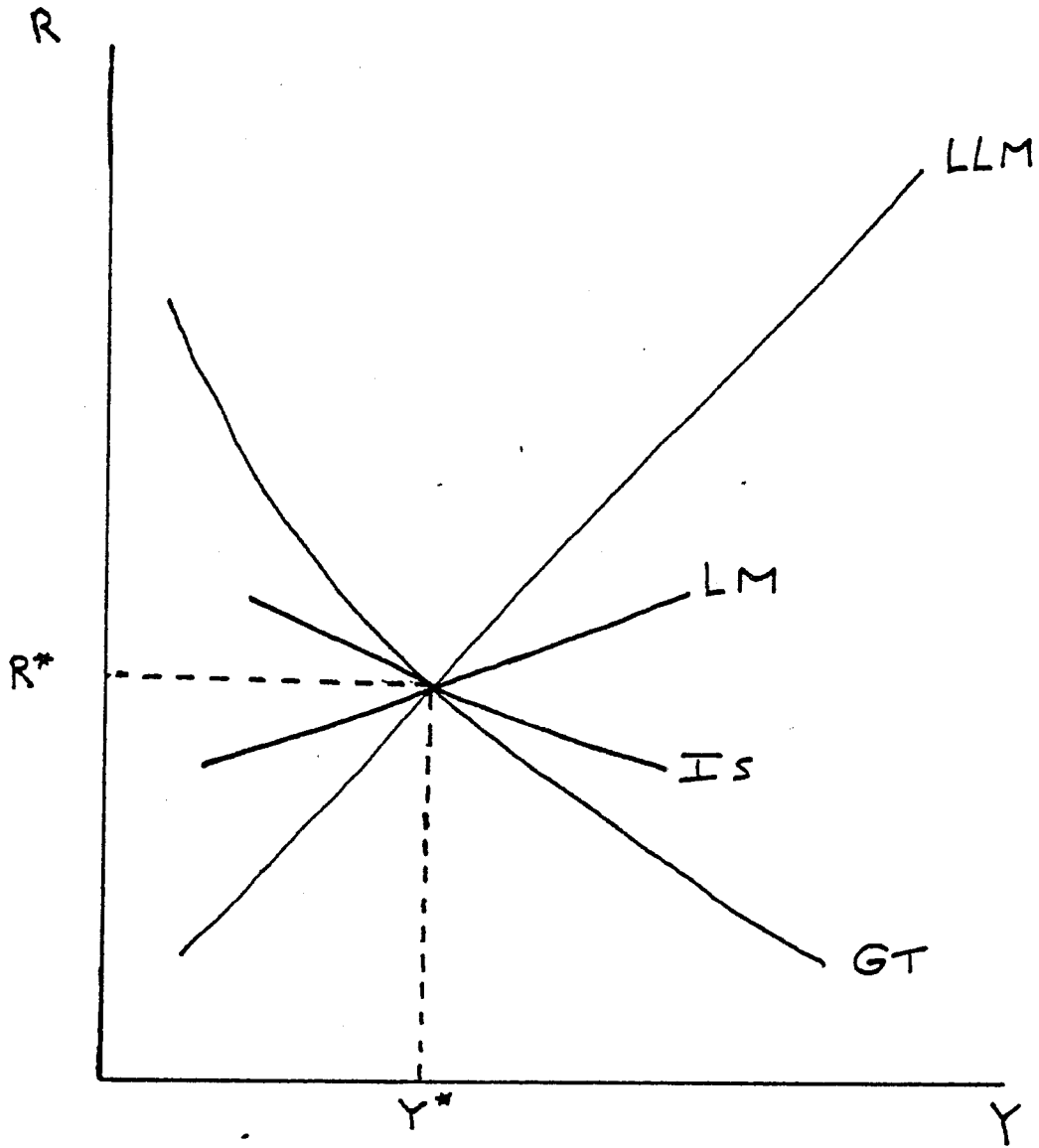
$$\dot{K} = I[H(F(K, D)) B(K, D) - K]$$

$$\dot{D} = G + (1-\theta) F(K, D) D - \theta B(K, D)$$

Linearizing at the long-run equilibrium  $(K^*, D^*, \bar{M})$  we find that necessary and sufficient local stability conditions for this model are<sup>10</sup>

$$(5a) \quad I' [H'(R)Y F_K + H(R)B_K - 1] + (1-\theta) [F_D D + R] - \theta B_D < 0$$

FIGURE 2



and

$$(5b) \quad (1-\theta)H'(R)YRF_K - \theta H'(R)YF_{K^B D} + (1-\theta)H(R)RB_K - (1-\theta)F_D - (1-\theta)R + \theta B_D \\ + (1-\theta)H(R)D B_{K^B D} - (1-\theta)H(R)D F_{K^B D} + \theta H'(R)YB_{K^B D} > 0 .$$

Figure 2 represents the long-run equilibrium in R-Y space. The equations are

$$L(R(1-\theta), \frac{1}{\mu} \hat{\mu} Y) = M \quad (LLM)$$

$$G + (1-\theta)(\hat{\mu} Y - M - H(R)Y) - \theta Y = 0 \quad (GT) .$$

For concreteness I have assumed  $\theta + (1-\theta)R[H(r) - \hat{\mu}] < 0$  .

### Short-Run and Long-Run Disequilibrium

The complete model, allowing simultaneously for short-run disequilibrium (positions off the intersection of an IS curve and an LM curve) and long-run disequilibrium is represented by the following system of four simultaneous differential equations:

$$\dot{Y} = \gamma_{11} [I(H(R)Y - K) + G + (1-\theta)RD - \theta Y - S(\hat{\mu} Y - M - D - H(R)Y)]$$

$$\dot{R} = \gamma_{22} [L(R(1-\theta), \frac{Y}{M+D+H(R)Y}) (M+D+H(R)Y) - M]$$

$$\dot{K} = I(H(R)Y - K)$$

$$\dot{D} = G + (1-\theta)RD - \theta Y$$

Figure 2 also represents the long-run equilibrium for this system.

The linear approximation at the equilibrium  $(Y^*, R^*, K^*, D^*; \bar{M})$  gives:

$$\begin{array}{c} \dot{Y} \\ \dot{R} \\ \dot{K} \\ \dot{D} \end{array} = \begin{array}{cc} \gamma_{11} [(I'+S')H(R) - \theta - S'\hat{\mu}] & \gamma_{11} [(I'+S')H'(R)Y + (1-\theta)D] \\ \gamma_{22} [L_2 + (L - \frac{L_2 Y}{W})H(R)] & \gamma_{22} [WL_1(1-\theta) + (L - \frac{L_2 Y}{W})H'(R)Y] \\ \hline I'H(R) & I'H'(R)Y \\ -\theta & (1-\theta)D \end{array} \begin{array}{cc} -I'\gamma_{11} & S'\gamma_{11} \\ 0 & (L - \frac{L_2 Y}{W})\gamma_{22} \\ -I' & 0 \\ 0 & (1-\theta)R \end{array} \begin{array}{c} Y-Y^* \\ R-R^* \\ K-K^* \\ D-D^* \end{array} \quad (6)$$

$$\text{or } \dot{X} = A(X-X^*) .$$

The short-run IS/LM disequilibrium model given by (3) is the  $2 \times 2$  upper-left submatrix of the larger system. It can easily be seen that while this submatrix is Q-stable, the larger matrix is at most P-stable, since one of its diagonal elements is positive  $[(1-\theta)R]$ . In the appendix it is shown that for systems such as (6), stronger results can be stated. It is possible for the short-run adjustment process to appear stable when considered in isolation (conditions such as (4a) and (4b) are satisfied) but for the complete system to be unstable when short-run and long-run adjustment processes are considered simultaneously (A is an unstable matrix.)<sup>11</sup> Conversely, (6) may be stable, yet (3) may appear unstable when considered in isolation. It is even possible for the short-run adjustment process to be stable and for the long-run adjustment process to be stable on the assumption that short-run adjustment occurs instantaneously (conditions such as (5a) and (5b) are satisfied), yet for basically the same system to be unstable when short-run adjustment is rapid but non-instantaneous

and both kinds of adjustment processes are allowed for simultaneously. The converse may also hold: both the short-run adjustment mechanism and the long-run stock adjustment process may appear unstable when considered in isolation, but a system incorporating them jointly and allowing for their full interaction may be stable.

### Conclusion

There are only two consistent ways of dealing with short-run and long-run disequilibrium processes.<sup>12</sup> The first is to assume that temporary equilibrium prevails at each point in time (that short-run adjustment occurs instantaneously). The second is to explicitly model price and quantity adjustment to market disequilibrium as a dynamic process taking place in real time, simultaneously with the changes in stocks and expectations that take place. Attempts to combine these two approaches and to separate the two processes may result in seriously misleading conclusions.



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## FOOTNOTES

1. Negishi, T., "The Stability of a Competitive Economy: A Survey Article," Econometrica, Vol. 30, No. 4 (Oct. 1962) pp. 635-69.
2. Negishi, T. Op. Cit. p. 637.
3. Negishi, T. Op. Cit. p. 637, footnote 6.
4. Negishi, T. Op. Cit. p. 637.
5. Tobin, J. and Buiter, W., "Long Run Effects of Fiscal and Monetary Policy on Aggregate Demand," Cowles Foundation Discussion Paper, No. 384, December 1974, forthcoming in Proceedings of the Conference on Monetarism, ed. J. Stein, Amsterdam, North Holland. See also Buiter, W. Temporary Equilibrium and Long-Run Equilibrium, Unpublished Ph.D. Thesis, Yale University, 1975, Chapter 4, pp. 151-202.
6. See Tobin and Buiter, op. cit. and Buiter, op. cit. for a detailed discussion.
7. This would be exactly correct, e.g., if the production function were Cobb-Douglas. The proposition that the market value of claims on the earnings of capital will vary directly with the level of effective demand and inversely with the rate of interest, does not require such a narrow interpretation. Nothing of importance would be changed if private wealth were defined as:  $W = M + D + K$  or  $W = M + D + q(R, Y)K$ .  
 $q_R < 0$ ,  $q_Y > 0$ .
8. An exhaustive study of this can be found in Chang, W.W. and Smyth, D.J., "Stability and Instability of the IS-LM Equilibrium," Oxford Economic Papers, Vol. 24, No. 3 (Nov. 1972) pp. 272-93.
9. The system  $\dot{y} = By$  is said to be Q-stable (qualitatively stable) i.f.f. the solution to  $y = By$  converges to the equilibrium position (0), given only the sign pattern of B. Thus, a matrix B is Q-stable i.f.f.

for all numerical values of the coefficients of  $B$  consistent with the sign pattern of  $B$  the solution of  $\dot{y} = By$  converges to zero. If the system converges to the equilibrium for some but not necessarily all numerical values of  $B$  consistent with the sign pattern of  $B$ ,  $B$  is said to be P-stable (potentially stable).

If  $B$  is Q-stable, then if  $D$  is a diagonal matrix with positive diagonal ( $d_{ii} > 0$  for all  $i$ ) then  $DB$  is also Q-stable. See Allingham, M.G. and Morishima, M., "Quantitative Economics and Comparative Statics," in Theory of Demand, Real and Monetary, ed. M. Morishima et. al. Oxford: Clarendon Press, 1973, pp. 3-69 and Arrow, K.J., "Stability Independent of Adjustment Speed." Trade, Stability and Macroeconomics, eds. G. Horwich and P.A. Samuelson. New York, Academic Press, 1974, pp. 181-203.

10. An almost identical model is analyzed in detail in Tobin and Buiter, Op. Cit. p. 24.
11. Let  $a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$ ,  $a_0 > 0$  be the characteristic equation of  $A$ . Necessary and sufficient conditions for all roots of this equation to have negative real parts are given by the following inequalities, which must hold simultaneously:

$$a_1 > 0; \quad a_1a_2 - a_0a_3 > 0; \quad a_1a_2a_3 - a_1^2a_4 - a_0a_3^2 > 0; \quad a_4 > 0.$$

12. I ignore the "stability analysis" of many general equilibrium models where market adjustment takes place before trading occurs without any guarantee that the price adjustment process will converge to the equilibrium within a reasonable period of time (or ever). To make sense out of this we have to assume that the price adjustment process takes place outside real (or calendar) time. Such exercises have little or no economic interest.

## APPENDIX

This appendix consists of 2 "non-theorems," i.e. proofs that certain conjectures are invalid.

Consider the linearized version of a complete dynamic model, written as a system of  $N$  simultaneous first order differential equations:

$$\text{a.1} \quad \dot{X} = AX$$

$X$  is an  $N \times 1$  vector  $(x_1, \dots, x_M, x_{M+1}, \dots, x_N)'$

$A$  is an  $N \times N$  matrix of full rank

$$A = [a_{ij}] \quad i, j=1, \dots, N.$$

The matrix  $A$  is partitioned into 4 submatrices

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$A_{11}$  is an  $M \times M$  matrix of full rank.  $M < N$

$A_{22}$  is an  $(N-M) \times (N-M)$  matrix.

$A_{21}$  is an  $(N-M) \times M$  matrix.

$A_{12}$  is an  $M \times (N-M)$  matrix.

The vector  $X$  is partitioned conformably:

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$X_1$  is an  $M \times 1$  vector.

$X_2$  is an  $(N-M) \times 1$  vector.

a.1 can therefore be written as

$$\text{a.2} \quad \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

Short-run disequilibrium analysis selects a subset of the variables of the complete system and analyses their behaviour over time holding the other variables constant.

Without loss of generality, the short-run endogenous variables will be labeled  $x_1, x_2, \dots, x_M$ .

The linearized version of the short-run disequilibrium model is

$$\text{a.3} \quad \dot{X}_1 = A_{11} X_1$$

If adjustment to short-run disequilibrium is instantaneous, the complete dynamic system can be written as

$$\text{a.4} \quad \begin{bmatrix} 0 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

Solving the first  $M$  equations for  $X_1$  as a function of  $X_2$  and substituting this into the last  $N-M$  equations, we get:

$$\text{a.5} \quad \dot{X}_2 = [-A_{21}A_{11}^{-1}A_{12} + A_{22}] X_2 .$$

The question now is, does stability of A.5 and stability of A.3 imply stability of A.2 and conversely? A  $2 \times 2$  counterexample will disprove this conjecture. Consider the 2 equation dynamic system

$$\text{a.6} \quad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

This is stable if and only if

$$\text{a.7} \quad a_{11} + a_{22} < 0$$

and

$$\text{a.7}'' \quad a_{11}a_{22} - a_{21}a_{12} > 0 .$$

Its short-run disequilibrium subsystem

$$\text{a.8} \quad \dot{x}_1 = a_{11}x_1$$

is stable i.f.f.

$$\text{a.9} \quad a_{11} < 0 .$$

The complete system with instantaneous adjustment to short-run disequilibrium

$$\dot{x}_2 = \left[ -\frac{a_{21}a_{12}}{a_{11}} + a_{22} \right] x_2 ,$$

or

$$a.10 \quad \dot{x}_2 = \left[ \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11}} \right] x_2$$

is stable i.f.f.

$$a.11 \quad \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11}} < 0 .$$

It is easy to see that a.9 and a.11 do not imply both a.7' and a.7''.

a.9 and a.11 do imply a.7'', but  $a_{22}$  could be positive and sufficiently large to violate a.7', i.e., take  $a_{22} > 0$ ,  $a_{22} > |a_{11}|$  and  $a_{22} < \frac{a_{12}a_{21}}{a_{11}}$ . This requires that  $a_{12}$  and  $a_{21}$  have

opposite signs. If  $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$  were a symmetric matrix, a.9 and

a.11 would imply a.7' and a.7''.

Conversely, a.7' and a.7'' are consistent with  $a_{11} > 0$  and

$$\frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11}} > 0 . \text{ (Take}$$

$$a_{22} < 0, |a_{22}| > a_{11} \quad \text{and} \quad a_{22} > \frac{a_{12}a_{21}}{a_{11}}$$

Again this requires that  $a_{12}$  and  $a_{21}$  have opposite signs. Similarly, comparing just a.6 and a.8, it can be shown easily that  $a_{11} > 0$  is consistent with  $a_{11} + a_{22} < 0$  and  $a_{11}a_{22} - a_{12}a_{21} > 0$ , and that  $a_{11} + a_{22} > 0$  and  $a_{11}a_{22} - a_{12}a_{21} < 0$  are consistent with  $a_{11} < 0$ .

Finally, we can see how in the simple  $2 \times 2$  model the complete model approaches the long-run disequilibrium model with instantaneous short-run adjustment, when the short-run speed of adjustment  $\gamma$  approaches infinity.

Decompose  $a_{11}$  and  $a_{12}$  into  $\gamma\alpha_{11}$  and  $\gamma\alpha_{12}$  respectively,  $\gamma > 0$ .

The short-run disequilibrium model is stable i.f.f.

$$\text{a.12} \quad \gamma\alpha_{11} < 0 \quad \text{i.e.} \quad \text{i.f.f.} \quad \alpha_{11} < 0.$$

The long-run disequilibrium model with instantaneous short-run adjustment is stable i.f.f.

$$\text{a.13} \quad \frac{-a_{21}\alpha_{12}}{\alpha_{11}} + a_{22} < 0 \quad \text{or} \quad \frac{\alpha_{11}a_{22} - a_{21}\alpha_{12}}{\alpha_{11}} < 0$$

The complete model is stable i.f.f.

$$\text{a.14}' \quad \gamma\alpha_{11} + a_{22} < 0$$

and

$$\text{a.14}'' \quad \alpha_{11}a_{22} - a_{21}\alpha_{12} > 0.$$



If  $a_{22}$  is finite,  $\lim_{\gamma \rightarrow \infty} [\gamma \alpha_{11} + a_{22}] < 0$  if  $\alpha_{11} < 0$ .

Under these circumstances, as the speed of adjustment becomes infinite,

a.12 and a.13 imply a.14' and a.14''.