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ABSTRACT

The paper provides a formalisation of the monetary folk proposition that fiat base money is an asset of the holder but not a liability of the issuer. The issuance of irredeemable fiat base money can have pure fiscal effects on private demand. With irredeemable fiat base money, weak restrictions on the monetary policy rule suffice to rule out liquidity trap equilibria - equilibria in which all current and future short nominal interest rates are at their lower bounds. In a model with flexible prices, liquidity trap equilibria cannot occur as long as the private sector does not expect the monetary authority to reduce the nominal money stock to zero in the long run. In a New-Keynesian model, they are ruled out provided the private sector expects the authorities not to reduce the nominal stock of base money below a certain finite level in the long run. Liquidity trap equilibria can exist if and for as long as the private sector expects that the monetary authorities will ultimately reverse any current expansion of the monetary base in present value terms.

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1 Introduction

"Let us suppose now that one day a helicopter flies over this community and drops an additional $1000 in bills from the sky, ... Let us suppose further that everyone is convinced that this is a unique event which will never be repeated," (Friedman [1969, pp 4-5].

This paper aims to provide a rigorous analysis of Milton Friedman's famous parable of the 'helicopter' drop of money. A related objective is to show that even when the economy is in a liquidity trap (when all current and future short nominal interests rates are at their lower bounds and there is satiation with real money balances), a helicopter drop of money will stimulate demand, unlike a helicopter drop of government bonds.

The argument involves a number of steps. The first is a formal statement, in an optimising dynamic general equilibrium model with money, of the 'folk proposition' that (fiat base) money is an asset to the private holder but not a liability of the issuer - the government. This is done through an asymmetric specification of the solvency constraints of the households and the government, motivated by the assumption that fiat base money is *irredeemable*.

The model has two financial stores of value: base money and one-period risk-free non-monetary interest-bearing nominal debt. The solvency constraint for the household is the requirement that the present discounted value of its terminal net financial wealth (the sum of its money holdings and its net non-monetary financial assets) be non-negative.

The solvency constraint for the government is the requirement that the present discounted value of its terminal *non-monetary* net financial debt be non-positive. The government does not view its monetary debt as a liability that will eventually have to be redeemed for something else of equal value. This formalisation of the concept of irredeemable base money is the central new idea in this paper.[1]

[1] The American Heritage Dictionary (2000) defines an *irredeemable* object as one 'that cannot be bought back or paid off' or 'not convertible into coin' The Concise Oxford Dictionary (1995) gives: '(of paper money) for which issuing authority does not undertake ever to pay coin.' The meaning attached to it here is 'does not represent a claim on the issuer for anything other than the equivalent amount of itself'.

From the point of view of the individual private holder of base money, it is an asset that can be realised, that is, exchanged for other goods, services or financial instruments of equal market value, at any time. The fact that the private sector as a whole cannot dispose of base money at its discretion is not a constraint that is internalised by the individual household. Each individual household believes and acts as if it can dispose of its holdings of base money at any time, at a price in terms of goods and services given by the reciprocal of the general price level at that time: the 'hot potato theory of money' is not perceived by any household as a constraint on its own, individual ability to get rid of money balances. The aggregation of the intertemporal budget constraints of the individual private households also does not incorporate the irredeemability property of fiat base money. Neither the individual household's intertemporal budget constraint nor the aggregated household sector's intertemporal budget constraint incorporate the constraint recognised by the government that the only claim a private holder of fiat base money has on the issuer is the right to exchange X units of fiat base money for X units of fiat base money.

The next step is to show how the existence of irredeemable fiat base money implies that fiat base money is, in a precise but non-standard way, net wealth to the consolidated private and public sectors, and that this bestows on monetary policy a *pure fiscal effect*: in a representative agent model with rational expectations, changes in the stock of nominal base money can change real private consumption demand by changing the permanent income or comprehensive (financial plus human) wealth of the household sector, after consolidation of the private and government sectors' intertemporal budget constraints. The definition of a pure fiscal effect holds constant the sequences of current and future price levels, nominal and real interest rates, real endowments and real public spending on goods and services.

This additional channel for monetary policy effectiveness is only operational, that is, only makes a difference to equilibrium prices and quantities, real and/or nominal, when the economy is in a liquidity trap, defined as an equilibrium when all current and future short

nominal interest rates are at their lower bounds (conventionally zero). Next I show that, with irredeemable fiat base money, liquidity trap equilibria cannot exist as rational expectations equilibria, provided the monetary policy rule satisfies some simple and intuitive properties. This is demonstrated both for a model with flexible prices and for a model with a New-Keynesian Phillips curve.[2] These same restrictions on the monetary policy rule also rule out the existence of deflationary bubbles in models where money is not the only financial asset (see Buiter and Sibert (2003)).[3]

Finally, the paper considers briefly the *practical modalities* of implementing a helicopter drop of base money in an economy in which monetary policy and fiscal policy are implemented through different institutions or agencies. How can the monetary authorities, either on their own or jointly with the fiscal authorities, implement a helicopter drop of money?

Central to the approach is the nature of government fiat base money as an *irredeemable final means of settlement* of private sector claims on the public sector. In what follows, 'irredeemable' is used as a shorthand for '*final* and *irredeemabl*e'.

To the best of my knowledge, the particular formalisation proposed in this paper of the notion that (fiat base) money is an asset to the private sector but not a liability to the public sector, has not been proposed before. It was not part of the transmission mechanism of a helicopter drop of money proposed by Friedman (1969). Whether and in what way 'outside' money is net wealth to the private sector or to the consolidated private sector and public sector was the subject of much discussion in some of the great treatises on monetary economics of the 1960s and 1970s, including Patinkin (1965), Gurley and Shaw (1960) and Pesek and Saving (1967). None of these contributions proposes the approach advocated in this paper, however. Clower (1967) stressed the unique properties of the monetary medium of

[2] It would also hold with an Old-Keynesian supply side, e.g. an accelerationist Phillips curve.

[3] Obstfeld and Rogoff (1983, 1986, 1996) have shown this for models in which money is the only financial asset. Without irredeemable base money, deflationary bubbles cannot be rules out in the models of this paper which have both fiat money and non-monetary, interest-bearing government debt as stores of value.

exchange (money buys goods and goods buy money but goods do not buy goods), but the cash-in-advance constraint he proposed did not require the monetary object to be an irredeemable IOU of the state. The discussions in Hall (1983), Stockman (1983), King (1983), Fama (1983), Sargent and Wallace (1984) and Sargent (1987) of outside money, private money and the payment of interest on money ask some of the same questions as this paper, but do not propose the same answer.

The endowment economy model with a flexible price level used in this paper is a deterministic version of that found in Lucas (1980), but Lucas's paper and the vast literature it inspired do not impose the asymmetric household and government solvency constraints proposed here. Variants of that same model have also been used in the debate on the Fiscal Theory of the Price Level (see e.g. Woodford (1995) and Buiter (2002)). This literature focused very closely on the government's intertemporal budget constraint, but the special significance of the irredeemability of government fiat money was not a theme that was developed. The closest approximations to explicit references to the irredeemability property of government fiat base money can be found in Sims (2000, 2003), Buiter (2003a,b), Eggertson (2003) and Eggertsson and Woodford (2003), but the idea is never developed formally and completely.

2 Fiat base money: the monetary authority's irredeemable, final means of settlement

Underlying the analysis that follows are the following three key primitive assumptions

Assumption 1: *Base money is perceived as an asset by each individual household. Each household believes it can always realise this asset in any period at the prevailing market price for money in that period.*

Assumption 2: *Base money does not have to be redeemed by the government – ever. It is the final means of settlement of government obligations vis-à-vis the private sector. It does not represent a claim on the issuer for anything other than the same amount of itself*.

Assumption 3: *Additional base money can be created at zero incremental cost by the government.*

Most monetary general equilibrium models with a non-predetermined general price level have at least one equilibrium in which the price of money is zero in each period. The flexible price level model considered in this paper is no exception. The interesting results of this paper apply only to equilibria in which there is a positive price of money in each period, and I will only consider such equilibria. For reasons of space, the analysis will be further restricted to fundamental equilibria; with one brief exception, bubbles and sunspots are not considered.

In most of the paper, a positive demand for real money balances is generated by the inclusion of real money balances as an argument in the direct utility function. A simple cash-in-advance model is also considered (in Section 4.4). Both these approaches to motivating a demand for fiat base money support equilibria in which the price of money is positive in each period.

In what follows, 'money' means *fiat base* money – the monetary liabilities of the sovereign (the state). The sovereign (the *government* in what follows) is the consolidated General Government and Central Bank. Real world base money today is currency (notes and coin which typically bear a zero nominal interest rate) plus commercial bank balances held with the Central Bank (which can be either interest-bearing or non-remunerated).[4] For the purpose of this paper, base money is the only form of money (there are no banks whose liabilities could serve as means of payment and medium of exchange) and all base money is assumed to have a government-determined nominal rate of interest, which is treated as exogenous (see Hall (1983) and Buiter and Panigirtzoglou (2001, 2003) for examples of models where the nominal interest rate on money is determined by a simple rule).

[4] Indeed it would be as easy to pay a negative nominal interest rate on bank reserves as a positive interest rate. Paying interest, negative or positive on currency would be more difficult and costly, but administratively and technically possible (see Porter (1999), Goodfriend (2000), Buiter and Panigirtzoglou (2001, 2003)).

Assumption 3 states that base money can be created at zero incremental cost by the government. For simplicity, in the formal model we assume that there also is no fixed cost of issuing money.

There are two key implications of Assumptions 1 and 2. First, once a private party has accepted base money from the government in payment for goods or services or in settlement of any other form of financial obligation of the government to the private party, there is no further claim by the private party on the government: base money is a (in our model the only) *final* means of settlement when the government settles an IOU with the private sector. Second, there is no redemption date for base money: base money does not have an infinite maturity - there is no redemption even in the infinitely distant future. As a financial instrument, currency is like a zero coupon perpetuity. With a non-zero nominal interest rate on base money, it would be like a positive or negative coupon perpetuity.

The property that base money is an *irredeemable final means of settlement* for obligations of the government to the private sector, is central for what follows. It is related to but not the same as *legal tender* status of money.[5] The popular definition of 'legal tender' is that of a means of payment than cannot be refused if it is offered (tendered) to settle a debt or other financial obligation.[6] The folk definition of legal tender is stronger than the legal definition. The folk definition of legal tender goes beyond the irredeemability property of base money that is central to this paper, insofar as it relates to the settlement of *all* claims, including claims among private parties. On the other hand, legal tender can be redeemable. Confederate fiat base money (Confederate Treasury notes) were convertible into Confederate bonds.[7]

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[5] US Federal Reserve notes carry the inscription: *"This note is legal tender for all debts, public and private"*. This is a short summary of the Section 102 of the Coinage Act of 1965 (Title 31 United States Code, Section 392), which contains the following text: *" All coins and currencies of the United States, regardless of when coined or issued, shall be legal tender for all debts, public and private, public charges, taxes, duties and dues."*

[6] The folk definition is the one found in generalist dictionaries. For instance, the Concise Oxford Dictionary defines *legal tender* as "currency that cannot legally be refused in payment of debt (usually up a limited amount for baser coins, etc.)".

[7] Three successive monetary reforms encouraged holders of Confederate Treasury notes to exchange these notes for Confederate bonds by imposing deadlines on their convertibility (see Burdekin and Weidenmier

The expression 'legal tender' has a technical meaning in relation to the settlement of a debt. The legal definition does *not* imply that legal tender is a means of payment that *must* be accepted by the parties to a transaction, but rather that it is a legally defined means of payment that *should not be refused* by a creditor in satisfaction of a debt. Thus, if a debtor pays in legal tender the exact amount he owes under the terms of a contract, he has a strong *prima-facie* defence in law, if he is subsequently sued for non-payment of the debt. However, in the US, there is no Federal statute which mandates that private businesses or individuals must accept Federal Reserve notes or coin or Treasury notes as a form of payment. Private businesses are free to develop their own policies on whether or not to accept cash unless there is a State law which says otherwise.

The feature of irredeemable base money that is key for this paper is that the acceptance of payment in base money by the government to a private agent constitutes a final settlement between that private agent (and any other private agent with whom he exchanges that base money) and the government. It leaves the private agent without any further claim on the government, now or in the future.

This irredeemability property is clearly attached to currency issued by the government (generally through the Central Bank, although the US even today has Treasury notes outstanding). It is brought out very nicely in the phrase *"...promise to pay the bearer the sum of..."* found on all Bank of England notes. It means that the Bank of England will pay out the face value of any genuine Bank of England note no matter how old. The promise to pay stands good for all time but simply means that the Bank will always be willing to exchange one (old, faded) £10 Bank of England note for one (new, crisp) £ 10 Bank of England note (or even for two £ 5 Bank of England notes). Because it promises only money in exchange for money, this 'promise to pay' is, in fact, a statement of the irredeemable nature of Bank of

(2003)).

England notes.[8] It is less clear whether the second conventional component of base money, commercial bank balances held with the central bank, are irredeemable in the same sense as currency. If they are not, the analysis that follows is applicable only to the currency component of base money.

Inside money is money that is a claim by one private agent on another private agent. *Strong* outside money is money that is an asset to the consolidated private and public sectors. Commodity money is strong outside money. *Weak* outside money is money that is not a claim by one private agent on another private agent. Thus all strong outside money is also weak outside money, but in addition monetary liabilities of the government held by the private sector are weak outside money. Fiat base money will be shown to be a limited form of *strong* outside money - perhaps the term *semi-strong* should be borrowed from the literature on the empirical testing of market efficiency to characterise it properly.

Some commodity monies have intrinsic, that is, non-monetary, value as a consumption, intermediate or capital good. Gold, salt, cattle and cigarettes are historical examples. I do not consider this kind of intrinsically valuable strong outside money. There is, however, a partial resemblance between the government-issued fiat base money considered this paper and commodity money that does not have any intrinsic value. Pet rocks, the candy wrappers that are part of many first expositions of Samuelson's pure consumption loans model (Samuelson (1958)), or the stone money used on the Micronesian island state of Yap are examples.

However, Yap stone money differs from the fiat base money considered in this paper in two ways. First, the stock outstanding can be varied only very slowly (if at all) and at (prohibitively) high marginal cost. The monetary policy rules that prevent liquidity traps

[8] Note also that the expression *"...promise to pay the bearer the sum of ..."* has nothing to do with the *legal tender* status of Bank of England notes.

would be hard to implement in a world with Yap-style outside money.[9] Second, during any period, t, say, strong outside money of the intrinsically worthless Yap variety constitutes net wealth to the consolidated private and public sectors in an amount given by the period t value of the period t stock.[10] Fiat base money issued by the government constitutes net wealth to the consolidated private and public sectors only in an amount given by the period t present discounted value of the terminal base money stock.

Except in Section 4.4, when I consider overlapping generations (OLG) models, the analysis uses a representative agent model in which debt neutrality or Ricardian equivalence apply: holding constant the sequences of current and future real public spending on goods and services and of nominal money stocks, the substitution of current borrowing for current lump-sum taxes will not affect the real or nominal equilibrium allocations, provided the government continues to satisfy its intertemporal budget constraint. Under the same conditions, a helicopter drop of non-monetary government debt will have no effect on any nominal or real equilibrium values. It seems preferable to establish the new transmission channel for monetary policy in a model where conventional *non-monetary* deficit financing does not have any real or nominal effects.

With irredeemable fiat base money, a current tax cut financed by printing money will not, *ceteris paribus*, that is, at given current and future prices, nominal and real interest rates, real activity and real public spending levels, have to be matched by a future increase in taxes of equal present value. A current tax cut financed by borrowing using non-monetary debt instruments, will, *ceteris paribus*, have to be matched by a future tax increase of equal present value.

This difference between the effects of monetising a government deficit and financing it by issuing non-monetary debt persists even if the interest rates on base money and on non-

[9] This is perhaps just as well. A helicopter drop of Yap-style stone money could be a serious health hazard for those on the ground.

[10] Yap money shares this property with intrinsically valuable commodity money.

monetary debt are the same (say zero), now and in the future. When both money and bonds bear a zero nominal interest rate, there remains a key difference between them: the principal of the bonds is redeemable, the principal of base money is not.

The paper shows how, with irredeemable fiat base money, the proper combination of monetary and fiscal policies can almost always, *ceteris paribus*, boost aggregate demand. The qualification *almost* reflects the existence of two possible exceptions. First, there is the possibility that perverse *future* policies (future reversals of current expansionary monetary policies) could, through the *present* rational anticipation of these policies, negate what would otherwise be the expansionary effect of the current policy measures on demand. Then there is the possibility that non-rational expectations of future offsetting policy actions could neutralise current expansionary measures for as long as the non-rational expectations persist.

The proposed expansionary policy measure is the mundane version of the helicopter drop of money proposed by Milton Friedman. The simplest example is a temporary tax cut or increased transfer payment by the government to the household sector, financed through an one-off, permanent expansion of the monetary base. It is key that the increase in the nominal stock of money associated with the helicopter drop be permanent, or at least that it is perceived as permanent. It will be effective only if it leads to an increase in the present discounted value of current and future net increases in the stock of base money. If the helicopter drop of money is expected to be reversed, in present value terms, in the future, the current helicopter drop will not raise aggregate demand.

3 Modeling household and government behaviour

I consider a simple dynamic competitive equilibrium model of a perishable endowment economy with a representative private agent and a government sector, consisting of a consolidated fiscal authority and monetary authority. In what follows, 'government' will

be used to refer to the consolidated fiscal and monetary authorities. The words *'state'* or *'sovereign'* are probably better than the word *'government'* to describe the consolidated General Government sector and Central Bank, but the usage 'government budget constraint' is by now too firmly established.[11] When the government is partitioned into distinct fiscal and monetary authorities in Section 5, the fiscal authority is called the *Treasury* and the monetary authority the *Central Bank*.

There is no uncertainty and markets are competitive.[12] Time, indexed by t, is measured in discrete intervals of equal length, normalised to unity. There is a countably infinite number of periods, indexed by $t \geq 0$. Household and government behaviour is modeled starting in period 1, taking as given the contractual financial obligations inherited from period 0.

3.1 Household behaviour

Households are price takers in all markets in which they transact. There is a continuum of households whose aggregate measure is normalised to unity. They receive an exogenous perishable endowment, $y_t > 0$, each period, consume $c_t \geq 0$ and pay net real lump-sum taxes τ_t. They have access to two stores of value: fiat base money, a liability of the government, a unit of which, if acquired during period t and held at the end of period t, $t \geq 0$, pays $1+i^M_{t+1} \geq 0$ of money in period $t+1$, and a nominal one-period bond which, in exchange for one unit of money in period t pays $1+i_{t+1} \geq 0$ units of money in period $t+1$..[13] The quantities of money and nominal one period bonds outstanding at the end of period t (and

[11] The General Government is the consolidated central (Federal) state (provincial, cantonal) and municipal (local) government sector. It also includes the off-budget or off-balance sheet public entities, like the social security funds, for whose liabilities the central, state or local government sector is ultimately responsible.
[12] Adding uncertainty and a richer menu of non-monetary securities would add notational complexity but would not alter any results as long as markets are complete.
[13] The payment of interest on base money has been studied by Hall (1983) and by Sargent and Wallace (1984) and Sargent (1987, Chapter 5.5)

the beginning of period $t+1$) are denoted M_t and B_t respectively. The money price of output in period t is $P_t \geq 0$. The government is assumed to have a monopoly of the issuance of base money, so $M_t \geq 0,\ 0 \leq t$. The nominal value of household net financial wealth at the beginning of period $t+1$ (inclusive of interest due) is denoted F^h_{t+1}, so

$$F^h_{t+1} \equiv (1+i^M_{t+1})M_t + (1+i_{t+1})B_t \tag{1}$$

The household objective function is given in equation (2); $m_j \equiv M_j/P_j$ is the stock of real money balances at the end of period t .

$$\begin{aligned} v_t &= \sum_{j=t}^{\infty} (1+\rho)^{-(j-t)} u(c_j,\ m_j) \\ \rho &> 0;\ c_j,\ m_j \geq 0;\ u_{cm} \geq 0 \end{aligned} \tag{2}$$

The period felicity function u is increasing, concave and twice continuously differentiable. I also will assume that consumption and real money balances are weak complements in the sense that the marginal utility of consumption is non-decreasing in the stock of real money balances, $u_{cm} \geq 0$. This is sufficient to rule out complications with the transversality condition when there is satiation with real money balances.

Money is held even when it is rate-of-return dominated by bonds as a store of value because end-of-period real money balances are an argument in the direct period felicity function. To permit the derivation of closed-form consumption rules, or consumption functions, and thus to enhance the transparency of the argument, the period felicity function in most of what follows is assumed to be double iso-elastic (with a constant intertemporal substitution elasticity, σ, and a constant static substitution elasticity between consumption and real money balances, φ (see equation (3)). All key propositions in this paper would go

through for more general functional forms and for most alternative ways of introducing money into the model including 'money in the shopping function', 'money in the production function' and 'cash-in-advance' models (Clower (1967)). A cash-in-advance model is considered in Section 4.4.

$$
\begin{aligned}
&\text{For } \sigma>0,\ \sigma\neq 1 \\
&v_t = \sum_{j=t}^{\infty}(1+\rho)^{-(j-t)}\sigma(\sigma-1)^{-1}z_j^{(\sigma-1)\sigma^{-1}} \\
&\text{For } \sigma=1 \\
&v_t = \sum_{j=t}^{\infty}(1+\rho)^{-(j-t)}\ln z_j \\
&\text{For } 0<\alpha<1,\ \varphi>0,\ \varphi\neq 1 \\
&z_j = \left[\alpha^{\frac{1}{\varphi}}c_j^{\frac{\varphi-1}{\varphi}}+(1-\alpha)^{\frac{1}{\varphi}}(M_j/P_j)^{\frac{\varphi-1}{\varphi}}\right]^{\frac{\varphi}{\varphi-1}} \\
&\text{For } \varphi=1 \\
&z_j = c_j^{\alpha}(M_j/P_j)^{1-\alpha}
\end{aligned}
\tag{3}
$$

The single-period household budget identity for $t \geq 1$ is given by[14]

$$F_{t+1}^h \equiv (1+i_{t+1})[F_t^h + P_t(y_t - \tau_t - c_t)] + (i_{t+1}^M - i_{t+1})M_t \tag{4}$$

Initial (period 0) financial asset stocks are predetermined:

$$
\begin{aligned}
B_0 &= \bar{B}_0 \\
M_0 &= \bar{M}_0 > 0
\end{aligned}
\tag{5}
$$

Let $b_t \equiv B_t/P_t$, $f_t^h \equiv F_t^h/P_t$ and $\pi_t \equiv (P_t/P_{t-1}) - 1$, where π_t is the rate of inflation.

The risk-free real interest rate r_t is defined by

[14] An equivalent representation is: $M_t + B_t \equiv (1 + i_t^M)M_{t-1} + (1 + i_t)B_{t-1} + P_t(y_t - \tau_t - c_t)$.

$$1 + r_{t+1} \equiv (1 + i_{t+1})P_t/P_{t+1} \tag{6}$$

Let $I_{t_0,t_1} \equiv \prod_{s=t_0}^{t_1}(1+i_s)^{-1}$, $t_1 \geq t_0$; $I_{t_0,t_0-1} \equiv 1$, be the nominal market discount factor between periods t_1 and t_0, and $R_{t_0,t_1} \equiv \prod_{s=t_0}^{t_1}(1+r_s)^{-1}$, $t_1 \geq t_0$; $R_{t_0,t_0-1} \equiv 1$, the corresponding real market discount factor. The solvency constraint of the household is that the present discounted value of terminal financial wealth must, in the limit as the time horizon goes to infinity, be non-negative, that is,

$$\lim_{N\to\infty} I_{t+1,N}F_N^h = \lim_{N\to\infty} I_{t+1,N}[(1+i_N^M)M_{N-1} + (1+i_N)B_{N-1}]^h \geq 0 \tag{7}$$

This no-Ponzi game condition in equation (7) incorporates the important assumption, stated explicitly as Assumption 1 in Section 2, that base money is (and is perceived to be by the household) an asset of the household that owns it.

Solving the household period budget identity forward yields

$$\begin{aligned} F_t^h &\equiv (1+i_t^M)M_{t-1} + (1+i_t)B_{t-1} \\ &= \sum_{j=t}^{\infty} I_{t+1,j}[P_j(c_j+\tau_j-y_j)+(i_{j+1}-i_{j+1}^M)(1+i_{j+1})^{-1}M_j] + \lim_{N\to\infty} I_{t+1,N}F_N^h \end{aligned} \tag{8}$$

The household period budget identity (4) can also be written as:

$$f_{t+1}^h \equiv (1+r_{t+1})(f_t^h + y_t - \tau_t - c_t) + (i_{t+1}^M - i_{t+1})(1+\pi_{t+1})^{-1}m_t \tag{9}$$

The associated solvency constraint is

$$\lim_{N\to\infty} R_{t+1,N} f_N = \lim_{N\to\infty} R_{t+1,N}[(1+i_N^M)(1+\pi_N)^{-1})m_{N-1} + (1+r_N)b_{N-1}]^h \geq 0 \tag{10}$$

Solving the period budget identity (9) forward yields

$$\begin{aligned} f_t^h &\equiv (1+i_t^M)(1+\pi_t)^{-1}m_{t-1} + (1+r_t)b_{t-1} \\ &= \sum_{j=t}^{\infty} R_{t+1,j}[c_j+\tau_j-y_j+(i_{j+1}-i_{j+1}^M)(1+i_{j+1})^{-1}m_j] + \lim_{N\to\infty} R_{t+1,N} f_N^h \end{aligned} \tag{11}$$

There obviously exists no equilibrium supporting a negative differential between the nominal interest on non-monetary financial claims and the nominal interest rate on money.[15] I therefore consider only monetary policy rules that support equilibria in which money is weakly dominated as a store of value, that is, equilibria supporting a non-negative differential between the nominal interest rate and the nominal interest rate on money.

The representative competitive consumer optimises in each period t the utility function given in equation (2) or (3) defined over non-negative sequences of consumption and end-of-period real money balances, subject to (7) and (8) (or equivalently, (10) and (11)), and the initial asset stocks given in (5). The household takes the tax sequence as given. The household's optimum programme is characterised, for $j \geq t$, by equations (12), (13) and (14).

$$u_c(c_j, m_j) = (1+r_{j+1})(1+\rho)^{-1}u_c(c_{j+1}, m_{j+1}) \tag{12}$$

$$u_c(c_j, m_j) = (1+i_{j+1})(i_{j+1}-i_{j+1}^M)^{-1}u_m(c_j, m_j) \tag{13}$$

[15] This follows from the simplest *no-arbitrage* argument. If the nominal interest rate on money were to exceed the nominal interest rate on bonds, households would have a 'risk-free pure profit machine' by borrowing through the issuance of non-monetary bonds and investing the proceeds in money.

$$\lim_{N\to\infty}(1+\rho)^{-(N-t)}u_c(c_N,\ m_N)f_N^h = 0 \tag{14}$$

The Euler equation (12) implies $u_c(c_N,\ m_N)=u_c(c_t,\ m_t)(1+\rho)^{N-t}R_{t+1,N}$. The transversality condition (14) can therefore also be written as

$$u_c(c_t,\ m_t)\lim_{N\to\infty}R_{t+1,N}f_N^h = 0 \tag{15}$$

As long as $u_c > 0$, that is, as long as there is economic scarcity, equation (15) is equivalent to the condition, given in (16), that the present discounted value of the terminal debt be zero exactly:

$$\lim_{N\to\infty}R_{t+1,N}f_N^h = P_t^{-1}\lim_{N\to\infty}I_{t+1,N}F_N^h = 0 \tag{16}$$

For the iso-elastic utility function in (3), the household's optimum programme satisfies:

$$\frac{c_{j+1}}{c_j} = \left(\frac{1+r_{j+1}}{1+\rho}\right)^{\sigma}\left[\left(1+\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+i_{j+2}}{i_{j+2}-i_{j+2}^M}\right)^{\varphi-1}\right)\left(1+\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+i_{j+1}}{i_{j+1}-i_{j+1}^M}\right)^{\varphi-1}\right)^{-1}\right]^{\frac{\sigma-\varphi}{\varphi-1}} \tag{17}$$

$$\begin{gathered} c_t = \alpha(1-\alpha)^{-1}[(i_{t+1}-i_{t+1}^M)(1+i_{t+1})^{-1}]^{\varphi}m_t \\ i_{t+1} \geq i_{t+1}^M \end{gathered} \tag{18}$$

and

$$\alpha^{\frac{\sigma-1}{(\varphi-1)\sigma}} c_t^{-\frac{1}{\sigma}} \left(1 + \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{1+i_{t+1}}{i_{t+1}-i_{t+1}^M} \right)^{\varphi-1} \right)^{\frac{\sigma-\varphi}{\sigma(\varphi-1)}} \lim_{N\to\infty} R_{t+1,N} f_N^h =$$

$$(1-\alpha)^{\frac{\sigma-1}{(\varphi-1)\sigma}} m_t^{-\frac{1}{\sigma}} \left(\frac{1+i_{t+1}}{i_{t+1}-i_{t+1}^M} \right) \left(1 + \left(\frac{\alpha}{1-\alpha} \right) [(i_{t+1}-i_{t+1}^M)(1+i_{t+1})^{-1}]^{\varphi-1} \right)^{\frac{\sigma-\varphi}{\sigma(\varphi-1)}} \lim_{N\to\infty} R_{t+1,N} f_N^h = 0 \qquad \textbf{(19)}$$

Equation (17) is the double iso-elastic version of the consumption Euler equation. Equation (18) is the optimality condition relating the money stock in period t to consumption in period t. Equation (19) is the transversality condition for the double isoelastic utility function. Note that the analysis is restricted to those double iso-elastic functions for which the assumption $u_{cm} \geq 0$ is satisfied. It is clear that there are such cases. First, consider $\varphi = 1$. In that case, $u_c = \frac{\alpha}{c^{\sigma^{-1}}} \left[\left(\frac{\alpha}{1-\alpha} \right) \left(\frac{i-i^M}{1+i} \right) \right]^{(\sigma-1)(\alpha-1)\sigma^{-1}}$. When $i = i^M$, that is, when real money balances are unbounded, $u_c = 0$ if $\sigma < 1$ (an intertemporal substitution elasticity less than one) and $c > 0$. When $\sigma \geq 1$, either u_c is independent of the real stock of money balances (when $\sigma = 1$), or u_c increases with the stock of real money balances (when $\sigma > 1$).

Next, consider $\sigma = \varphi$. In this case $u_c = (\alpha/c)^{1/\varphi}$. It is clear that u_c is independent of the stock of real money balances and that $u_c > 0$ for all bounded values of c.

As long as $u_c > 0$, the household solvency constraint binds (holds with equality) and equation (16) is part of the optimal programme. Under the conditions imposed on the household optimisation programme, the transversality condition (14) or (19) is sufficient (together with the other optimality conditions (12) or (17) and (13) or (18)) for an optimum.[16]

[16] There is an extensive literature on the necessity and sufficiency of the transversality condition in infinite horizon optimisation problems, see e.g. Arrow and Kurz (1970), Weitzman (1973), Araujo and Scheinkman (1983), Stokey and Lucas with Prescott (1989), Michel (1990) and Kamihigashi (2001, 2002). The conditions

If the money-in-the-direct-utility function of this Section is replaced a cash-in-advance model of money demand (see Section 4.4), the condition that $u_c > 0$ is always satisfied for bounded value s of c.

Note that the household's optimisation problem contains but one *state variable*, the net financial wealth of the household, f_t^h. The stock of real money balances, m_t, is not, from a formal mathematical point of view, a state variable, although it is, economically, a durable good. Formally, m_t is like c_t, a *control* variable. Efficient financial markets turn the individual portfolio allocation decision between money and nominal bonds into a decision rule with a single state variable: the predetermined inherited net financial wealth of the economic agent, f_t^h.[17]

When (16) holds, the household's intertemporal budget constraint becomes

$$F_t^h \equiv (1+i_t^M)M_{t-1} + (1+i_t)B_{t-1} = \sum_{j=t}^{\infty} I_{t+1,j}[P_j(c_j+\tau_j-y_j)+(i_{j+1}-i_{j+1}^M)(1+i_{j+1})^{-1}M_j]$$

or **(20)**

$$f_t^h \equiv (1+i_t^M)(1+\pi_t)^{-1}m_{t-1}+(1+r_t)b_{t-1}=\sum_{j=t}^{\infty} R_{t+1,j}[c_j+\tau_j-y_j+(i_{j+1}-i_{j+1}^M)(1+i_{j+1})^{-1}m_j]$$

From the optimality conditions (17) and (18) and the intertemporal budget constraint of the household (20), we obtain the household consumption function for $t \geq 1$ given in equations (21) to (25). Real comprehensive household wealth in period t is denoted w_t; it is the sum of real household financial wealth held at the beginning of period t, f_t^h, and

stated in the body of the text are sufficient for the set-up under consideration.

[17] If we set up the household optimisation problem as an infinite horizon discrete time Hamiltonian problem to which the discrete time maximum principle can be applied, there is no co-state variable associated with the stock of real money balances. Nor is there a transversality condition associated with the terminal value of the stock of real money balances. The stock of net financial wealth does have a co-state variable associated with it, and it also has the standard transversality condition (14) or (19) associate with its terminal value.

household real human wealth (the present discounted value of current and future after-tax endowment income), h_t.[18]

$$c_t = \mu_t w_t \tag{21}$$

$$w_t = f_t^h + h_t \tag{22}$$

$$f_t^h \equiv (1+r_t)\left[(1+i_t^M)(1+i_t)^{-1}m_{t-1}+b_{t-1}\right] = (1+i_t^M)M_{t-1}/P_t + (1+i_t)B_{t-1}/P_t \tag{23}$$

$$h_t = \sum_{j=t}^{\infty} R_{t+1,j}(y_j - \tau_j) \tag{24}$$

[18]The expression in (25) for the marginal propensity to consume out of comprehensive wealth simplifies when future real and nominal interest rates are expected to be constant. In that case, we get:

$\mu = \left[1 + (1-\alpha)\alpha^{-1}\left(\frac{1+i}{i-i^M}\right)^{\varphi-1}\right]^{-1}\left[\frac{(1+\rho)^{\sigma}-(1+r)^{\sigma-1}}{(1+\rho)^{\sigma}}\right]$. From this equation, the *steady state* marginal propensity to consume out of comprehensive wealth is independent of the nominal interest rate only if the elasticity of substitution between consumption and real money balances is one ($\varphi = 1$). In that case the steady-state marginal propensity to consume becomes $\mu = \alpha\left(\frac{(1+\rho)^{\sigma} - (1+r)^{\sigma-1}}{(1+\rho)^{\sigma}}\right)$. However, $\varphi = 1$ is not sufficient for the marginal propensity to consume to be independent of the sequence of current and future nominal interest rates outside steady state. For that to be true we require both φ and σ, the intertemporal substitution elasticity, to be equal to unity (see Fischer (1979a,b) and Buiter (2003)). When $\varphi = \sigma = 1$ (logarithmic intertemporal preferences and a unitary elasticity of substitution between the composite consumption good and real money balances), the marginal propensity to consume out of comprehensive wealth simplifies to the expression given below. It is now also independent of the real interest rate: $\mu = \frac{\alpha\rho}{1+\rho}$.

$$\mu_t \equiv \left\{ \sum_{j=t}^{\infty} \left[1 + \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{1+i_{j+1}}{i_{j+1} - i_{j+1}^{M}} \right)^{\varphi-1} \right] \prod_{s=t+1}^{j} \frac{(1+r_s)^{\sigma-1}}{(1+\rho)^{\sigma}} \left(\frac{1 + \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{1+i_{s+1}}{i_{s+1} - i_{s+1}^{M}} \right)^{\varphi-1}}{1 + \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{1+i_s}{i_s - i_s^{M}} \right)^{\varphi-1}} \right)^{\frac{\sigma-\varphi}{\varphi-1}} \right\}^{-1} \quad \textbf{(25)}$$

Monetary policy is said to have a *pure fiscal effect* on aggregate demand if changes in the sequence of current and future nominal money stocks can change aggregate demand holding constant the initial financial asset stocks and inherited financial obligations, the sequences of current and future values of nominal and real interest rates, money prices, real government spending on goods and services, and before-tax endowments.

From the consumption function given in equations (21) to (25), it follows that monetary policy can have a *pure fiscal effect* on aggregate demand (in our model, on private consumption) if and only if it affects the present discounted value of current and future taxes. The key policy question thus becomes: holding constant the sequence of real public spending on goods and services, can the authorities change the present discounted value of the sequence of current and future expected real taxes and thus change real consumption demand? In this representative agent model which exhibits debt neutrality or Ricardian equivalence, the answer turns out to be *'no'* unless base money is irredeemable.[19]

3.2 The government

The government sector is the consolidated fiscal and monetary authorities (the Treasury and Central Bank). Its decision rules are exogenously given, and like the household

[19]In overlapping generations models without operative intergenerational gift or bequest motives, postponing taxes while keeping their present discounted value constant will boost consumption demand if this fiscal action redistributes resources from households with long remaining time horizons (the young and the unborn) to households with short remaining time horizons (the old). See Section 4.

sector, it is subject to a solvency constraint or intertemporal budget constraint, which has to hold identically, that is, for all feasible values of the variables entering into the intertemporal budget constraint that are not choice variables of the government.

The government solvency constraint is the requirement that the present discounted value of the government's *non-monetary* debt must be non-positive in the limit as the time horizon goes to infinity. This follows from Assumption 2. Unlike bonds, base money, by assumption, does not have to be redeemed *ever* by the government. This means that while money is in the legal sense a liability of the government, it does not represent an effective liability of the government, in the sense that there is no obligation for the issuer ever to redeem it (to extinguish it by forcing the issuer to exchange it for something else with an equal market value). This is the key feature of the model that, even in a liquidity trap, (almost) always (barring (expectations of) perverse policies that the government will, in the long run, redeem its base money stock) makes a helicopter drop of money a means of boosting aggregate demand - unlike a helicopter drop of bonds.[20]

The aggregate financial liability of the government at the beginning of period t (principal plus interest) is

$$F_t^g \equiv (1+i_t^M)M_{t-1} + (1+i_t)B_{t-1}$$

The government's single-period budget identity for $t \geq 1$ is given in (26), its solvency constraint in (27).[21]

$$F_{t+1}^g \equiv (1+i_{t+1})[F_t^g + P_t(g_t - \tau_t)] + (i_{t+1}^M - i_{t+1})M_t \qquad \textbf{(26)}$$

[20] Note that even when the nominal interest rate on bonds equals the nominal interest rate on money and even if both interest rates are zero, the principal of the bond has to be redeemed by the government, but not the principal of the base money 'liability'.

[21] An equivalent expression to (26)) would be $A_t+B_t\equiv(1+i_t^M)M_{t-1}+(1+i_t)B_{t-1}+P_t(g_t-\tau_t$.

$$\lim_{N\to\infty} I_{t+1,N}[F_N^g - (1+i_N^M)M_{N-1}] = \lim_{N\to\infty} I_{t+1,N}(1+i_N)B_{N-1} \leq 0 \qquad \textbf{(27)}$$

Consider the case where both the government-determined nominal interest rate on base money and the nominal interest rate on non-monetary financial claims are zero, now and in the future ($i_t^M = i_t = 0,\ t \geq 1$). The government solvency constraint implies that, in this case, the net non-monetary government debt has to be retired in the long run: $\lim_{N\to\infty} B_{N-1} \leq 0$. From a financial point of view, bonds are in this case like base money in that both have a zero nominal interest rate. Bonds are, however, unlike base money in that their principal must be redeemed, while there is no redemption obligation for base money. From the point of view of the government's solvency constraint or intertemporal budget constraint, there is no requirement that, $\lim_{N\to\infty} I_{t+1,N}(1+i_N^M)M_{N-1} \leq 0$, regardless of whether the interest rate on non-monetary financial instruments exceeds or equals the interest rate on base money.[22]

Using the definition $f_t^g \equiv F_t^g/P_t$, We can rewrite (26) and (27) as

$$f_{t+1}^g \equiv (1+r_{t+1})(f_t^g + g_t - \tau_t) + (i_{t+1}^M - i_{t+1})(1+\pi_{t+1})^{-1}m_t \qquad \textbf{(28)}$$

and

$$\lim_{N\to\infty} R_{t+1,N}[f_N^g - (1+i_N^M)(1+\pi_N)^{-1}m_{N-1}] = \lim_{N\to\infty} R_{t+1,N-1}\, b_{N-1} \leq 0 \qquad \textbf{(29)}$$

[22]The government's solvency constraint in (27) in principle would also permit the government to be, in the long run and in present discounted value, a net creditor in non-monetary financial claims, that is, $\lim_{N\to\infty} I_{t+1,N}(1+i_N)B_{N-1} < 0$, but this possibility is not pursued further in this paper. If government behaviour instead of being characterised by a number of *ad-hoc* policy rules, were to be derived from the optimisation of a reasonable objective function (e.g. that of the representative household), and if there were any real resource costs associated with raising tax revenues (or if taxes were distortionary), the government would always satisfy its solvency constraint with equality. Here the requirement that the government solvency constraint binds is a primitive assumption.

From (28) and (29), assumed to hold with equality, we obtain the following intertemporal budget constraint for the government:[23]

$$f_t^g \equiv (1+i_t^M)M_{t-1}/P_t + (1+i_t)B_{t-1}/P_t \equiv (1+i_t^M)(1+\pi_t)^{-1}m_{t-1} + (1+r_t)b_{t-1}$$
$$= \sum_{j=t}^{\infty} R_{t+1,j}[\tau_j - g_j + (i_{j+1} - i_{j+1}^M)(1+\pi_{j+1})^{-1}m_j] + \lim_{N\to\infty} R_{t+1,N}m_{N-1} \tag{30}$$

3.3 Consolidating the household and government accounts

The final step in the formal argument is that conditions (31) and (32) or, equivalently, conditions (33) and (34) hold.

$$F_t^g = F_t^h = F_t, \quad t \geq 0 \tag{31}$$

$$\lim_{N\to\infty} I_{t+1,N} F_N^h = 0$$
$$\lim_{N\to\infty} I_{t+1,N} F_N^g = \lim_{N\to\infty} I_{t+1,N}(1+i_N^M)M_{N-1} \tag{32}$$

$$f_t^g = f_t^h = f_t, \quad t \geq 0 \tag{33}$$

[23] Note that a non-zero value for the discounted terminal money stock satisfies the homogeneous equation of the government's period budget identity (28): $\lim_{N\to\infty} R_{t+2,N}(1+i_N^M)(1+\pi_N)^{-1}m_{N-1} = (1+r_{t+1})\lim_{N\to\infty} R_{t+1,N}(1+i_N^M)(1+\pi_N)^{-1}m_{N-1}$.

$$\lim_{N\to\infty} R_{t+1,N} f_N^h = 0$$

$$\lim_{N\to\infty} R_{t+1,N} f_N^g = \lim_{N\to\infty} R_{t+1,N}(1+i_N^M)(1+\pi_N)^{-1} m_{N-1} \tag{34}$$

The last term of equations (32) and (34) need not equal zero. In particular, if there is a liquidity trap, that is, $i_t = i_t^M$, $t \geq 1$ with, say, a constant nominal interest rate on base money, then $\lim_{N\to\infty} I_{t+1,N}(1+i_N^M)M_{N-1} = \lim_{N\to\infty}(1+\bar{i}^M)^{-(N-1)} M_{N-1}$. As long as the initial stock of base money is positive, if the growth rate of the nominal base money stock is not less than the nominal interest rate on base money, that is, $M_t/M_{t-1} \geq (1+\bar{i}^M)^{-1}$, then $\lim_{N\to\infty}(1+\bar{i}^M)^{-(N-1)} M_{N-1} > 0$. For instance, in the empirically relevant case where the nominal interest rate on base money is zero, $\lim_{N\to\infty}(1+\bar{i}^M)^{-(N-1)} M_{N-1} = \lim_{N\to\infty} M_{N-1}$, so in this example any non-negative long-run growth rate of the nominal base money stock will produce a positive value for $\lim_{N\to\infty} I_{t+1,N}(1+i_N^M)M_{N-1}$. This suggests a simple design feature for monetary policy rules to rule out liquidity trap equilibria, as shown in Section 4.

We can write the government period budget identity (28) and solvency constraint (29) in the following equivalent form:

$$b_t \equiv (1+r_t)b_{t-1} + g_t - \tau_t - [M_t - (1+i_t^M)M_{t-1}]P_t^{-1} \tag{35}$$

$$\lim_{N\to\infty} R_{t+1,N} b_N \leq 0 \tag{36}$$

Solving (35) recursively forward and using (36), assumed to hold with equality, we get the intertemporal budget constraint in (37), which is equivalent to the one in (30):

$$(1+r_t)b_{t-1} \equiv (1+i_t)B_{t-1}P_t^{-1} \equiv \sum_{j=t}^{\infty} R_{t+1,j}\left(\tau_j - g_j + [M_j - (1+i_j^M)M_{j-1}]P_j^{-1}\right) \tag{37}$$

We can use the government's intertemporal budget constraint, (37), to eliminate non-monetary government debt from the household's comprehensive wealth, given in (22), (23) and (24), and thus from the household consumption function, given in (21) through (25). This produces:

$$w_t \equiv (1+i_t^M)M_{t-1}P_t^{-1} + \sum_{j=t}^{\infty} R_{t+1,j}\left(y_j - g_j + [M_j - (1+i_j^M)M_{j-1}]P_j^{-1}\right) \tag{38}$$

Note that

$$\begin{aligned}&(1+i_t^M)M_{t-1}P_t^{-1} + \lim_{N\to\infty}\sum_{j=t}^{N} R_{t+1,j}[M_j - (1+i_{j-1}^M)M_{j-1}]P_j^{-1} \\ &\equiv \lim_{N\to\infty}\sum_{j=t+1}^{N} R_{t+1,j}(i_j - i_j^M)M_{j-1}P_j^{-1} + \lim_{N\to\infty} R_{t+1,N}(1+i_N^M)M_{N-1}P_N^{-1}\end{aligned} \tag{39}$$

Substituting (39) into (38), and using the money demand first-order condition (18), the consumption function can be written as follows:

$$c_t = \hat{\mu}_t \hat{w}_t \tag{40}$$

$$\hat{w}_t = \sum_{j=t}^{\infty} R_{t+1,j}(y_j - g_j) + \lim_{N\to\infty} R_{t+1,N}(1+i_N^M)(1+\pi_N)^{-1} m_{N-1}$$
$$= \sum_{j=t}^{\infty} R_{t+1,j}(y_j - g_j) + P_t^{-1} \lim_{N\to\infty} I_{t+1,N}(1+i_N^M) M_{N-1} \tag{41}$$

$$\hat{\mu}_t \equiv \left\{ \sum_{j=t}^{\infty} \prod_{s=t+1}^{j} \frac{(1+r_s)^{\sigma-1}}{(1+\rho)^{\sigma}} \left(\frac{1+\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+i_{s+1}}{i_{s+1}-i_{s+1}^M}\right)^{\varphi-1}}{1+\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+i_s}{i_s-i_s^M}\right)^{\varphi-1}} \right)^{\frac{\sigma-\varphi}{\varphi-1}} \right\}^{-1} \tag{42}$$

Equations (40) to (42) differ from the consumption function that would be obtained without irredeemable base money (that is, with the solvency constraint for the government specified symmetrically to that of the private sector) because of the presence of the present discounted value of the terminal money stock, $\lim_{N\to\infty} R_{t+1,N}(1+i_N^M)(1+\pi_N)^{-1} m_{N-1} \equiv$ $P_t^{-1} \lim_{N\to\infty} I_{t+1,N}(1+i_N^M) M_{N-1}$ in the expression for the intertemporal budget constraint of the consolidated private and government sectors. Irredeemable base money is net wealth to the consolidated private and public sectors in the limited sense that the present discounted value of the terminal stock of base money is perceived as part of the consolidated resource base for private consumption, alongside the present discounted value of the sequence of real endowments net of real government spending, $\sum_{j=t}^{\infty} R_{t+1,j}(y_j - g_j)$. In Section 4, it is shown that the present discounted value of the terminal money stock will be zero in well-behaved (non-

liquidity trap) equilibria. It can, however, be positive in liquidity trap equilibria. This will suggest ways of specifying monetary policy rules in such a way that liquidity trap equilibria cannot exist.

The intuition behind equation (42) is straightforward.[24] From the perspective of the government, there is no requirement that the present discounted value of its aggregate terminal financial liabilities be non-positive. While the present discounted value of the government's terminal non-monetary liabilities must be non-positive, there is no non-positivity constraint on the present discounted value of its irredeemable monetary liabilities. At the beginning of period t, for given sequences of prices, real and nominal interest rates, real public spending and real before-tax endowments, $\{P_j, r_j, i_j, i_j^M, g_j, y_j; j \geq t\}$, the government can therefore, as far as its perception of its own intertemporal budget constraint (30) or (37) is concerned, reduce the present discounted value of its current and future real tax sequence by increasing the present discounted value of its sequence of real 'net' monetary issuance (or real net seigniorage) $\{(M_j-(1+i_j^M)M_{j-1})P_j^{-1}; j \geq t\}$ by that same amount.[25] From (39), the sum of the real value of the initial stock of money balances plus the present value of current and future real net monetary issues is given by $\lim_{N\to\infty} R_{t+1,N}(1+i_N^M)\frac{M_{N-1}}{P_N} - \sum_{j=t+1}^{\infty} R_{t+1,j}\left(\frac{i_j-i_j^M}{1+\pi_j}\right)m_{j-}$. We will see in Section 4 that $\lim_{N\to\infty} R_{t+1,N}(1+i_N^M)M_{N-1}P_N^{-1}=0$ in well-behaved equilibria (that is, non-

[24]Equation (35) could have been obtained more directly by combining the household intertemporal budget constraint (12) and the government intertemporal budget constraint (25) and using the first-order conditions (13) and (14).

[25]It is important that, in this statement, the current and future values of the general price level are held constant. From the government's intertemporal budget constraint, equation (37), any change in the initial general price level, P_t, would change the real value of the initial stock of non-monetary debt, if $B_t \neq 0$. Equilibrium changes in real taxes and net seigniorage would have to allow for this.

liquidity trap equilibria, with $i_t > i_t^M$). If follows that the sum of the present discounted value of current and future real net seigniorage plus the real value of the stock of initial money balances equals the present discounted value of the interest bill saved because private agents hold base money rather than non-monetary debt, $\sum_{j=t+1}^{\infty} R_{t+1,j}(i_j - i_j^M)(1+\pi_j)^{-1} m_{j-1}$. The sequences of nominal and real interest rates and, therefore also of real money demands (when the lower bounds on nominal interest rates are not binding, see (18)) are held constant in the characterisation of the pure fiscal effect of monetary policy. It follows that in well-behaved equilibria, there is no pure fiscal effect of monetary policy.

Equations(40), (41) and (42) do not say that, if $\lim_{N\to\infty} R_{t+1,N}(1+i_N^M)M_{N-1}P_N^{-1}=0$, monetary policy cannot affect real consumption demand (it is clear that they say nothing about the ability of monetary policy to influence nominal consumption demand). If the present value of the terminal base money stock is zero, monetary policy can still affect *real* consumption if it can affect either current or anticipated future real endowment income, or (more plausibly) current and anticipated real and nominal interest rates.

Equations (40), (41) and (42) have nothing at all to say about the effect of monetary policy on the current and future values of the general price level. Other equilibrium conditions, including the monetary equilibrium condition, are required to determine these. The intertemporal budget constraints then show what kind of restrictions must be imposed on the government's fiscal, financial and monetary programme (FFMP) to support the equilibrium in question.

In a world with strong outside money, say Yap money, and without fiat base money issued by the government, the analogue to the comprehensive wealth of the private sector given in (42) would be [26]:

$$\hat{w}_t = \sum_{j=t}^{\infty} R_{t+1,j}(y_j - g_j) + P_t^{-1} M_t^y \tag{43}$$

For simplicity, I have assumed that Yap money does not bear interest and that nature is not expected to provide future additions to the stock of Yap money. An endowment of intrinsically valuable commodity money would be represented in a similar manner.

3.4 The Fiscal-Financial-Monetary Programme (FFMP) of the government

Real public spending on goods and services is assumed constant:

$$0 \leq g_t = \bar{g} \qquad t \geq 1 \tag{44}$$

Unless an alternative tax rule is specified, real lump-sum taxes are assumed to vary endogenously to keep constant the real stock of government financial debt (monetary and non-monetary). Using (28) and $f_{t+1} = f_t = f_0$, $t \geq 0$, it follows that:

$$\tau_t = \bar{g} + \frac{r_{t+1}}{1+r_{t+1}} f_0 + \left(\frac{i_{t+1}^M - i_{t+1}}{1+i_{t+1}} \right) m_t = \bar{g} + \frac{r_{t+1}}{1+r_{t+1}} f_0 + \left(\frac{\alpha - 1}{\alpha} \right) \left(\frac{i_{t+1} - i_{t+1}^M}{1+i_{t+1}} \right)^{1-\varphi} c_t \tag{45}$$

We assume that the nominal interest rate on base money is exogenous and constant, that is,

$$i_t^M = \bar{i}^M, \qquad t \geq 1 \tag{46}$$

[26] M_t^y is the stock of Yap money. Yap money is the numeraire and P is the price level in units of Yap money.

I shall consider two kinds of monetary policy rules. The first is a constant growth rate for the nominal stock of base money:

$$M_{t+1} = (1 + \bar{\nu})M_t, \qquad t \geq 1 \; ; \; M_0 > 0; \;\; \bar{\nu} \geq (1+\bar{i}^M)(1+\rho)^{-1} \tag{47}$$

This is the only monetary rule considered for the flexible price level version of the model. The second kind of monetary rule is the combination of a simplified Taylor rule for the short nominal interest rate (when application of this rule does not cause the short nominal interest rate to violate the lower bound constraint), and a constant growth rate of the nominal money stock, when the application of the Taylor rule would cause the short nominal interest rate to violate the lower bound.[27] The nominal interest rate in that case is kept at the lower bound. This rule is given in (48). It will be the rule considered for the New-Keynesian version of the model.

$$\begin{aligned}
&\text{If } 1+\pi^* + \gamma(\pi_{t+1}-\pi^*) > (1+\bar{i}^M)(1+\rho)^{-1}, \quad \gamma > 1 \\
&\qquad 1+i_{t+1}=(1+\rho)[1+\pi^* + \gamma(\pi_{t+1}-\pi^*)] \\
&\text{If } 1+\pi^* + \gamma(\pi_{t+1}-\pi^*) \leq (1+\bar{i}^M)(1+\rho)^{-1} \\
&\qquad 1+i_{t+1}=1+\bar{i}^M \\
&\text{and} \\
&\qquad M_{t+1}/M_t = 1 + \bar{\nu}, \qquad 1 + \bar{\nu} > 1+\bar{i}^M
\end{aligned} \tag{48}$$

Equation (48) says that, as long as the lower bound constraint on the short nominal interest rate is not binding, the short nominal interest rate rises more than one-for-one with the (expected) inflation rate. When $1+\pi_{t+1} > \gamma^{-1}[(1+\bar{i}^M)(1+\rho)^{-1}+(\gamma-1)(1+\pi^*)]$ (the 'normal'

[27] It is a simplified Taylor rule, because the nominal interest rate does not respond to the output gap, $y_t - y^*$. Nothing significant depends on this simplification.

region) the nominal money stock is endogenously determined through the money demand function:

$$M_t/P_t \equiv m_t = (1-\alpha)\alpha^{-1}[(1+i_{t+1})(i_{t+1}-\bar{i}^M)^{-1}]^{\varphi} c_t \qquad i_{t+1} \geq \bar{i}^M \tag{49}$$

The behaviour of the stock of non-monetary public debt follows from the constancy of f_t and the behaviour of the endogenous stock of real money balances:

$$b_t \equiv B_t/P_t = f_0 - (1+\bar{i}^M)(1+i_{t+1})^{-1}m_t \tag{50}$$

Equation (48) also says that, if the application of the Taylor rule implies a value for the short nominal rate of interest that is less than the value of the nominal interest rate on base money, the short nominal interest rate instead is set equal to $\bar{i}^M$. From the money demand equation (49) it then follows that, with the demand for real money balances unbounded when $i = \bar{i}^M$, the authorities can choose any sequence for the nominal money stock, provided they adjust the nominal stock of non-monetary debt appropriately (see (50)). This is consistent, when $i=\bar{i}^M$, with the tax rule in (45) which keeps real net financial debt constant. For concreteness, I assume that the authorities choose a constant proportional growth rate $\bar{\nu}$ for the nominal money stock.[28]

4. Irredeemable money in general equilibrium

I consider two alternative supply-side specifications: a flexible price or New-Classical model with an exogenous and constant level of capacity output, $y^* > 0$, and a New-Keynesian

[28]An alternative FFMP would keep the real value of non-monetary public debt constant, that is, total real financial wealth constant, rather than the real stock of non-monetary debt, that is, $B_t = (1 + \pi_t)B_{t-1}$. Taxes in that case would be given by: $\tau_t = \bar{g} + r_t b_0 - [M_t - (1+\bar{i}^M)M_{t-1}]P_t^{-1}$.

Phillips curve, with a predetermined general price level but a non-predetermined, forward-looking rate of inflation. For both models, actual output always equals demand, so

$$y_t = c_t + g_t \qquad t \geq 1 \tag{51}$$

4.1 A flexible price level

With a flexible price level actual output always equals the exogenous level of capacity output:

$$y_t = y^* \tag{52}$$

For an equilibrium to exist, public spending must be less than capacity output:

$$0 \leq g_t = \bar{g} < y^*$$

It follows that any equilibrium must satisfy the following:

$$c_t = y^* - \bar{g} \tag{53}$$

$$\begin{aligned} c_t &= (y^* - \bar{g})\hat{\mu}_t \sum_{j=t}^{\infty} R_{t+1,j} + \hat{\mu}_t \lim_{N\to\infty} R_{t+1,N}(1+\bar{i}^M)(1+\pi_N)^{-1} m_{N-1} \\ &= (y^* - \bar{g})\hat{\mu}_t \sum_{j=t}^{\infty} R_{t+1,j} + \hat{\mu}_t P_t^{-1} \lim_{N\to\infty} I_{t+1,N}(1+\bar{i}^M) M_{N-1} \end{aligned} \tag{54}$$

$$\begin{aligned} 1 + r_j &= (1+\rho)\left(\frac{1+(1-\alpha)\alpha^{-1}[(1+i_{j+2})(i_{j+2}-i_{j+2}^M)^{-1}]^{\varphi-1}}{1+(1-\alpha)\alpha^{-1}[(1+i_{j+1})(i_{j+1}-i_{j+1}^M)^{-1}]^{\varphi-1}}\right)^{\frac{\sigma-\varphi}{\sigma(1-\varphi)}} \\ &= 1+\rho \text{ if either } \sigma = \varphi \text{ or } \varphi = 1 \end{aligned} \tag{55}$$

$$M_t/P_t = (1-\alpha)\alpha^{-1}\left((1+r_{t+1})P_{t+1}P_t^{-1}[(1+r_{t+1})P_{t+1}P_t^{-1}-(1+\bar{i}^M)]^{-1}\right)^{\varphi}(y^*-\bar{g})$$
$$1+i_{t+1} \equiv (1+r_{t+1})P_{t+1}P_t^{-1} \geq 1+\bar{i}^M \tag{56}$$

The propensity to consume out of consolidated comprehensive wealth, $\hat{\mu}_t$, is defined in (42). Consider the case where the authorities fix the growth rate of the nominal stock of base money (equation (47)). First, note that when $1+\bar{v} > (1+\bar{i}^M)(1+\rho)^{-1}$ there exists a stationary equilibrium, the *fundamental* equilibrium, with $\pi = \bar{v}$.[29] The stock of real money balances is constant and finite. In this stationary equilibrium, the constant real interest rate is positive ($r = \rho > 0$), and the discounted value of the terminal money stock is therefore zero. The question of interest here is, does there exist a fundamental equilibrium that is also a liquidity trap, that is, can equations (42), (47) and (52) to (56) be satisfied for all $t \geq 1$ with $1 + i_t = 1 + \bar{i}^M$?

Proposition 1.

A liquidity trap equilibrium does not exist in the flexible price model if the growth rate of the stock of nominal base money is equal to or greater than the nominal interest rate on base money, that is, if $\bar{v} \geq \bar{i}^M$.

The proof of Proposition 1 is trivial. In a fundamental equilibrium, $\hat{\mu}_t = (\sum_{j=t}^{\infty} R_{t+1,j})^{-1} = \rho(1+\rho)^{-1}$. Equations (53) and (54) can both hold only if

$$\lim_{N\to\infty} R_{t+1,N}(1+\bar{i}^M)(1+\pi_N)^{-1}m_{N-1} \equiv P_t^{-1}\lim_{N\to\infty} I_{t+1,N}(1+\bar{i}^M)M_{N-1} = 0 \tag{57}$$

[29] There also exists an equilibrium with a zero price of money in each period. In addition there are non-fundamental or bubble equilibria (see Buiter and Sibert (2003)). No comprehensive taxonomy and treatment is attempted here.

A liquidity trap is defined as a situation where $i_t = \bar{i}^M$, $t \geq 1$. It follows that in a liquidity trap equation (57) can hold only if

$$\lim_{N\to\infty}(1+\bar{i}^M)^{-(N-1-t)} \prod_{s=t+1}^{N-1} (1+\pi_s)m_{N-1} = P_t^{-1} \lim_{N\to\infty}(1+\bar{i}^M)^{-(N-1-t)}M_{N-1} = 0$$

If the initial nominal stock of base money (at time t say) is positive and the growth rate of the nominal stock of base money is not less than the nominal interest rate on base money, then $\lim_{N\to\infty}(1+\bar{i}^M)^{-(N-1-t)}M_{N-1} > 0$. Equation (57) can then only be satisfied if $P_t = +\infty$. However, if $P_t = +\infty$ and the period t nominal money stock is finite, then the period t value of the real stock of base money is zero. If $M_t/P_t = 0$, then the monetary equilibrium condition (56) can only be satisfied with an infinite nominal interest rate. By assumption, $i = \bar{i}^M$. With the iso-elastic money demand function of (56), the demand for real money balances becomes unbounded when $i_t = \bar{i}^M$. This unbounded demand for real money balances when $i_t = \bar{i}^M$ is not necessary for Proposition 1 to hold, however. All that is required is that the demand for real money balances be positive when $i_t = \bar{i}^M$. Proposition 1 therefore applies equally when the money demand function is derived from a strict cash-in-advance constraint (see Section 4.4).

Proposition 1 has the following implication for monetary policy in the practically relevant case where the nominal interest rate on base money is zero.

Corollary 1

When the nominal interest rate on base money is zero, there can only be a liquidity trap equilibrium in the flexible price level model if, in the long run, the

authorities (are expected to) reduce the nominal stock of base money to zero. Any monetary rule that does not lead to eventual demonetisation of the economy rules out a liquidity trap equilibrium.

Proposition 1 also has obvious implications for the existence of deflationary bubbles in the flexible price level model.

Corollary 2

In the flexible price level model, deflationary bubbles do not exist when base money is irredeemable, even though base money is not the only financial asset. Without the irredeemability of base money, deflationary bubbles would exist in models with non-monetary financial instruments in the private portfolio.

Proof: see Buiter and Sibert (2003) (see also. Obstfeld and Rogoff (1983, 1986, 1996)).

4.2 The helicopter drop of money

How should one represent a 'helicopter drop of base money' in this model? It is apparent from the equilibrium conditions (42) and (52) to (56), that the neutral monetary operation described by Friedman in the introductory quote to this paper can represent both an a-historical ('parallel universes') counterfactual or a real-time (that is, calendar-time) counterfactual. The a-historical interpretation of the neutral monetary operation requires a change in the predetermined initial money stock. As long as the arrow of time moves just in one direction, initial conditions cannot be varied in a real-time counterfactual: at the beginning of period *t*, M_{t-1} is given. One can compare, starting in period *t*, two alternative economies that are identical in all but two respects. First, in the second economy (indexed with superscript 2) the path of the nominal stock of base money lies $\bar{\kappa} > 0$ percent above that in the first economy (indexed with superscript 1), that is, $M_j^2 = (1 + \bar{\kappa})M_j^1$, $j \geq t-1$. Second, the governments in both economies choose sequences for non-monetary debt issuance and lump-sum taxes that satisfy their intertemporal budget constraints (real government spending sequences and the nominal interest rates on base

money are the same in the two economies). An example would be the tax rule (45). The non-monetary debt and tax sequences can therefore be different in the two economies.[30] [31]

Consider an equilibrium for the benchmark economy given by $\{P_j^1;\ i_j^1;\ r_j^1;\ c_j^1; j \geq t\}$. Provided current and future equilibrium nominal interest rates are not at their lower bounds ($i_j^1 > \bar{i}^M$, $j \geq t$), there exists an equilibrium in the counterfactual economy given by $\{P_j^2 = (1+\bar{\nu})P_j^1;\ i_j^2 = i_j^1;\ r_j^2 = r_j^1;\ c_j^2 = c_j^1;\ j \geq t\}$: money is neutral across these two equilibria.[32]

In general, an equiproportional increase in current and future nominal prices will reduce the real value of any net non-monetary nominally denominated financial assets or liabilities the private and public sectors may have. In the model this will be the case if $B_t \neq 0$, for some $j \geq t-1$. Because of the government's intertemporal budget constraint and the debt-neutrality properties implied by the use of a representative agent, any change in the real value of the net non-monetary debt of the government will be matched by a change in current and expected future lump sum taxes of equal present discounted value.

The interpretation of Friedman's helicopter drop of money as an event taking place in real or calendar time formalises it as an unanticipated, temporary tax cut (or transfer payment increase) in period t, financed through a permanent increase, starting in period t, in the nominal stock of base money. For simplicity I consider again a world in which, before and after the increase in the stock of base money, the stock of nominal base money is expected to stay

[30] They would be the same if there were no nominally denominated non-monetary debt.

[31] The literal version of Friedman's experiment has a constant nominal money stock in the benchmark economy (economy 1), that is, $M_j^1 = \bar{M}^1$, $j \geq t-1$. The counterfactual economy (economy 2) has $M_j^2 = \bar{M}^2 = (1+\bar{\kappa})\bar{M}^1$, $j \geq t-1$.

[32] There may exist other equilibria also, including non-monetary and sunspot or bubble equilibria. That is not the focus of this paper.

constant forever.[33] I again consider only equilibria in which the lower bound on the nominal interest rate is not a binding constraint. In this historical or real-time counterfactual, there again exists an equilibrium in which money is neutral, even though the initial, predetermined stock of money is not increased when the current, period t, nominal money stock and the nominal money stocks expected beyond t increase.

It is clear, however, that, right from the initial period, t, on, the equilibrium price sequence is the same as in the a-historical counterfactual, and that the same holds for the equilibrium sequences of real and nominal interest rates and consumption; money is neutral despite the fact that the increase in the period t price level by $\bar{\kappa}$ percent reduces the real value of the initial, predetermined, stock of base money, M_{t-1}/P_t (in addition to the real value of the initial, predetermined, stock of nominal government debt, B_{t-1}/P_t. The initial money stock does not figure in any of the equilibrium conditions (52) to (56) and only plays a role 'in the background' through the government's and the household sector's intertemporal budget constraints, and the government's FFMP. With the tax rule given in (45) (or with the alternative tax rule described in footnote (28), current and future lump-sum taxes adjust to absorb any impact on the intertemporal budget constraints of a lower real value of the inherited stocks of nominal base money and nominal non-monetary public debt. This suggests the following proposition:

Proposition 2:

> *In the representative agent model, it does not matter how money gets into the system: Because of Ricardian equivalence, helicopter money drops have the same effect on real and nominal equilibrium prices and quantities as open market purchases.*

[33] With a time-varying nominal money stock, the exercise would become an unanticipated equiproportional increase in the nominal money stocks in period t and beyond.

Compare two economies, indexed by superscripts 1 and 2. Initial conditions are identical. In one economy the government increases the current, period t, nominal stock of base money by an amount ΔM_t, through a period t tax cut. In the other economy the same increase in the nominal stock of base money in period t is achieved through the purchase in period t of non-monetary debt by the government (a so-called 'open market purchase'). The sequences of real public spending on goods and services are the same in the two economies, and so are the nominal money stock sequences in period t and later. The government satisfies its intertemporal budget constraint in both economies, for instance by applying the tax rule (45) after period t. In follows that the equilibrium sequences for all nominal and real endogenous variables (except, possibly, the real values of current and future nominal non-monetary debt stocks and the real value of current and future lump-sum taxes) are the same in the two equilibria: $\{P_j^1 = P_j^2;\ i_j^1 = i_j^2;\ r_j^1 = r_j^2; c_j^1 = c_j^2; j \geq t\}$.

Proposition (2) is a direct implication of debt neutrality or Ricardian equivalence, and many versions of it are around (see e.g. Wallace (1981) and Sargent (1987)). Proof is by inspection of the equilibrium conditions (52) to (56) and the government's intertemporal budget constraint (37). Debt neutrality or Ricardian equivalence means that a helicopter drop of government non-monetary debt makes no difference to any real or nominal equilibrium values, except of course for the present value of current and future lump-sum taxes. Non-monetary debt is redeemable, so the present value of the terminal stock of non-monetary debt is zero. Since

$$(1+i_t)B_{t-1}P_t^{-1} + \lim_{N\to\infty}\sum_{j=t}^{N} R_{t+1,j}[B_j - (1+i_j)B_{j-1}]P_j^{-1} \equiv \lim_{N\to\infty} R_{t+1,N-1}\, b_{N-1} \qquad \textbf{(58)}$$

and $\lim_{N\to\infty} R_{t+1,N-1}\, b_{N-1} = 0$, it follows that

$$(1+i_t)B_{t-1}P_t^{-1} + \lim_{N\to\infty}\sum_{j=t}^{N} R_{t+1,j}[B_j - (1+i_j)B_{j-1}]P_j^{-1} \equiv 0 \qquad \textbf{(59)}$$

The ability to issue non-monetary debt does not relax the government's intertemporal budget constraint in any way: the sum of the value of the outstanding stock of non-monetary debt and the present discounted value of net future non-monetary debt issuance is zero. Because of debt neutrality, the timing of lump-sum taxes does not matter, only their present discounted value. Therefore, issuing money by lowering taxes today by an amount x has the same effect on the real and nominal equilibrium as issuing the same amount of money today by purchasing (retiring) non-monetary debt today and cutting future taxes by the same amount, x, in present discounted value.[34]

4.3 A New-Keynesian Phillips curve

The validity of the result that liquidity trap equilibria can be ruled out once some mild and sensible restrictions are imposed on permissible monetary policies, is not restricted to the flexible price level model. Consider instead the New-Keynesian Phillips curve due to Calvo (1983). Calvo's model has monopolistically competitive price setting firms facing randomly timed opportunities for changing the nominal price of their products. The timing of opportunities to change the price is governed by a Poisson process with parameter δ where $1-\delta$ is the probability that a firm's price set in period t will still be in effect the next period. There is a continuum of price setters distributed evenly on the unit circle. The parameter δ therefore measures not only the probability of any price setter's contract being up for a change the next period, but also the fraction of the population of price setters changing their prices during any given period. The simplest version of the model specifies the natural logarithm of the current contract price of the ith firm, $p_t^i = \ln P_t^i$, as a forward-looking moving average with exponentially declining weights, of the logarithm of the expected future general price level, $p_t = \ln P_t$, and of expected future excess demand, that is

[34]Both ways of increasing the nominal stock of base money today are likely to represent an incomplete characterisation of set of changes in current and future lump-sum taxes that are necessary, in both cases, for the government to satisfy its intertemporal budget constraint.

$$p_t^i = (1-\delta)p_{t+1}^i + \delta[p_t + \eta(y_t - y^*)]$$
$$0 < \delta < 1;\ \eta' > 0;\ \eta(0) = 0;\ y \le \bar{y} < +\infty;\ \lim_{y \uparrow \bar{y}} \eta(y - y^*) = +\infty \tag{60}$$

The current value of the general price level is a weighted average of the past general price level and the current contract price, with the weights reflecting the shares of old and new contracts in the population of firms:

$$p_t = (1-\delta)p_{t-1} + \delta p_t^i \tag{61}$$

The specification in (60) of the effect of current and future excess demand on the current contract price implies a finite limit, $\bar{y} > y^*$, to the amount of output that can be produced with existing, finite resources. This restriction is *a-priori* plausible, and resonates well with the version of the New-Keynesian Phillips curve proposed by Woodford (1996). In that approach, the price set in period t by firms that are free to do so is a weighted average of current and future mark-ups over marginal cost.[35] The assumption that there is an absolute ceiling, $\bar{y}$, to the amount that can be produced from existing finite resources is the same as the assumption that the marginal cost curve becomes vertical at $\bar{y}$.

The specific functional form chosen for the η function is:

$$\eta(y - y^*) = (1-\delta)^{-1}\delta^2\left[\eta_0 - \eta_1\left(y^* - y_t + \eta_1\eta_0^{-1}\right)^{-1}\right]$$
$$\eta_0,\ \eta_1 > 0\ ;\ \bar{y} \equiv y^* + \eta_1\eta_0^{-1}$$

Using the approximation $\pi_t \approx p_t - p_{t-1}$, the New-Keynesian Phillips curve is given by:

$$\pi_{t+1} = \pi_t + \eta_0 - \eta_1\left(y^* - y + \eta_1\eta_0^{-1}\right)^{-1} \tag{62}$$

[35] Woodford (1996) derives the marginal cost based New-Keynesian Phillips curve for an endowment economy. Chadka and Nolan (2002) extend it to an economy with capital accumulation.

The discrete-time approach used in the paper thus far is certainly the most suitable vehicle for the analysis of some of some of the finer points of intertemporal budget constraints and solvency issues. However, a complete qualitative analysis of the dynamics of the New-Keynesian model using the convenient two-dimensional phase diagram is possible only for the continuous time analogue of the model. To keep the number of state variables down to 2, the model is further restricted to the case of a unitary elasticity of substitution between consumption and real money balances ($\varphi = 1$) and a unitary intertemporal substitution elasticity ($\sigma = 1$). The details of the derivation of the continuous time version of the model are available on request.[36] The key relations are the following:

$$\begin{aligned} \dot{c} &= (r - \rho)c \\ \rho &> 0;\ c \geq 0 \end{aligned} \tag{63}$$

$$\begin{aligned} c(t) &= \rho\left(\int_t^{\infty} e^{-\int_t^v r(u)du}[y(v)-g(v)]dv + \lim_{v\to\infty} e^{-\int_t^v r(u)du}\frac{M(v)}{P(v)}\right) \\ &= \rho\left(\int_t^{\infty} e^{-\int_t^v r(u)du}[y(v)-g(v)]dv + P(t)^{-1}\lim_{v\to\infty} e^{-\int_t^v i(u)du}M(v)\right) \end{aligned} \tag{64}$$

$$\begin{aligned} m \equiv M/P &= (1-\alpha)\alpha^{-1}(i-\bar{i}^{M})^{-1}c \\ 0 < \alpha < 1;\ i &\geq \bar{i}^{M};\ M,\ P \geq 0 \end{aligned} \tag{65}$$

[36] At http://www.nber.org/~wbuiter/heliap2.pdf

$$r \equiv i - \pi \tag{66}$$

$$\pi \equiv \frac{\dot{P}}{P} \tag{67}$$

$$\begin{aligned} &\text{If} \quad \rho + \pi^* + \gamma(\pi - \pi^*) \geq \bar{i}^M \\ &\qquad i = \rho + \pi^* + \gamma(\pi - \pi^*) \qquad\qquad \gamma > 1 \\ &\text{If} \quad \rho + \pi^* + \gamma(\pi - \pi^*) < \bar{i}^M \\ &\qquad i = \bar{i}^M \\ &\qquad \text{and} \\ &\qquad \dot{M}/M = \bar{\nu} > \bar{i}^M \end{aligned} \tag{68}$$

$$\begin{aligned} y &= c + g \\ 0 &\leq g \leq y^* \end{aligned} \tag{69}$$

$$\begin{aligned} \dot{\pi} &= \eta_0 - \eta_1\left(y^* - y + \eta_1\eta_0^{-1}\right)^{-1} \\ \eta_0, \eta_1 &> 0; \; 0 \leq y \leq \bar{y} = y^* + \eta_1\eta_0^{-1} \end{aligned} \tag{70}$$

The state-space representation of this dynamic system consists of two first-order non-linear differential equations, (72) and (73) the first of which has a regime switch when one of the state variables, the rate of inflation, π, crosses a threshold value, $\hat{\pi}$, given by

$$\hat{\pi} = \gamma^{-1}[(\gamma-1)\pi^* + \bar{i}^M - \rho] \tag{71}$$

$$\begin{aligned} \dot{c} &= (\gamma - 1)(\pi - \pi^*)c \qquad \text{if } \pi > \hat{\pi} \\ \dot{c} &= (\bar{i}^M - \pi - \rho)c \qquad \text{if } \pi \leq \hat{\pi} \end{aligned} \tag{72}$$

$$\dot{\pi} = \eta_0 - \eta_1\left(y^* + \eta_1\eta_0^{-1} - g - c\right)^{-1} \tag{73}$$

The steady states are:

$$c = y^* - g \tag{74}$$

and

$$\begin{aligned} \pi &= \pi^* \\ &\text{or} \\ \pi &= \bar{i}^M - \rho \end{aligned} \tag{75}$$

I assume that $\pi^* > \bar{i}^M - \rho$, or, equivalently, that $\hat{\pi} < \pi^*$. This means that the steady state rate of inflation when the lower bound on the short nominal interest rate is not binding (and which equals the 'target rate of inflation', π^*, implicit in the Taylor rule), exceeds the steady

state of inflation when the lower bound on the short nominal interest rate is binding, $\bar{i}^M - \rho \equiv \pi^{**}$, say. The behaviour of the system when this condition is violated is not hard to analyse but is not economically interesting. The dynamic system given in (72) and (73), does not incorporate the constraint on consumption implied by (64) and the assumption of that the growth rate of the nominal money stock exceeds the nominal interest rate on money when the short nominal interest rate is at its lower bound (see equation (68)). Both consumption, c, and the rate of inflation, π, are non-predetermined state variables. The price level, P, is predetermined, however.

Figure 1 shows the behaviour of the system near the two steady states.

FIGURE 1 HERE

The normal steady state is Ω^N and the liquidity trap steady state is Ω^L. The boundary between the range of (high) inflation rates for which the lower bound is not a binding constraint on the implementation of the Taylor rule and the range of (low) inflation rates where it is a binding constraint is given by $\pi = \hat{\pi}$, where $\pi^{**} < \hat{\pi} < \pi^*$. The motion along each of the solution orbits to (72) and (73) is counter-clockwise. Any solution that starts on an orbit outside the closed orbit, centered on Ω^N that just touches Ω^L from the right (the *normal* solution region), will sooner or later end up in and forever after stay inside, the region where the lower bound on the nominal interest rate is binding (to the left of $\hat{\pi}$).

Figure 2 shows the global behaviour of the system, with consumption constrained to lie between 0 and $\bar{y}$.

FIGURE 2 HERE

Ultimately, every solution trajectory outside the normal solution region will end up moving in a north-westerly direction towards the upper boundary for consumption.[37] When the

[37] Note that from a welfare point of view, these explosively deflationary solutions are not obviously undesirable: consumption ends up at the physical maximum and there is satiation with real money balances.

implication for private consumption demand of the irredeemability of base money is added as a constraint on permissible solution trajectories, it is clear that none of the solution orbits that start outside the normal solution region are permissible. At some point, say at time $t = t'$, the nominal interest rate sequence along such an explosive orbit would reach the lower bound and stay there forever after. Base money growth at a rate no less than the nominal interest rate at the lower bound (that is, $\bar{v} > \bar{i}^M$) would ensure that the present discounted value of the nominal money stock would grow without bound. With the price level predetermined at $t = t'$, real consumption demand would be unbounded and would exceed the physical upper bound $\bar{y}$. Such solutions to the system of differential equations (72) and (73), do therefore not satisfy (64). A growth rate of the nominal stock of base money *permanently* higher than the nominal interest rate on base money is sufficient but not necessary for this result. As long as the long-run nominal stock of base money expected at $t = t'$ satisfies $\lim_{s \to \infty} M(s) > P(t')\rho^{-1}(\bar{y} - \bar{g})$, it cannot be a rational expectations solution to (72), (73) and (64).

The foregoing discussion implies that the following proposition holds:

Proposition 3.

In the New-Keynesian model, the augmented Taylor rule given in (68), which states that, when the nominal interest rate is at its lower bound, the growth rate of the nominal stock of base money exceeds the nominal interest rate on base money, suffices to rule out liquidity trap equilibria that are also rational expectation equilibria.

Corollary.

When the nominal interest rate on base money is zero, any monetary rule that prescribes a positive growth rate of the nominal stock of base money when the nominal interest rate is at its zero lower bound, suffices to rule out liquidity trap equilibria that are also rational expectations equilibria.

The analysis also suggests a reason why liquidity traps may occur despite the authorities' intent to follow a monetary rule such as (68):

Proposition 4:

Perverse expectations can cause a liquidity trap.

The earlier argument that the adoption of the rule given in (68) would preclude liquidity trap equilibria relied on the effect on current consumption of private agent's expectations concerning the behaviour of the nominal money stock in the long run, should the economy land in a liquidity trap. Specifically, the private sector is assumed to believe that, whenever the current short nominal interest rate is at its lower bound, the growth rate of the nominal stock of base money will exceed the nominal interest rate on base money. With a zero nominal interest rate on base money, the private sector expects expansion of the nominal stock of base money whenever the current short nominal interest rate is zero. The present discounted value of the terminal stock of base money would be infinite in a liquidity trap under this rule, and consumption would exceed the physical capacity of the economy to produce.

A liquidity trap therefore cannot be a rational expectations equilibrium. It can, however, be a non-rational expectations equilibrium. If the private sector expects that any present increase in the stock of nominal base money will eventually be reversed, the long-run expected value of the nominal money stock would not become unbounded, and could be too low to rule out liquidity trap equilibria. If the authorities persist in their expansionary monetary policies and maintain the growth rate of the nominal money stock at of base money at a level in excess of the nominal interest rate on base money, presumably learning would eventually take place and eventually expectations concerning the long-run behaviour of the nominal stock of base money would be revised upwards. This could, in the long run, provide an exit out of the liquidity trap.

4.4 Irredeemable money in overlapping generations models

In many ways, the asymmetric perception of base money by the private and public sectors is more easily rationalised in an overlapping generations (OLG) model than in the representative agent model used thus far.[38] In overlapping generations models, new private agents emerge over time if the birth rate, β, is positive. Older generations will disappear if there is a positive death rate, θ. With new households/consumers being born, it is quite reasonable for each individual household to hold the belief that base money is an asset that it can realise at any time, now or in the future. Therefore, from the point of view of each household's solvency constraint, base money should be treated in the same way as non-monetary financial (or real) claims.[39] In the finite and certain horizon OLG models of Allais (1947) and Samuelson (1958), each generation will aim for a life-time consumption profile that implies zero aggregate financial wealth at the end of the last period of its life.

The Yaari-Blanchard model

In the 'new generations and uncertain lifetimes' version of the OLG model (see Blanchard (1985), Yaari (1965), Buiter (1988, 1991, 2003a), Frenkel and Razin (1987) and Weil (1989)), $\beta \geq 0$ is the constant number of new households born in each period for every household alive at the beginning of that period and θ, with $1 > \theta \geq 0$ is the constant probability that a household alive at the beginning of period t will be dead by period $t+1$. Each generation has a continuum of households, and θ is both the individual period probability of death and the fraction of each generation (and therefore of the population as a whole) that dies each period. All households receive the same (age-independent) endowments and pay the same taxes. Each household is born with zero initial financial wealth and has zero financial wealth when it dies. This is consistent with uncertain lifetimes (if $\theta > 0$) because of perfect annuities markets with free entry. The representative household of each generation solves the same optimisation

[38] I only consider OLG models without intergenerational gift and bequest motives.
[39] Due allowance will of course have to be made for the fact that base money and other financial claims may carry different pecuniary rates of return.

problem as the earlier representative household, with two modifications. The effective subjective period discount factor is $[(1+\rho)(1+\theta)]^{-1}$ instead of $(1+\rho)^{-1}$ to allow for uncertain lifetimes. The risk-free rates of return earned by surviving households are $(1+i)(1+\theta)$ rather than $1+i$ for bonds and $(1+i^M)(1+\theta)$ rather than $1+i^M$ for base money, to allow for the actuarial premium earned by surviving households. Following aggregation and substitution of the unchanged government's intertemporal budget constraint into the household consumption function, we get the following aggregate household consumption function:

$$c_t = \breve{\mu}_t \breve{w}_t \tag{76}$$

$$\breve{\mu}_t \equiv \left\{ \sum_{j=t}^{\infty} \left[1 + \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{1+i_{j+1}}{i_{j+1} - i_{j+1}^M} \right)^{\varphi-1} \right] \prod_{s=t+1}^{j} \frac{(1+r_s)^{\sigma-1}}{(1+\theta)(1+\rho)^{\sigma}} \left[\frac{1 + \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{1+i_{s+1}}{i_{s+1} - i_{s+1}^M} \right)^{\varphi-1}}{1 + \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{1+i_s}{i_s - i_s^M} \right)^{\varphi-1}} \right]^{\frac{\sigma-\varphi}{\varphi-1}} \right\}^{-1} \tag{77}$$

$$\begin{aligned} \breve{w}_t \equiv\ & (1+i_t^M) M_{t-1} P_t^{-1} + \sum_{j=t}^{\infty} R_{t+1,j} \left(1 - (1+\beta)^{-(j-t)} \right) \tau_j \\ & + \sum_{j=t}^{\infty} R_{t+1,j} \left((1+\beta)^{-(j-t)} y_j - g_j + [M_j - (1+i_j^M) M_{j-1}] P_j^{-1} \right) \end{aligned} \tag{78}$$

Using (39) and the money demand function, which is unchanged from the representative agent specification given in (18), we get:

$$c_t = \hat{\mu}_t \hat{w}_t \tag{79}$$

$$\beta_t \equiv \left\{ \sum_{j=t}^{\infty} \prod_{s=t+1}^{j} \frac{(1+r_s)^{\sigma-1}}{(1+\theta)(1+\rho)^{\sigma}} \left(\frac{1+\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+i_{s+1}}{i_{s+1}-i_{s+1}^{M}}\right)^{\varphi-1}}{1+\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+i_s}{i_s-i_s^{M}}\right)^{\varphi-1}} \right)^{\frac{\sigma-\varphi}{\varphi-1}} \right\}^{-1} \tag{80}$$

$$\begin{aligned} \hat{w}_t \equiv & \sum_{j=t}^{\infty} R_{t+1,j}[(1+\beta)^{1(j-t)}y_j - g_j] + \lim_{N\to\infty} R_{t+1,N}(1+\bar{i}^{M})(1+\pi_N)^{-1}m_{N-1} \\ & + \sum_{j=t}^{\infty} R_{t+1,j}[1-(1+\beta)^{-(j-t)}]\tau_j \end{aligned} \tag{81}$$

In equations (76) to (78) or from equations (79) till (81), we find the familiar property of this type of OLG model, that postponing taxes by issuing non-monetary debt while keeping constant the present discounted value of current and future taxes will, provided the birth rate, β, is strictly positive, boost the comprehensive wealth of the generations currently alive through a boost to their human wealth. With taxes in each period by assumption falling equally on everyone alive in the period, postponing taxes means that part of the postponed taxes will be paid by future generations - the unborn. There is absence of debt neutrality or Ricardian equivalence. The potential pure fiscal effect of irredeemable money, represented by the presence of the term $\lim_{N\to\infty} R_{t+1,N}(1+\bar{i}^{M})(1+\pi_N)^{-1}m_{N-1} = P_t^{-1}\lim_{N\to\infty} I_{t+1,N}(1+\bar{i}^{M})M_{N-1}$ in the definition of consolidated comprehensive private wealth in equations (78) and (81) is the same in the OLG model as in the representative agent model.

Because the OLG model does not exhibit Ricardian equivalence, a helicopter drop of money (a permanent increase in the nominal money stock financed though a current tax cut) no

longer has the same effect as the same increase in the money stock financed through an open market purchase of non-monetary debt (accompanied by cut in future taxes of equal present discounted value, to satisfy the government's intertemporal budget constraint). The key assumptions that drive this are (1) given sequences of nominal base money and real public spending on goods and services, and (2) the government satisfies its intertemporal budget constraint under both policies. Since the deferral of taxes boosts current consumption demand, the open market purchase will (at given nominal and relative prices, including intertemporal prices) boost consumer demand less than the helicopter drop of money financed by a contemporaneous tax cut. An open market purchase of non-monetary debt in period t is formally the same as the combination of a helicopter drop of money in period t, that is, a tax cut in period t to finance an increase in the money stock in period t, and a simultaneous reversal of that tax cut to retire an amount of non-monetary debt equal to the increase in the stock of base money. Postponing a tax cut, for given sequences of nominal base money and real public spending, dampens demand.

The Allais-Samuelson OLG model

Probably the most convincing argument for the asymmetric treatment of terminal base money balances in the solvency constraints of the government and the households comes from the familiar finite-horizon OLG model. A simple example follows. Households live for 2 periods. All households of a given generation are identical. There is a continuum of households

held be a member of generation t at the end of the ith period of its life are M^{i}_{t-1+i} and B^{i}_{t-1+i}, $i=1,2$.

Money demand is motivated through a strict cash-in-advance constraint on private and government consumption (see Lucas (1980, 1982) or Sargent (1987, Chapter 5, pp. 156-162)). The unit period, t, say, is partitioned into three distinct sub-periods, each of which contains one trading session. Households/portfolio holders in each generation, are divided into shoppers and workers who do not communicate until the third sub-period. No household can consume its own endowment. Purchases of consumption goods by households and by the government are subject to a Clowerian cash-in-advance constraint (Clower (1967)). In sub-period one, only securities are traded. During that sub-period, the consumption good cannot be traded and firms cannot pay out the period-t endowments to households. The securities traded in sub-period one include asset stocks, base money and interest-bearing non-monetary debt or bonds, carried over from the previous period. Without affecting any of the results of this paper, the asset menu could be extended to include tradable equity - ownership claims to the endowment streams - although in our formal model, equity is not included in the tradable asset menu. The endowment is therefore rather like labour time or labour services. There is a market for current labour services but, in the absence of slavery, no market in ownership claims to future labour services.[40]

The government announces its taxes, public spending, net new money issuance and net new debt issuance for period t at the beginning of the period, before the securities markets open and pays interest and principal due on its outstanding stocks of debt instruments. Both the household sector and the government are subject to a cash-in-advance constraint on their purchases of the perishable consumption good. In sub-period one, when the financial markets are open, the household sector (young and old) and the government have to acquire the money

[40] The assumption that the labour endowments of future generations are not owned by anyone alive today (that is, the absence of hereditary slavery) is a key feature of the OLG model. The further simplifying assumption made here that the labour endowment when old is zero, is immaterial.

balances each needs to purchase period t's planned consumption. The amounts of money a member of generation t acquires during the first sub-period of periods t and $t+1$ when the financial markets are open are z_t^1 and z_{t+1}^2 respectively. Real financial wealth (including interest paid) held at the beginning of the ith period of its life by a member of generation t is f_{t-1+i}^i, $i=1,2,3$. It follows that

$$\begin{aligned} z_t^1 P_t^{-1} + \tau_t^1 + B_t^1 P_t^{-1} &\leq f_t^1 \\ z_{t+1}^2 P_{t+1}^{-1} + \tau_{t+1}^2 + B_{t+1}^2 P_{t+1}^{-1} &\leq f_{t+1}^2 \end{aligned} \tag{82}$$

$$\begin{aligned} f_{t+1}^2 &= (1+i_{t+1}^M)(P_t y_t^1 + z_t^1 - P_t c_t^1) P_{t+1}^{-1} + (1+i_{t+1}) B_t^1 P_{t+1}^{-1} \\ f_{t+2}^3 &= (1+i_{t+2}^M)(z_{t+1}^2 - P_{t+1} c_{t+1}^2) P_{t+2}^{-1} + (1+i_{t+2}) B_{t+1}^2 P_{t+2}^{-1} \end{aligned} \tag{83}$$

Households are born without financial assets or liabilities:

$$f_t^1 = 0 \tag{84}$$

The solvency constraint of generation t is

$$f_{t+2}^3 \geq 0 \tag{85}$$

A household of generation t maximises

$$\begin{aligned} v_t &= \sigma(\sigma-1)^{-1}(c_t^1)^{(\sigma-1)\sigma^{-1}} + (1+\rho)^{-1}\sigma(\sigma-1)^{-1}(c_{t+1}^2)^{(\sigma-1)\sigma^{-1}}, \quad \sigma > 0;\ \sigma \neq 1 \\ &= \ln c_t^1 + (1+\rho)^{-1}\ln c_{t+1}^2, \ \sigma = 1 \end{aligned} \tag{86}$$

subject to (82) to (85) and the cash-in-advance constraints (87)

$$z_t^1 \geq P_t c_t^1$$
$$z_{t+1}^2 \geq P_{t+1} c_{t+1}^2 \tag{87}$$

Under the optimal programme, (82) and (85) bind and optimal consumption is given by

$$c_t^1 = (1+\rho)^{\sigma}[(1+\rho)^{\sigma}+(1+r_{t+1})^{\sigma-1}]^{-1}[(1+i_{t+1}^M)(1+i_{t+1})^{-1}y_t^1 - \tau_t^1 - \tau_{t+1}^2(1+r_{t+1})^{-1}] \tag{88}$$

$$\begin{aligned} c_{t+1}^2 &= (1+r_{t+1})^{\sigma}[(1+\rho)^{\sigma}+(1+r_{t+1})^{\sigma-1}]^{-1}[(1+i_{t+1}^M)(1+i_{t+1})^{-1}y_t^1 - \tau_t^1 - \tau_{t+1}^2(1+r_{t+1})^{-1}] \\ &= (1+i_{t+1}^M)M_t^1 P_{t+1}^{-1} + (1+i_{t+1})B_t^1 P_{t+1}^{-1} - \tau_{t+1}^2 \end{aligned} \tag{89}$$

with

$$\begin{aligned} z_t^1 &\geq P_t c_t^1 \\ &= P_t c_t^1 \text{ if } i_{t+1} > i_{t+1}^M \\ z_{t+1}^2 &\geq P_{t+1} c_{t+1}^2 \\ &= P_{t+1} c_{t+1}^2 \text{ if } i_{t+2} > i_{t+2}^M \end{aligned} \tag{90}$$

Aggregate private consumption, aggregate output and aggregate taxes are given by

$$c_t = c_t^1 + c_t^2$$
$$y_t = y_t^1$$

$$\tau_t = \tau_t^1 + \tau_t^2$$

The government's period budget identity, solvency constraint and intertemporal budget constraint (assuming its solvency constraint holds with equality) are assumed to be the same as before (equations (35), (36) and (37) respectively),[41] and the government's cash-in-advance constraint, assumed to be binding, is given by

$$z_t^g = P_t g_t$$

At the end of each period, t say, and at the beginning of the next period, $t+1$, all financial assets are owned by those born at the beginning of period t, that is,

$$M_t = M_t^1$$

$$B_t = B_t^1$$

Monetary equilibrium implies that

$$M_t = z_t^1 + z_t^2 + z_t^g$$

Aggregate consumption demand, after substituting out for the initial non-monetary public debt using (37) is given by:

$$c_t = \left(\frac{(1+\rho)^{\sigma}}{(1+\rho)^{\sigma}+(1+r_{t+1})^{\sigma-1}}\right)\left[\left(\frac{1+i_{t+1}^{M}}{1+i_{t+1}}\right)y_t-\tau_t^1-\frac{\tau_{t+1}^2}{1+r_{t+1}}\right] + \sum_{j=t}^{\infty} R_{t+1,j}(\tau_j^1+\tau_j^2-g_j) - \tau_t^2$$

$$+ (1+i_t^M)M_{t-1}P_t^{-1}+\sum_{j=t}^{\infty} R_{t+1,j}[M_j-(1+i_j^M)M_{j-1}]P_j^{-1}$$

Using (39) and noting that $(i_j-i_j^M)m_{j-1}=(i_j-i_j^M)(c_{j-1}+g_{j-1})=(i_j-i_j^M)y_{j-1}$ both when the cash-in-advance constraint binds and when it does not, we can rewrite the aggregate consumption function as:

$$c_t = (1+\rho)^{\sigma}[(1+\rho)^{\sigma}+(1+r_{t+1})^{\sigma-1}]^{-1}[(1+i_{t+1}^M)(1+i_{t+1})^{-1}y_t-\tau_t^1-\tau_{t+1}^2(1+r_{t+1})^{-1}] - \tau_t^2$$

$$+ \sum_{j=t}^{\infty} R_{t+1,j}(\tau_j^1+\tau_j^2-g_j) + \sum_{j=t+1}^{\infty} R_{t+1,j}(i_j-i_j^M)(1+\pi_j)^{-1}y_{j-1} + P_t^{-1}\lim_{N\to\infty} I_{t+1,N}(1+i_N^M)M_{N-1} \quad \textbf{(91)}$$

The aggregate consumption function for period t depends on current and future endowments, current and future real public spending, current and future taxes (because of the absence of debt neutrality) and the present discounted value of the terminal stock of base money: $P_t^{-1}\lim_{N\to\infty} I_{t+1,N}(1+i_N^M)M_{N-1}$.

In this infinite-lived economy with overlapping finite-lived households, each of which disposes of all its financial wealth, including base money, at the end of its life, the consolidated aggregate consumption function very naturally includes the present value of the terminal stock of base money as an argument. This provides further motivation for the asymmetric treatment of base money in the solvency constraints of the household and government sectors in the infinite-lived representative agent model.[42]

[42] A cash-in-advance version of the representative agent model can be found in http://www.nber.org/~wbuiter/heliap1.pdf

5. Institutional arrangements for a helicopter drop of base money: how much can the central bank do on its own?

To evaluate implementation issues, I break down the consolidated period budget identity and solvency constraint of the General Government and Central Bank into separate accounts each for the General Government (the Treasury) and the Central Bank.

The Central Bank has the monetary base, M, on the liability side of its financial balance sheet. On the asset side it has the stock of domestic credit.[43] For simplicity, domestic credit is restricted to Central Bank credit to the General Government, that is, Central Bank holdings of nominally denominated General Government interest-bearing debt or Treasury debt, B^{cb}.

Equation (92) is the period budget identity of the Treasury and equation (93) that of the Central Bank. B is the stock of Treasury interest-bearing debt held outside the Central Bank; τ^h is real value of the tax payments by the domestic private sector (the household sector here) to the Treasury; τ^{cb} is the real value of taxes paid by the Central Bank to the Treasury and h the real value of the transfer payments made by the Central Bank to the private sector (the instrument through which the Central Bank itself can engage in 'helicopter drops'). For simplicity the Central Bank is assumed not to spend anything on real goods and services.[44]

$$(B_t + B_t^{cb})P_t^{-1} \equiv g_t - \tau_t^h - \tau_t^{cb} + (1 + i_t)(B_{t-1} + B_{t-1}^{cb})P_t^{-1} \tag{92}$$

$$(M_t - B_t^{cb})P_t^{-1} \equiv \tau_t^{cb} + h_t - [(1+i_t)B_{t-1} + (1+\bar{i}^M)M_{t-1}]P_t^{-1} \tag{93}$$

[43]The model is of a closed economy, so I ignore official foreign exchange reserves, which are often held by the Central Bank.

[44]It is common practice for the Treasury to appropriate the profits of the Central Bank. This would mean in our model that $\tau_t^{cb} = -h_t + (i_t B_{t-1} - \bar{i}^M M_{t-1})P_t^{-1}$. Under this tax rule, the change in the stock of base money would equal domestic credit expansion by the Central Bank: $\Delta M_t = \Delta B_t^{cb}$. There is an obvious generalisation to the case of an open economy where the Central Bank holds international reserves as well as domestic debt on the asset side of its balance sheet.

The solvency constraints of the Treasury and the Central Bank are given in (94) and (95) respectively.[45]

$$\lim_{N\to\infty} R_{t+1,N}\, b_N^{cb} \geq 0 \tag{94}$$

$$\lim_{N\to\infty} R_{t+1,N}(b_N + b_N^{cb}) \leq 0 \tag{95}$$

Note that the Central Bank views its monetary liabilities as irredeemable. These solvency constraints imply the following intertemporal budget constraints for the Treasury (equation (96)) and the Central Bank (equation (97)).

$$(1+i_t)(B_{t-1} + B_{t-1}^{cb})P_t^{-1} \leq \sum_{j=t}^{\infty} R_{t+1,j}\left(\tau_j^h + \tau_j^{cb} - g_j\right) \tag{96}$$

$$(1+i_t)B_{t-1}^{cb}P_t^{-1} \geq \sum_{j=t}^{\infty} R_{t+1,j}\left(\tau_j^{cb} + h_j - [M_j - (1+\bar{i}^M)M_{j-1}]P_j^{-1}\right) \tag{97}$$

Adding (92) and (93) together and noting that $\tau \equiv \tau^h - h$ gives the period budget identity of the state, that is, the consolidated General Government (Treasury) and Central Bank, in (28). Subtracting (97) from (96) and assuming both of them to hold with equality, gives the intertemporal budget constraint of the state in (37).

Consider the financial balance sheet of the Central Bank in Table 1.

[45] $b_t^{cb} \equiv B_t^{cb}/P_t$.

Table1 Central Bank Balance Sheet	
Assets	Liabilities
B^{cb}	M
	N^{cb}

Central Bank financial net worth, N^{cb}, is the excess of the value of its financial assets, Treasury debt, B^{cb}, over its monetary liabilities, M. In principle, there is nothing to prevent N^{cb} from being negative. There are two reasons for this. The first applies to any economic agent, (including, in our case, the Treasury or the household sector). Financial net worth excludes the present value of anticipated or planned future non-contractual outlays and revenues. It is therefore perfectly possible, in principle, for an economic agent, including the Central Bank, to survive and thrive with negative financial net worth, provided the present discounted value of its future primary surpluses is sufficient to cover the negative financial net worth. The second reason is specific to the Central Bank. Financial net worth includes the stock of base money as a Central Bank liability. Since base money is irredeemable, the Central Bank does not need to generate future primary surpluses to service the outstanding stock of base money. Indeed, the current and future primary surpluses of the Central Bank (whose present discounted value should at least be equal to the value of its outstanding non-monetary debt $-B^{cb}$) include the sequence of future net seigniorage, $M_j - (1+\bar{i}^{M})M_{j-1}$.

Can the Central Bank perform a 'helicopter money drop' on its own?

Technically, if the Central Bank could make transfer payments to the private sector, the entire (real-time) Friedmanian helicopter money drop could be implemented by the Central Bank without Treasury assistance. At time t there would be a (temporary) increase in h_t (a transfer payment to the household sector by the Central Bank) financed by increasing the monetary base

('printing money'). An example would be for the Governor of the Central Bank to issue a $1,000 cheque, drawn upon the Central Bank's account, to every man, woman and child in the country. After the event, this would show up the financial balance sheet of the Central Bank as an increase in the stock of base money and a corresponding reduction in the financial net worth of the Central Bank. In its budget identity and profit and loss account the monetary injection would be recorded as a transfer payment from the Central Bank to the private sector.

The legality of such an implementation of the helicopter drop of money by the Central Bank on its own would be doubtful in most countries with clearly drawn boundaries between the Central Bank and the Treasury. The Central Bank would be undertaking an overtly fiscal act, something which is normally the exclusive province of the Treasury.[46]

An economically equivalent (albeit less entertaining) implementation of the helicopter drop of money would be a tax cut (or a transfer payment) implemented by the Treasury (a cut in τ_t^h), financed through the sale of Treasury debt to the Central Bank (an increase in ΔB_t^{cb}), which would then monetise the transaction ($\Delta(\Delta M_t) = \Delta(\Delta B_t^{cb})$). If the direct sale of Treasury debt to the Central Bank (or direct Central Bank lending to the Treasury) is prohibited (as it is for the countries that belong to the Euro area), the monetisation of the tax cut could be accomplished by the Treasury financing the tax through the sale of Treasury debt to the domestic private sector (or overseas), with the Central Bank purchasing that same amount of non-monetary interest-bearing debt in the secondary market, thus expanding the base money supply.

6. Conclusion

[46]In the US, the Treasury could, in principle, undertake both the fiscal and the monetary sides of the helicopter drop of money, because the Treasury has the power to issue irredeemable Treasury notes, which are interchangeable with Federal Reserve notes. US Treasury notes, issued by the Department of the Treasury since the Legal Tender Act of 1862 are part of the stock of US currency. Like Federal Reserve notes (authorised by the Federal Reserve Act of 1913), they are non-interest-bearing irredeemable bearer notes and constitute legal tender. They were issued until January 21, 1971. Those that remain in circulation are obligations of the U.S. government.

The paper provides a formalisation of the monetary 'folk' proposition that fiat (base) money is an asset of the holder (the private sector) but not a liability of the issuer (the monetary authority, as agent of the state). Earlier versions of the paper had the subtitle "A suggestion for slightly complicating the theory of money", but fear of hubris caused me to drop this.[47] I believe that this paper shows that the irredeemable nature of the monetary liabilities of the state should and can be incorporated into otherwise conventional approaches to monetary economics.

Following the proposed approach, fiat base money constitutes net wealth to the consolidated private and public sectors in the sense that the present discounted value of the terminal stock of base money, when that terminal date is infinitely far away, is part of the consolidated comprehensive wealth that drives private consumption. The issuance of irredeemable base money can therefore have pure fiscal effects. In well-behaved equilibria, that is, equilibria that are not liquidity traps, this paper's asymmetric treatment of the solvency constraints of the private sector and the monetary authority has no implications for the behaviour of nominal or real equilibrium prices and quantities. It does play a role when all current and future risk-free nominal interest rates are at their lower bound, which is the definition of a liquidity trap used in this paper.

When the monetary authorities view money as irredeemable but the private sector treats it as a realizable financial asset, simple and plausible restrictions on the monetary policy rule suffice to rule out liquidity trap equilibria as rational expectations equilibria. In the flexible price level variant of the model, a liquidity trap equilibrium is only possible if the monetary authorities are expected to de-monetise the economy in the long run, by reducing the nominal money stock to zero. In the New-Keynesian variant, a liquidity trap equilibrium is ruled out whenever the authorities are believed to keep the nominal stock of base money above a certain, finite, threshold level in the long run. Any positive long-run growth rate for the nominal stock of

[47]See Hicks (1935), Bryant and Wallace (1980) and Wallace (1990) for some of the important contributions to monetary theory that were published under the "A suggestion for .." heading.

base money is sufficient to rule out a liquidity trap equilibrium when the nominal interest rate on base money is zero, the institutionally most relevant case. Liquidity trap equilibria are possible as rational expectations equilibria only if monetary policies are perverse. With non-rational expectations - e.g. the belief that the monetary authorities will, in the long run, reverse and undo any current and past increases in base money - liquidity trap equilibria can exist for as long as these perverse expectations persist.

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Figure 1

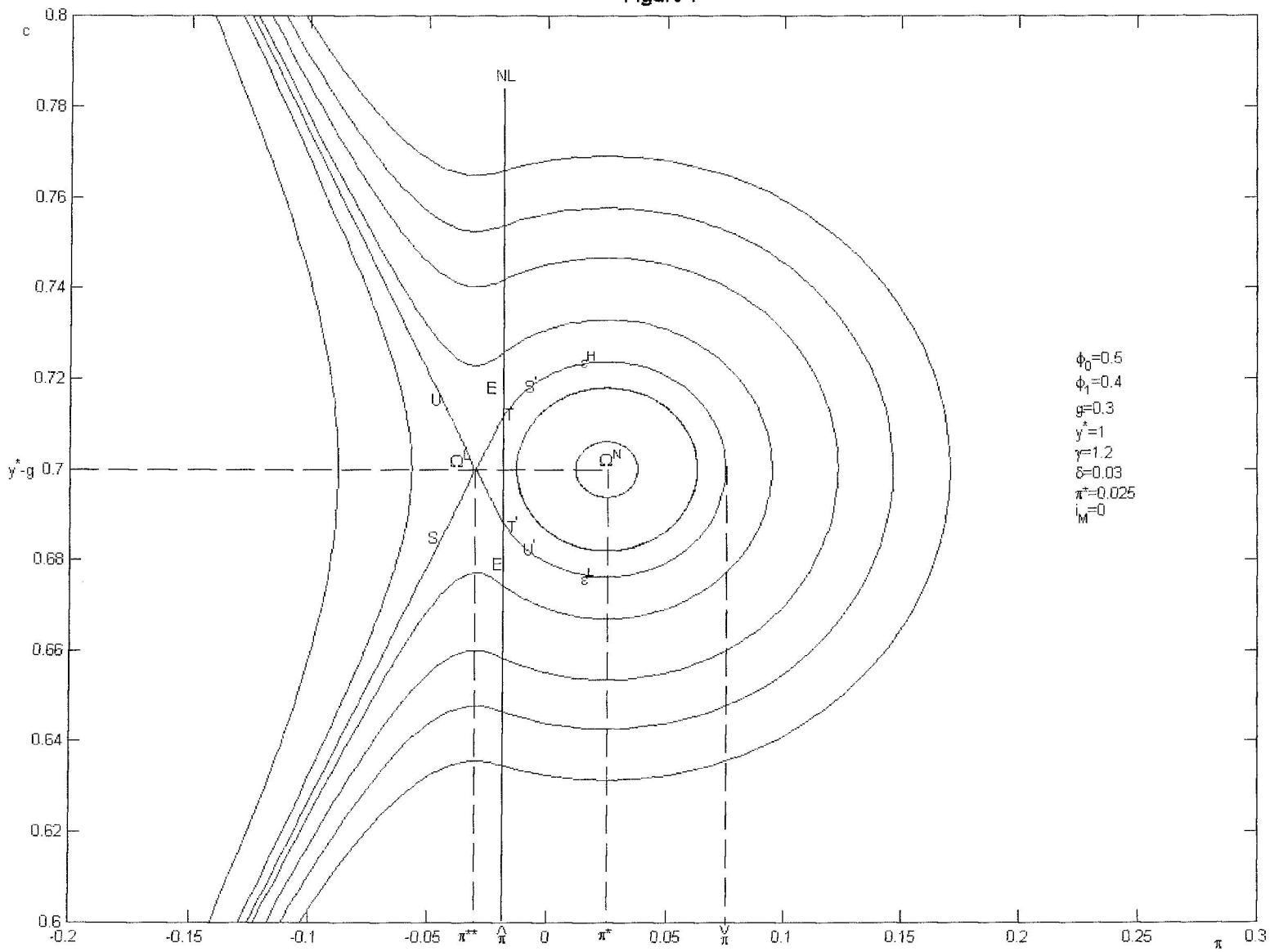
c
0.8
0.78
0.76
0.74
0.72
y*-g 0.7
0.68
0.66
0.64
0.62
0.6
-0.2
-0.15
-0.1
-0.05
π**
π̂
0
π*
0.05
π̌
0.1
0.15
0.2
0.25
π
0.3
NL
E
S'
U
T
Ω^A
Ω^N
S
T'
U'
E
ε^H
ε^L
φ0=0.5
φ1=0.4
g=0.3
y*=1
γ=1.2
δ=0.03
π*=0.025
iM=0

Figure 2