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ABSTRACT

This paper analyzes Krugman's contention that there is a "gold standard paradox" in the speculative attack literature. The paradox occurs if a country's currency appreciates after it runs out of gold or equivalently if a speculative attack can happen only after the country "naturally" runs out of reserves. We first show that Krugman's paradox is a very general phenomenon which does not require mean reverting processes for the fundamentals and which can be present in discrete time models as well as in continuous time models. We present several specific cases in which the paradox occurs i.e. environments which do not support an equilibrium. Next we show that, contrary to Krugman's conjecture, it is not necessary to abandon the assumption of a perfectly fixed exchange rate in favor of a band system in order to recover a well-defined equilibrium. We propose two alternative ways of amending the model which produce an equilibrium and preserve the fixed exchange rate assumption.

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1. INTRODUCTION

In a recent paper Paul Krugman [1989] has argued that there is a "gold standard (or more generally a fixed exchange rate regime) paradox." Reasonable specifications of the processes governing the fundamentals that drive the shadow floating exchange rate and the stock of international reserves result in anomalous behavior: a speculative attack can occur only after the country has already run out of reserves without a speculative attack i.e. without a sudden run on its currency that strips the monetary authority of its remaining international reserves in instantaneous "stock–shift" fashion. Such a "natural" collapse will be associated with a jump– (discontinuous) appreciation of its currency and the expectation of gradual (or smooth) appreciation of its currency immediately following the attack and jump–appreciation.

In Krugman's words, the gold standard paradox is that ". . . it is not possible for the public actually to expect zero change in the exchange rate while it is fixed" (p. 18). In addition "when a country runs out of gold [as a result of a natural collapse], its currency appreciates" (p. 20). Finally ". . . a speculative attack, if it occurs, will happen only after the country would have run out of gold in the absence of a speculative attack" (p. 21).

In this paper we show first that Krugman's paradox is a phenomenon that is much more general than his paper suggests. It is in particular not dependent on the presence of mean reversion in the fundamentals process. To rule it out requires either that the fundamental and the shadow exchange rate follow a random walk without drift or that they be weakly monotonic over time i.e. either strictly nondecreasing or strictly nonincreasing. What must be ruled out is the possibility that a high value of the shadow exchange rate (a weak currency) be associated with an expectation of future exchange rate appreciation (a strengthening of the currency) and that a low value of the shadow exchange rate be associated with expected future depreciation.

A slightly generalized version of Krugman's model is developed in Section 2. After demonstrating the full thrust of Krugman's critique of the speculative attack literature in Section 3 for the continuous time case and in Section 4 for the discrete time case, we offer our proposals for a resolution of the paradox in Section 5.

2. THE MODEL

The model is given in equations (1) through (6).

(1) $$s(t) = m(t) - m^{*}(t) + v(t) + \gamma E_t \dot{s}(t) \qquad \gamma > 0$$

(2) $$m = \ln(D + R)$$

(3) $$m^{*} = \ln(D^{*} + R^{*})$$

(4) $$R + R^{*} = G$$

(5) $$R > \underline{R} \text{ and } R^{*} > \underline{R}^{*}$$

(6) $$G > \underline{R} + \underline{R}^{*}$$

s is the logarithm of the spot exchange rate, the price of foreign currency in terms of domestic currency. m is the logarithm of the domestic nominal money stock and m^* the logarithm of the foreign nominal money stock. $v(t)$ measures the logarithm of foreign money demand (at a given foreign price level) relative to domestic money demand (at a given domestic price level). In the continuous time model the sample paths of v will be assumed to be continuous functions of time. E_t is the expectation operator conditional on information at time t. D is the stock of home country domestic credit, assumed to be exogenous and constant, and D^* the exogenous and constant stock of foreign domestic credit.[1] R is the home country stock of reserves and R^* the foreign stock of reserves.[2] The total world stock of reserves G is constant. Equation (5) define the values of R and R^* for which the fixed exchange rate regime is viable. Each country establishes a constant reserve floor ($\underline{R}$ for the home country and $\underline{R}^*$ for the foreign country). When reserves fall below these floors the fixed exchange rate regime collapses, and a permanent free float begins. Equation (6) states that global gold reserves are sufficient to satisfy the minimal requirements for reserves of both countries simultaneously. Without loss of generality we set $\underline{R} = \underline{R}^* = 0$, so (6) becomes

$$(6') \qquad G > 0 .$$

We shall consider two classes of structural models that yield equation (1). The first is a model with two monies, home and foreign, held by private agents, and gold which is held only by the two national monetary authorities. The second model adds two bonds—both of the fixed–price, single–period kind—one denominated in home currency and the other in foreign currency.

The two monies (and gold) model is given in equations (7) through (9). It is the familiar imperfect direct currency substitution model.

$$(7) \qquad m(t) - p(t) = -\tfrac{1}{2}\gamma E_t \dot{s}(t) + ky(t) \qquad \gamma > 0,\ k > 0$$

(8) $$m^*(t) - p^*(t) = \tfrac{1}{2}\gamma E_t\dot{s}(t) + ky^*(t)$$

(9) $$p(t) = p^*(t) + s(t)$$

p and p^* are the logarithms of the domestic and foreign price levels respectively, and y and y^* domestic and foreign output. Equations (7) and (8) are the demand functions for domestic and foreign currency. Equation (9) is the condition for purchasing power parity (P.P.P.). Equations (7) to (9) yield (1) with $v = k(y^* - y)$.

The two monies, two bonds (and gold) model is given in equations (10) through (13).

(10) $$m(t) - p(t) = -\gamma i(t) + ky(t) \qquad \gamma > 0, k > 0$$

(11) $$m^*(t) - p^*(t) = -\gamma i^*(t) + ky^*(t)$$

(12) $$i(t) = i^*(t) + E_t\dot{s}(t)$$

(13) $$p(t) = p^*(t) + s(t)$$

i and i^* are the domestic and foreign single–period riskless nominal interest rates. Equation (12) is the condition for uncovered interest parity (U.I.P).

Solving equation (1) forward in time and choosing the unique continuously convergent solution we get

$$s(t) = \frac{1}{\gamma}\int_t^\infty e^{-\frac{1}{\gamma}(s-t)} E_t[m(s) - m^*(s) + v(s)]ds .$$

The shadow floating exchange rate at t that will describe the economy if the home country were to run out of reserves at time t, $\hat{s}(t)$, is defined by:

(14a) $$\hat{s}(t) = lnD - ln(D^* + G) + \frac{1}{\gamma}\int_t^\infty e^{-\frac{1}{\gamma}(s-t)} E_t v(s)ds \ .$$

The shadow floating exchange rate at time t that will prevail if the foreign country were to run out of reserves at time t, $\hat{s}^*(t)$, is defined by:

(14b) $$\hat{s}^*(t) = ln(D + G) - lnD^* + \frac{1}{\gamma}\int_t^\infty e^{-\frac{1}{\gamma}(s-t)} E_t v(s)ds \ .$$

Note that

(15a) $$\hat{s}^*(t) = \hat{s}(t) + K$$

where

(15b) $$K = ln(D + G) + ln(D^* + G) - lnD - lnD^* > 0 \ .$$

The world starts at $t = 0$. If at $t = t_1 \geq 0$ a successful speculative attack is launched against the home country's currency, it must be true that

(16a) $$\hat{s}(t_1) = s_0 \ ,$$

(16b) $$\hat{s}(t) < s_0 \qquad \text{for all } t < t_1 \ .$$

If at $t = t_2 \geq 0$ a successful speculative attack is launched against the foreign currency, it must be true that

(17a) $$\hat{s}^*(t_2) = s_0 \ ,$$

(17b) $\hat{s}^*(t) > s_0$ for all $t < t_2$.

The range of values of the shadow floating exchange rates for which there is no risk of speculative attack can be expressed as follows in terms of these two shadow floating exchange rates:

(18a) $\hat{s}(t) < s_0$,

(18b) $\hat{s}^*(t) = \hat{s}(t) + \mathit{I} > s_0$

or

$$s_0 - \mathit{I} < \hat{s}(t) < s_0 .$$

We call (18a, b) the condition for S–viability (i.e. for speculative viability).

Let $\hat{v}$ denote that value of v for which $\hat{s} = s_0$. Similarly let $\hat{v}^*$ be the value of v for which $\hat{s}^*(t) = s_0$. Note that $\hat{v}$ and $\hat{v}^*$ will depend on the nature of the (stochastic) process governing v.

The criteria for S–viability given in (18a, b) can be written in terms of v (in "primal" form) as

(19) $\hat{v}^* < v < \hat{v}$ (S–viability) .

Myopic shadow floating exchange rates will be used to define the value of v at which a natural collapse will occur. If the exchange rate is expected to remain constant, the myopic shadow floating exchange rate at time t—if a natural collapse of the fixed exchange rate regime were to occur at that instant because the home country runs out of reserves—is denoted $\bar{s}(t)$ i.e

(20a) $$\bar{s}(t) = lnD - ln(D^* + G) + v(t) \; .$$

The myopic floating exchange rate at time t, if a natural collapse occurs because the foreign country runs out of reserves, is denoted $\bar{s}^*(t)$ i.e.

(20b) $$\bar{s}^*(t) = ln(D + G) - lnD^* + v(t) \; .$$

Note again that

(21) $$\bar{s}^*(t) = \bar{s}(t) + K \; .$$

Let $\bar{v}$ be the value of v for which $\bar{s} = s_0$ and $\bar{v}^*$ the value of v for which $\bar{s}^* = s_0$. It is clear that whatever the v process,

(22a) $$\bar{v} = s_0 - \left[lnD - ln(D^* + G) \right]$$

and

(22b) $$\bar{v}^* = s_0 - \left[ln(D + G) - lnD^* \right] \; .$$

The system will not suffer a natural collapse as long as the condition given in equation (23) (which we refer to as reserve viability or R–viability) holds

(23) $$\bar{v}^* < v < \bar{v} \qquad \text{(R–viability)} \; .$$

The combined criteria for the system not to suffer either speculative or natural attacks is therefore

(24) $$max(\bar{v}^* , \hat{v}^*) < v < min(\hat{v} , \bar{v}) \qquad \text{(S\&R–viability)} \; .$$

While the fixed exchange rate regime endures, i.e. right up to the instant at which either a natural collapse occurs or a successful speculative attack occurs, the expected rate of exchange rate depreciation is zero. This is so because v, the exogenous forcing variable, was assumed to have continuous sample paths. If the stock of reserves exceeds by any amount, however small, the larger of zero and the value of the reserve stock that would be withdrawn in speculative stock–shift fashion in the case of a successful speculative attack, then the instantaneous probability of a collapse is zero, and the exchange rate is expected to remain constant this instant.

A "correct" speculative attack is a speculative attack in the right direction. A correct speculative attack against the home currency when $\hat{s} = s_0$ for the first time requires that the expected rate of depreciation of the exchange rate at the moment of the attack should increase from zero (which is the correct expectation while the fixed exchange rate regime prevails) to some positive value. Only then will there be the stock–shift reduction in the relative demand for home country money that, with D and D^* given, achieves the stock–shift reduction in home country reserves to its critical threshold (zero). If the (rationally) expected rate of depreciation remains zero after the collapse, there is no (stock–shift) speculative attack: natural and speculative attacks coincide.

Analogously a correct speculative attack against the foreign currency requires that at the moment of the attack and collapse the expected proportional rate of depreciation of the home currency falls from zero to some negative value. Again, if the (rationally) expected rate of depreciation remains at zero, there is no (stock–shift) speculative attack.

A moment's reflection will confirm that the speculative attack at the upper bound of the S&R viable range will be correct (i.e. involve a speculative run against the home currency which strips the home country monetary authorities of their remaining reserves) if and only if

(25a) $\hat{v} \leq \bar{v}$ (correct attack at upper boundary) .

The speculative attack at the upper boundary should occur at a value of v no greater than than the value of v at which a natural attack occurs. Loosely speaking this can be rephrased as "the speculative attack should occur before the natural attack."

Similarly the criteria for a correct speculative attack at the lower boundary (a speculative run stripping the foreign monetary authority of its reserves) is

(25b) $\quad \bar{v}^* \leq \hat{v}^* \quad$ (correct attack at upper boundary) .

The relative money demand term $v(t)$ is assumed to be governed by the following stochastic process:

(26) $\quad dv = \mu dt - \rho(v - v_0)dt + \sigma dz \qquad \sigma \geq 0 .$

$z(t)$ is standardized Brownian motion i.e. the increments dz are identically, independently, and normally distributed with zero mean and unit variance. Equation (26) is a slight generalization of Krugman's equation because a drift or trend term μ is included. Values of $\rho > 0$ indicate mean reversion in the autoregressive component of (26); $\rho < 0$ indicates nonstationary behavior of the autoregressive component of (26).

Given (7) and assuming $1 + \gamma\rho > 0$ which is required for convergence of the integrals in equations (14a) and (14b) when v is governed by (21), the four shadow floating exchange rates are given by:

(27a) $\quad \bar{s} = lnD - ln(D^* + G) + \frac{\gamma}{1 + \gamma\rho}(\rho v_0 + \mu) + \frac{1}{1 + \gamma\rho}v_t$

with

(27b) $\quad E_t d\hat{s}(t) = \frac{1}{1 + \gamma\rho}[\mu - \rho(v(t) - v_0)]dt ,$

(28a) $\quad \hat{s}^* = ln(D + G) - lnD^* + \frac{\gamma}{1 + \gamma\rho}(\rho v_0 + \mu) + \frac{1}{1 + \gamma\rho}v_t$

with

(28b) $$E_t d\hat{s}^*(t) = \frac{1}{1 + \gamma\rho}\,[\mu - \rho(v(t) - v_0)]\,dt \ ,$$

(29a) $$\bar{s}(t) = lnD - ln(D^* + G) + v(t) \ ,$$

(29b) $$\bar{s}^*(t) = ln(D + G) - lnD^* + v(t) \ .$$

We now have the information to determine $\hat{v}$ and $\hat{v}^*$. ($\bar{v}$ and $\bar{v}^*$ are always given by (22a, b).)

(30a) $$\hat{v} = [1 + \gamma\rho][s_0 - (lnD - ln(D^* + G))] - \gamma\rho(v_0 + \frac{\mu}{\rho})$$

(30b) $$\hat{v}^* = [1 + \gamma\rho][s_0 - (ln(D + G) - lnD^*)] - \gamma\rho(v_0 + \frac{\mu}{\rho})$$

Note from (30a, b), (22a, b) and (15b) that

(31a) $$\hat{v} = \hat{v}^* + [1 + \gamma\rho]K \ ,$$

(31b) $$\bar{v} = \bar{v}^* + K \ .$$

Therefore we have, since $[1 + \gamma\rho] > 0$ and $K > 0$,

$$\hat{v} > \hat{v}^* \ ,$$

$$\bar{v} > \bar{v}^* \ .$$

The criterion for a correct speculative attack at $\hat{v}$, the upper boundary of the S–viable range, can be restated for this particular v process as:

(32a) $$E_t\dot{s}(\hat{v}) = (1 + \gamma\rho)^{-1}[\mu - \rho(\hat{v} - v_0)] \geq 0 \ .$$

The criterion for a correct speculative attack at $\hat{v}^*$, the lower boundary of the S–viable range can be restated as:

(32b) $$E_t\dot{s}(\hat{v}^*) = (1+\gamma\rho)^{-1}[\mu - \rho(\hat{v}^* - v_0)] \leq 0\,.$$

In Table 1 we summarize the various viability and correctness conditions. When an economy has a nonzero S&R viable range of v values and when the speculative attacks at the upper and lower bounds are in the correct direction, the economy has achieved perfection as defined in equation (33).

(33) $$\bar{v}^* \leq \hat{v}^* < v < \hat{v} \leq \bar{v}$$

3. THE PARADOX STATED AND ILLUSTRATED

v is a random walk without drift

Consider the case when $\mu = \rho = 0$. The shadow exchange rates too will be random walks without drift in this case:

(34) $$\hat{s} = \bar{s} = lnD - ln(D^* + G) + v\;;$$

(35) $$\hat{s}^* = \bar{s}^* = ln(D + G) - lnD^* + v\,.$$

It follows immediately that

(36) $$\hat{v} = \bar{v} = s_0 - (lnD - ln(D^* + G))\,,$$

and

(37) $$\hat{v}^* = \bar{v}^* = s_0 - (ln(D + G) - lnD^*)\,.$$

Figure 1 shows the characterization of this economy when v follows a random walk without drift. $\hat{s}$ and $\bar{s}$ coincide as do $\hat{s}^*$ and $\bar{s}^*$, $\hat{v}$ and $\bar{v}$ as well as $\hat{v}^*$ and $\bar{v}^*$. The S&R–viable range of v is between $\hat{v}^*(= \bar{v}^*)$ and $\hat{v}(= \bar{v})$. It has length I and is independent of s_0. As $\hat{v}$ is approached from below, the fixed exchange rate regime collapses as the home country runs out of reserves. Since the postcollapse expected rate of exchange rate depreciation is zero (see (32a)), there is no stock–shift loss of reserves when the collapse occurs. Speculative and natural collapses coincide.

As $\hat{v}^*$ is approached from above, the foreign country runs out of reserves, again without a stock–shift speculative attack. Note that the S&R–viability criterion is satisfied (equation (24)) as well as the two criteria for correct attacks at the upper and lower boundaries.

v is a random walk with drift

Now consider the case where $\rho = 0$, and $\mu > 0$ [3] shown in Figure 2. In this case we have

$$\hat{s} = lnD - ln(D^* + G) + v + \gamma\mu \ , \tag{38}$$

$$\hat{s}^* = ln(D + G) - lnD^* + v + \gamma\mu \ , \tag{39}$$

$$E_t\dot{s}(\hat{v}) = \mu \ , \tag{40}$$

$$E_t\dot{s}(\hat{v}^*) = \mu \ . \tag{41}$$

$\tilde{s}$ and $\tilde{s}^*$ are always as in (29a, b).

Note that the S–viable range of v between $\hat{v}^*$ and $\hat{v}$ has length $\mathit{\Gamma}$. The S&R viable range is between $\tilde{v}^*$ and $\hat{v}$, and has length $\mathit{\Gamma} - \gamma\mu$. Clearly μ can be so large that $\mathit{\Gamma} < \gamma\mu$. In that case there is no S&R viable range since $\tilde{v}^* > \hat{v}$. In what follows we assume $\mathit{\Gamma} - \gamma\mu > 0$. The criterion for a correct speculative attack at the upper boundary is satisfied: $\hat{v} < \tilde{v}$ or $E_t\dot{s}(\hat{v}) = \mu > 0$. This speculative attack will involve a stock–shift loss of reserves for the home country. The criterion for a correct speculative attack at the lower boundary fails, however: $\hat{v}^* < \tilde{v}^*$ or $E_t\dot{s}(\hat{v}^*) = \mu > 0$. It does not make sense to launch a speculative attack against the foreign country currency through a stock–shift **increase** in relative demand for foreign money and consequently a stock–shift **inflow** of reserves into the foreign country.

Note that the failure of the fixed exchange rate regime to make sense at the lower boundary has nothing to do with mean reversion. The relative money demand process is nonstationary in our example.

If the v process is deterministic ($\sigma = 0$), any v process starting above $\bar{v}^*$ (and below $\hat{v}$) would result in a finite life for the fixed exchange rate regime and a correct collapse at the upper boundary. If the v process had nonnegative increments, there also would be no risk of running into the incorrect attack problem at the lower boundary even if the increments were stochastic.

With $\sigma > 0$ however there is a positive probability that with v driven by Brownian motion (i.e. normal, identically distributed independent increments), any process starting off at $\bar{v}^* < v < \hat{v}$ will reach $\bar{v}^*$ in finite time. (Note that since $\mu > 0$ the probability that v will reach any lower bound in finite time is strictly less than 1.)

What happens when v falls to $\bar{v}^*$? If agents were truly myopic even after the natural collapse and continued to expect $E_t\dot{s} = 0$ even for $v \leq \bar{v}^*$, the economy would move along the myopic or static exchange rate expectations curve $\bar{s}^*$ after v reaches $\bar{v}^*$ for the first time. There is no speculative attack. Private agents are uninformed but satisfied with their money holdings.

Suppose instead that when v reaches $\bar{v}^*$ from above, private agents correctly realize that following the collapse of the fixed exchange rate regime (for whatever reason) there will be a free float with a positive expected rate of depreciation of the home country's currency. In this case there will at $v = \bar{v}^*$ be a stock–shift increase in the demand for foreign money and a stock–shift reduction in the demand for home country money. There would be a stock–shift rush of reserves into the foreign country. Another way to look at this is to note that for $\hat{v}^* < v \leq \bar{v}^*$ in Figure 2, $\hat{s}^* > s_0$. The postnatural collapse exchange rate represents a finite jump depreciation of the home country's exchange rate. This makes foreign currency a great investment, so reserves rush into the foreign country. This "collapse scenario" therefore makes no sense. No equilibrium exists at $\bar{v}^*$.

v is a nonstationary, first-order autoregressive process without drift

When $\mu = 0$ but $\rho \neq 0$, v follows what is sometimes called the "Ornstein–Uhlenbeck (O.U.)" process. With $\rho < 0$, the process is nonstationary. We have

$$\hat{s} = lnD - ln(D^* + G) + v_0 + \frac{1}{1 + \gamma\rho}(v - v_0) , \tag{42}$$

$$E_t\dot{s}(\hat{v}) = \frac{-\rho}{1 + \gamma\rho}(\hat{v} - v_0) , \tag{43}$$

$$\hat{s}^* = ln(D + G) - lnD^* + v_0 + \frac{1}{1 + \gamma\rho}(v - v_0) , \tag{44}$$

$$E_t\dot{s}(\hat{v}^*) = \frac{-\rho}{1 + \gamma\rho}(\hat{v}^* - v_0) . \tag{45}$$

$\bar{s}$ and $\bar{s}^*$ are given in (29a, b).

With $\rho < 0$ but $\frac{1}{1 + \gamma\rho} > 0$, the $\hat{s}$ schedule and $\hat{s}^*$ schedule have a common slope in v,s space $\left[\frac{1}{1 + \gamma\rho}\right]$ which exceeds the unitary slopes of the $\bar{s}$ and $\bar{s}^*$ curves. The $\hat{s}$ and $\bar{s}$ curves intersect at $v = v_0$. So too do the $\hat{s}^*$ and the $\bar{s}^*$ curves. As always, the $\hat{s}^*$ curve lies a vertical distance K above the $\hat{s}$ curve, and the $\bar{s}^*$ curve lies a vertical distance K above the $\bar{s}$ curve.

The configuration drawn in Figure 3 exhibits perfection. There is a finite S&R viable range $(\hat{v}^*, \hat{v})$, and $\bar{v}^* < \hat{v}^*$ while $\hat{v} < \bar{v}$.

While this case is in some ways rather like that of a random walk with positive drift, the difference here is that there is a correct speculative attack at the lower boundary. Since $v_0 > \hat{v}^*$, at the lower boundary $\hat{v}^*$ the informed speculator, knowing that $Edv = -\rho(v - v_0)$, expects v to fall further (the model is unstable). The expected rate of change of s at $\hat{v}^*$ is therefore negative, and a speculative attack is launched against the foreign currency at $\hat{v}^*$.

There are however many other configurations. They can be characterized graphically by moving the fixed exchange rate s_0 up or down. When s_0 is above $\bar{\bar{s}}$ (the value of s at which the $\hat{s}^*$ and $\bar{s}^*$ curves cross), we lose the correct speculative attack at the lower boundary: $\hat{v}^* < \bar{v}^*$, and $v_0 < \hat{v}^*$. When s_0 rises to or above $\bar{\bar{s}}$ (the value of s at which the $\hat{s}$ and $\bar{s}^*$ curves cross) the S&R viable range vanishes altogether.

When s_0 is below $\underline{s}$ (the value of s at which the $\hat{s}$ and $\bar{s}$ curves cross), we lose the correct speculative attack at the upper boundary. When s_0 is at or below $\underline{\underline{s}}$ (the value of s at which the $\hat{s}^*$ and the $\bar{s}$ curves cross), the S&R viable range again vanishes altogether.

v is a stationary, first–order autoregressive process without drift

The case $\mu = 0$, $\rho > 0$ is the stationary (or mean reverting) first–order *AR* process analyzed by Krugman. The equations for $\hat{s}$, $\hat{s}^*$, $E_t\dot{s}(\hat{v})$ and $E_t\dot{s}(\hat{v}^*)$ are as in equations (42)–(45). With $\rho > 0$, the $\hat{s}^*$ and $\hat{s}$ curves have less than unitary slopes. The configuration analyzed by Krugman is shown in Figure 4. While there is a finite S&R viable range $(\bar{v}^*, \bar{v})$ we have incorrect speculative attacks both at the upper and the lower boundaries: $\hat{v} > \bar{v}$, and $\hat{v}^* < \bar{v}^*$. With $\hat{v}^* < \bar{v}^* < v_0 < \bar{v} < \hat{v}$, the expected rate of change of s is negative at $\bar{v}$ (and a fortiori at $\hat{v}$) and positive at $\bar{v}^*$ (and a fortiori at $\hat{v}^*$). When v is large private agents expect it to decline towards v_0, and when s is high private agents expect it to fall. When v is low private agents expect it to rise towards v_0. And when s is low private agents expect it to rise.

Raising s_0 above $\bar{s}$, the value of s at which the $\bar{s}^*$ and $\hat{s}^*$ curves cross, eliminates the incorrect attack at the lower boundary. We now have $v_0 < \bar{v}^* < \hat{v}$. Raising s_0 further above $\bar{\bar{s}}$, the value of s at which the $\hat{s}^*$ and $\bar{s}$ curves cross, causes the S&R viable range to vanish.

Lowering s_0 below $\underline{s}$, the value of s at which the $\hat{s}$ and $\bar{s}$ curves cross, eliminates the incorrect attack at the upper boundary with $\hat{v} < \bar{v} < v_0$. Reversion to v_0 now means that when v is large (but still less than v_0) private economic agents expect a further rise. When s is high private economic agents expect a further increase. The speculative attack at the upper bound of the S&R viable range $(\hat{v})$ is correct: a stock–shift loss of reserves for the home country.

When s_0 is at or below $\bar{\bar{s}}$, the value of s at which the $\bar{s}^*$ and $\hat{s}$ curves cross, the S&R viable range again vanishes. With $\rho > 0$, there is therefore no value of s_0 for which perfection prevails.

4. THE GOLD STANDARD PARADOX IN A DISCRETE TIME MODEL

In order to ensure that the paradox is not an artifact of continuous time models driven by Brownian motion, we reformulate the model of equations (1) through (6) as a discrete time model. This will also facilitate the interpretation of our resolution of the paradox in Section 5.

(46) $$s_t = m_t - m_t^* + v_t + \gamma E_t(s_{t+1} - s_t) \qquad \gamma > 0$$

(47a) $$v_t = \mu + \rho v_0 + (1 - \rho) v_{t-1} + z_t$$

(47b) $$E z_t = 0 \; ; \; E z_t z_s = 0 \quad if \; t \neq s \; , \qquad = \sigma^2 \geq 0 \quad if \; t = s$$

(48a) $$m_t = ln(D + R_t)$$

(48b) $$m_t^* = ln(D^* + R_t^*)$$

(40a) $$R_t + R_t^* = G$$

(40b) $$G > 0$$

(50) $$R_t > 0 \text{ and } R_t^* > 0$$

The last two equations again define the conditions under which the fixed exchange rate regime will survive.

We define the following variables:

$$\Delta = lnD - ln(D^* + G) \; ,$$

$$\Delta^* = ln(D + G) - lnD^* .$$

Note that

$$I = \Delta^* - \Delta .$$

The two shadow exchange rates $\hat{s}$ and $\hat{s}^*$ are given by

(51a) $$\hat{s}_t = \Delta + \frac{\gamma}{1 + \gamma\rho}(\rho v_0 + \mu) + \frac{1}{1 + \gamma\rho}v_t ,$$

(51b) $$\hat{s}^*_t = \Delta^* + \frac{\gamma}{1 + \gamma\rho}(\rho v_0 + \mu) + \frac{1}{1 + \gamma\rho}v_t .$$

$\hat{s}_t$ is the exchange rate that prevails in period t if the gold standard collapses that period because the home country runs out of reserves. $\hat{s}^*_t$ is the exchange rate that prevails in period t if the gold standard collapses in that period because the foreign authority runs out of reserves. During a period in which a collapse occurs, reserves can be bought and sold at s_0 and $\hat{s}_t$ (or at s_0 and $\hat{s}^*_t$). Thus for a collapse to occur because the home country authority runs out of reserves, it is necessary and sufficient that $\hat{s}_t \geq s_0$. Consider for simplicity a two monies and gold world i.e. a world without interest–bearing assets. Arbitrage here means direct currency arbitrage. If $\hat{s}_t < s_0$ and the home authorities were running out of reserves, private agents pursuing pure arbitrage profits would sell reserves to the domestic authority in exchange for home currency at s_0 and would instantaneously sell the home currency thus acquired at the postcollapse price of foreign exchange $\hat{s}_t$. Any private agent with access to gold or foreign currency could engage in this profitable set of riskless transactions, say by buying gold from the foreign authority in exchange for foreign currency and presenting the gold thus obtained to the home authority in exchange for domestic currency at the fixed rate s_0. Note that private agents need only use gold as a means of trading between national monies. They need not hold any gold stocks, and the model indeed assumes they do not.

Any incipient exhaustion of the home currency stock of reserves if $\hat{s}_t < s_0$ would therefore be reversed by arbitrage–induced private portfolio transactions. Home country reserves would be replenished instantaneously, and the collapse would be avoided. The same holds *mutatis mutandis* for incipient reserve exhaustion in the foreign country when $\hat{s}^*_t > s_0$.

In a world with two monies and two bonds, no arbitrageur with costless access to bonds would ever hold money if the nominal interest rates on the fixed price home currency bond and the fixed price foreign currency bond were positive. Arbitrage would involve sales and purchases of bonds denominated in different national currencies. We postpone the discussion of this case to Section 5.3.

As before we define $\hat{v}$ as the minimal value of v consistent with $\hat{s} \geq s_0$. Similarly, $\hat{v}^*$ is defined as the maximal value of v consistent with $\hat{s}^* \leq s_0$. It is easily checked that

(52a) $$\hat{v} = (1 + \gamma\rho)(s_0 - \Delta) - \gamma(\rho v_0 + \mu) \ ,$$

(52b) $$\hat{v}^* = (1 + \gamma\rho)(s_0 - \Delta^*) - \gamma(\rho v_0 + \mu) \ .$$

Note that

$$\hat{s}^* = \hat{s} + \Gamma \ ,$$

and

$$\hat{v} = \hat{v}^* + (1 + \gamma\rho)\Gamma \ .$$

Assuming convergence we have $1 + \gamma\rho > 0$.

Speculative attack viability or S–viability again requires that

(53) $$\hat{v}^* < v < \hat{v} \qquad \text{(S–viability)} \ .$$

While the fixed exchange rate regime survives, the behavior of reserves is governed by

$$s_0 = ln(D + R_t) - ln(D^* + G - R_t) + v_t + \gamma E_t(s_{t+1} - s_0) \ .$$

In order to have $R_t > 0$, it is therefore necessary and sufficient that

$$v_t < -\Delta + s_0 - \gamma E_t(s_{t+1} - s_0) \ .$$

In order to have $R_t^* > 0$, it is necessary and sufficient that

$$v_t > -\Delta^* + s_0 - \gamma E_t(s_{t+1} - s_0) \ .$$

The minimal value of v_t for which $R_t = 0$ is given by

(54a) $$\bar{v}_t = s_0 - \Delta - \gamma E_t(s_{t+1} - s_0) \ .$$

The maximal value of v_t for which $R_t^* = 0$ is given by

(54b) $$\bar{v}_t^* = s_0 - \Delta^* - \gamma E_t(s_{t+1} - s_0) \ .$$

Reserve viability or R–viability therefore requires

(55) $$\bar{v}^* < v < \bar{v} \qquad \text{(R–viability)} \ .$$

Note that as before

$$\bar{v} = \bar{v}^* + \Gamma \ .$$

Let

$$\pi_t \equiv \text{probability}(\hat{s}_{t+1} \geq s_0 \mid \hat{s}_t < s_0) \ ;$$

$$\pi_t^* \equiv \text{probability}(\hat{s}^*_{t+1} \leq s_0 \mid \hat{s}^*_t > s_0) ;$$

$$E_t\hat{s}_{t+1} \equiv E_t(\hat{s}_{t+1} \mid \hat{s}_{t+1} \geq s_0 , \hat{s}_t < s_0) ;$$

and

$$E_t\hat{s}^*_{t+1} \equiv E_t(\hat{s}^*_{t+1} \mid \hat{s}^*_{t+1} \leq s_0 , \hat{s}^*_t > s_0) .$$

It follows that the <u>unconditional</u> future expected exchange rate $E_t s_{t+1}$ is given by

(56) $$E_t s_{t+1} = \pi_t E_t \hat{s}_{t+1} + \pi_t^* E_t \hat{s}^*_{t+1} + (1 - \pi_t - \pi_t^*) s_0 .$$

We now specialize the stochastic process z_t given in (47b) as follows.

(57) $$\begin{aligned} z_{t+1} &= \delta \text{ with probability } 0.5 \\ &= -\delta \text{ with probability } 0.5 \\ \delta &\geq 0 \end{aligned}$$

The variance of z, σ^2 in (47b), is δ^2 in this case. We define

(58a) $$\eta_t = (1 + \gamma\rho)(s_0 - \Delta) - (1 + \gamma)(\rho v_0 + \mu) - (1 - \rho) v_t ,$$

(58b) $$\eta_t^* = (1 + \gamma\rho)(s_0 - \Delta^*) - (1 + \gamma)(\rho v_0 + \mu) - (1 - \rho) v_t .$$

Note that

(59) $$\eta = \eta^* + (1 + \gamma\rho) K .$$

By inspection of (51a), (47a) and (50a) (and of (51b), (47a) and (58b)) it follows that π_t = probability$(z_{t+1} \geq \eta_t)$, and π_t^* = probability$(z_{t+1} \leq \eta_t^*)$. We therefore can establish the following.

(60a) $$\pi_t = 0 \quad \text{if } \eta_t > \delta$$

(60b) $$\pi_t = 0.5 \text{ if } -\delta < \eta_t \leq \delta$$

(60c) $\pi_t = 1 \quad \text{if } \eta_t < -\delta$

(60d) $\pi^*_t = 0 \quad \text{if } \eta^*_t < -\delta \text{ i.e. if } \eta_t < -\delta + (1 + \gamma\rho)\Gamma$

(60e) $\pi^*_t = 0.5 \text{ if } -\delta \leq \eta^*_t < \delta \text{ i.e. if } -\delta + (1 + \gamma\rho)\Gamma \leq \eta_t < \delta + (1 + \gamma\rho)\Gamma$

(60f) $\pi^*_t = 1 \quad \text{if } \eta^*_t \geq \delta \text{ i.e. if } \eta_t \geq \delta + (1 + \gamma\rho)\Gamma$

If $(1 + \gamma\rho)\Gamma \leq 2\delta$, we have:

(60g) $1 - \pi - \pi^* = 0.5 \text{ if } -\delta < \eta < -\delta + (1 + \gamma\rho)\Gamma$,

(60h) $1 - \pi - \pi^* = 0.5 \text{ if } \delta < \eta < \delta + (1 + \gamma\rho)\Gamma$,

(60i) $1 - \pi - \pi^* = 0 \quad \text{if } -\delta + (1 + \gamma\rho)\Gamma < \eta \leq \delta$,

(60j) $1 - \pi - \pi^* = 0 \quad \text{if } \eta < -\delta$,

(60k) $1 - \pi - \pi^* = 0 \quad \text{if } \eta > \delta + (1 + \gamma\rho)\Gamma$.

If $(1 + \gamma\rho)\Gamma > 2\delta$, we have:

(60l) $1 - \pi - \pi^* = 1 \quad \text{if } \delta < \eta < -\delta + (1 + \gamma\rho)\Gamma$,

(60m) $1 - \pi - \pi^* = 0.5 \text{ if } -\delta < \eta < \delta$,

(60n) $1 - \pi - \pi^* = 0.5 \text{ if } -\delta + (1 + \gamma\rho)\Gamma < \eta < \delta + (1 + \gamma\rho)\Gamma$,

(60o) $1 - \pi - \pi^* = 0 \quad \text{if } \eta < -\delta$,

(60p) $1 - \pi - \pi^* = 0 \quad \text{if } \eta > \delta + (1 + \gamma\rho)\Gamma$.

Figure 5 illustrates these probabilities. Note that there is no range of values of η_t for which the survival until the next period of the fixed exchange rate regime is certain if the world stock of reserves isn't large enough ($(1 + \gamma\rho)\Gamma \leq 2\delta$). This case is drawn in Figure 5a. Figure 5b has a range of η_t values ($\delta < \eta_t < -\delta + (1 + \gamma\rho)\Gamma$) for which the gold standard is certain to survive until the next period. This requires $(1 + \gamma\rho)\bar{I} > 2\delta$.

It is possible that a negative realization of z_{t+1} (i.e. $z_{t+1} = -\delta$) nevertheless pushes z_{t+1} above η_t for the first time. In that case a positive realization ($z_{t+1} = \delta$) would of course also have pushed z_{t+1} above η_t. In period t it would therefore have been certain that during the next period there was going to be a collapse with the home country authority running out of reserves. Formally the inequalities in (61a, b) can both hold.

(61a) $$-\delta \geq \eta_t \,,$$

and

(61b) $$\eta^*_{t-1} < z_t < \eta_{t-1} \,.$$

Using (58a) and (47a) we can rewrite (61a, b) as

(61a′) $$-\delta \geq (1 + \gamma\rho)(s_0 - \Delta) - (1 + \gamma)(\rho v_0 + \mu) - (1 - \rho)v_{t-1}$$
$$- z_t + \rho v_t - (\mu + \rho v_0)$$

and

(61b′) $$(1 + \gamma\rho)(s_0 - \Delta^*) - (1 + \gamma)(\rho v_0 + \mu) - (1 - \rho)v_{t-1}$$
$$< z_t < (1 + \gamma\rho)(s_0 - \Delta) - (1 + \gamma)(\rho v_0 + \mu) - (1 - \rho)v_{t-1} \,.$$

If v is a random walk without drift, ($\rho = \mu = 0$), equation (61a′) and the second inequality in equation (61b′) are inconsistent. If however $\rho v_t - (\mu + \rho v_0)$ is a sufficiently negative number, a guaranteed one–sided collapse next period can occur. In this case the expected future shadow exchange rate conditional on the occurrence of a collapse is

(62a) $$E_t\hat{s}_{t+1} = \Delta + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t] \,.$$

In exactly the same way it is possible that a positive realization of z_{t+1} can push z_{t+1} below η_t^* for the first time. Then there will be certainty in period t that the foreign authority will be stripped of reserves in period $t+1$. The expected future shadow exchange rate, conditional on a collapse occurring, for this case is

(62b) $$E_t\hat{s}^*_{t+1} = \Delta^* + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t] .$$

In all other cases only a positive realization $(+\delta)$ can push z beyond η for the first time, and only a negative $(-\delta)$ realization can push z below η^* for the first time. We therefore have

(63a) $$E_t\hat{s}_{t+1} = \Delta + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t + \delta] ,$$

and

(63b) $$E_t\hat{s}^*_{t+1} = \Delta^* + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t - \delta] .$$

From (56), (60), (62a, b) and (63a, b) we finally obtain $E_t s_{t+1}$. First consider the case illustrated in Figure 5a where $(1 + \gamma\rho)\bar{R} \leq 2\delta$.

(64a) $$E_t s_{t+1} = 0.5s_0 + 0.5\{\Delta + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t + \delta]\}$$

if $-\delta < \eta_t < -\delta + (1 + \gamma\rho)\bar{R}$. In this case there is a fifty percent chance of a collapse of the home currency in the next period.

(64b) $$E_t s_{t+1} = 0.5s_0 + 0.5\{\Delta^* + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t - \delta]\}$$

if $\delta < \eta_t < \delta + (1 + \gamma\rho)\bar{R}$. In this case there is a fifty percent chance of a collapse of the foreign currency in the next period.

(64c) $E_t s_{t+1} = 0.5\{\Delta + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t + \delta]\}$

$+ 0.5\{\Delta^* + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t - \delta]\}$

if $-\delta + (1 + \gamma\rho)K < \eta_t < \delta$. In this case a collapse the next period is certain, and it is equally likely to be a collapse of the home currency as a collapse of the foreign currency. This implies $2\delta > K(1 + \gamma\rho)$, so the fixed exchange rate regime would only last for one period.

(64d) $E_t s_{t+1} = \Delta + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t]$

if $\eta_t < -\delta$. In this case there is a certain collapse of the home country currency in the next period.

(64e) $E_t s_{t+1} = \Delta^* + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t]$

if $\eta_t > \delta + (1 + \gamma\rho)K$. In this case there is a certain collapse next period of the foreign currency.

Next consider the case illustrated in Figure 5b where $(1 + \gamma\rho)K > 2\delta$.

(65a) $E_t s_{t+1} = s_0$

if $\delta < \eta_t < -\delta + (1 + \gamma\rho)K$. In this case the fixed exchange rate regime is certain to survive until the next period.

(65b) $E_t s_{t+1} = 0.5s_0 + 0.5\{\Delta + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t + \delta]\}$

if $-\delta < \eta_t < \delta$. There is a fifty percent chance of a collapse of the home currency during the next period.

$$(65c)\quad E_t s_{t+1} = 0.5 s_0 + 0.5\{\Delta^* + (1+\gamma\rho)^{-1}[(1+\gamma)(\rho v_0 + \mu) + (1-\rho)v_t - \delta]\}$$

if $-\delta + (1+\gamma\rho)\Gamma < \eta_t < \delta + (1+\gamma\rho)\Gamma$. There is a fifty percent chance of a collapse of the foreign currency next period.

$$(65d)\quad E_t s_{t+1} = \Delta + (1+\gamma\rho)^{-1}[(1+\gamma)(\rho v_0 + \mu) + (1-\rho)v_t]$$

if $\eta_t < -\delta$. There is a certain collapse of the home currency in the next period.

$$(65e)\quad E_t s_{t+1} = \Delta^* + (1+\gamma\rho)^{-1}[(1+\gamma)(\rho v_0 + \mu) + (1-\rho)v_t]$$

if $\eta_t > \delta + (1+\gamma\rho)\Gamma$. There is a certain collapse of the foreign currency in the next period.

Now that we know $E_t s_{t+1}$ from equations (64a–e) and (65a–e), we can calculate $\bar{v}_t$ and $\bar{v}^*_t$ from equations (54a, b). With $\hat{v}$ and $\hat{v}^*$ given in equations (52a,b) we can establish whether the gold standard paradox arises in this model. There is a paradox if $\bar{v} < \hat{v}$ or if $\bar{v}^* > \hat{v}^*$. In that case a country can run out of reserves without a speculative attack even though the economy possesses a speculative collapse point.[4] For brevity's sake we focus on $\bar{v} < \hat{v}$. The $\bar{v}^* > \hat{v}^*$ case is symmetric.

The easiest example proving that the paradox can occur is the special case of the discrete time model in which it replicates exactly the key features of the continuous time model of Section 3. This is the case where the survival of the gold standard into the next period is guaranteed i.e. the case with $E_t s_{t+1} = s_0$ given in equation (65a). It requires $(1 + \gamma\rho)\Gamma > 2\delta$ and $\delta < \eta_t < -\delta + (1+\gamma\rho)\Gamma$. In this case $\bar{v} = s_0 - \Delta$, and since we always have $\hat{v} = (1+\gamma\rho)(s_0 - \Delta) - \gamma(\rho v_0 + \mu)$ the analysis of Section 3 is directly transferable.

For instance, we have $\bar{v} < \hat{v}$ if $\rho = 0$ and $\mu < 0$. If v follows a random walk with negative drift, there will be a natural collapse at the upper boundary (the home country runs out of reserves) before a speculative attack occurs. Alternatively if $\mu = 0$ and $0 < 1 - \rho < 1$ (v is a first-order stationary autoregression without drift), then $\tilde{v} < \hat{v}$ if $s_0 > v_0 + \Delta$.

The occurrence of the paradox is not dependent on the special feature of the previous two examples that $E_t s_{t+1} = s_0$. One further illustration should suffice to make this point. Consider the case where there is a fifty percent chance of a collapse of the foreign currency in the next period and no chance at all of a home currency collapse. We choose $(1 + \gamma\rho)K < 2\delta$ so the relevant equation for $E_t s_{t+1}$ is (64b) with $\delta < \eta_t < \delta + (1 + \gamma\rho)K$. For simplicity consider the case where v follows a random walk with drift ($\rho = 0$). In this case $\hat{v} = s_0 - \Delta - \gamma\mu$, and $\tilde{v}$ is solved from equations (64b) and (54a). This yields

$$\tilde{v} = s_0 - (1 + 0.5\gamma)^{-1}\{\Delta + 0.5\gamma[\Delta^* + (1 + \gamma)\mu - \delta]\} .$$

It follows that

$$\hat{v} - \tilde{v} = 0.5\gamma(1 + 0.5\gamma)^{-1}[K - (\mu + \delta)] > 0 \text{ if } K > \mu + \delta .$$

There is a difference between the continuous time case with its continuous sample paths of v and the discrete time case where v makes discrete, discontinuous steps. First given the bounded support of our random shocks it is possible for a fixed exchange rate regime to survive forever. If v is governed by a stationary *AR1* process without drift ($|1 - \rho| < 1$ and $\mu = 0$), then a sufficiently small value of δ relative to K ensures that if the stock of reserves starts off in the interior of the viable zone, it will stay there forever, barring regime switches (see Obstfeld [1986a]).

Second even if $\tilde{v} < \hat{v}$, it need not be the case that from a value of v less than $\tilde{v}$ the economy would have to experience a natural collapse before it could reach the speculative attack point. A sufficiently large positive realization of z_{t+1} can put v_{t+1} above $\hat{v}$ even though v_t is below $\bar{v}$. Such a large shock would "by–pass" the natural collapse point and give us $\hat{s}_{t+1} = s_{t+1} > s_0$ i.e. a well–behaved speculative attack. (In the same manner a sufficiently large negative realization of z_{t+1} can put v_{t+1} below $\hat{v}^*$ even though $\hat{v}^* < \bar{v}^* < v_t$.) The similarity to the case of continuous time and continuous sample paths of v is that for a given shock z_{t+1} there will be a range of initial values v_t such that if $v_t < \bar{v} < \hat{v}$, we have $\bar{v} \leq v_{t+1} < \hat{v}$.[5]

Apart from demonstrating the analytical advantages of working with continuous time models, Section 4 proves that the gold standard paradox is not an artifact of the special class of continuous time processes studied by Krugman, and in Sections 2 and 3 of this paper. It also supplies the convenience of a finite length unit period which may facilitate the interpretation of our resolution of the paradox in Section 5.

5. THE PARADOX RESOLVED

5.1 The Scope of the Paradox

The reason for the occurrence of the paradox should by now be clear: if a low value of the currency is associated with expectations of appreciation (a higher value of $\hat{s}(t)$ is associated with a negative value of $E_t\dot{s}(t)$ following a collapse in the continuous time model and with a negative value of $E_t(s_{t+1} - s_t)$ in the discrete time model), there will not be a correct speculative attack at the upper boundary of the S&R viable zone. If a high value of the currency is associated with expectations of depreciation (a lower value of $\hat{s}(t)$ is associated with a positive value of $E_t\dot{s}(t)$ following a collapse or a positive value of $E_t(s_{t+1} - s_t)$), there will not be a correct speculative attack at the lower boundary of the S&R viable zone. This means there will never be any trouble when the shadow exchange rates are weakly monotonic over time. In that case the movement of the actual shadow exchange rate is always in the same direction as the change in the expected rate of change of the exchange rate at the moment a collapse occurs: if the actual and expected shadow exchange rates always rise or remain constant (fall or remain constant), there will always be a correct speculative attack at the upper (lower) boundary of the S&R viable zone. If we start above (below) the lower (upper) boundary, the system will never descend (rise) towards it.

Deterministic models with a constant rate of change of the fundamental ($\rho = 0$, $\sigma = 0$) therefore never present any problems. If the drift is zero ($\mu = 0$), the fixed exchange rate regime will survive forever provided the initial value of v is in the interior of the S&R viable zone. Positive drift ($\mu > 0$) means a certain collapse at the upper boundary (a selling attack against the home country currency). Negative drift means a certain collapse at the lower boundary (a selling attack against the foreign currency). Such models were considered in Krugman [1979], Flood and Garber [1984], and Buiter [1987].

Stochastic models in which the increment of v is always nonnegative (say because they are drawings from an exponential distribution) will also display a monotonically nondecreasing shadow exchange rate over time. The only possible collapse is a correct collapse at the upper boundary. Such models were considered by Flood and Garber [1984], and Buiter[1987].

The random walk without drift ($\mu = \rho = 0$) is trouble free not because its actual movement over time is monotonic but because for every value of $v(t)$, $E_t dv(t) = 0$, and for every value of $s(t)$, $E_t ds(t) = 0$. There are never any (stock–shift) speculative attacks in that model.

For all other models in the class of v processes given in (26) there are parameter configurations with incorrect attacks at the lower boundary and/or at the upper boundary. Examples are Grilli [1986], Obstfeld [1986b] and Buiter [1989]. An S&R viable range may also fail to exist when $\mu \neq 0$ and/or $\rho \neq 0$, but that poses no paradox. The paradox is the existence of an S&R feasible range with perverse attacks at one or both boundaries.[6] In other words, the paradox is the failure of a well–defined equilibrium to exist for some range of values of the fundamentals.

Krugman conjectures that there can be no nonparadoxical fixed exchange rate regime or gold standard. He abandons the assumption of a fixed exchange rate regime and replaces it with an exchange rate "target zone" with hard boundaries as long as reserve thresholds are not broken.

Within such a target zone where the exchange rate floats freely between the boundaries, expectations of depreciation can and will in general be nonzero. Appropriate monetary interventions prevent the exchange rate from breaking through the upper or lower limits of the target zone as long as reserves last. Equilibrium is well–defined in these types of models. There is now a vast literature on this subject. (See e.g. Dixit [1988], Krugman [1987, 1988, 1989], Miller and Weller [1988(a, b), 1989], Flood and Garber [1989], Froot and Obstfeld [1989(a, b)], and Dumas [1989].)

The "smooth pasting" and related techniques used to solve for the behavior of the exchange rate between the bounds permit the proper implications to be drawn from assumptions about efficient intertemporal speculation. No "target zone paradox" to match the gold standard paradox has as yet been identified.

While the transformation of the gold standard into a band system delivers a proper equilibrium, to write off the fixed exchange rate regime in this way as a figment of the mathematician's imagination is to throw out the baby with the bath water. The argument that even the historical gold standard was not a truly fixed exchange rate regime because of the existence of the "gold points" misses the point. The gold points wedge reflected the real cost of shipping gold (mainly between the U.S. and Britain). These transportation and transactions costs are surely negligible today. Charles de Gaulle may have insisted on physical shipment of gold from Fort Knox to Paris, but efficient business practice today means the (virtually costless) exchange of paper ownership claims to gold rather than physical transshipment. Most foreign exchange reserves today are paper claims rather than heavy physical objects anyway. Reserve gains and losses are bookkeeping entries that can be effected virtually instantaneously and at negligible cost.

Clearly a truly fixed exchange rate regime is an abstraction that very closely approximates some historical international monetary arrangements as well as some prospective future arrangements (e.g. those that are emerging for the European

Community). In the next two subsections we present two alternative ways of guaranteeing the existence of an equilibrium without abandoning the assumption of fixed exchange rates.

5.2 The Missing Equation and the Missing Money Holdings of Arbitrageurs In the Two–Monies–and–Gold Model

Unlike the exposition of the gold standard paradox in the previous two Sections, its resolution is a brief affair. It is implicit in our discussion at the beginning of Section 4 of what happens during a period in which a country runs out of official reserves and a floating exchange rate is adopted. During that period, currencies (and, in models with interest bearing assets, bonds denominated in different currencies) can be sold in exchange for reserves and repurchased instantaneously at potentially distinct prices: s_0 and $\hat{s}$ in the case of a collapse of the home currency; s_0 and $\hat{s}^*$ in the case of a collapse of the foreign currency. This possibility of risk free arbitrage profits, or rather the market response that eliminates this possibility, is not part of the formal structure of either the continuous time model (equations (1) through (6)) or the discrete time model (equations (46) through (50)). Inclusion of the missing "no arbitrage profits" condition (given as equation (68a, b) below) and inclusion of the missing economic agents (currency arbitrageurs) dissolves the paradox and confirms the validity of the key conclusions of the conventional approach.

Without loss of generality we focus in what follows on a threatening home currency collapse in period t with $\bar{v} < \hat{v}$. If $\bar{v} < \hat{v}$, it follows that when $v_t = \bar{v}$, we have $\hat{s}_t < s_0$. We also restrict our discussion here to the direct currency substitution model of equations (7)–(9). The case of bond arbitrage is discussed in Section 5.3

The money demand functions represented in equations (1) and (46) only represent the demands for home and foreign currency **excluding** any demand reflecting international currency arbitrage. Let m and m^* denote, as before, the stocks of home and foreign currency. Money holdings of international currency arbitrageurs are denoted m^{*a} and m^{*a}. The monetary equilibrium condition including the money holdings of arbitrageurs is, in continuous time

(67a) $$s_t = m_t - m_t^* - (m_t^a - m_t^{*a}) + v_t + \gamma E_t \dot{s}(t) \ .$$

The nonarbitrage demand for home currency (relative to foreign currency), $m^n - m^{*n}$ is of course given by

(67b) $$m_t^n - m_t^{*n} - s_t = -v_t - \gamma E_t \dot{s}(t) \ .$$

The presence of efficient arbitrageurs ready to avail themselves of opportunities for riskless profit means that we can impose the no arbitrage profits assumption given in equations (68a, b).

Assumption 1: No Arbitrage Profits

(68a) $$\hat{s}_t - s_0 \geq 0 \text{ i.f.f. } R_t = 0 \ ,$$

and

(68b) $$\hat{s}_t^* - s_0 \leq 0 \text{ i.f.f. } R_t^* = 0 \ .$$

To remove a major indeterminacy from the model we assume that if there are no pure arbitrage profits to be earned, arbitrageurs will reduce to zero $m^a - m^{*a}$ their relative holdings of home currency to foreign currency (henceforth to be referred to as relative currency holdings).

Assumption 2: Minimal Efficient Size Arbitrage Portfolios

(69) If $m_t^a - m_t^{*a} = 0$ implies $R(t) > 0$ and $R^*(t) > 0$, then

$$m_t^a - m_t^{*a} = 0 .$$

If $\ell_t = 0$ and $\hat{s}_t \geq s_0$, then $m_t^a - m_t^{*a} = 0$.

If $\ell_t^* = 0$ and $\hat{s}_t^* \leq s_0$, then $m_t^a - m_t^{*a} = 0$.

Second–order costs of managing any portfolio other than $m^a - m^{*a} = 0$ could be used to rationalize (69).

While it would be interesting to consider a model in which arbitrageurs hold gold as well as home and foreign currency, we wish to modify the model used by Krugman in as few ways as possible. We therefore assume that gold is only used by arbitrageurs to switch between home and foreign currency, and that their gold holdings are zero. Equations (4) and (49a) are therefore maintained.

There will be pure arbitrage profit opportunities whenever $\ell_t = 0$ (the home country abandons the fixed exchange rate standard) and $\hat{s}_t < s_0$. By using reserves to purchase home currency at s_0 and instantaneously reselling that home currency at $\hat{s}$, arbitrageurs can earn riskless positive profits. The same holds when the foreign country abandons the fixed exchange rate regime ($\ell_t^* = 0$) and $\hat{s}_t^* > s_0$.

When $\ell_t = 0$ and $\hat{s}_t < s_0$ the opportunity cost variable governing nonarbitrage relative money demands $m_t^n - m_t^{*n} - s_t$ is still the intertemporal relative price $-E_t ds(t)$ (or $-E_t(s_{t+1} - s_t) = -E_t(s_{t+1} - s_0)$ in the discrete time case). The relevant opportunity cost variable governing relative arbitrage demand for money $m^a - m^{*a}$ is $-(\hat{s}_t - s_0)$, and the sensitivity of relative arbitrage demand for money to this opportunity cost variable is infinite. Since the market cannot eliminate this pure arbitrage profit opportunity once the contingency triggering it has occurred (i.e. once $\ell_t = 0$ and the home country abandons the fixed exchange rate regime with $(s_t = \hat{s}_t)$), the market instead prevents the contingency that triggers the pure arbitrage opportunity from occurring: it removes the threat of a home country collapse with $\hat{s} < s_0$ by ensuring that home country official reserves stay above the critical threshold level of zero as long as $\hat{s} < s_0$. The mechanism that brings this about is that arbitrageurs replenish home country official reserves when $v \geq \bar{v}$ (and $v < \hat{v}$), or equivalently that for $\bar{v} \leq v < \hat{v}$, changes in v are absorbed into equivalent changes in relative money holdings of arbitrageurs.

What this means is that when $\bar{v} \leq v < \hat{v}$, the stock of international reserves R is no longer governed by v. Reserves are kept just above the minimal threshold level by the missing actors in Krugman's account of the paradox, the international currency arbitrageurs.

It is important to note that the arbitrageurs' reverse flow of gold to the home country authorities (their absorption of relative home currency) is never more nor less that the amount required to restore reserves to a positive level. That it is never less follows from equations (68a, b). That it is never more follows from equation (69). In the continuous time case where dz is an infinitesimal, the arbitrageurs' response at each instant will have the same infinitesimal magnitude. As soon as $R(t) > 0$, the arbitrageurs' incentive to sell reserves in exchange for home country currency vanishes as the probability of an immediate collapse with $\hat{s} < s_0$ disappears. If however equation (69) did not hold, then arbitrageurs—indifferent between holding home and foreign currency when $R > 0$ and $R^* > 0$—could arbitrarily set reserves at any level by choosing arbitrary values of $m^a - m^{*a}$.

Recapitulating if $R_t = 0$ and $\hat{s}_t < s_0$, arbitrageurs would buy up the entire domestic money stock at s_0 in order to get rid of it again that same instant (in the same period) at $\hat{s}$. The fact that if $\hat{s}_t < s_0$ a large (stock–shift) inflow of reserves driven by arbitrage would take place bounds R_t away from (above) zero. This will occur for as long as $\hat{s} < s_0$. For values of v such that $\bar{v} \leq v < \hat{v}$ arbitrageurs will increase or reduce their relative money holdings according to $d(m^a - m^{*a}) = dv$.[7] This will preserve money market equilibrium at the fixed exchange rate, since with $s = s_0$ and $E_t ds(t) = 0$ (in the continuous time case—the discrete time case is slightly more complex) we have $d(m^n - m^{*n}) = -dv$.

Thus in Figure 4 when $\bar{v} \leq v < \hat{v}$, as v increases from $\bar{v}$ we move horizontally to the right along the s_0 schedule from Ω_1 to Ω_3. At Ω_3 there is a collapse of the home currency with $\ell = 0$, but since $s_0 = \hat{s}$ this creates no problem for the conventional analysis. As the fixed exchange rate regime collapses at Ω_g because $\ell = 0$, the expected rate of depreciation becomes negative after being equal to zero (in the continuous time model). There is a stock–shift **increase** in the relative nonarbitrage demand for home currency.

Where does the money come from that satisfies the increased (relative) nonarbitrage demand for home currency when the expected rate of depreciation falls from zero to $E_t ds(\hat{v}) < 0$? Not out of domestic credit—by assumption D and D^* are constant. Not out of the official reserves of the home country—which are given at zero. It comes out of money balances released by arbitrageurs who, since $s_0 = \hat{s}$, now reduce their relative money demand to zero. The (relative) home country currency accumulated by arbitrageurs at $\hat{v}$ (at Ω_g) in Figure 4 is $m^a - m^{*a} = \hat{v} - \bar{v}$. This is also equal to the stock–shift increase in the relative demand for home currency by nonarbitrageurs at $\hat{v}$.

In the continuous time model, for any v satisfying $\bar{v} \leq v < \hat{v}$, the relative demand for home currency by nonarbitrageurs is given by

$$m^n - m^{*n} = s_0 - v \; . \tag{70a}$$

For the same range of values of v, the relative holdings of home currency by arbitrageurs are given by

$$m^a - m^{*a} = v - \bar{v} \; . \tag{70b}$$

When $v = \hat{v}$ the relative demand for home currency by arbitrageurs falls to zero i.e is reduced by $\hat{v} - \bar{v}$. The relative demand for home currency by nonarbitrageurs increases by $-\gamma E_t ds(\hat{v})$. From equations (30a), (22a) and (27a) it follows that

$$\hat{v} - \bar{v} = -\gamma E_t \dot{s}(\hat{v}) = \gamma\rho[s_0 - (lnD - ln(D^* + G)) - (v_0 + \tfrac{\mu}{\rho})] \; . \tag{71}$$ [8]

Thus the increase in the relative demand for home country currency by nonarbitrageurs at $\hat{v}$ can be met and is met exactly out of the money holdings released by arbitrageurs who have no longer any riskless profit motive for holding on to home currency.

The gold standard therefore collapses in Figure 4 at Ω_3 as the traditional analysis asserted. Unlike what was suggested by the traditional analysis however there is no speculative attack on the remaining home country reserves at Ω_3. Instead the increased nonarbitrage demand for relative home country currency associated with the fall in the expected depreciation rate at Ω_3 is met out of the accumulated money balances of arbitrageurs who maintained the gold standard between Ω_1 and Ω_3.

In a similar manner in Figure 4 to the left of Ω_2 (for $\hat{v}^* < v \leq \bar{v}^*$), arbitrageurs will be building up relative holdings of foreign currency thus preventing foreign reserves R^* from falling to zero. At Ω_4 where $v = \hat{v}^*$ there is a collapse of the gold standard as R^* falls to zero (after being infinitesimally above zero for all v between $\hat{v}^*$ and $\bar{v}^*$). There is a stock–shift increase in the relative nonarbitrage demand for foreign currency at $\hat{v}^*$ which is associated with the increase in the expected rate of depreciation of the home currency from zero to $E_t ds(\hat{v}^*) > 0$. This increase in the relative nonarbitrage demand for foreign currency is met out of the now redundant holdings of relative foreign currency by arbitrageurs. From Ω_2 to Ω_4 the exchange rate stays at s_0. Once Ω_4 is reached, $\hat{s}^*$ takes over. Exactly the same can be said about the behavior of the economy between Ω_2 and Ω_4 in Figure 2.

5.3 The Resolution of the Paradox when Bond Arbitrage Occurs

In the case where arbitrage is conducted proximately through interest–bearing bonds denominated in different currencies, the argument of Section 5.2 is applicable only if it can be shown that this bond arbitrage spills over into the monetary equilibrium condition. That is it must affect either relative money demands or relative stocks of domestic credit. This case can be made under the following sets of conditions.

1. *There is no equilibrium with interest-bearing assets.*

In two–period overlapping generations (OLG) models with each generation consisting of identical individuals and without firms whose "life span" exceeds that of individuals, there can be no private lending or borrowing. If in addition there is no interest–bearing public debt outstanding and no real assets but there are noninterest–bearing "outside" national money stocks, we have reconstructed the model of Section 5.2. There are no equilibria with debt of any kind. Further restrictions on the international uses of national currencies would have to be imposed to obtain limited substitutability between currencies and a determinate exchange rate (see Kareken and Wallace [1981]).

2. *Interest is paid on currency.*

If interest is paid on currency and if the currency interest rates mimic the bond interest rates, we again could have direct currency arbitrage. This amounts to assuming the problem away.

3. *Money demand and bond demand are segmented.*

A certain subset of agents may be constrained not to hold bonds but can hold the two currencies. The single–country version of such a model can be found in Sargent and Wallace's "Unpleasant Monetarist Arithmetic" OLG model (Sargent and Wallace[1981]). Small bills–type arguments and the assumption that the poor cannot pool resources to invest in interest–bearing assets that can be acquired only in large denominations leads to an equilibrium in which the poor hold only money and (if the nominal interest rate is positive) the rich hold only bonds. An obvious two–country extension results in one set of agents (the poor) holding only the two currencies while the rich hold the two bonds. The poor would supply the direct currency arbitrage that would make the argument of the previous subsection applicable.

4. *There is a zero nominal interest rate equilibrium.*

Can it happen that for values of v satisfying $\bar{v} \leq v < \hat{v}$ we have $i = i^* = 0$, even if $i > 0$ and $i^* > 0$ for $v < \bar{v}$ and $v > \hat{v}$? It is true that the equilibrium conditions of the model given in equations (1) through (6) (after substituting equation (67a) for equation (1)) have nothing to say about the levels of the nominal interest rates. The model of equations (10)–(13) that generates equation (1) and (*mutatis mutandis*) equation (67a) does however seem to rule out a sudden decrease of i and i^* at $\bar{v}$ (or an increase at $\hat{v}$). With both y and y^* given at a point in time, with $p = p^* + s_0$, $m = ln(D + R)$ and $m^* = ln(D^* + R)$, a common fall in i and i^* requires a discontinuous fall in p and p^* to provide the higher level of real money balances demanded at the lower nominal interest rates. Even if direct currency arbitrageurs existed, they could not supply an increase in the nominal stocks of both currencies to satisfy the increased nonarbitrage demand for money at given price levels as this cannot be achieved through a profitable zero net worth transaction.

A discontinuous fall in p and p^* which would be fully anticipated is inconsistent with an optimizing intertemporal equilibrium unless there are no means of intertemporal transformation of real resources. If the single global good were perishable, then discontinuous anticipated changes in p and p^* would be consistent with intertemporal efficiency (see Drazen and Helpman [1985]).

Assuming this to be the case, a quasidynamic argument can be made that the attempts by bond arbitrageurs at $\bar{v}$ to unload foreign bonds and acquire home bonds would end up driving the bond arbitrageurs and their transactions into the currency markets. At $\bar{v}$ no private agent wishes to purchase foreign currency denominated bonds. Arbitrageurs will be unable to borrow privately in the foreign bond market in order to invest in home currency bonds. With D and D^* given, the two governments also do not purchase foreign bonds or sell home bonds.

If a discontinuous drop in p and p^* (by the same amount) is consistent with intertemporal equilibrium, a zero nominal interest rate equilibrium would satisfy the remaining equilibrium conditions and send us back to the scenario of Section 5.2. Bond arbitrageurs now are happy to become currency arbitrageurs, and because R is endogenous they can build up m^a and reduce m^{*a} simultaneously.

The stability of this equilibrium seems a bit suspect: one can see how the attempt to increase home bond holdings by all arbitrageurs as long as $i > 0$ could drive i to zero. Why the other half of the attempted bond reshuffle—the attempt to sell foreign bonds or even to go short in them—should result in a zero foreign interest rate rather than an infinite one (as the foreign bond price collapses) is not obvious. Given the assumptions made here and in Section 5.2, $i = i^* = 0$ would be an equilibrium: no private agent would have an incentive to change his behavior. How to get there from $v < \bar{v}$ is another matter.

5. *Saving the bond market.*

Where the previous three arguments had the bond arbitrageurs' demand spill over into the relative demands for currency, our final argument assumes an impact of bond arbitrage on the stocks of domestic credit, D and D^*.

Assume that a sudden decline in i and i^* at $\bar{v}$ to zero is not consistent with an optimizing intertemporal equilibrium say because there are sufficiently rich opportunities for the intertemporal transformation of real goods. In that case there would, with D and D^* constant, be a collapse of the market for foreign currency bonds: every private arbitrageur would try to sell foreign currency denominated debt (and indeed go short in it) and buy home currency denominated debt.

If the authorities responded to this threatened collapse of the foreign bond market by undertaking the minimal open-market operations required to prevent a bond market collapse, they would choose dD and dD^* such that

$$\frac{1}{D^* + G}dD^* - \frac{1}{D}dD = dv \; . \tag{72}$$

If $dv > 0$, this could involve the foreign country government switching from borrowing from the private sector to borrowing from its central bank. In addition the home government could move in the opposite direction and engage in an open–market sale. If the assumption of exogenous domestic credit were replaced by equation (72) when $\bar{v} \leq v < \hat{v}$, the two governments would effectively take over from the private currency arbitrageurs described in the previous subsection. The prevention of a collapse of the fixed exchange rate regime would be a by–product of policies aimed at preventing a bond market collapse. At $\hat{v}$ there would be a stock–shift reversal of the cumulative flow open–market operations that brought the economy from $\bar{v}$ to $\hat{v}$. There would be an open–market purchase by the home government and/or an open–market sale by the foreign government to provide private agents with the relatively larger stocks of home money demanded at the negative postcollapse expected rate of exchange rate depreciation.

It is hardly surprising that (relative) domestic credit expansion policies can be used to stabilize reserves in the face of exogenous relative money demand shifts. It is nevertheless interesting that such a policy can be rationalized as a response to a threatened bond market collapse. If government debt is denominated in the national currency, a forward looking foreign government may well have a strong incentive to prevent a collapse in the market for its debt.

This last approach implies a change in the structure of the model which is comparable to Krugman's proposal for remodeling the gold standard as a band system. In both cases the policies pursued by the government have been altered in order to achieve an equilibrium. Our solution however has the advantage of showing that the management of a gold standard or a fixed exchange rate regime is feasible. Moreover, historical evidence exists that is consistent with our interpretation —

for example the gold standard crisis during the 1890s. As analyzed in Grilli [1989], between 1983 and 1896 there was a widespread fear that the U.S. Treasury would run out of gold reserves and that the gold standard would have to be abandoned. During this period the financial markets were very unstable, and this instability reached a peak with the panic of 1893. To ensure the viability of the gold standard, on four occasions the U.S. Treasury issued bonds. These operations were exactly of the kind illustrated above: a swap of gold for domestic bonds. In one instance (the issue of February 1895), in addition to issuing bonds the Treasury also "subsidized" speculators (the Belmont–Morgan syndicate) in order for them to hold domestic currency. In this way they transformed money into an interest–bearing asset—a measure which would also provide a remedy to the crisis—as illustrated in Section 5.3.2 above.[9]

6. *Other approaches to direct currency arbitrage in the presence of debt with positive nominal interest rates.*

In Section 5 and especially in Section 5.3 the importance has become apparent of the detailed specification of the choice problem that generates the demands for the dominated assets home and foreign currency. Without explicit microfoundations of the demands for the two national currencies, the case for and against direct currency arbitrage in the presence of rate of return dominating assets cannot be resolved conclusively. We hope to compare various alternative approaches (e.g. cash–in–advance, Allais–Baumol–Tobin and precautionary demand for money models) in future work. It seems likely however that it will be possible to generate plausible conditions under which private agents who hold various national currencies for transactions or precautionary reasons can be induced to depart from their normal cash holdings in response to opportunities for pure arbitrage profits.

6. CONCLUSION

The gold standard paradox turns out not to be a gold standard contradiction or inconsistency. Krugman's critical probing of the speculative attack literature has brought out serious weaknesses in the way in which these models were interpreted and described, but not in the formal analyses of when and under what circumstances fixed exchange rate regimes collapse or of how the postcollapse exchange rate behaves.

The explicit recognition in speculative attack models of the role of arbitrageurs faced with the prospect of an imminent collapse of the fixed exchange rate regime and the associated possibility of riskless profits permits us to rule out the possibility of a natural collapse occurring before the speculative attack takes place. Speculative collapses occur where and when the traditional literature says they will.

The (stock–shift) changes in nonarbitrage money demand associated with collapses are however in the paradoxical cases identified by Krugman not accommodated by (stock–shift) changes in official international reserves but instead come out of the accumulated money balances of arbitrageurs who release them when a conventional collapse (which eliminates their opportunity for riskless profits) occurs. When nominal riskless debt instruments with positive interest rates can be held in addition to money, one has to work quite hard to generate direct currency arbitrage rather than (or in addition to) bond arbitrage. This is of course a general problem in monetary economics, and we cannot offer a fully satisfactory solution.

Between a paradoxical natural collapse point and the proper speculative collapse point private arbitrageurs keep the government whose reserves are about to be exhausted supplied with the minimal amount of reserves required for the survival of the fixed exchange rate regime. If the fundamentals drive the shadow exchange rate to the conventional speculative attack point, the arbitrageurs release the money holdings they accumulated while keeping the threatened government supplied with reserves and thus satisfy the increased nonarbitrage demand for money. A conventional transition to a free float (or to some other postcollapse regime not analyzed in this paper) then takes place.

NOTES

1 A nonconstant but still exogenous $D(t)$ or $D^*(t)$ can be subsumed under $v(t)$.

2 The purist will note some untidiness as regards the composition of the stock of reserves. If reserves are gold, let p^G be the domestic currency price of gold and p^{*G} the foreign currency price of gold. Then $m = ln(D + p^G R)$ and $m^* = ln(D^* + p^{*G} R^*)$. The exchange rate $S \equiv e^s = \frac{p^G}{p^{*G}}$. Without loss of generality we can choose units such that $p^G = p^{*G} = 1$, but in that case we must also set $s_0 = 1$.

3 The case $\mu < 0$ is conceptually identical.

4 The condition for S&R viability, for correct attacks at the boundaries and for perfection are the same in the discrete time model as those for the continuous time model summarized in Table 1 except that equations (32a) and (32b) are replaced respectively by (32a′) and (32b′). If the fixed exchange rate regime collapses in period $t+1$ because the home country runs out of reserves, then

(32a′) $\quad E_t S_{t+1} \leq E_t \hat{S}_{t+1}$.

If the fixed exchange rate regime collapses in period $t+1$ because the foreign country runs out of reserves, then

(32b′) $\quad E_t S_{t+1} \geq E_t \hat{S}^*_{t+1}$.

5 Or, for a given initial value $v_t < \bar{v} < \hat{v}$, there will be a range of realizations of z_{t+1} such that $\bar{v} \leq v_{t+1} < \hat{v}$.

6 Note that if we consider arbitrary (nonlinear) v processes the shadow exchange rate curves may intersect the s_0 line more than once, and we could get several S&R viable ranges with different combinations of correct and incorrect speculative attacks at the boundaries!

7 Note that arbitrageurs can finance an increase in m^a with a reduction in m^{*a} such that $dexp(m^a) + S_0 dexp(m^{*a}) = 0$. The arbitrageurs can be the nonarbitrage money holders wearing different hats. It is not necessary that they be able to borrow (at a zero interest rate) to finance an increase in m^a.

8 $\hat{v} - \bar{v} = -\gamma E_t \dot{s}(\hat{v})$ holds for all continuous forcing processes v. At $\bar{v}$ with m^a –

$m^{*a} = 0$, we have $s_0 = \Delta + \tilde{v}$; just below $\hat{v}$ we have $s_0 = \Delta + m^a - m^{*a} + \hat{v}$; at $\hat{v}$ we have $s_0 = \Delta + \hat{v} + \gamma E_t \dot{s}(v)$.

9 This example of course does not prove that without the intervention an equilibrium would not have existed. In fact the open-market intervention could have been used to forestall a "proper" speculative attack and collapse. Indeed, this is the interpretation given in Grilli [1989].

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Table 1

Summary of Viability and Correctness Criteria

S–viability:

(21) $\hat{v}^* < v < \hat{v}$.

R–viability:

(23) $\bar{v}^* < v < \bar{v}$.

S&R–viability:

(24) $max(\bar{v}^*, \hat{v}^*) < v < min(\hat{v}, \bar{v})$.

Correct attack at upper boundary:

(25a) $\hat{v} \leq \bar{v}$

or

(32a) $E_t\dot{s}(\hat{v}) = (1 + \gamma\rho)^{-1}[\mu - \rho(\hat{v} - v_0)] \geq 0$.

Correct attack at lower boundary:

(25b) $\bar{v}^* \leq \hat{v}^*$

or

(32b) $E_t\dot{s}(\hat{v}^*) = (1 + \gamma\rho)^{-1}[\mu - \rho(\hat{v}^* - v_0)] \leq 0$.

Perfection (S&R viability and correct speculative attacks at both boundaries):

(33) $\bar{v}^* \leq \hat{v}^* < v < \hat{v} \leq \bar{v}$.

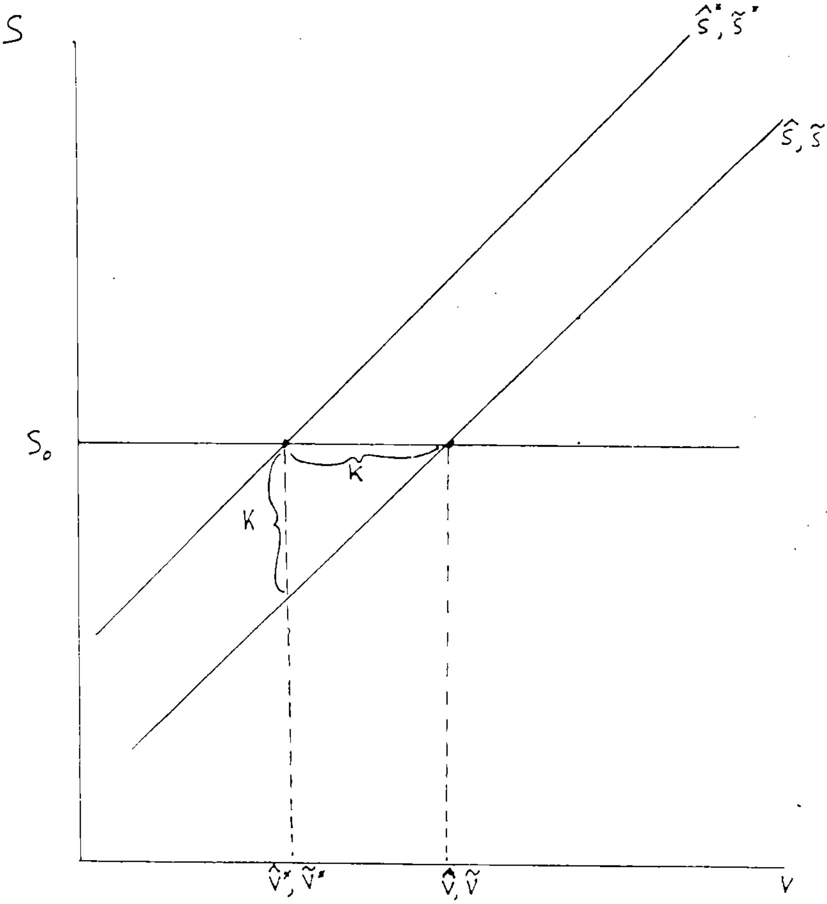

FIGURE 1

$\rho = 0 ; \mu = 0$

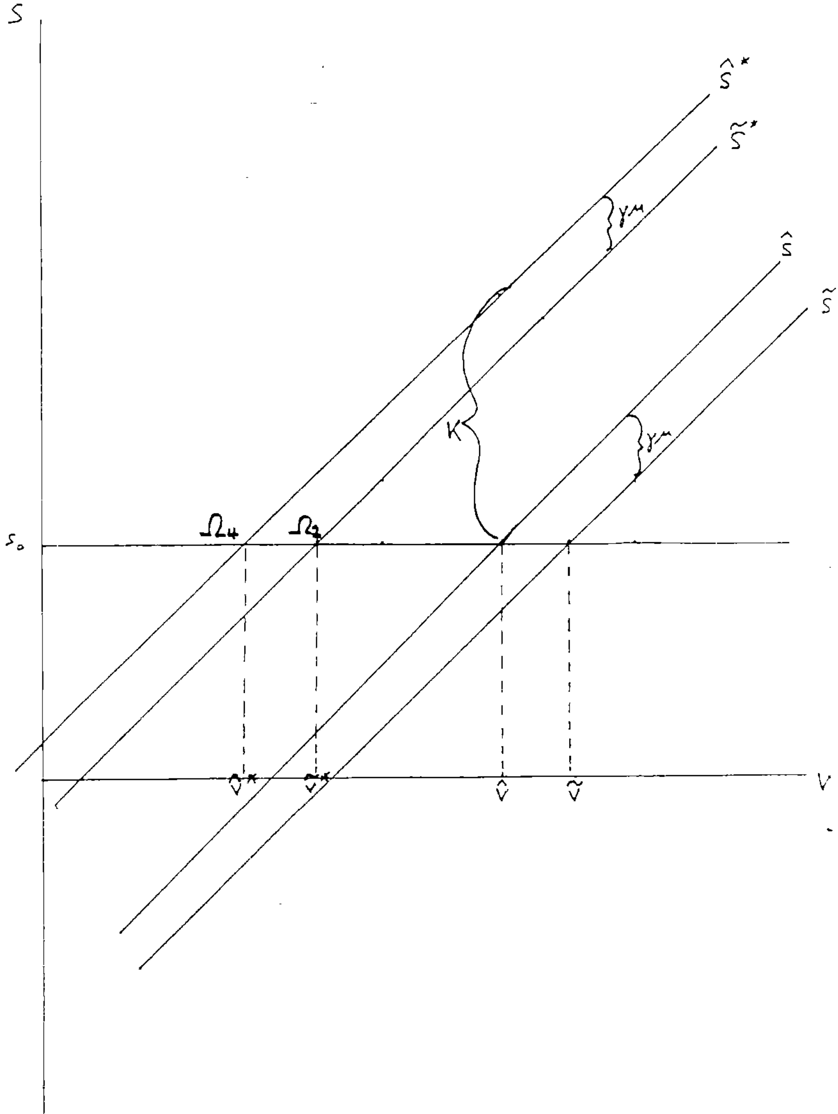

FIGURE 2,

$\rho = 0,\ \mu > 0$

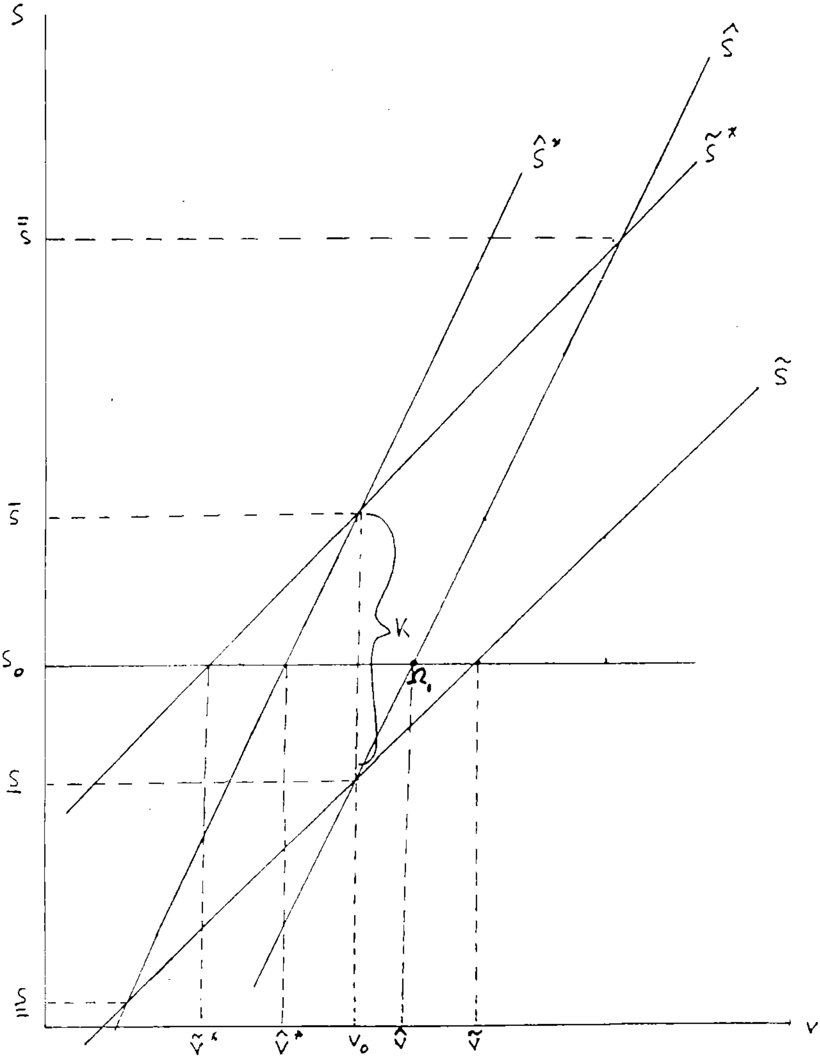

FIGURE 3

$\mu = 0$; $\rho < 0$

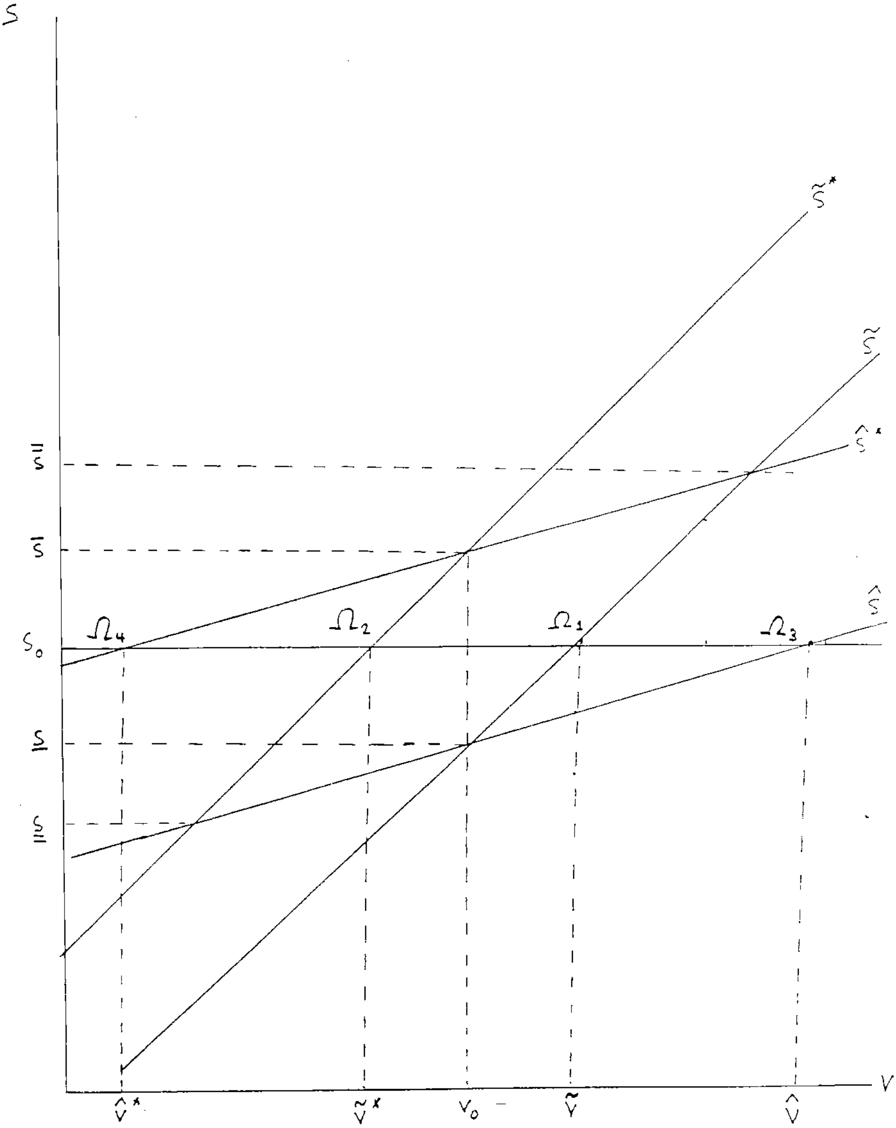

FIGURE 4: $\mu = 0$; $\rho > 0$

FIGURE 5A

$[(1+\gamma\varsigma)K < 2\delta]$

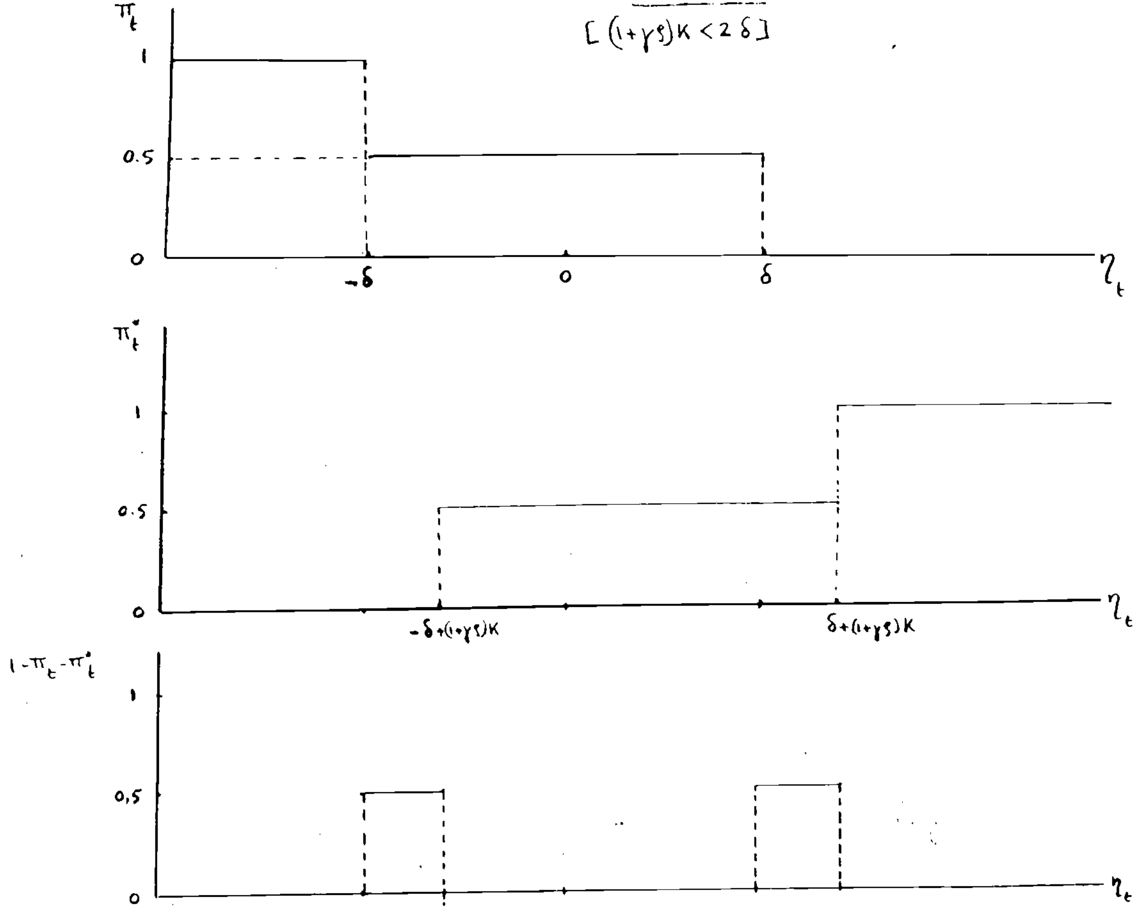

Figure 5b

$[(1+\gamma\varsigma)K > 2\delta]$