The Fallacy of the Fiscal Theory of the Price Level*

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Abstract

This paper argues that a recent line of literature, the 'fiscal theory of the price level' developed by Woodford, Cochrane, Sims and others, is based on a fallacy.

The source of the fallacy is an *economic* misspecification. The proponents of the fiscal theory of the price level do not accept the fundamental proposition that the government's intertemporal budget constraint is a constraint on the government's instruments that must be satisfied for all admissible values of the economy-wide endogenous variables. Instead they require it to be satisfied only *in equilibrium*.

This economic misspecification has implications for the mathematical or logical properties of the equilibria supported by models purporting to demonstrate the properties of the fiscal approach. These include: overdetermined (internally inconsistent) equilibria; anomalies like the apparent ability to price things that do not exist; the need for arbitrary restrictions on the exogenous and predetermined variables in the government's budget constraint; and anomalous behaviour of the 'equilibrium' price sequences, including behaviour that will ultimately violate physical resource constraints.

The issue is of more than academic interest. Policy conclusions could be drawn from the fiscal theory of the price level that would be harmful if they influenced the actual behaviour of the fiscal and monetary authorities. The fiscal theory of the price level implies that a government could exogenously fix its real spending, revenue and seigniorage plans, and that the general price level would adjust the real value of its contractual nominal debt obligations so as to ensure government solvency. When reality dawns, the result could be painful fiscal tightening, government default or unplanned recourse to the inflation tax.

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I. Introduction

It is not common for an entire scholarly literature to be based on a fallacy, that is, "on faulty reasoning; misleading or unsound argument".[1] The recently revived 'fiscal theory of the price level' is an example of a research programme that is fatally flawed, conceptually and logically.[2]

The source of the fallacy is an *economic* misspecification. The proponents of the fiscal theory of the price level do not accept the fundamental proposition that the government's intertemporal budget constraint is a constraint on the government's instruments that must be satisfied for all admissible values of the economy-wide endogenous variables. Instead they require it to be satisfied only *in equilibrium*.

This economic misspecification has implications for the mathematical or logical properties of the equilibria supported by models purporting to demonstrate the properties of the fiscal approach. These include: overdetermined (internally inconsistent) equilibria; anomalies like the apparent ability to price things that do not exist; the need for arbitrary restrictions on the exogenous and predetermined variables in the government's budget constraint; and anomalous behaviour of the 'equilibrium' price sequences, including behaviour that may ultimately violate physical resource constraints.

The purpose of this paper is to put an end to this fruitless line of enquiry by demonstrating the nature and origins of the fallacy.

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[1] In the Concise Oxford Dictionary [1995], the following definitions of a fallacy can be found. (1) a mistaken belief, esp. based on unsound argument; (2) faulty reasoning; misleading or unsound argument; (3) (Logic) a flaw that vitiates an argument.

[2] The seminal contribution is Begg and Haque [1984]. The recent revival includes Leeper [1991], Leeper and Sims [1994], Woodford [1994, 1995, 1996, 1998a,b], Auernheimer and Contreras [1995] Sims [1994, 1997], Cochrane [1996, 1999a,b], Dupor [1997], Loyo [1997a,b],

The issue is of more than academic interest. Policy conclusions could be drawn from the fiscal theory of the price level that would be harmful if they influenced the actual policy behaviour of the fiscal and monetary authorities. Specifically, a government might infer from the fiscal theory of the price level, that there is never any threat of default, no matter how it sets its fiscal-financial-monetary programme. For instance, even when it fixes exogenously the real sequence of primary surpluses plus seigniorage, the general price level will always adjust the real value of its non-monetary nominal debt so as to ensure its solvency.

The fiscal theory of the price level is based on the distinction between two kinds of budgetary rules for the government. Following Woodford (and with apologies to the memory of David Ricardo) I shall refer to these as *Ricardian* and *non-Ricardian* fiscal rules. In what follows, the government is to be interpreted as the consolidated general government and central bank. The government spends on goods and services (exhaustive public spending), makes transfer payments, raises current revenues and, either meets its debt obligations (interest payments and repayment of principal) or defaults. Its financial deficit is financed either by issuing non-interest-bearing fiat money or interest-bearing non-monetary liabilities (bonds).

A *fiscal rule* (a better but excessively long label would be a *fiscal-financial-monetary programme*) is a complete sequence of rules specifying public spending, taxes net of transfers, money issuance (seigniorage) and bond issuance in each period and in each state of nature. Exhaustive public spending is not the issue in this literature. It is therefore helpful to keep the sequence of real public spending on goods and services constant throughout and to focus on net taxes, money issuance and government borrowing.

Luttmer [1997], Olivei [1997]; critical evaluations include Canzoneri, Cumby and Diba [1998a,b], Buiter [1998], McCallum [1998] and Clements, Herrendorf and Valentinyi [1998].

Like every agent in a multi-period economy, the government faces an intertemporal budget constraint or solvency constraint. A *Ricardian* fiscal rule requires that the government's solvency constraint holds for all admissible sequences of the endogenous variables. That is, the government's intertemporal budget constraint holds identically, not just in equilibrium. A *non-Ricardian* fiscal rule requires the government's solvency constraint to hold only for equilibrium sequences of the endogenous variables.

For expositional simplicity, this paper models a world without uncertainty. With a Ricardian fiscal rule, there is never any default on the public debt: contractual debt obligations are met.[3] Loosely speaking, with a Ricardian fiscal rule, there has to remain one degree of freedom in the fiscal-financial-monetary programme. The government cannot, for instance, given its inherited debt obligations, specify exogenous sequences for real public spending, real taxes net of transfers and real monetary issuance, and still expect, except by chance, to meet its contractual obligations in full. Either public spending, or taxes or seigniorage would have to be residually determined during at least one period (as a function of the economy-wide endogenous variables in the model and of the government's other policy instruments) to ensure that the intertemporal budget constraint is satisfied for all admissible sequences of the economy-wide endogenous variables.

A non-Ricardian fiscal rule permits what, from a Ricardian perspective, would be overdetermined fiscal-financial-monetary programmes. The solvency constraint is required to hold only in equilibrium, not for all admissible sequences of the endogenous variables. An

[3] With uncertainty, a strict Ricardian fiscal rule, that is, one which rules out government default with certainty, is only possible if there is a complete set of contingent spot and futures markets. In an incomplete markets world, default is in general a possibility. Specifying an appropriate intertemporal budget constraint (or sequence of budget constraints) in a world with incomplete markets is beyond the scope of this paper.

example of a non-Ricardian fiscal rule would be one which fixes exogenously the entire sequence of real primary surpluses[4] plus real seigniorage.

The Ricardian view implies that if such a non-Ricardian fiscal rule is adopted, the government will not, except in very special circumstances, be able to meet its contractual debt obligations exactly: there will either be default on the public debt, or the government could be 'super-solvent': after all contractual obligations are met, there are surplus resources left. From a Ricardian perspective, the over-determined non-Ricardian fiscal rule therefore requires the addition of another endogenous variable to the model: the default discount factor on the notional or contractual value of the government's non-monetary debt instruments.

The non-Ricardian view maintains that despite the *prima-facie* overdetermined fiscal-financial-monetary programme, there will be no discount (or premium) on the notional or contractual value of the government's non-monetary debt instruments. Instead the *equilibrium* general price level adjusts the real value of the government's *nominal* contractual debt obligations to a level that is consistent with the government solvency constraint - the requirement that the aggregate real value of all the government's outstanding contractual debt obligations be equal to the present discounted value of future real primary surpluses plus seigniorage. It views this as an alternative theory of price determination to the Ricardian one, a 'fiscal theory of the price level'.

There are two ways of refuting the fiscal theory of the price level. The first is based on *a-priori* economic considerations. I consider it to be axiomatic that only those models of a market economy are well-posed, in which, if default is ruled out, budget constraints (including the government budget constraint), must be satisfied for all admissible values of the economy-wide

[4] The government's primary surplus is its financial surplus, excluding net interest receipts.

endogenous variables. It does not matter whether the government (or the private agents) are small (price-taking) or large (monopolistic or monopsonistic). It does not matter whether the government optimises (or what it optimises), satisfices or acts according to ad-hoc decision rules.

According to this Ricardian postulate about the proper specification of budget constraints, a non-Ricardian fiscal rule that rules out default, is ill-posed. Any model that incorporates a non-Ricardian fiscal rule, yet assumes that all contractual debt obligations are met, does not make *economic* sense.[5]

The second way to refute the fiscal theory of the price level applies even if one does not accept the *a-priori* assertion or postulate that budget constraints must be satisfied always, not only in equilibrium, and that consequently a non-Ricardian fiscal rule only makes sense if we explicitly introduce an endogenous default discount factor on the public debt. This second approach involves the demonstration of a number of mathematical (or logical) and conceptual anomalies that characterise equilibria purported to be supported by non-Ricardian fiscal rules without default.

The paper shows that it is not true that the general price level can mimic, generically, the role of a default discount factor on the public debt, even if we only consider the government's budget identity and solvency constraint. Because the general price level cannot be negative, an arbitrary restriction on the *predetermined* stocks of government debt instruments and on the *exogenous* sequences of public spending and revenues (including seigniorage), is necessary for the general price level to mimic the role of a default discount factor on the public debt, even when we consider only the government's accounts.

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[5] Note that there is no disagreement about the fact, that, if private default is to be ruled out, households and businesses must satisfy their budget constraints for all admissible sequences of

When this arbitrary restriction is satisfied, a further conceptual anomaly arises. The fiscal theory of the price level can determine the price of money in a model without money, that is, in a world where money does not exist as a physical commodity or financial claim (private or public, 'inside' or 'outside') and does not function as a medium of exchange, means of payment or store of value. In such a world 'money' only serves as a unit of account or numeraire, and there exists a financial claim committing the issuer to make a payment whose value is fixed in terms of this pure numeraire 'money'. The fiscal theory of the price level would appear to be able to determine the price of phlogiston if phlogiston were used as the numeraire in a general equilibrium model and if some agent had issued phlogiston-denominated financial claims.[6]

Mathematical/logical anomalies include the following. In any finite-horizon economy (with flexible nominal prices or with nominal price rigidities), a non-Ricardian fiscal rule without an endogenous default discount factor on the public debt leads to an overdetermined (internally inconsistent) price level when the government follows a monetary rule[7]. A finite-horizon economy with a non-Ricardian fiscal rule and nominal price rigidities also is overdetermined when the government adopts a nominal interest rate rule.

In an infinite–horizon economy with flexible nominal prices, there is an overdetermined price level under a non-Ricardian monetary rule if the velocity of circulation of money is constant (as in simple cash-in-advance models). When velocity is endogenous, there is no price level overdeterminacy, but explosive or implosive behaviour of the 'equilibrium' price sequence is possible, even with constant fundamentals. In many monetary models, such implosive or

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the economy-wide endogenous variables. It is my maintained hypothesis that the government is in no way different.

[6] Phlogiston is an imaginary element formerly believed to cause combustion.

explosive price level behaviour would imply the eventual violation of economy-wide real resource constraints. When there are nominal price rigidities, there is overdeterminacy in the infinite-horizon economy (as in the finite-horizon economy) under both monetary rules and nominal interest rate rules.

By introducing the government debt default discount factor as an additional endogenous variable, I present a Ricardian resolution of the fiscal fallacy. The over-determined non-Ricardian fiscal rule implies, generically, a non-unitary equilibrium value for the default discount factor. This Ricardian resolution of the fiscal fallacy also destroys the fiscal theory of the price level. In an economy with flexible nominal prices, money is now neutral when the government follows a monetary rule and there is price level indeterminacy when the government follows a nominal interest rate rule.

II. The Model

John Cochrane [1999a] provides a published example of a model purporting to support the fiscal theory of the price level, whose inadequacy can be established simply by counting equations and unknowns. According to Cochrane *"Fiscal price determination is easiest to see in a terminal period, or a period in which the government sells no new debt"*(Cochrane [1999a, p. 327]). A key advantage of the finite-horizon, complete markets specification is that the appropriate intertemporal budget constraints are self-evident.

The model that follows differs from Cochrane's in two ways. The first is the addition of an index-linked one-period government bond to the asset menu. This is not essential for proving

[7] A monetary rule in this paper is an exogenous sequence of positive nominal money stocks. The sequence is such as to support a non-negative nominal interest sequence as an equilibrium. An interest rate rule is a non-negative sequence of period nominal interest rates.

the internal inconsistency of the fiscal approach, but it helps bring out more clearly an anomaly in the fiscal approach that should have given its proponents cause for concern, even without counting equations and unknowns.

The second difference from Cochrane's presentation is that I present a fully specified general equilibrium model. Cochrane presents just the government's single period budget identity and its intertemporal budget constraint. The rest of the model (household behaviour, production and equilibrium conditions), is omitted. Counting equations and unknowns in his government sector sub-model (and making the innocuous simplifying assumption that the real interest rate sequence is exogenous), Cochrane concludes that this sub-model alone determines the equilibrium price level sequence. Had he written down the missing bits of the general equilibrium model, he would have concluded that the model is overdetermined under a monetary rule, or whenever the price level is a predetermined variable.

Time, indexed by t, is measured in discrete intervals of equal length, normalised to unity. There are N periods indexed by t, $1 \leq t \leq N$. Initial asset stocks are inherited from period 0.

There is no uncertainty, markets are complete and contracts are enforced costlessly. Default therefore does not occur. In Section III, I will propose an interpretation of the fallacy involving default on public debt. For that reason I will refer to the prices of public debt instruments in the absence of default as *notional* prices and to the prices of public debt instruments when there is default as *effective* prices.

II1. Household Behaviour

Households act as price takers in all markets in which they operate. They receive an exogenous perishable endowment, $y_t > 0$, each period, consume c_t and pay real lump-sum taxes

t_t.[8] They have access to three stores of value: non-interest-bearing fiat money; a nominal one-period bond with a *notional* money price P_t^B in period *t*, which entitles the buyer to a single contractual nominal coupon payment $\Gamma > 0$ in period *t+1*; and a real or index-linked one-period bond with a *notional* money price P_t^b in period *t*, which entitles the buyer to a single contractual real coupon payment $g > 0$ in period *t+1*.

The notional bond prices are the prices that will prevail if there is no default, that is, if the contractual payments (Γ or g) are made with certainty. The effective bond prices are the market prices that actually prevail, if there is (a risk of) partial or complete default. Until further notice in Section III, default is ruled out and the effective prices therefore equal the notional prices. A richer menu of liabilities (longer maturities, contingent coupon payments) could be included, but would not add to the analysis. The quantities of money, nominal bonds and real bonds outstanding at the end of period *t* (and the beginning of period *t+1*) are denoted M_t, B_t and b_t, respectively. The money price of output in period *t* is P_t.

We will only consider equilibria in which money is weakly dominated as a store of value, that is equilibria supporting a non-negative nominal interest rate sequence. The motive for holding money is that end-of-period real money balances are an argument in the direct utility function. To keep the analysis as transparent as possible, the period felicity function is assumed to be iso-elastic and money is assumed to enter the period felicity function in an additively separable manner. All key propositions in this paper would go through for more general functional forms and for most alternative ways of introducing money into the model including 'money in the shopping function' and 'money in the production function'. For the strict Clower [1967] cash-in-advance models, there exists no finite-horizon equilibrium with a positive price of

[8] They take the sequence of taxes as given.

money unless one introduces another ‘closure rule’ to ensure that money is accepted in exchange for goods and services in the last period of the model. The fiscal theory of the price level fails in (infinite-horizon) cash-in-advance models as well, however, as shown in Section IV.

The representative competitive and tax-taking consumer maximises the following utility functional, defined over non-negative sequences of consumption and end-of-period real money balances

$$u_t = \sum_{j=0}^{N-t} \left[\frac{1}{1-h} c_{t+j}^{\;1-h} + f \frac{1}{1-h} \left(\frac{M_{t+j}}{P_{t+j}} \right)^{1-h} \right] \left(\frac{1}{1+d} \right)^{j}$$

$$c_{t+j},\, M_{t+j} \geq 0;\;\; h,\; f,\; d > 0 \tag{1}$$

The single-period household budget identity is, for $1 \leq t \leq N$,

$$M_t - M_{t-1} + P_t^B B_t - \Gamma B_{t-1} + P_t^b b_t - P_t\, g b_{t-1} \equiv P_t (y_t - t_t - c_t) \tag{2}$$

The intertemporal budget constraint or solvency constraint is that at the end of period N, the household cannot have positive debt, that is,

$$P_N^B B_N + P_N^b b_N \geq 0 \tag{3}$$

Since utility is increasing in consumption and real balances, (3) will hold with equality.

Initial financial asset stocks are predetermined, that is,

$$\begin{aligned} B_0 &= \bar{B}_0 \\ b_0 &= \bar{b}_0 \\ M_0 &= \bar{M}_0 > 0 \end{aligned} \tag{4}$$

The riskless one-period nominal interest rate set in period t is $i_{t,t+1}$ and the riskless one-period real interest rate in period t is $r_{t,t+1}$. Simple arbitrage ensures that

$$1 + i_{t,t+1} = (1 + r_{t,t+1}) \frac{P_{t+1}}{P_t} = \frac{\Gamma}{P_t^B} = \frac{P_{t+1}\, g}{P_t^b} \tag{5}$$

Let $R_{t-1,t+j}$ be the nominal discount factor between periods *t-1* and *t+j*, that is,

$$
\begin{aligned}
R_{t-1,t+j} &\equiv \prod_{k=0}^{j} \frac{1}{1+i_{t-1+k,t+k}} \qquad \text{for } j \geq 0 \\
&\equiv 1 \qquad\qquad\qquad\qquad \text{for } j = -1
\end{aligned}
\tag{6}
$$

Solving the household budget identity (2) forward recursively, using (5) yields

$$
\begin{aligned}
\Gamma B_{t-1} + P_t\, g b_{t-1} \equiv \sum_{j=0}^{N-t} R_{t,t+j} \left[P_{t+j}(c_{t+j} + t_{t+j} - y_{t+j}) + (M_{t+j} - M_{t+j-1}) \right] \\
+ R_{t,N}(P_N^B B_N + P_N^b b_N)
\end{aligned}
\tag{7}
$$

The household solvency constraint (3), holding with equality, implies that

$$
\Gamma B_{t-1} + P_t\, g b_{t-1} \equiv \sum_{j=0}^{N-t} R_{t,t+j} \left[P_{t+j}(c_{t+j} + t_{t+j} - y_{t+j}) + (M_{t+j} - M_{t+j-1}) \right] \tag{8}
$$

The household optimal consumption programme is characterised by

$$
\left(\frac{c_{t+1}}{c_t} \right)^{h} = (1 + r_{t,t+1})(1 + d)^{-1} \qquad\qquad 1 \leq t \leq N-1 \tag{9}
$$

$$
\frac{M_t}{P_t} = c_t \left[f \left(\frac{1+i_{t,t+1}}{i_{t,t+1}} \right) \right]^{\frac{1}{h}} \qquad\qquad 1 \leq t \leq N-1 \tag{10}
$$

$$
\frac{M_N}{P_N} = c_N\, f^{\frac{1}{h}} \tag{11}
$$

Equation (10) is the familiar optimality condition relating the optimal money stock in period *t* to optimal consumption in that period. The money-in-the direct utility function approach views money as a consumer durable yielding a flow of unspecified liquidity services each period. In the last period, *N*, money only has value because of the liquidity services it yields that period. Effectively, real money balances in period N become a perishable commodity, as shown in equation (11), which does not involve any intertemporal relative price.

II2. The Government

Government decision rules are exogenously given, subject only to some basic feasibility conditions, including the government's intertemporal budget constraint or solvency constraint. The government's single-period budget identity is given in (12), its solvency constraint in (13).

$$M_t - M_{t-1} + P_t^B B_t - \Gamma B_{t-1} + P_t^b b_t - P_t \, g b_{t-1} \equiv P_t (g_t - t_t) \tag{12}$$

$$P_N^B B_N + P_N^b b_N \leq 0 \tag{13}$$

$$\begin{aligned} M_0 &= \overline{M}_0 > 0 \\ B_0 &= \overline{B}_0 \\ b_0 &= \overline{b}_0 \end{aligned}$$

The government's solvency constraint is analogous to that of the private sector: at the end of the last period (N) the government cannot have a positive stock of debt outstanding. If the government solvency constraint is to hold identically, government spending, money issuance or taxes (or some combination of these three financing modes) must adjust in such a way as to ensure that the government always reaches the end of period N with non-positive non-monetary debt. In Section III, I allow for breaches of contract or default. In this case the government solvency constraint helps determine the appropriate default discount on the notional value (or default-risk-free value) of the public debt.

Equations (12) and (5) imply

$$\begin{aligned} \Gamma B_{t-1} + P_t \, g b_{t-1} \equiv & \sum_{j=0}^{N-t} R_{t,t+j} \left[P_{t+j} (t_{t+j} - g_{t+j}) + (M_{t+j} - M_{t+j-1}) \right] \\ & + R_{t,N} (P_N^B B_N + P_N^b b_N) \end{aligned} \tag{14}$$

Equations (13) and (14) imply the present value budget constraint given in (15)

$$\Gamma B_{t-1} + P_t\, g b_{t-1} \leq \sum_{j=0}^{N-t} R_{t,t+j} \left[P_{t+j}(t_{t+j} - g_{t+j}) + (M_{t+j} - M_{t+j-1}) \right] \tag{15}$$

In equilibrium, equation (15) will hold with equality, because the household solvency constraint will also have to be satisfied.

I will consider two monetary ‘regimes’, an (open-loop) monetary rule and an (open-loop) nominal interest rate rule.

The *monetary rule* specifies an exogenous positive sequence for the nominal money stock,

$$\{ M_t ; 1 \leq t \leq N \} = \{ \overline{M}_t > 0 ; 1 \leq t \leq N \} . \tag{16}$$

The nominal money stock sequences considered are restricted to those supporting a non-negative nominal interest rate sequence.

The *nominal interest rate rule* specifies an exogenous non-negative sequence for the nominal interest rate,

$$\{ i_{t-1,t} ; 1 \leq t \leq N-1 \} = \{ \bar{i}_{t-1,t} \geq 0 ; 1 \leq t \leq N-1 \} \tag{17}$$

II2A. Ricardian fiscal-financial-monetary programmes

The real government spending sequence is exogenous,[9]

$$\begin{aligned} g_t &= \bar{g}_t \qquad\qquad 1 \leq t \leq N \\ 0 &\leq \bar{g}_t < y_t \end{aligned} \tag{18}$$

[9] One should not use general equilibrium considerations to impose a-priori restrictions on the instrument choices of the government. The government could, in principle, set $g > y$. In that case no equilibrium exists.

With the nominal money stock sequence and the real spending sequence exogenous, the sequence of taxes becomes endogenous: any rule for taxes that permits the government's intertemporal budget constraint (15) to be satisfied is appropriate (and equivalent) for our purposes.

The reason that, given the nominal money stock sequence[10] $\{\bar{M}_t\},\ 0 \le t \le N$, and given the real public spending sequence, $\{\bar{g}_t\}, 1 \le t \le N$, the borrowing mix and tax policy (the sequences $\{B_t\},\ \{b_t\},\ 1 \le t \le N-1$ and $\{t_t\}, 1 \le t \le N$) do not matter for either real or nominal equilibrium as long as the government's solvency constraint is satisfied, is that the model exhibits debt neutrality or Ricardian equivalence. The representative agent assumption and the lump-sum nature of net taxes generate the debt neutrality. The inflation tax is in general a non-lump-sum tax, so debt neutrality only applies to changes in the sequences of debts and taxes, holding constant the nominal money stock sequence or the nominal interest rate sequence.

Since any lump-sum tax rule that satisfies the government solvency constraint is equivalent, I shall for concreteness assume that taxes are set to achieve a zero nominal non-monetary debt from the end of period *1* on, that is,

$$
\begin{aligned}
t_1 &= \bar{g} - \frac{\bar{M}_1 - \bar{M}_0}{P_1} + \frac{\Gamma \bar{B}_0}{P_1} + \phi \bar{b}_0 \\
t_t &= \bar{g} - \frac{\bar{M}_t - \bar{M}_{t-1}}{P_t} \qquad 1 < t \le N
\end{aligned}
\tag{19}
$$

Equations (16), (18) and (19) define our Ricardian monetary rule. Equations (17), (18) and (19) define our Ricardian nominal interest rate rule.

[10] or given the nominal interest rate sequence

Many other Ricardian fiscal-financial-monetary programmes are possible, including programmes based on ad-hoc or optimising feedback rules for the government's instruments. The simple exogenous or open-loop monetary rule and nominal interest rate rule that the paper will focus on can be described as being characterised by 'dominant' or 'active' monetary policy and 'subordinate', 'accommodating' or 'passive' fiscal policy. Exogenous sequences are specified in each period for either the nominal money stock or the nominal interest rate, and fiscal policy (taxes in our model) accommodates so as to ensure that the government's solvency constraint is always satisfied, through equation (19).

Another Ricardian fiscal rule would specify exogenous sequences for real public spending and real taxes net of transfers, and have the government borrow or lend just enough to keep the real stock of public debt constant at its initial value, until the final period when it would have to go to zero, that is,

$$
\begin{aligned}
g_t &= \bar{g}_t \ , \\
t_t &= \bar{t}_t \ , \qquad 1 \le t \le N
\end{aligned}
\tag{20a}
$$

$$
\begin{aligned}
\frac{\Gamma B_t + P_{t+1} \phi_t}{P_{t+1}} &= \frac{\Gamma B_{t-1} + P_t \phi_{t-1}}{P_t} \qquad 1 \le t \le N-1 \\
P_N^B B_N + P_N^b b_N &\le 0
\end{aligned}
\tag{20b}
$$

Under this Ricardian fiscal rule, fiscal policy (taxes, spending and borrowing) is dominant or active and monetary policy is subordinate, accommodating or passive. Seigniorage in at least one period is endogenous and adjusts passively to ensure that the government solvency constraint is satisfied for all admissible values of the economy-wide endogenous variables.[11]

Note that, from a Ricardian perspective, once we have specified (20a,b), it is self-evident, *from a consideration of the government's intertemporal budget constraint alone*, that the sequence of nominal money stocks, $M_t,\ \ 1 \le t \le N$ cannot be exogenous, but has to contain at least one endogenous element. No general equilibrium considerations are required to conclude that monetary policy cannot be set independently once it is recognised that the solvency constraint must be satisfied for all admissible values of the economy-wide endogenous variables, including the general price level.

II2B. Non-Ricardian fiscal-financial-monetary programmes

Woodford, Sims, Leeper, Cochrane and other proponents of the fiscal theory of the price level propose an alternative fiscal-financial closure rule for dynamic general equilibrium models with a government, called non-Ricardian rules by Woodford. While there are by now a very large variety of non-Ricardian rules, they all suffer from the same fatal flaw. The following very simple rule proposed in Woodford [1995] is the most convenient vehicle for making this clear. It is also a rule consistent with Cochrane [1999a].

Woodford proposes the following tax rule:

$$t_t = \bar{s}_t - \frac{M_t - M_{t-1}}{P_t} \qquad\qquad 1 \le t \le N \qquad\qquad (21)$$

where $\{\bar{s}_t\}$, $1 \le t \le N$ is an exogenously given real sequence of taxes plus seigniorage.

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[11] This is the policy in 'stage 2' of the Sargent and Wallace [1981] 'unpleasant monetarist arithmetic model'. In our finite, horizon model, we could fix exogenously at most *N-1* elements of the nominal money stock sequence, *{M_t , 1# t # N}*. At least one element would have to be

Our *non-Ricardian monetary rule* will be defined by an exogenous sequence of real public spending, an exogenous sequence of $\bar{s}_t$ and an exogenous strictly positive sequence of nominal money stocks, that is, by equations (16), (18) and (21).

Our *non-Ricardian nominal interest rate rule* will be defined by an exogenous sequence of real public spending, an exogenous sequence of $\bar{s}_t$ and an exogenous non-negative sequence of nominal interest rates, that is, by equations (17), (18) and (21).

II3. Market Clearing

The goods market clears each period, that is,

$$y_t = c_t + g_t \qquad 1 \le t \le N$$

For simplicity, I assume in what follows that the real fundamentals are constant, that is,

$$\begin{aligned} y_t &= \bar{y} \\ g_t &= \bar{g} \qquad 1 \le t \le N \end{aligned}$$

Only non-negative equilibrium price sequences are permissible.

II4. Equilibrium Under the Ricardian Monetary Rule

Provided the exogenous and strictly positive nominal money stock sequence satisfies $\frac{\bar{M}_{t+1}}{\bar{M}_t} \ge \frac{1}{1+d}$, the equilibrium nominal interest rate sequence will be non-negative. The equilibrium is characterised by equation (16) and

$$c_t = c = \bar{y} - \bar{g} \qquad 1 \le t \le N \tag{22}$$

---

determined endogenously. Sargent and Wallace determine a constant proportional growth rate of the nominal money stock.

$$r_{t,t+1} = d \qquad 1 \le t \le N-1 \qquad (23)$$

$$1 + i_{t,t+1} = (1+d)\frac{P_{t+1}}{P_t} \qquad 1 \le t \le N-1 \qquad (24)$$

$$\frac{M_t}{P_t} = (\bar{y} - \bar{g})\left[\frac{f(1+d)\frac{P_{t+1}}{P_t}}{(1+d)\frac{P_{t+1}}{P_t} - 1}\right]^{\frac{1}{h}} \qquad 1 \le t \le N-1 \qquad (25a)$$

$$\frac{M_N}{P_N} = (\bar{y} - \bar{g})\, f^{\frac{1}{h}} \qquad (25b)$$

$$\frac{\Gamma B_0}{P_1} + g b_0 \equiv \sum_{j=0}^{N-1}\left(\frac{1}{1+d}\right)^j \left[t_{1+j} + \left(\frac{M_{1+j} - M_j}{P_{1+j}}\right) - \bar{g}\right] \qquad (26)$$

$$\begin{aligned} t_1 &= \bar{g} - \frac{M_1 - M_0}{P_1} + \frac{\Gamma B_0}{P_1} + g b_0 \\ t_t &= \bar{g} - \frac{M_t - M_{t-1}}{P_t} \qquad 1 < t \le N \end{aligned} \qquad (27)$$

$$\begin{aligned} M_0 &= \bar{M}_0 > 0 \\ B_0 &= \bar{B}_0 \\ b_0 &= \bar{b}_0 \end{aligned} \qquad (28)$$

Note that the government solvency constraint (26) has to hold in equilibrium (even if it did not have to hold as an identity), because of the household solvency constraint and the goods market equilibrium condition.

The key point to note is that the monetary equilibrium conditions (25a,b) provide N equations that uniquely[12] determine the N (non-negative) equilibrium prices P_t , $t = 1, \ldots, N$. Equation (25b) alone determines P_N as a function of the nominal stock of money in the last period, $\overline{M}_N$. The remaining $N-1$ monetary equilibrium conditions given by (25a) determine P_{N-1} down to P_1 , given the solution for the price level in period N and the exogenous values of the nominal money stocks in periods 1 to $N-1$. The tax rule given in (27) then determines the N values of the lump-sum tax sequence. Given that tax rule, the government's solvency constraint holds identically.

Another way of putting this is that the government's solvency constraint (and the assumed exogeneity of the real public spending sequence and the nominal money stock sequence) forces the tax sequence to become endogenous.

The equilibrium real and nominal interest rate sequences and the equilibrium consumption sequence are also uniquely determined.

Under this Ricardian monetary rule, money is *conditionally neutral* (see Buiter [1998]). Holding constant the initial stock of nominal non-monetary debt, B_0, equal proportional changes in the sequence of nominal money stocks (including the initial nominal stock of money), $\{\overline{M}_t\}, 0 \le t \le N$, and in the sequences of all endogenous nominal prices $\{P_t, P_t^B, P_t^b\}, 1 \le t \le N$ leave the real equilibrium unchanged. If the initial stock of non-monetary nominal debt is non-zero, the sequence of (endogenous) real lump-sum taxes will change (according to (27)), because

[12] This economy also has a non-monetary equilibrium, in which the value of money is zero in each period. As this is not relevant for the fiscal theory of the price level, this barter equilibrium is not considered here.

of the change in the real value of the initial stock of nominal non-monetary government debt, $\frac{B_0}{P_1}$, when the initial price level changes. The proof is by inspection.

Under the Ricardian monetary rule, money and the initial stock of nominal non-monetary debt are *jointly unconditionally neutral* (see Buiter [1998]). Equal proportional changes in the sequence of nominal money stocks (including the initial nominal stock of money), $\{\overline{M}_t\}, 0 \le t \le N$, in the initial stock of nominal non-monetary debt, B_0, and in the sequences of all endogenous nominal prices $\{P_t, P_t^B, P_t^b\}, 1 \le t \le N$ leave the real equilibrium unchanged. The (endogenous) sequence of real lump-sum taxes will not change (again according to (27)). The proof is again by inspection.

I summarise this as Proposition 1.

Proposition 1.

Under the Ricardian monetary rule, money is neutral.

II5. Equilibrium Under the Ricardian Nominal Interest Rate Rule

With an exogenous non-negative nominal interest rate sequence (and endogenous nominal money stocks), the equilibrium is characterised by equations (17), (22), (23), (24), (25a,b), (26), (27) and (28). It is helpful to rewrite, using (24), the two monetary equilibrium conditions (25a,b) as (29a,b).

$$\frac{M_t}{P_t} = (\bar{y} - \bar{g})\left[\frac{f(1+\bar{i}_{t,t+1})}{\bar{i}_{t,t+1}}\right]^{\frac{1}{h}} \qquad 1 \le t \le N-1 \qquad (29a)$$

$$\frac{M_N}{P_N} = (\bar{y} - \bar{g})\, f^{\frac{1}{h}} \qquad (29b)$$

The key point to note here is that the monetary equilibrium conditions (29a,b) provide N equations that uniquely[13] determine the N equilibrium real money stocks, M_t/P_t , $t = 1, \ldots, N$. The endogenous equilibrium nominal money stock sequence $\{M_t\}$, $1 \leq t \leq N$ and the equilibrium price sequence $\{P_t\}$, $1 \leq t \leq N$ are indeterminate. The tax rule given in (27) then determines the N values of the lump-sum tax sequence. If the initial stock of nominal non-monetary debt, B_0 , is non-zero, the first term in the equilibrium real tax sequence, t_1, which depends on $\frac{B_0}{P_1}$, is also indeterminate. However, it continues to be the case that, given that tax rule in (27), the government's solvency constraint holds identically. The equilibrium real interest rate sequence, the equilibrium inflation rate sequence and the equilibrium consumption sequence are also uniquely determined.

Price level indeterminacy under a Ricardian nominal interest rate rule is a familiar result. In a frictionless economy, with flexible, market-clearing nominal prices, an exogenous nominal interest rate sequence does not provide a nominal anchor for the system. The reason is that, despite its name, the short *nominal* interest rate is a *real* variable, the real pecuniary opportunity cost of holding money balances.

I summarise this as Proposition 2.

Proposition 2.

[13] See footnote 12.

Under the Ricardian nominal interest rate rule, all nominal equilibrium values are indeterminate in our flexible price model. Only real equilibrium values are (uniquely) determined by the model.

Price level indeterminacy under Ricardian monetary rules is a feature of the class of flexible price level, general equilibrium models considered in this paper rather than a problem for monetary policy in the real world. More policy-relevant models would view the price level (and/or the money wage) in any given period as predetermined. With such ‘Keynesian’ money wage or price rigidities, price level indeterminacy is automatically eliminated (see Section II8).

II6. Equilibrium Under the Non-Ricardian Monetary Rule

Substituting the rule given by (18) and (21), that real tax revenue plus the real value of seigniorage is exogenously given, into the government solvency constraint (26), we get

$$\frac{\Gamma \bar{B}_0}{P_1} + \gamma \bar{b}_0 \equiv \sum_{j=0}^{N-1} \left(\frac{1}{1+\delta} \right)^j \left[\bar{s}_{1+j} - \bar{g} \right] \tag{30}$$

The right-hand side of (30) is exogenous. Everything on the left-hand-side of (30) except for P_1 is exogenous or predetermined. The government solvency constraint (30) under the non-Ricardian rule therefore determines the equilibrium value of P_1. This is the fiscal theory of the price level.

Assume again that the exogenous and strictly positive nominal money stock sequence satisfies $\frac{\bar{M}_{t+1}}{\bar{M}_t} \geq \frac{1}{1+\delta}$. The putative equilibrium under the non-Ricardian monetary rule is characterised by the equations (16), (22), (23), (24), (25a,b), (28) and (30).

The key point to note is that, just as with the Ricardian monetary rule, the monetary equilibrium conditions (25a,b) again provide N equations that uniquely[14] determine the N equilibrium prices P_t , $t = 1, \ldots, N$.

Unfortunately for the fiscal theory of the price level, P_1 is also determined by equation (30). Except for a set of parameter values of measure zero, the system is overdetermined.[15] There is no non-Ricardian or fiscal theory of the price level under a monetary rule.

I summarise this as Proposition 3.

Proposition 3.

Under the non-Ricardian monetary rule, the price level sequence is overdetermined.

Corollary

There is no 'alternative' fiscal theory of the price level under a monetary rule. When there is a determinate price level under a Ricardian rule, imposing a non-Ricardian rule results in an internally inconsistent model.

Note that Proposition 3 and its Corollary are logical propositions, involving no more than a counting of equations and unknowns. The relevance of the fiscal theory of the price level is therefore not an empirical issue. There is no internally consistent fiscal theory of the price level. Of course there may well be empirical regularities linking the general price level, anticipated and unanticipated inflation, and measures of nominal public debt or deficits. 'Ricardian' models like Sargent and Wallace's 'Unpleasant Monetarist Arithmetic' model (Sargent and Wallace [1981]),

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[14] See footnote 12.

which do not fall foul of the fallacy of the fiscal theory of the price level, offer one possible motivation for such empirical regularities.

Adherents of the fiscal approach might recognise the inconsistency of the fiscal approach and our exogenous monetary rule, but conclude that the incompatibility of the non-Ricardian fiscal rule and our open-loop monetary policy only shows that it is this particular monetary rule that has to be discarded, rather than the non-Ricardian fiscal rule.[16] The argument goes as follows: under a non-Ricardian fiscal rule, fiscal policy is dominant and monetary policy must be subordinate or accommodating. Subordinate and accommodating monetary policy means that the nominal money stock sequence cannot be specified exogenously. The nominal money stock in at least one period must be endogenous.[17] This would resolve the inconsistency.

This argument is flawed. It uses general equilibrium considerations to restrict the a-priori permissible instrument choices of the government. If a government does not have to worry about its solvency constraint being satisfied for all permissible values of the economy-wide endogenous variables (the central assumption of the fiscal approach), it can certainly choose both sequences for its *real* primary surplus including seigniorage and its *nominal* money stock. As the following two equations, representing the non-Ricardian monetary rule make clear, real taxes net of transfers, t_t, can adjust passively to allow the government to follow the non-Ricardian fiscal rule and select an exogenous non-negative sequence for the nominal money stock.

$$\Gamma\frac{B_t}{P_{t+1}} + \phi_t \equiv (1+d)\left[\Gamma\frac{B_{t-1}}{P_t} + \phi_{t-1} + \bar{g} - \bar{s}_t\right]$$

---

[15] Except of course for certain ('measure zero') parameter configurations.
[16] I am indebted to John Vickers who, by playing devil's advocate most effectively, induced me to address this issue.
[17] Surprisingly, however, accommodating or subordinate monetary policy would be consistent with an entirely exogenous sequence of nominal interest rates.

$$\overline{M}_t - \overline{M}_{t-1} = P_t(\bar{s}_t - t_t)$$

Of course, the behaviour of the public debt under this rule will, in general, be inconsistent with government solvency. There may exist a unique sequence of values for the general price level that makes this non-Ricardian monetary rule consistent with government solvency. However, that sequence of general price level values does not, generically, satisfy the remaining equilibrium conditions. To proscribe the exogenous monetary rule because of its (general) equilibrium implications would compound the original conceptual error of the fiscal theory of the price level, which was to impose government solvency only in equilibrium. Two wrongs do not make a right.

Passive monetary policy certainly can be a logical implication of a particular parameterisation of spending, tax and borrowing policies in a Ricardian framework. An example was given in Section II2A and equations (20a,b). The logical necessity for monetary policy to be subordinated to fiscal policy in that example follows directly and exclusively from the recognition of the fact that the government's solvency constraint has to hold identically. It did not involve general equilibrium considerations.

II7. Equilibrium Under the Non-Ricardian Nominal Interest Rate Rule

With an exogenous non-negative nominal interest rate sequence (and endogenous nominal money stocks), the equilibrium is characterised by equations (17), (22), (23), (24), (25a,b), (28) and (30).

The monetary equilibrium conditions (25a,b) provide N equations that uniquely[18] determine the N equilibrium real money stocks, M_t/P_t , $t = 1, \ldots, N$. The equilibrium real interest rate, inflation and consumption sequences are also uniquely determined.

Under a non-Ricardian nominal interest rate rule, the price level indeterminacy characteristic of the Ricardian nominal interest rate rule can be resolved, provided a key condition, explained below, is satisfied. Note that the government solvency constraint (30) determines P_1. Given P_1 equation (24) determines all future price levels P_2 to P_N. Equations (25a,b) then uniquely determine the endogenous nominal money stock sequence.

The key caveat is that, since the general price level cannot be negative, a necessary condition for the government solvency constraint under the non-Ricardian rule (equation (30)) to make sense is that condition (31) be satisfied.

$$\operatorname{sgn}\left(\bar{B}_0\right) = \operatorname{sgn}\left\{ \sum_{j=0}^{N-1} \left(\frac{1}{1+d} \right)^j \left[\bar{s}_{1+j} - \bar{g} \right] - g\bar{b}_0 \right\} \qquad (31)$$

This says that the initial stock of non-monetary nominal public debt must be positive (negative) if the excess of the present discounted value of future primary government surpluses (including seigniorage revenues) over the value of the initial stock of index-linked government debt is positive (negative).[19]

Everything on either side of equation (31) is exogenous or predetermined. There is no reason why arbitrary configurations of $B_0, b_0, d, \bar{g}$ and $\{\bar{s}_t\}, 1 \leq t \leq N$ would always satisfy (31), although they may, fortuitously, do so.

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[18] See footnote 12.

[19] For instance, if all public debt is index-linked, ($B_0 = 0$), condition (31) is not satisfied and (30) cannot determine the initial price level. In an open economy version of this model, the same would be true if all public debt were denominated in foreign currency.

The most that the fiscal theory of the price level therefore could aspire to, when the arbitrary restriction (31) is satisfied, is to be a way of removing the price level indeterminacy characteristic of equilibria under a Ricardian nominal interest rate rule, when nominal prices are flexible. As Section II10 makes clear, accepting this interpretation would result in a major conceptual anomaly. I therefore reject it.

Note that the fiscal or non-Ricardian theory of the price level is never an *alternative* to a monetary or Ricardian theory of the price level. When the monetary or Ricardian theory gives a determinate equilibrium price level sequence (as it does under our Ricardian monetary rule), the imposition of a non-Ricardian fiscal rule leads to an overdetermined system.

Generically[20], that is, without imposing the arbitrary restriction (31), an admissible (non-negative) initial value for the general price level can therefore not be determined from the government's intertemporal budget constraint, even under a nominal interest rate rule.[21] The need for arbitrary restrictions such as (31) is inherent in the fiscal approach. Consider, for instance, an alternative non-Ricardian fiscal rule under which the government fixes its sequences of public spending and taxes (plus seigniorage) in nominal terms rather than in real terms, e.g.

[20] I am using the word 'generic' or 'generical' in the standard logical sense of *'of, applicable to, or referring to all the members of a genus, class, group or kind; general'*.

[21] There are other, and more robust, ways of removing the nominal indeterminacy. Consider, for instance, the equilibrium configuration under the Ricardian nominal interest rule, given in equations (17), (22), (23), (24), (25a,b), (26), (27) and (28). A determinate price level sequence exists, for instance, if the authorities fix just one of the elements in the sequence of nominal money stocks $\{M_t\}, 1 \leq t \leq N$. This way of removing the nominal indeterminacy is not subject to arbitrary restrictions such as (31). Finally, consider a world with a more Keynesian determination of nominal prices. Nominal wage or price stickiness means that the price level in any given period is predetermined and updated through a dynamic money wage or price level adjustment mechanism like the Phillips curve. In such a world, a Ricardian nominal interest rule supports a determinate equilibrium price level sequence, while a non-Ricardian nominal interest rule would result in an overdetermined price level, even if (31) were satisfied (see Section II.8).

$$P_t g_t = \overline{G}_t$$

$$P_t s_t = \overline{S}_t$$

$$1 \leq t \leq N$$

The government's solvency constraint (assumed to hold with equality) can be written as

$$\Gamma B_{t-1} + P_t\, g b_{t-1} \leq \sum_{j=0}^{N-t} R_{t,t+j} \left(\overline{S}_{t+j} - \overline{G}_{t+j} \right) \qquad 1 \leq t \leq N \tag{32}$$

Consider the case where the government fixes the sequence of short nominal interest rates. The $R_{t,t+j}$ are therefore exogenous.

Again this non-Ricardian rule cannot generically resolve the price level indeterminacy characteristic of equilibria under a Ricardian nominal interest rate rule. The reason is, again, that equation (32) cannot be generically solved for a non-negative initial value of the price level. It can determine a non-negative initial value of the price level if and only if the arbitrary restriction in (33) is satisfied.

$$\text{sgn}\{b_{t-1}\} = \text{sgn}\{\sum_{j=0}^{N-t} R_{t,t+j} \left(\overline{S}_{t+j} - \overline{G}_{t+j} \right) - \Gamma B_{t-1}\} \qquad 1 \leq t \leq N \tag{33}$$

Thus, if there is no index-linked debt, this example of a non-Ricardian nominal interest rate rule would not be able to produce a determinate price level.

This is the counterpart, when the government fixes the nominal sequence of primary surpluses (plus seigniorage), of the result that there can be no fiscal theory of the price level, when the sequence of *real* primary surpluses (plus seigniorage) is given exogenously, unless there is a non-zero stock of nominal public debt (see equation (31)). Arbitrary restrictions on the predetermined and exogenous variables in the government solvency constraint are always required to support a non-negative equilibrium price level sequence.

II.8 Non-Ricardian Fiscal Rules in the Presence of Nominal Price Rigidities

The case for rejecting the non-Ricardian fiscal rule is strengthened when it can be shown that it leads to internal inconsistencies under more general monetary policy rules, such as nominal interest rate rules, even when (31) is satisfied. It this sub-section I show that any model in which the price level is *predetermined* will result in an overdetermined equilibrium when a non-Ricardian fiscal rule is imposed, regardless of the specification of the monetary policy rule. I will illustrate this with a simple ‘Keynesian’ modification of the earlier model.

Assume that, instead of a flexible price level and market clearing in the goods market, we have a predetermined price level and demand-determined output. Actual output, y, is endogenous and demand-determined. It can differ from the exogenous level of capacity output, $\bar{y}$. The price level is updated through a simple accelerationist Phillips curve.[22] Any old-Keynesian or new-Keynesian model of nominal price or wage rigidities would have the same implications for the fiscal theory of the price level as our simple accelerationist Phillips curve.

The ‘Keynesian’ equilibrium conditions are (34), (35) and (36):

$$y_t = c_t + \bar{g} \qquad 1 \le t \le N \tag{34}$$

$$\begin{gathered} \frac{P_{t+1}}{P_t} = j\,(y_t - \bar{y}) + \frac{P_t}{P_{t-1}} \qquad 1 \le t \le N-1 \\ j > 0 \end{gathered} \tag{35}$$

[22] The simple accelerationist Phillips curve has the inflation rate predetermined, and not just the price level. In period *1*, both P_1 and P_0 are predetermined.

$$
\begin{aligned}
P_1 &= \bar{P}_1 > 0 \\
P_0 &= \bar{P}_0 > 0
\end{aligned}
\tag{36}
$$

The remaining equilibrium conditions are given by (37) to (43). Equation (40) is obtained by substituting the household first-order conditions (9), (10) and (11) into the household solvency constraint (8) for period *1*. Note that (43) imposes the non-Ricardian fiscal rule.

$$
\left(\frac{c_{t+1}}{c_t}\right)^h = (1+r_{t,t+1})(1+d)^{-1} \qquad 1 \le t \le N-1 \tag{37}
$$

$$
\frac{M_t}{P_t} = c_t \left[f \left(\frac{1+i_{t,t+1}}{i_{t,t+1}} \right) \right]^{\frac{1}{h}} \qquad 1 \le t \le N-1 \tag{38}
$$

$$
\frac{M_N}{P_N} = c_N f^{\frac{1}{h}} \tag{39}
$$

$$
\begin{aligned}
&\frac{\bar{M}_0 + \Gamma \bar{B}_0}{P_1} + g\bar{b}_0 + \sum_{j=0}^{N-1} \bar{R}_{1,1+j}(y_{1+j} - t_{1+j}) \equiv \\
&+ c_1 \left\{ \sum_{j=0}^{N-2} \left(\frac{1}{1+d} \right)^j \left[1 + f^{\frac{1}{h}} \left(\frac{1+i_{1+j,2+j}}{i_{1+j,2+j}} \right)^{\frac{1-h}{h}} \right] + \left(\frac{1}{1+d} \right)^{N-1} (1 + f^{\frac{1}{h}}) \right\}
\end{aligned}
\tag{40}
$$

$$
1 + i_{t,t+1} = (1 + r_{t,t+1}) \frac{P_{t+1}}{P_t} \qquad 1 \le t \le N-1 \tag{41}
$$

$$
\begin{aligned}
\bar{R}_{t,t+j} &= \prod_{k=0}^{j} \frac{1}{1 + r_{t+k,t+k+1}} && 1 \le j \le N-1 \\
&= 1 && j = 0
\end{aligned}
\tag{42}
$$

$$
\frac{\Gamma \bar{B}_0}{P_1} + g\bar{b}_0 \equiv \sum_{j=0}^{N-1} \bar{R}_{1,1+j} \left[\bar{s}_{1+j} - \bar{g} \right] \tag{43}
$$

Under a nominal interest rate rule, equations (34) to (43) provide *6N+1* equations to determine *6N* unknowns (*N* values for each of y_t, c_t and M_t, $t = 1, ..., N$; $N-1$ values for $r_{t,t+1}$, $t = 1, ..., N-1$; *N* values for $\bar{R}_{1,1+j}$, $j = 0, \dots, N-1$ and $N+1$ values for P_t, $t = 0, ..., N$. With the price level in period *1* predetermined, it cannot do the job of mimicking a default discount factor on the public debt in equation (43). The putative equilibrium under the non-Ricardian nominal interest rule is always overdetermined. Overdeterminacy also characterises the system under the non-Ricardian monetary rule. This overdeterminacy under both the non-Ricardian monetary rule and the non-Ricardian fiscal rule, remains when the economy has an infinite horizon.

I summarise the discussion in the following proposition.

Proposition 4.

Non-Ricardian fiscal-financial-monetary rules never provide an alternative theory of the price level to the conventional Ricardian rules. When Ricardian rules support a determinate equilibrium price level sequence (under the Ricardian monetary rule in the flexible price model and under the Ricardian monetary rule and the Ricardian nominal interest rule in the model with nominal price rigidity), imposing the non-Ricardian rule results in overdeterminacy.

When Ricardian rules result in price level indeterminacy (as with the Ricardian nominal interest rate rule and flexible nominal prices), non-Ricardian rules do not generically support a non-negative initial price level. An arbitrary restriction on the predetermined public debt stocks and the exogenous public spending and revenue sequences must be satisfied.

II9. What Went Wrong (1): Incomplete Model Specification

The source of Cochrane's error is easily located. Cochrane presents just the government's single period budget identity and the government solvency constraint.

The single-period government budget identity can be rewritten, for $1 \le t \le N$, as

$$\Gamma \frac{B_t}{P_{t+1}} + \phi b_t \equiv (1+d)\left(\Gamma \frac{B_{t-1}}{P_t} + \phi b_{t-1} + \bar{g} - \bar{s}_t\right) \tag{44}$$

This updates the stocks of public debt each period.[23] The government's solvency constraint is updated each period, $1 \le t \le N$ as in (45).

$$\frac{\Gamma B_{t-1}}{P_t} + \phi b_{t-1} \equiv \sum_{j=0}^{N-t} \left(\frac{1}{1+d}\right)^j \left[\bar{s}_{t+j} - \bar{g}\right] \tag{45}$$

The initial stocks of nominal and real public debt, $\bar{B}_0$ and $\bar{b}_0$, and their coupon payments Γ and g are given.[24] In Cochrane's example, as in the flexible price model of this paper, the sequence of equilibrium real rates of return is exogenous and constant. The fiscal theory of the price level then assumes that the entire sequence of real primary surpluses (including real seigniorage), $\bar{s}_t - \bar{g}$, *t = 1, ..., N*, is also exogenously given. It follows that the initial price level, P_1, is determined from (30) (or from (45) for *t = 1*) alone. Given P_1, the rest of the equilibrium price sequence, P_2 to P_N, is determined from equations (44) and (45) applied in each successive period *t = 2, ..., N*. The equilibrium money price sequence is determined from the government's intertemporal budget constraint alone.

[23] The choice between issuing index-linked or nominal bonds is treated as exogenous.
[24] Cochrane does not include index-linked debt and assumes that $\Gamma = 1$. Nothing essential depends on this.

Cochrane then offers the following reflection: *"The reader may be uncomfortable that the rest of the economy is not specified - where are preferences, technology and shocks? The answer is that a wide specification of models includes equations such as* ...{(44) and (45)}...; *those equations will determine the price level no matter what the rest of the economy looks like, so we don't have to spell them out". (Cochrane [1999a, p. 328])*[25]. This is wrong for two reasons.

First, even if we restrict ourselves to a consideration of the government sector alone (equations (44), (45) and the initial debt stocks, B_0 and b_0), there is nothing to guarantee that condition (31) is satisfied. Only a *non-negative* equilibrium price sequence makes economic sense.

Second, assume (31) is satisfied. A unique, non-negative value of the initial price level, P_1, satisfies (45) in period *1*. Under that condition, the initial value of the general price level can indeed mimic the operation of a public debt default discount factor in period *1*, *as regards the government solvency constraint.* The problem then becomes that, while the initial price level may indeed do the job of a period *1* public debt default discount factor as regards the government's solvency constraint, it is in fact likely to be 'occupied elsewhere'.

When the rest of the economy *is* spelled out, it is obvious that the equilibrium price level sequence is overdetermined under a non-Ricardian monetary rule even when the price level is flexible. When the price level is predetermined, the equilibrium is overdetermined both under a non-Ricardian monetary rule and under a non-Ricardian nominal interest rate rule. This rejection

[25] The equation number references in Cochrane's quote have been changed to refer to the equivalent equations in this paper.

of the validity of the fiscal theory of the price level on grounds of its internal inconsistency, is a mathematical, logical issue, not an empirical matter.[26]

II10. What is the Price of Phlogiston? The Price of Money in an Economy Without Money

Taken at face value, the fiscal theory of the price level can determine the price of money (the reciprocal of P) when (31) is satisfied, even in a world in which money does not exist either as a physical object or as a financial claim. The price of money in a world without money can be determined from the government budget alone[27], under a non-Ricardian nominal interest rate rule, if three conditions are satisfied. First, there is something called "money" which serves as the numeraire or unit of account. Second, there is a zero-net-supply financial claim that is denominated in this "money", that is, the value of the coupon payments associated with this financial claim is specified in terms of "money". Third, a condition like (31) or (33) is satisfied.

Such a demonetised world is a special case of our model. So far the analysis has been restricted to the case where $f > 0$. Real money demand was therefore always positive in equilibrium.[28] Now consider the case where $f = 0$. The demand for real money balances will be zero in equilibrium. Households don't demand money and the government does not issue

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[26] Consider a system of $N \geq 2$ simultaneous equations. If one can establish that one of the equations determines the value of one endogenous variable, x, say, one does not yet conclude that one has a satisfactory theory of x. The remaining $N-1$ equations could also determine an equilibrium value for that same variable x. If these two values for x are not identical, the system is overdetermined, that is, internally inconsistent. This is what happens under a non-Ricardian monetary rule and, when nominal price rigidity is present, also under the non-Ricardian nominal interest rate rule.

[27] that is, from equations (44) and (45) for $t = 1,..., N$ and the initial stocks of public debt, B_0 *and* b_0.

[28] See footnote 12.

money. Money is dropped from the asset menu. It is no longer used as a store of value, medium of exchange or means of payment. Something called 'money' could still be the numeraire, but is has no existence other than as a numeraire or unit of account. In addition, there is a government debt instrument that has its coupon payment, Γ, specified in terms of money.[29] Like phlogiston, it exists only as a name. Note that the payment on the nominal debt instrument, Γ, cannot be made in money, since there is no monetary asset in the economy. The nominal bond can only promise to make a payment worth Γ units of money. The payment would have to be made in some good or financial claim that actually exists.

However, the government solvency constraint under the non-Ricardian nominal interest rate rule, given by (45) or (32) still appears capable of pricing money (provided the condition given in equation (31) or (33) is satisfied). Consider an economy satisfying (45) and (31). As long as a stock of public debt (with the appropriate sign) is outstanding, which is denominated in terms of this pure numeraire 'money', a determinate non-negative price level sequence is generated. A theory capable of pricing something that does not exist is quite an achievement. This economic non-sense should have rung alarm bells.[30] The re-interpretation of the non-

[29] To emphasise the point, money does not exist as a scarce physical commodity (an intrinsically useful consumer good, factor of production or produced productive input) or as a financial claim.

[30] Consider a *K* good Arrow-Debreu general equilibrium model with a government (but without money)). "Phlogiston", which only exists as a name, is the numeraire. In addition, one or more agents in the economy may issue securities promising to make a (future) payment worth Γ units of phlogiston. The Arrow-Debreu model determines *K-1* relative prices in equilibrium. (There can be multiple equilibria, each one of which determines *K-1* relative prices). Equivalently, each equilibrium is a *K* vector of prices in terms of the numeraire, phlogiston, that is unique only up to a positive multiplicative constant. The fiscal theory of the price level claims to be able to determine, for each Arrow-Debreu equilibrium, a unique *K*-vector of phlogiston prices.

Ricardian rules in terms of a government default discount equation in Section III, resolves this paradox.[31]

II11. What Went Wrong (2): The Role of Budget Constraints in a Market Economy

Budget constraints are *the* defining characteristic of a market economy. They should be equally central in formal models of a market economy. For a consumer or household, the budget

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Some of the contributors to the fiscal theory of the price level (notably Woodford [1998a] and Sims [1994]) appear to regard the ability of the fiscal theory to price money in an economy without money as a virtue rather than an embarrassment.

There clearly are interesting economic questions about what happens to an economy in which money initially has some essential transactions role, when, due to technical change and/or deregulation, the economy demonetises. In our simple economy, we can model the process of demonetisation of our 'fiat outside money' economy, by considering the limiting behaviour of an economy for which $f > 0$ as $f \to 0$. Similar limiting behaviour can be analysed in shopping technology models such as Woodford [1998a] and Buiter [1998]. However, it is important that this limit be taken in such a way that the mathematics adequately represents the economic process of demonetisation.

Viewing the equilibrium of a demonetised economy ($f = 0$) as the limit, as $f \to 0$ of a sequence of equilibria of economies indexed by f > 0, does not represent an economically interesting exercise. This 'cross sectional' limit of a sequence of parallel universes, does not, generically, capture the economically relevant 'time series' process of a single economy gradually demonetising over time. The economically interesting experiment would consider a single economy in which the money demand parameter is indexed by time, e.g.: $f_t \geq 0$, $f_{t+1} \leq f_t$ and $\lim_{t\to\infty} f_t = 0$ (see Buiter [1998]).

In an economy without money, the price of money, that is, the value of the numeraire, is inherently arbitrary. Any positive number will do. This goes beyond the indeterminacy of the price of money in a monetary economy ($f > 0$) when the authorities pursue a Ricardian nominal interest rate rule. In an economy without money the price of money is not just indeterminate, it is conceptually undefined.

Searching for limiting behaviour of the equilibrium price of money sequence in a demonetising monetary economy, in the hope of finding one which converges to an equilibrium price of money sequence in a non-monetary economy, is therefore bound to be a will-o'-the wisp.

The issue of how an economy with essential 'outside' fiat money behaves in the limit as technological progress makes outside fiat money redundant, is an important and interesting one.

constraint is the requirement that the value of consumption expenditure cannot exceed the value of the consumer's endowment. For a business, the budget constraint is that profits should be non-negative. It is no different for the government: the value of its outstanding debt (priced at default-free or notional bond prices) cannot exceed the present discounted value of its current and future primary surpluses (including seigniorage). Budget constraints hold for optimising, satisficing or ad-hoc households, firms and governments. It is a minimal feasibility condition or internal consistency condition of any agent's plan in a market economy.[32]

Budget constraints apply in static models without uncertainty and in dynamic models, with or without uncertainty. If markets are incomplete, it is not in general possible to reduce the budget constraint to a single weak inequality, pricing contingent endowments and consumption at state-contingent intertemporal prices. Sequential budget constraints and more subtle notions of insolvency and default are required. This paper only considers market economies with complete markets, so these issues do not concern us.

Special issues arise in models with an infinite number of time periods. Most of this paper has focussed on the case of a finite number of periods, because the concept of an intertemporal budget constraint is non-controversial in such an economy. For completeness, the infinite horizon case is considered in Section IV below. This permits us to analyses the fiscal fallacy using the cash-in-advance model, something that could not be done in a finite-horizon economy.

So is the issue of the behaviour of economies with pure private money or monies. Neither issue has anything to do, however, with the fiscal theory of the price level (see Buiter [1998]).

[32] Take a competitive consumer in a static two commodity endowment economy. This consumer cannot arbitrarily choose the amounts of both goods he consumes (or the real value of the consumption of both goods in terms of some appropriate price index). If he fixes the consumption of one good, his consumption of the second good is determined endogenously or residually by the requirement that he has to satisfy his budget constraint. Even a monopolist is not able to rely on prices assuming the values required for him to be able to execute an arbitrary

I take is as axiomatic that the solvency constraint of the government, like the solvency constraint of the representative household, has to hold both in and out of equilibrium. It has to hold for all admissible sequences of prices even though the government may be (and may take advantage of the fact that it is) a 'large' agent, capable of influencing equilibrium real and nominal variables. It holds both in a monetary economy and in a non-monetary or barter economy (the special case of our model with $\phi = 0$ and with money dropped from the asset menu and from the private and public sector budget constraints). It holds as an identity in an economy with a finite time horizon and in the infinite-horizon case considered in Section IV below.

Summing up: *the government budget constraint requires that the notional or contractual value*[33] *of the government's outstanding stocks of non-monetary debt instruments be no greater than the present discounted value of the government's primary (non-interest) surpluses plus seigniorage (money issuance). It represents a constraint on the government's choice variables that must be satisfied for all admissible values of those variables appearing in the government's budget constraint that are not government choice variables but that are endogenous in the model as a whole.*[34]

This basic feature of budget constraints is denied by all contributors to the fiscal theory of the price level. A clear statement can be found in Woodford [1998a].

"Note that our argument does not involve any denial that the value of the public debt must actually equal the present value of future government budget surpluses, in equilibrium.

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production plan while still making a non-negative profit. Things are no different for the government.

[33] That is, the value at default risk-free prices.

[34] These 'economy-wide' endogenous variables will, of course, in general be influenced by the values assigned by the government to its choice variables or instruments. If the government perceives itself to be 'large', it may indeed set its choice variables to deliberately influence the

What we deny is that condition (1.14) [that the initial value of the public debt equals the present discounted value of current and future primary surpluses and seigniorage (WHB)] *is a constraint upon government fiscal policy, that must be expected to hold regardless of the evolution of goods prices and asset prices. Instead of a "government budget constraint", the condition is properly viewed as an equilibrium condition, that follows from the joint requirements of private sector optimisation and market clearing. But as an equilibrium condition rather than an implication of the fiscal policy rule, it can play a role in equilibrium inflation determination*. (Woodford [1998a, pp. 17-18]).

The main argument in defence of this proposition appears to be that the government is a 'large' agent:

"Furthermore, even in the case of an optimising government, the government should not optimise subject to given market prices and a given budget constraint as private agents are assumed to do in a competitive equilibrium. For the government is a large agent, whose actions can certainly change equilibrium prices, and optimising government surely should take account of this in choosing its actions". (Woodford [1998, p. 30]).

The 'large agent' argument is spurious. First, there is no logical connection between an agent's ability to influence market prices and his being subject to a budget constraint. A monopolist or monopsonist still faces a budget constraint. Second, it is not true that governments are necessarily 'large' in the sense of being able to influence equilibrium prices or quantities. There is a vast literature on small open economies (for instance the class of models characterised by uncovered interest parity, a fixed nominal exchange rate and purchasing power parity), in which governments cannot, by assumption, influence any nominal or real prices.

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values of the economy-wide endogenous variables. Nothing in the definition of the government

The *a-priori* argument, postulate or primitive, that governments, like all other agents in a market economy, are subject to a budget constraint that must be satisfied both in and out of equilibrium, should really be the end of the story. Any model of government behaviour that does not require the government budget constraint to hold identically is, for that reason alone, economically ill-posed and therefore inadmissible.

It is nevertheless instructive to follow the argument one step further and to see, as was done in the earlier parts of this Section, and in Sections III and IV below, how, if one persists with this a-priori inadmissible approach, one gets either a contradiction (in the shape of an overdetermined equilibrium price system) or other anomalous behaviour.

II12. Beyond the fiscal theory of the price level: the household budget constraint theory of the price level?

Substituting the household first-order conditions (9), (10) and (11) into the household solvency constraint (8) for period *1* yields equation (40). When the household intertemporal budget constraint is viewed as constraint that must be satisfied not only in equilibrium, but for all admissible values of the economy-wide endogenous variables, (40) represents a constraint on c_1 , that is, it determines the optimal value of c_1 as a function of the economy-wide endogenous variables that appear in (40).

Applying the logic of the fiscal theory of the price level to the household sector, however, I can overdetermine the household's optimal consumption and portfolio allocation programme and fix c_1 at some arbitrary positive level, $c_1 = \bar{c}_1$, say. The household solvency constraint then

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budget constraint requires the government to act as a 'small' or competitive agent.

determines the period *1* price level. This gives us the 'household budget constraint theory of the price level':

$$\frac{M_0+\Gamma B_0}{P_1}+\gamma b_0 \equiv \sum_{j=0}^{N-1}\bar{R}_{1,1+j}(t_{1+j}-y_{1+j}) \\ +\bar{c}_1\left\{\sum_{j=0}^{N-2}\left[\left(\frac{1}{1+\delta}\right)^{j}\left(1+\varphi^{\frac{1}{\eta}}\left(\frac{1+i_{1+j,2+j}}{i_{1+j,2+j}}\right)^{\frac{1-\eta}{\eta}}\right)\right]+\left(\frac{1}{1+\delta}\right)^{N-1}(1+\varphi^{\frac{1}{\eta}})\right\} \tag{46}$$

For the initial price level to be non-negative, it would of course have to be the case that

$$\operatorname{sgn}\left(\frac{M_0+\Gamma B_0}{P_1}\right) \\ \equiv \operatorname{sgn}\left\{\begin{array}{l}\sum_{j=0}^{N-1}\bar{R}_{1,1+j}(t_{1+j}-y_{1+j}) \\ +\bar{c}_1\left\{\sum_{j=0}^{N-2}\left[\left(\frac{1}{1+\delta}\right)^{j}\left(1+\varphi^{\frac{1}{\eta}}\left(\frac{1+i_{1+j,2+j}}{i_{1+j,2+j}}\right)^{\frac{1-\eta}{\eta}}\right)\right]+\left(\frac{1}{1+\delta}\right)^{N-1}(1+\varphi^{\frac{1}{\eta}})\right\}-\gamma b_0\end{array}\right\} \tag{47}$$

This 'household budget constraint theory of the price level' would be recognised immediately as a complete economic non-sense, even if we restricted ourselves to a consideration of the household sector alone.

The economy-wide equilibrium implications would also be rather disconcerting. For starters, an exogenous private consumption level would violate the goods market equilibrium condition, $\bar{y}_1=\bar{c}_1+g_1$, unless government spending passively accommodated the private consumption programme. If, by chance, the resource constraint were not violated, the price level would be overdetermined under a monetary rule.

The household budget constraint theory of the price level is no different from the fiscal theory of the price level. The individual household could be ‘large’ rather than competitive and ‘tax-taking’, leading to first-order conditions for the household optimal consumption programme that are different from the competitive ones used in (40) and (46). Indeed the household could have its behaviour specified in a completely ad-hoc manner (like the government in this paper), subject only to non-negativity constraints on consumption and money holdings.

The source of the fallacy in the household budget constraint theory of the price level is the failure to recognise that the household solvency constraint has to hold identically, both in and out of equilibrium.

III. Understanding and Resolving the Fallacy: the Government Solvency Constraint as a Default Discount Equation

It is clear by now what the cause of the error in the fiscal theory of the price level is. When the government fixes the sequences of real spending and real tax revenues including seigniorage (and sticks to these sequences), it can no longer guarantee that its debt will be serviced according to letter of the debt contract.

The arbitrage equation (5) prices government debt in terms of notional bond prices. The contractual future payments, Γ and g, associated with the two debt instruments are discounted at the riskless rate, reflecting the assumption that these payments will indeed be made in full, that is, on the assumption that there is no default. P_t^B and P_t^b are the notional or default-free prices of the two government debt instruments.

When the discounted value of the primary surpluses plus seigniorage differs from the default-free or notional value of the outstanding stock of public debt, the debt should be valued at effective or default-risk adjusted prices, $\tilde{P}_t^B$ and $\tilde{P}_t^b$ respectively. Assume that all debt has equal seniority and that any difference between the present discounted value of the primary surpluses plus seigniorage and the default-free value of the outstanding stock of public debt is pro-rated equally over all outstanding debt. Let D_{t+1} denote the fraction of the contractual payments due in period *t+1* that is actually paid. That is, the actual payments in period *t+1* on the two debt-instruments issued in period *t* are (with certainty, in this simple model),

$$\tilde{\Gamma}_{t+1} = D_{t+1}\Gamma$$

and

$$\tilde{g}_{t+1} = D_{t+1}\, g \qquad (47)$$

$$0 \leq t \leq N-1$$

It follows immediately that

$$\tilde{P}_t^B = D_{t+1} P_t^B \qquad (48a)$$

$$\tilde{P}_t^b = D_{t+1} P_t^b\,, \qquad (48b)$$

I shall refer to D_{t+1} as the default discount factor on the notional value of the debt outstanding at the end of period *t*.

Note that

$$1+i_{t.t+`1} = \left(1+r_{t,t+1}\right)\frac{P_{t+1}}{P_t} = \frac{\tilde{\Gamma}_{t+1}}{\tilde{P}_t^B} = \frac{P_{t+1}\tilde{g}_{t+1}}{\tilde{P}_t^b} \qquad (49)$$

In principle, households or firms can default as well as the government. However, except in Section II12, households have been assumed to follow 'Ricardian' budgetary strategies. The only rules for household consumption, money demand and borrowing that are permitted are rules that ensure the household always meets its contractual debt obligations. Even though the household never defaults, the household singe-period budget identity and solvency constraint must allow for the possibility of government default.

With government default, the single-period household budget identity is, for *1* $\leq t \leq N$.

$$M_t - M_{t-1} + \tilde{P}_t^B B_t - D_t \Gamma B_{t-1} + \tilde{P}_t^b b_t - P_t D_t \phi_{t-1} \equiv P_t(y_t - t_t - c_t) \qquad (50)$$

The intertemporal budget constraint or solvency constraint of the household remains that at the end of period N, the household cannot have positive debt,

$$\tilde{P}_N^B B_N + \tilde{P}_N^b b_N \geq 0 \qquad (51)$$

With default, the budget identity of the government becomes

$$M_t - M_{t-1} + \tilde{P}_t^B B_t - D_t \Gamma B_{t-1} + \tilde{P}_t^b b_t - P_t D_t \phi_{t-1} \equiv P_t(g_t - t_t) \qquad (52)$$

and the government's solvency constraint becomes

$$\tilde{P}_N^B B_N + \tilde{P}_N^b b_N \leq 0 \qquad (53)$$

The default discount factors, D_t , are determined as follows. In the last period, (52) and (53) imply

$$D_N \leq \frac{M_N - M_{N-1} + P_N(t_N - g_N)}{\Gamma B_{N-1} + P_N \phi_{N-1}} \qquad (54)$$

In general, for $1 \leq t \leq N$

$$D_t \leq \frac{\sum_{j=0}^{N-t} R_{t,t+j} \left[P_{t+j}(t_{t+j} - g_{t+j}) + (M_{t+j} - M_{t+j-1}) \right]}{\Gamma B_{t-1} + P_t \phi_{t-1}} \qquad (55)$$

The default discount factor is the ratio of the present discounted value of the resources that will actually be available for servicing the public debt (on the assumption that the government sticks to its future plans for public spending, taxes net of transfers and seigniorage) to the notional or contractual value of the public debt.

It is slightly awkward to talk of default in a model without uncertainty (or in any complete markets model). In this setting, unanticipated default could occur only in the initial period. In period *1* it is known with certainty what the future payments associated with the non-monetary debt instruments will in fact be. Without loss of generality, we can therefore restrict the analysis to the case where only the period *1* default discount factor, D_1, can differ from *1*. In all subsequent periods, effective debt prices would equal notional prices.

For simplicity, assume (55) holds with equality (as it will in equilibrium). The default discount factor equation or effective public debt pricing equation is given in (56).

$$
\begin{aligned}
& D_1(\Gamma B_0 + P_1 b_0) = \sum_{j=0}^{N-1} R_{1,1+j}\left[P_{1+j}(t_{1+j} - g_{1+j}) + (M_{1+j} - M_j)\right] \\
& D_t = 1 \qquad\qquad 2 \le t \le N
\end{aligned}
\tag{56}
$$

Should we impose the constraint that $0 \le D_1 \le 1$? This rules out both $D_1 < 0$ (notional debtors can be effective creditors and vice versa) and $D_1 > 1$ (the default discount factor can be a 'super-solvency premium'). Consider first the case for ruling out $D_1 < 0$ *a-priori*. In that case equation (56) applies only if

$$
\begin{aligned}
& \operatorname{sgn}\left[\Gamma B_0 + P_1 b_0\right] \\
& = \operatorname{sgn}\left\{ \sum_{j=0}^{N-1} R_{1,1+j}\left[P_{1+j}(t_{1+j} - g_{1+j}) + (M_{1+j} - M_j)\right] \right\}
\end{aligned}
\tag{57}
$$

Consider the case where (57) is violated. For instance, let the private sector hold a positive stock of public debt in period *1* (at default-free prices), although the government's present discounted value of future primary surpluses plus seigniorage is negative. If one did insist on imposing (57), it would follow that there exists no feasible fiscal-financial monetary programme and therefore no equilibrium.

On the other hand, one could argue that a government that is really committed to the spending, tax and monetary issuance sequences on the RHS of (56) would, if the RHS of (56) were to be negative, have no option but to impose an immediate capital levy on the private sector in period *1*, large enough to create a stock of public sector credit (negative public debt) equal in value to the present discounted value of the excess of current and future public spending over taxes plus seigniorage. Thus a positive notional or contractual value of the public debt would have to be transformed, in period *1,* into a negative effective value of the public debt. A negative value of D_1, the initial public debt default discount factor would, on this interpretation, make perfect economic sense, unlike a negative value of the general price level. I will adopt this second approach and admit negative values of D_1.

The constraint $D_t \leq 1$ implies that the public debt can trade at a *discount* on its notional value if the present value of future primary surpluses and seigniorage falls short of its default-free value, but not at a *premium* if the opposite applies. If $D_t > 1$ is ruled out, government bond-holders do not get more than the government is contractually obliged to pay them, if the present discounted value of future primary surpluses and seigniorage exceeds the default-free value of the public debt. This means that the effective public debt pricing equation (56) should be replaced by (58).

$$D_1 = \min\left\{ 1 \, , \frac{\sum_{j=0}^{N-1} R_{1,1+j} \left[P_{1+j}(t_{1+j} - g_{1+j}) + (M_{1+j} - M_j) \right]}{\Gamma B_0 + P_1 g b_0} \right\} \tag{58}$$

If one chose to impose (58), one would also need a theory for determining how any surplus of the present discounted value of future primary surpluses and seigniorage over the notional value of the outstanding stock of public debt, is disbursed or disposed of. Permitting $D_1 > 1$ means that any surplus is given to the initial bond holders. Since this simplifies the exposition considerably, I will permit $D_1 > 1$.

The government solvency constraint in period *1* (holding with equality) can be rewritten as[35]

$$D_1\left(\frac{\Gamma\bar{B}_0}{P_1}+\gamma\bar{b}_0\right)\equiv\sum_{j=0}^{N-1}\left(\frac{1}{1+\delta}\right)^j\left[t_{1+j}+\left(\frac{M_{1+j}-M_j}{P_{1+j}}\right)-g_{1+j}\right] \tag{59}$$

The government solvency constraint can therefore be viewed as an effective public debt pricing equation or default discount factor equation.[36] The present discounted value of future primary surpluses and seigniorage equals (‘determines’, if seigniorage and primary surplus sequences are taken as given) the effective value of the initial government debt. If the notional or contractual value of the initial debt differs from its effective value, the government solvency constraint determines the ratio of the effective to the notional value, that is, the public debt default discount factor.

In the case without uncertainty[37], the treatment of the default discount factor defines the Ricardian and the non-Ricardian fiscal rules.

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[35] For notational simplicity we impose the equilibrium condition that the real interest rate equals the pure rate of time preference.

[36] In general, the default discount factor is determined simultaneously with all other endogenous variables by the complete set of equilibrium conditions. Only in very special cases (such as a constant real interest rate and the absence of nominal government debt), can the equilibrium be represented in such a way that the government solvency constraint alone determines the default discount factor.

[37] Or, more generally, with complete markets.

A Ricardian fiscal rule is defined by the requirement that $D_1 \equiv 1$.[38] With a Ricardian rule, there can be no default premium or discount.[39] In that case either taxes, or public spending or monetary growth during at least one period have to be determined residually so as to satisfy the solvency constraint at default-free or notional debt prices. In the simple example used in this paper, lump-sum taxes adjust to ensure government solvency (equation (19).

With a non-Ricardian rule, the government is permitted to *overdetermine* its fiscal-financial-monetary programme. The default discount factor in the initial period, D_1, is now endogenously determined. Unlike the general price level, it appears in the government solvency constraint only. In general, the present value of future primary surpluses plus seigniorage now is not equal to the value of the outstanding debt at default-free or notional debt prices. The effective value of the initial stock of debt then becomes endogenous and will be equated to the present discounted value of the future primary surpluses and seigniorage. The government's fiscal-financial-monetary programme is overdetermined if all contractual debt obligations are required to be met, but an additional endogenous variable, the initial period default discount factor, D_1, removes the inconsistency and ensures that the general equilibrium problem remains well-posed.

The analysis of Ricardian rules in Section III is therefore unaffected by the introduction of the default discount factor. This is because the Ricardian rule is defined by the condition that $D_1 \equiv 1$, and because this was assumed in the analysis of both the Ricardian monetary rule and the Ricardian nominal interest rate rule.

The analysis of non-Ricardian rules is, however, changed in a crucial way by the introduction of the default discount factor. For sake of brevity, I will only consider the flexible

[38] Remember that, without loss of generality, we assume that $D_t = 1,\ 2 \le t \le N$.

price level model in the remainder of this paper. If the government follows the non-Ricardian rule used in this paper, the amended government solvency constraint becomes

$$D_1\left(\frac{\Gamma\overline{B}_0}{P_1}+g\bar{b}_0\right)\equiv\sum_{j=0}^{N-1}\left(\frac{1}{1+d}\right)^j\left[\bar{s}_{1+j}-\bar{g}\right] \tag{60}$$

The equilibrium conditions under the non-Ricardian monetary rule are now given by equations (16), (22), (23), (24), (25a,b) (28) and (60). Note that the original government solvency constraint (30) is replaced by (60).

The overdeterminacy of the equilibrium under the original Ricardian monetary rule is removed through the introduction of the addition endogenous variable D_1. With P_1 determined by the monetary equilibrium conditions (25a,b), the solvency constraint (60) determines the default discount (or premium) on the public debt, D_1, in the initial period.

Note that, with the introduction of the default discount factor, money is neutral under the Ricardian monetary rule. While there now is a well-defined equilibrium under the non-Ricardian monetary rule, there is no fiscal theory of the price level.

Provided condition (31) is satisfied, the initial general price level, P_1, can play a similar role in satisfying the government's solvency constraint as the initial default discount factor, D_1. If there is a positive (negative) stock of nominal non-monetary debt outstanding and if the present discounted value of future primary surpluses and seigniorage is positive (negative), there exists a value of the initial general price level that will equate the real value of the outstanding non-monetary public debt to the real present discounted value of future primary surpluses and seigniorage. If there is no index-linked debt, equation (60) determines $\frac{D_1}{P_1}$: from the point of

[39] As was pointed out earlier, the household budget rules are always required to be Ricardian.

view of the government solvency constraint, the general price level works like the reciprocal of the default discount factor.

However, while D_1 and P_1 are interchangeable as regards satisfying the government solvency constraint (if (31) holds), they are not interchangeable as regards the remaining equilibrium conditions (25a,b). Only the public debt default discount factor can therefore 'clear' the government's intertemporal budget constraint in a well-posed general equilibrium problem with an overdetermined fiscal-financial-monetary programme.

With the addition of the default discount factor, the equilibrium under the non-Ricardian nominal interest rate rule is given by equations (17), (22), (23), (24), (25a,b), (28) and (60). Again the old government solvency equation (30) is replaced by (60). Price level indeterminacy now is present under the non-Ricardian nominal interest rate rule. If $B_0 \neq 0$, price level indeterminacy also implies indeterminacy of the default discount factor. The government solvency condition does, however, always determine the effective real value of the total public debt, $D_1\left(\frac{\Gamma B_0}{P_1} + b_0\right)$, even when it does not determine the default discount factor and the initial price level separately.

I summarise this discussion with the following proposition.

Proposition 5

The introduction of a default discount factor on government debt makes models with non-Ricardian fiscal-financial-monetary programmes mathematically consistent as well as economically well-posed. It also destroys the fiscal theory of the price level. When the price level is flexible, money is neutral with a non-Ricardian monetary rule and there is price level indeterminacy with a non-Ricardian nominal interest rate rule.

The default discount factor approach to the public sector solvency constraint is valid for monetary economies and for non-monetary economies. It holds regardless of the composition and magnitude of the outstanding stocks of public debt instruments and regardless of the way in which the government determines its spending, tax and money issuance. It applies to large and small, optimising, satisficing or ad-hoc governments. It applies when nominal prices are flexible and when there are nominal price rigidities. As shown below, it applies to infinite-horizon as well as to finite-horizon economies.

The price of phlogiston again

Introducing the default discount factor permits the resolution of the conceptual anomaly that the fiscal theory of the price level appears to be able to price money in an economy without money.[40]

Note that (60) determines the effective real value of the total public debt even in a non-monetary economy (the case where $f = 0$ and money is dropped from the asset menu). It does so regardless of what the debt instruments are denominated in.

Consider an economy without money. Let phlogiston be the numeraire, and let the government have an outstanding stock B_0 of phlogiston-denominated liabilities, that is, contractual claims to a payment worth Γ units of phlogiston in period *1*. If all debt is index-linked ($\overline{B}_0 = 0$), equation (60) determines the default discount factor. If there is no index-linked debt ($\overline{b}_0 = 0$), equation (60) still defines the effective real value of the public debt, $\frac{D_1}{P_1}\Gamma\overline{B}_0$. However, both D_1 and P_1 are indeterminate. Only their ratio, D_1 / P_1, is determinate. The real

[40] If (31) is satisfied.

effective value of the debt is independent of how the debt is denominated. It depends exclusively on the real resources 'backing' the debt, that is, on the RHS of (60).

The RHS of (60), the present discounted value of future real primary surpluses is exogenous.[41] The government is assumed to stick to this sequence of real primary surpluses. Consequently, the effective real value of its initial debt must equal the present discounted value of this sequence of real primary surpluses.

It is clear that we cannot generically impose the constraint $D_1 = 1$ on the government solvency constraint for this case and solve for P_1, as this would require the arbitrary restriction $\operatorname{sgn}\{\overline{B}_0\} = \operatorname{sgn}\left\{ \sum_{j=0}^{N-1} \left(\frac{1}{1+d} \right)^j \left[\overline{s}_{1+j} - \overline{g} \right] \right\}$.[42]

In an economy without money, the general price level does not appear in any equilibrium condition other than the government solvency constraint (and it appears in the government solvency constraint only if the government has issued nominal debt).[43] Likewise, in our example, the price of phlogiston in terms of output, $\frac{1}{P}$, does not appear in any equilibrium condition other than the government solvency constraint, and it appears in the government solvency constraint only if the government has issued phlogiston-denominated debt. The government solvency constraint only determines $\frac{D_1\Gamma}{P_1}$, the effective real payment made to a holder of one unit of government debt. The recipient of the payment will not be able to determine whether he received a given real effective payment with, say, $D_1 = 1$ and $P_1 = 2$, or the same real effective payment with $D_1 = 0.5$ and $P_1 = 1$. He would only be able to tell the

[41] There is no seigniorage is the demonetised economy.
[42] I am assuming that even the price of phlogiston, $1/P$, cannot be negative.

difference if payment really were *made* in phlogiston (as opposed to being merely *denominated* in phlogiston), since in that case he could verify whether he really did receive Γ units of phlogiston for each government bond he held. Because phlogiston does not exist, except as a pure numeraire, he cannot verify how many units of phlogiston have been transferred to him or credited to his account. All he can verify is that he has received real resources worth $\frac{D_1\Gamma}{P_1}$ for each government bond he holds.

With a pure numeraire phlogiston (money), to have a claim worth x units of phlogiston (money), *means* no more than that one has a claim to $\frac{x}{P}$ units of the real good. It cannot be a claim to x units of phlogiston, as there is no such thing. While both the price of phlogiston and the default discount factor are indeterminate, the real effective value of the debt *is* determinate.

Thus, in a non-monetary economy, the price level itself is (of course), indeterminate, under a non-Ricardian fiscal rule and under a Ricardian rule. Under a non-Ricardian fiscal rule, the real effective value of the public debt is determinate, regardless of the denomination of the contractual debt obligations.[44] The paradox of the fiscal approach appearing to determine the price of money in an economy without money is therefore resolved.

[43] It appears also in the household solvency constraint.
[44] With a Ricardian rule and phlogiston-denominated public debt, both B_0 / P_1 and t_1 would be in addition be indeterminate (see (19)).

IV. The Infinite Horizon Case

I briefly consider the infinite horizon case. The question of how to write an intertemporal budget constraint for an infinite-lived agent is a difficult, and I believe not yet fully resolved, issue. I view this as more of a mathematical conundrum than a substantive economic problem.[45]

IV1. The Model

To obtain the infinite horizon case, we let $N \to \infty$ in (1) and follow the standard procedure of replacing the household solvency constraint (3) by the 'no Ponzi-finance' condition:[46]

$$\lim_{N\to\infty} R_{t,N}\left[\tilde{P}_N^B B_N + \tilde{P}_N^b b_N\right] \geq 0 \tag{61}$$

This produces the following present value household budget constraint

$$D_t\left[\Gamma B_{t-1} + P_t \gamma b_{t-1}\right] \geq \lim_{N\to\infty} \sum_{j=0}^{N-t} R_{t,t+j}\left[P_{t+j}(c_{t+j} + \tau_{t+j} - y_{t+j}) + (M_{t+j} - M_{t+j-1})\right] \tag{62}$$

Equation (62) is qualitatively similar to the finite-horizon intertemporal budget constraint of the household. This is the only rationalisation the literature offers for (61).

The household optimal consumption and money holdings programme is characterised by equations (9) and (10) for all time.

While equation (11), which characterises monetary equilibrium in the finite terminal period, N, applies when N is very large but finite, it cannot hold if the economy truly has no

[45] Where an equilibrium property depends crucially on the horizon being infinite rather than long but finite, that equilibrium property is not, in my view, of economic interest. This is of some importance when moving to an infinite horizon specification adds a whole plethora (sometimes a continuum) of possible solutions over and above the solutions corresponding to the long-but-finite horizon case. The only solutions that are economically relevant are those that preserve the key properties of the long-but-finite horizon case.

[46] We now also have to impose $\delta > 0$.

terminal period. Taking the limit as $N \to \infty$ of (11) and of (10), we get an inconsistency unless $\lim_{t\to\infty}\left(\frac{P_{t+1}}{P_t}\right) = \pm\infty$, which would not make economic sense. If we take the infinite horizon specification literally, (11) does not apply in the limit as $N \to \infty$.

The government's solvency constraint is (again following standard practice) also given by a 'no-Ponzi finance' condition:

$$\lim_{N\to\infty} R_{t,N}\left[\tilde{P}_N^B B_N + \tilde{P}_N^b b_N\right] \le 0 \tag{63}$$

Again the only good reason for this is that it produces the government intertemporal budget constraint (64), which is qualitatively similar to the finite horizon case.[47]

$$D_t\left[\Gamma B_{t-1} + P_t \gamma b_{t-1}\right] \le \lim_{N\to\infty} \sum_{j=0}^{N-t} R_{t,t+j}\left[P_{t+j}(\tau_{t+j} - g_{t+j}) + (M_{t+j} - M_{t+j-1})\right] \tag{64}$$

Without loss of generality, we again impose that default occurs only in the initial period.

$$D_t = 1, \quad 2 \le t$$

In the infinite-horizon case, equilibrium is characterised by the following equations (for reasons of space, I only consider the flexible price level case).

$$1 + i_{t,t+1} = (1+\delta)\frac{P_{t+1}}{P_t} \qquad\qquad 1 \le t \tag{65}$$

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[47] In Buiter and Kletzer [1998], it is shown, in the context of an overlapping generations model with an infinite number of generations, that (63) cannot be derived from reasonable primitive constraints on the government's fiscal-financial programme, when the government has the use of unrestricted lump-sum taxes and transfers. As I am now no longer impressed by infinite-horizon equilibrium properties that are qualitatively different from the equilibrium properties of models with long but finite horizons, I am quite happy to impose (63).

$$\frac{M_t}{P_t} = (\bar{y} - \bar{g})\left[\frac{f(1+d)\frac{P_{t+1}}{P_t}}{(1+d)\frac{P_{t+1}}{P_t} - 1}\right]^{\frac{1}{h}} \qquad 1 \le t \tag{66}$$

$$D_1\left(\frac{\Gamma\bar{B}_0}{P_1} + g\bar{b}_0\right) \equiv \lim_{N\to\infty}\left\{\sum_{j=0}^{N-1}\left(\frac{1}{1+d}\right)^j\left[t_{1+j} + \left(\frac{M_{1+j} - M_j}{P_{1+j}}\right) - \bar{g}\right]\right\} \tag{67}$$

Under the Ricardian fiscal rule we have in addition

$$D_1 \equiv 1 \tag{68}$$

and

$$\begin{aligned} t_1 &= \bar{g} - \frac{M_1 - M_0}{P_1} + \frac{\Gamma\bar{B}_0}{P_1} + g\bar{b}_0 \\ t_t &= \bar{g} - \frac{M_t - M_{t-1}}{P_t} \qquad 1 < t \end{aligned} \tag{69}$$

Under the non-Ricardian fiscal rule we have

$$t_t + \frac{M_t - M_{t-1}}{P_t} = \bar{s}_t \qquad 1 \le t \tag{70}$$

The fiscal theory of the price level imposes (70) and $D_1 = 1$. In my modification of the non-Ricardian fiscal rule, (70) is imposed but D_1 is endogenous.

Under a monetary rule we have

$$\begin{aligned} &\{M_t;\ 1 \le t\} = \Big\{\bar{M}_t > 0;\ 1 \le t \\ &\Big|\ \frac{\bar{M}_{t+1}}{\bar{M}_t} \ge \frac{1}{1+d};\ t \ge 0\Big\} \end{aligned} \tag{71}$$

Under a nominal interest rate rule we have

$$\{i_{t-1,t}; 1 \leq t\} = \{\bar{i}_{t-1,t} \geq 0; 1 \leq t\} \tag{72}$$

Again, only non-negative equilibrium price level sequences are admissible.

IV2. Equilibrium Under Ricardian and Non-Ricardian Monetary Rules

For expositional simplicity, consider the logarithmic special case of the household period felicity function with $h = 1$. The monetary equilibrium condition (66) implies the following first-order difference equation for the general price level.

$$P_{t+1} = \frac{M_t P_t}{(1+d)[M_t - f(\bar{y} - \bar{g})P_t]} \qquad t \geq 1 \tag{73}$$

For the equilibrium to be well-defined, we require $\frac{M_t}{P_t} > f(\bar{y} - \bar{g})$.

Under a Ricardian fiscal rule, there is no obvious boundary condition (either an initial or a terminal condition for the price level) to pin down a unique solution sequence for the price level. P_1 is a 'free' variable and there is a continuum of equilibrium price level sequences. Since when N is large but finite, the price level sequence is well-behaved and unique, I would wish to impose the condition that the price level continues to be well-behaved, and unique, in the infinite horizon case also.

When the model has a unique steady state, it is easy to operationalise the concept of being 'well-behaved'. Consider the case where the nominal money stock is kept constant: $M_t = \overline{M} > 0$. The unique steady-state price level, $\overline{P}$, is given by [48]

$$\overline{P} = \left(\frac{d}{1+d}\right)\left(\frac{\overline{M}}{f(\bar{y} - \bar{g})}\right)$$

For the constant nominal money stock case, equation (73) can be rewritten as

$$P_{t+1} = \left[\frac{1}{1 + \frac{d}{\overline{P}}(\overline{P} - P_t)} \right] P_t$$

Thus, when $P_t > \overline{P}$, the price level will be rising further away from its steady state value. When $P_t < \overline{P}$, the price level will be falling further away from its steady state value. The steady state is unstable. If we impose the requirement that the equilibrium be well-behaved, the steady state is the unique equilibrium.

According to the fiscal theory of the price level, P_1 is determined, with a non-Ricardian fiscal rule (from (67), (70) and $D_1 = 1$, provided condition (74), the infinite horizon analogue of (31), is satisfied).

$$\operatorname{sgn}\left(\overline{B}_0\right) = \operatorname{sgn}\left\{ \lim_{N\to\infty} \left\{ \sum_{j=0}^{N-1} \left(\frac{1}{1+d} \right)^j \left[\overline{s}_{1+j} - \overline{g} \right] \right\} - g\overline{b}_0 \right\} \qquad (74)$$

Under these conditions, there is a unique equilibrium price level sequence. From equation (66) or (73), it is clear that, although in each period, t, the current price level, P_t, is determined by the government solvency constraint for that period, there is always next period's price level, P_{t+1}, to restore balance between period t money demand and money supply. This escape from overdeterminacy does not vanish in the final period, as it does when N is finite, because by assumption there is no final period. While the mathematics is correct, from an economic point of view I consider this to be an 'abuse of infinity'. Not surprisingly, the equilibrium price sequence determined in this manner can behave anomalously.

[48] See footnote 9.

With a logarithmic period felicity function, the equilibrium behaviour of the price level under the non-Ricardian monetary rule will be explosive (or implosive) unless the value of the initial price level determined from (67), (70) and $D_1 = 1$ happens to be the steady state price level, that is, unless $P_1 = \bar{P}$.

If, with a constant nominal money stock, the price level were to start off below its steady state value, it would decline towards zero. The stock of real money balances would go to infinity.

In many monetary models, infinite real money balances would cause private consumption to become unbounded, which would violate the economy's real resource constraint. The equilibrium would not just fail to be well-behaved, it would not exist.

In the simple 'money in the direct utility function' model of this paper, unbounded real money balances do not violate the equilibrium conditions, because the nominal interest rate would go to zero, creating an unbounded equilibrium demand for real money balances without consumption becoming unbounded.[49] Obstfeld and Rogoff [1983, 1986, 1996] discuss plausible restrictions on the utility function that would rule out deflationary bubbles.[50]

Infinite real money balances would violate the economy-wide real resource constraint in the 'unpleasant monetarist arithmetic' model of Sargent and Wallace [1981] (see also Buiter [1987]). When the period felicity function of the 'poor' households who hold all the money (and for whom money is the only store of value) in this model generates a demand for real money

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[49] The resource constraint and the money demand function imply that

$$\bar{y} - \bar{g} = \frac{1}{f}\left(\frac{i_{t,t+11}}{1+i_{t,t+1}}\right)\frac{M_t}{P_t}.$$

This condition can be satisfied even if $\frac{M_t}{P_t} \to \infty$, as long as $i_{t,t+1} \to 0$ at the appropriate rate.

balances that is sensitive to the (expected) rate of inflation, the adoption of a non-Ricardian fiscal rule with $D_1 = 1$ in this model can force the economy to follow a solution trajectory along which real money balances increase without bound, even though real endowments are finite.

In the 'money in the shopping function' model used in Buiter [1998], a similar real resource constraint violation problem would arise if the price level were to *rise* without bound with a constant nominal money stock. Such behaviour could be implied in that model by the adoption of a non-Ricardian fiscal rule with $D_1 = 1$. As the stock of real money balances goes to zero, the real resources used up in the shopping function technology would go to infinity, violating the economy's real resource constraint.

These problems do not arise under the Ricardian monetary rule or under the modified non-Ricardian monetary rule with the endogenous default discount factor, since the initial price level, P_1, is non-predetermined in these cases.

In a Clower cash-in-advance model (which would be well-posed in an infinite horizon economy), an overdeterminacy problem arises under non-Ricardian monetary rule, even though the horizon is infinite. The reason is that the simple-cash-in-advance model has the period t price level determined by consumption in period t and the (initial) money stock in period t.

Consider the simple cash-in-advance-model due to Helpman [1981] and Lucas [1982]. Each unit period is subdivided into two sub-periods. In the first sub-period, the market for consumer goods is closed, but financial markets are open. The asset menu is the same as before. During this first sub-period, households collect interest and principal from last period's bond purchases, receive their endowments and pay taxes or receive transfers. They allocate these resources among money and new bond purchases. In the second sub-period financial markets

[50] They conclude that there are no plausible a-priori restrictions on the utility function that would

are closed, but the market for consumer goods is open. Consumption must be paid for with money: $\overline{M}_t \geq P_t c_t$. Assume that the equilibrium short nominal interest rate sequence is strictly positive (equation (71) is satisfied). The cash-in-advance constraint for consumption purchases will be binding. Monetary equilibrium is characterised by[51]

$$\overline{M}_t = P_t c_t \qquad t \geq 1 \tag{75}$$

Using the same endowment economy used in the rest of this paper, we have

$$c_t = \bar{y} - \bar{g}$$

Therefore,

$$P_t = \frac{\overline{M}_t}{\bar{y} - \bar{g}} \qquad t \geq 1 \tag{76}$$

This applies in all periods, including the initial period, $t = 1$. However, according to the fiscal theory of the price level, the period *1* price level is already determined by the government solvency constraint under the non-Ricardian fiscal rule (given by (67) and (70) and $D_1 = 1$, if (74) is satisfied) (see also Buiter [1998]). The price level is overdetermined.

With an infinite horizon, the non-Ricardian monetary rule without default therefore no longer results in an overdetermined price level sequence with the money-in-the-direct-utility function model, when the price level is flexible. It does result in an overdetermined price level in the simple cash-in-advance model. The reason for the difference is that the simple cash-in-advance model generates a constant (unitary) consumption velocity of circulation of money.

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rule out hyperinflationary bubbles.

[51] In the Helpman [1981]-Lucas [1982] variant of the cash-in-advance model, consumers can use cash acquired at the beginning of period t for consumption later in that period. The producers who receive the cash must hold it between periods. An alternative model constrains the household to use only money acquired in period *t-1* for consumption purchases in period *t*. The cash-in-advance constraint for that variant would be $M_{t-1} \geq P_t c_t$. Overdeterminacy under a monetary rule characterises this variant also.

With money demand independent of the opportunity cost of holding money, period t money demand only depends (given c_t) on the period t price level, and not on the (expected) period $t+1$ price level.[52] The escape route from overdeterminacy, available with the money-in-the-direct utility function model through its variable, (expected) inflation-dependent consumption velocity of circulation of money, is therefore not present in the simple cash-in-advance model.

Even when the non-Ricardian monetary rule without default does not result in an overdetermined price level, there are still two reasons for rejecting it. The first is the *a-priori* argument, that a government intertemporal budget constraint which holds identically (both in and out of equilibrium), is a defining property of any well-posed model of a market economy. The second reason is that the equilibrium price level sequence supported by the economically ill-posed equilibrium is not well-behaved. The anomalies in question can include violations of economy-wide real resource constraints, in which case no equilibrium would exist.

If nominal prices are predetermined, a non-Ricardian fiscal rule without an endogenous default discount factor always results in an overdetermined equilibrium.

In the infinite horizon case too, the fiscal theory of the price level permits the price of money to be determined in an economy without money (if (74) holds).

By freeing up the first-period public debt default discount factor (that is, by letting D_1 be determined endogenously), the non-Ricardian fiscal rule becomes economically well-posed in the infinite-horizon economy, as it does in the finite-horizon case. Again this means that there is no fiscal theory of the price level.

[52] It does not matter for the overdeterminacy result that the monetary equilibrium condition in period t involves the beginning-of-period money stock in the cash-in-advance model, but the end-of-period stock in most representations of the money-in-the-direct-utility function model.

IV3. Equilibrium Under Ricardian and Non-Ricardian Nominal Interest Rate Rules

Under a Ricardian nominal interest rate rule, price level indeterminacy continues to be a property of the equilibrium when nominal prices are flexible. Under a non-Ricardian nominal interest rate rule without default, a unique equilibrium can only be supported only if (74) is satisfied. Again, by allowing the first period default discount factor to become endogenous, the non-Ricardian fiscal rule becomes well-posed. Price level indeterminacy then characterises the equilibrium under the non-Ricardian fiscal rule when nominal prices are flexible.

V. Conclusion

The fiscal theory of the price level has fatal conceptual (or economic) and mathematical (or logical) flaws. The non-Ricardian fiscal rules that are the key element in the fiscal theory, deny the most fundamental property of budget constraints in a market economy: if default is to be ruled out, the budget constraint must apply not only in equilibrium, but for all admissible values of prices and other endogenous variables. This is as true for the government as it is for households and firms. Consequently not all elements in the sequences of real public spending, real net revenues and real seigniorage can be fixed exogenously.

This misunderstanding of the government's intertemporal budget constraint or solvency constraint leads the proponents of the fiscal theory of the price level to specify over-determined (non-Ricardian) fiscal-financial-monetary policy rules. They nevertheless insist that, in equilibrium, the government exactly meets its contractual debt obligations, that is, that the default discount factor on the public debt is unity.

The fiscal approach amounts to the assertion that the general price level can perform the role of the default discount factor on the public debt. The general price level is assumed to adjust the real value of the government's *nominally denominated* contractual debt obligations in such a way that the aggregate real value of all the government's contractual debt obligations equals the present discounted value of current and future real primary surpluses and seigniorage.

In this paper I show that, even if one considers just the government's intertemporal budget constraint in isolation, the proposition that the general price level can, generically, mimic the government debt default discount factor, is incorrect. When one considers the full set of equilibrium conditions, and not just the government's solvency constraint, the fiscal theory of the price level invariably leads either to mathematical inconsistencies or to conceptual anomalies. There is no internally consistent fiscal theory of the price level.

By explicitly introducing an endogenous public debt default discount factor into the model, I show how the overdetermined fiscal-financial monetary programme that is central to the fiscal theory of the price level, determines (jointly with the other equilibrium conditions that characterise a well-posed dynamic general equilibrium model) the *effective* value of the initial public debt (simultaneously with the other endogenous variables). In general, this effective value will be different from the *notional or contractual* value of the public debt. The endogenous default discount factor renders an economy with non-Ricardian fiscal rules economically well-posed and mathematically consistent. It also destroys the fiscal theory of the price level.

The public debt default discount factor approach to the government solvency constraint always produces logically consistent and economically sensible outcomes. Unlike the fiscal theory of the price level, it works the same way when nominal prices are flexible and when nominal price rigidities are present; with finite-horizon and with infinite-horizon models; under any configuration of initial contractual debt obligations; under monetary rules and under nominal interest rate rules; in economies with essential money and in barter economies.

The fiscal theory of the price level is a potentially dangerous fallacy. A theory purporting to show that a government can fix arbitrary real revenue, spending and money issuance programmes, and that the general price level will somehow adjust to make these programmes consistent with the contractual obligations represented by the outstanding stocks of public debt, is not conducive to good policy design.

Adoption of a non-Ricardian fiscal rule could have painful consequences when the Ricardian reality dawns. Consider the case where the original public spending, revenue and seigniorage plans are revealed to imply a discount on the contractual value of the public debt. Something will have to give. If government default does not occur and real primary surpluses are not boosted sharply, the two familiar Ricardian mechanisms linking public debt and inflation, the anticipated and unanticipated inflation taxes, will come into play.

Perhaps scholars cannot be held responsible for the use others make of their ideas. I am not so sure, especially if the ideas can be demonstrated to be wrong. At the very least, great care should be taken to ponder the economic meaning and to check the internal logic of propositions that have powerful policy implications. The potential extra-academic, public policy externalities of the fallacy of the fiscal theory of the price level make it imperative that the debunking of the fallacy be an audible and public affair.

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