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and Unanticipated Money  
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Hypothesis Testing

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## REAL EFFECTS OF ANTICIPATED AND UNANTICIPATED MONEY

### Some Problems of Estimation and Hypothesis Testing

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The paper addresses two issues that arise in estimation and testing of the real effects of anticipated and unanticipated money. First it is shown that identification of the effects of unanticipated (or unperceived) monetary growth on real output is possible only if the *a priori* restriction is imposed that monetary growth does not depend on unanticipated (or unperceived) output. Second, the existing empirical work of Barro and others does not allow for three known channels through which money can affect real variables. These are (1) past and present anticipations of future monetary growth (the inflation tax channel), (2) expectations of monetary growth in a given period conditioned at various preceding dates (the Fischer-Phelps-Taylor effect) and (3) past and present revisions in forecasts of future monetary growth. The presence of the first of these would mean that alternative open-loop monetary growth rules have real effects. The presence of the other two implies that monetary feedback rules can have real effects. Omission of the first channel can lead to biased estimates of the effects of past anticipated monetary growth. Potentially serious observational equivalence problems are associated with the other two.

### 1. Introduction

This paper addresses two issues that arise in the estimation of models of the real effects of anticipated and unanticipated changes in the money supply and in tests of hypotheses in such models. It is motivated by Barro's seminal empirical work on the role of anticipated and unanticipated monetary growth [Barro (1978, 1979), Barro and Rush (1980); see also Gordon (1979), Attfield, Demery and Duck (1981a, b)]. The first issue concerns an identification problem. The observational equivalence is established of models in which a real variable (say, output) is a function of unanticipated

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money growth and models in which money growth is a function of unanticipated output, e.g., via a policy reaction function or through a response of the private banking system. Identification of the effects of current unanticipated (or unperceived) monetary growth on real output is possible only if the *a priori* restriction is imposed that monetary growth does not depend on current unanticipated (or unperceived) output. The restriction that there be no effects of unanticipated output on money growth is quite distinct from the restriction(s) required for the identification of the effects of past anticipated money on output [see, e.g., Barro (1978, 1979)].

The second issue remains even if reliable time series on anticipated and unanticipated monetary growth are somehow available. It concerns the ways in which anticipated and unanticipated monetary growth can enter a 'semi-reduced form'<sup>1</sup> equation for output or any other real variable. Barro's specification of the output equation includes only current and lagged actual monetary growth and a distributed lag on unperceived contemporaneous monetary growth.<sup>2</sup> The literature suggests at least three further channels through which money affects real output. (1) Past and present anticipated *future* monetary growth. (2) Past and present revisions in forecasts of monetary growth. (3) A more general specification of unanticipated money, with expectations of monetary growth in a given period conditioned at various preceding dates — a Fischer (1977)-type hypothesis.

If anticipated future monetary growth affects real output, alternative fixed (open-loop) monetary growth paths will be associated with different paths for real output. If one of the other two channels is operative, alternative flexible (closed-loop) money supply rules or feedback rules will be associated with different distribution functions for real output. Omission of the first channel may lead to biased estimates of the output effects of past anticipated monetary growth. The other two channels are likely to be very difficult to identify. The issue of the observational equivalence of natural and unnatural (*sic*) rate theories of macroeconomics was first addressed by Sargent (1976). McCallum (1979) showed that there were circumstances in which Sargent's general observational equivalence proposition did not rule out the construction of tests capable of discriminating between classical and Keynesian hypotheses. Barro (1979) provides a penetrating discussion of the observational equivalence problem and of the kinds of *a priori* information (generally cross-equation overidentifying restrictions on the reduced form parameters) required for testing classical against Keynesian hypotheses. This paper confirms the conclusion reached by Sargent (1976) and others that a change, during the sample period or the forecasting interval, in the stochastic

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process governing the policy variables will often be required in order to establish the presence or absence of a role for stabilization policy.

## 2. The identification of the effects of anticipated and unanticipated monetary growth on output

### 2.1. Monetary growth independent of anticipated and unanticipated output

A simplified version of Barro's model of the effect of anticipated and unanticipated money on output is given in eqs. (1), (2) and (3),

$$Y_t = a_1 X_t + b_1 (\hat{m}_t - \hat{m}_{1,t}) + c_1 \hat{m}_t + u_{1,t} \quad (1)$$

$$\hat{m}_t = a_2 X_t + u_{2,t} \quad (2)$$

$$\Sigma = E((u_{1,t}, u_{2,t})(u_{1,t}, u_{2,t})) = \begin{bmatrix} \sigma_{u_1}^2 & 0 \\ 0 & \sigma_{u_2}^2 \end{bmatrix} \quad (3)$$

$Y$  is real output,  $X$  a vector of regressors and  $\hat{m}_t$  the actual rate of growth of money.

$u_1$  and  $u_2$  are serially uncorrelated normally distributed random variables with means zero and constant variances. They are also assumed to be contemporaneously independent — a necessary condition for the identification of the effect of anticipated monetary growth. The  $X$  variables may be stochastic but are distributed independently of the current and future values of the vector of disturbances  $u_t = (u_{1,t}, u_{2,t})'$ , in the sense that  $\text{cov}(u_t, X_{t-s}) = 0$  for all  $t$  and all  $s \geq 0$ . Thus  $X_t$  cannot include current endogenous variables but may include lagged endogenous variables (predetermined variables) as well as exogenous variables. Lagged actual and anticipated monetary growth rates will always be entered as separate arguments in the equations although they constitute proper regressors according to our definition. Thus  $X_t$  could include such variables as *FEDV*<sup>3</sup> and *ML*<sup>4</sup> and *MINW*<sup>5</sup> in the original Barro papers. Agents forming expectations are assumed to know the true structure of the model ( $a_1, a_2, b_1, c_1$  and  $\Sigma$  in the current example).  $X_t$  is also assumed to be known when expectations of  $\hat{m}_t$  are formed in period  $t$  but  $Y_t, \hat{m}_t, u_{1,t}$  and  $u_{2,t}$  are unobserved until period  $t+1$ .  $\hat{m}_{1,t}$  is the rate of growth of money in period  $t$ , anticipated in period  $\tau$ .  $\hat{m}_t - \hat{m}_{1,t}$  is therefore the currently unperceived part of current period monetary growth. Eq. (1) is the output equation; (2) describes

<sup>3</sup>Real Federal expenditure relative to normal.

<sup>4</sup>The military personnel/conscription variable.

<sup>5</sup>The minimum wage variable.

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the money supply process. For simplicity only the current innovation in the money supply process and current actual monetary growth are assumed to be arguments in the output equation.<sup>6</sup>

The assumption of rational expectations means that eqs. (1), (2) and (3) are known to private agents when they infer the growth of the money supply.  $u_{1t}$ ,  $u_{2t}$ ,  $Y_t$  and  $\dot{m}_t$  are assumed unknown when the current expectations of money supply growth is formed. Thus, assuming that anticipations are mathematical expectations conditional on the available information set,  $I_t$ , which consists of the model and  $X_t$ , we have

$$m_{t|t} = E(\dot{m}_t | I_t) = a_2 X_t \quad (4)$$

The reduced form of the model of (1), (2) and (3) is

$$Y_t = \beta_1 X_t + v_{1t} \quad (5)$$

$$\dot{m}_t = \beta_2 X_t + v_{2t} \quad (6)$$

$$\beta_1 = a_1 + c_1 a_2 \quad (7)$$

$$\beta_2 = a_2 \quad (8)$$

$$v_{1t} = (b_1 + c_1)u_{2t} + u_{1t} \quad (9a)$$

$$v_{2t} = u_{2t} \quad (9b)$$

$$\begin{aligned} \Omega_1 &= E((v_{1t}, v_{2t})(v_{1t}, v_{2t})) \\ &= \begin{bmatrix} \sigma_{v_1}^2 & \sigma_{v_1 v_2} \\ \sigma_{v_1 v_2} & \sigma_{v_2}^2 \end{bmatrix} = \begin{bmatrix} (b_1 + c_1)^2 \sigma_{u_2}^2 + \sigma_{u_1}^2 & (b_1 + c_1) \sigma_{u_2}^2 \\ (b_1 + c_1) \sigma_{u_2}^2 & \sigma_{u_2}^2 \end{bmatrix}. \end{aligned} \quad (9c)$$

The effect of anticipated money on output,  $\partial y_t / \partial \dot{m}_t + \partial y_t / \partial \dot{m}_{t-1}$ , is given by  $c_1$ , the effect of unanticipated money on output,  $\partial y_t / \partial \dot{m}_t | \dot{m}_{t-1}$ , by  $b_1 + c_1$ . The effect of unanticipated money can be obtained from  $\Omega_1$ , the variance-

<sup>6</sup>Barro includes a distributed lag function on actual and unanticipated monetary growth in (1), i.e., (1) is replaced by

$$Y_t = a_1 X_t + \sum_{i=0}^T b_{1,i} (\dot{m}_{t-i} - \dot{m}_{t-i|t-i}) + \sum_{i=0}^T c_{1,i} \dot{m}_{t-i} + u_{1t}. \quad (1)$$

It is true that even in a model in which only unanticipated money has real effects, these effects can be distributed over time, e.g., because the monetary surprises are 'built into' changes in the capital stock or in inventories. If such were the case and if neither the lagged capital stock nor lagged inventories are included as arguments in the reduced form equation for output, this equation should include an infinite distributed lag on past monetary innovations, not a finite order lag as in Barro.

covariance matrix of the reduced form disturbances:  $b_1 + c_1 = \sigma_{v_1 v_2} / \sigma_{v_2}^2$ .<sup>7</sup> Note that this requires independence of the structural disturbances  $u_{1t}$  and  $u_{2t}$ . To identify the effect of anticipated money, however, further *a priori* restrictions are required. Since we can obtain consistent and asymptotically efficient estimates of  $\beta_1 = a_1 + c_1 a_2$  and  $\beta_2 = a_2$ , an exclusion restriction permitting the identification of  $c_1$  is that  $a_1 = 0$ . Barro's exclusion restriction that government purchases do not affect real output falls in this category [Barro (1979)].<sup>8</sup>

## 2.2. Monetary growth dependent on anticipated and unanticipated output

The structure of eqs. (1), (2) and (3) is observationally equivalent to the model of eqs. (10), (11) and (3),

$$Y_t = a_1 X_t + u_{1t} \quad (10)$$

$$\dot{m}_t = a_2 X_t + b_2 (Y_t - \hat{Y}_{t|t}) + c_2 Y_t + u_{2t}. \quad (11)$$

Neither anticipated nor unanticipated money affect output but monetary growth responds both to anticipated and unanticipated output. Such a money supply response could reflect either the behaviour of the authorities through a monetary policy reaction function or the response of the private banking system to changes in the demand for money due to anticipated and unanticipated changes in income and associated changes in interest rates. A positive value of  $b_2 + c_2$  or of  $c_2$  can be interpreted as 'leaning with the wind' by the monetary authorities: with an unchanged monetary policy stance an (unanticipated) increase in output would tend to raise interest rates. The money supply expands to counteract this. Negative values of  $b_2 + c_2$  or of  $c_2$  could indicate a policy of 'leaning with the wind'. Note that  $\hat{Y}_{t|t} \equiv E(Y_t | I_t) = a_1 X_t$ . The reduced form of (10), (11) and (3) is

$$Y_t = \alpha_1 X_t + \eta_{1t} \quad (12)$$

$$\dot{m}_t = \alpha_2 X_t + \eta_{2t} \quad (13)$$

$$\alpha_1 = a_1 \quad (14a)$$

$$\alpha_2 = a_2 + c_2 a_1 \quad (14b)$$

<sup>7</sup>Consistent and asymptotically efficient estimates of  $\beta_1$ ,  $\beta_2$  and  $b_1 + c_1$  can be obtained by estimating (5) and (6) with an unrestricted variance-covariance matrix of the reduced form disturbances.

<sup>8</sup>It is of course not necessary that the entire vector  $a_1$  equal zero. Let  $X_t$  be an  $N$ -component vector  $X_t = [X_{t1}, \dots, X_{tN}]'$  and let  $a_1 = [a_{11}, \dots, a_{1N}]'$  and  $a_2 = [a_{21}, \dots, a_{2N}]'$ . Given estimates of  $\beta_1 = [a_{11} + c_1 a_{21}, \dots, a_{1N} + c_1 a_{2N}]'$  and  $\beta_2 = [a_{21}, \dots, a_{2N}]'$ ,  $c_1$  is identified provided  $a_{1j} = 0$  for some  $j = 1, \dots, N$ .

$$\Psi_{1t} = [1 - (b_1 + c_1)(b_2 + c_2)]^{-1} [u_{1t} + (b_1 + c_1)u_{2t}], \tag{17c}$$

$$\Psi_{2t} = [1 - (b_1 + c_1)(b_2 + c_2)]^{-1} [(b_2 + c_2)u_{1t} + u_{2t}], \tag{17d}$$

$$\Omega_3 = E((\Psi_{1t}, \Psi_{2t})'(\Psi_{1t}, \Psi_{2t}))$$

$$= \begin{bmatrix} \sigma_{\Psi_1}^2 & \sigma_{\Psi_1\Psi_2} \\ \sigma_{\Psi_1\Psi_2} & \sigma_{\Psi_2}^2 \end{bmatrix} = [1 - (b_1 + c_1)(b_2 + c_2)]^{-2} \times \begin{bmatrix} \sigma_{u_1}^2 + (b_1 + c_1)^2\sigma_{u_2}^2 & (b_2 + c_2)\sigma_{u_1}^2 + (b_1 + c_1)\sigma_{u_2}^2 \\ (b_2 + c_2)\sigma_{u_1}^2 + (b_1 + c_1)\sigma_{u_2}^2 & (b_2 + c_2)^2\sigma_{u_1}^2 + \sigma_{u_2}^2 \end{bmatrix}. \tag{17e}$$

The general structural model of eqs. (1), (11) and (3) in which both unanticipated and anticipated money affect real output ( $b_1 + c_1 \neq 0$  and  $c_1 \neq 0$  respectively) is observationally equivalent to a large set of sub-models with widely differing implications for the conduct of monetary policy. While these sub-models are special cases of the general structural model they are not nested either in each other or in the general model. The most interesting cases are the following:

- (a) Only anticipated money affects real output ( $c_1 \neq 0$  and  $b_1 + c_1 = 0$ ).
- (b) Anticipated money has the same effect on output as unanticipated money ( $c_1 = b_1 + c_1$  or  $b_1 = 0$ ).
- (c) Only unanticipated money affects real output ( $c_1 = 0$  and  $b_1 + c_1 \neq 0$ ).
- (d) Neither anticipated nor unanticipated money affect real output ( $b_1 = c_1 = 0$ ).

For simplicity and without loss of generality for the observational equivalence propositions, we assume in what follows that  $X_t$  (and therefore  $a_1$  and  $a_2$ ) is a scalar. If both anticipated and unanticipated output can affect money ( $c_2 \neq 0$ ,  $b_2 + c_2 \neq 0$ , a model in which both anticipated and unanticipated money can have (possibly distinct) effects on output ( $c_1 \neq 0$ ,  $b_1 + c_1 \neq 0$ ) is observationally equivalent to a model in which only anticipated money affects output ( $c_1 \neq 0$ ;  $b_1 + c_1 = 0$ ) and to a model in which anticipated and unanticipated money have the same effect on output ( $b_1 = 0$ ). A model in which only anticipated money affects output ( $c_1 \neq 0$ ,  $b_1 + c_1 = 0$ ) is observationally equivalent to a model in which neither anticipated nor unanticipated money have real effects ( $b_1 = c_1 = 0$ ).

*A priori* knowledge of the value of  $a_1$  is in some cases sufficient to discriminate between, on the one hand, 'Both anticipated and unanticipated money matter' ( $c_1 \neq 0$ ,  $b_1 + c_1 \neq 0$ ) or 'Only anticipated money matters'

$$\eta_{1t} = u_{1t}, \tag{14c}$$

$$\eta_{2t} = (b_2 + c_2)u_{1t} + u_{2t}, \tag{14d}$$

$$\Omega_2 = E((\eta_{1t}, \eta_{2t})'(\eta_{1t}, \eta_{2t})) = \begin{bmatrix} \sigma_{\eta_1}^2 & \sigma_{\eta_1\eta_2} \\ \sigma_{\eta_1\eta_2} & \sigma_{\eta_2}^2 \end{bmatrix} = \begin{bmatrix} \sigma_{u_1}^2 & (b_2 + c_2)\sigma_{u_1}^2 \\ (b_2 + c_2)\sigma_{u_1}^2 & (b_2 + c_2)^2\sigma_{u_1}^2 + \sigma_{u_2}^2 \end{bmatrix}. \tag{14e}$$

The model of (1), (2) and (3) is observationally equivalent to the model of (10), (11) and (3): their reduced forms cannot be distinguished [compare (5)–(9) with (12)–(14)] and will yield identical likelihood functions for the endogenous variables.

For the model of (10), (11) and (13) the effect of unanticipated output on money growth is identified *via*  $\Omega_2$  as  $b_2 + c_2 = \sigma_{\eta_1\eta_2}/\sigma_{\eta_1}^2$ . The effect of anticipated output on money can then be identified from  $\alpha_1$  and  $\alpha_2$  if, e.g., an exclusion restriction is imposed on one of the elements of  $a_2$ . If the true model is given by (10) and (11) but the econometrician mistakenly believes the true model to be (1) and (2), what is thought to be an estimate of  $b_1 + c_1$ , using the estimated variance-covariance matrix of the reduced form  $\hat{\sigma}_{\eta_1\eta_2}/\hat{\sigma}_{\eta_1}^2$  is neither that nor an estimate of  $b_2 + c_2$ , which would be given by  $\hat{b}_2 = \hat{\sigma}_{\eta_1\eta_2}/\hat{\sigma}_{\eta_1}^2$ .

### 2.3. Observational equivalence in a more general model

Consider the general model that permits, in principle, dependence of output on both anticipated and unanticipated money and dependence of monetary growth on both anticipated and unanticipated output,

$$Y_t = a_1 X_t + b_1(\hat{m}_t - \hat{m}_{t|t-1}) + c_1 \hat{m}_t + u_{1t}, \tag{1}$$

$$\hat{m}_t = a_2 X_t + b_2(Y_t - \hat{Y}_{t|t-1}) + c_2 Y_t + u_{2t}. \tag{11}$$

The disturbance terms are again as in (3).

With rational expectations the reduced form of this model is given by

$$Y_t = \delta_1 X_t + \Psi_{1t}, \tag{15}$$

$$\hat{m}_t = \delta_2 X_t + \Psi_{2t}, \tag{16}$$

$$\delta_1 = (a_1 + c_1 a_2)/(1 - c_1 c_2), \tag{17a}$$

$$\delta_2 = (a_2 + c_2 a_1)/(1 - c_1 c_2), \tag{17b}$$

( $c_1 \neq 0, b_1 + c_1 = 0$ ) and, on the other hand, 'Only unanticipated money matters' ( $c_1 = 0, b_1 + c_1 \neq 0$ ) or 'Neither anticipated nor unanticipated money matter' ( $b_1 = c_1 = 0$ ). If the hypothesis  $\delta_1 = a_1$  is consistent with the data, ( $c_1 \neq 0, b_1 + c_1 = 0$ ), ( $c_1 \neq 0, b_1 + c_1 = 0$ ) or ( $b_1 = 0, c_1 \neq 0$ ) can be correct only if either  $a_1 = -a_2/c_2$  or  $c_1 = 0$ . If  $c_1 = 0$ , anticipated money does not affect real output. If  $a_1 = -a_2/c_2, \delta_2 = 0$ , a hypothesis that can be tested. Thus if  $\delta_1$  is not significantly different from  $a_1$  and  $\delta_2$  is significantly different from zero,  $c_1 = 0$  is accepted. Even if this hypothesis is accepted, we cannot further discriminate between the hypothesis that only unanticipated money affects real output ( $c_1 = 0, b_1 + c_1 \neq 0$ ) and the hypothesis that neither anticipated nor unanticipated money affect real output ( $b_1 = c_1 = 0$ ) unless we impose the *a priori* constraint that unanticipated output does not affect monetary growth ( $b_2 + c_2 = 0$ ). Given that further restriction the variance-covariance matrix of the reduced form disturbances (17e), will be a diagonal matrix if  $b_1 + c_1 = 0$ .

Thus, starting from the general model of eqs. (1), (11) and (3) we can identify both the effect of anticipated money on output ( $c_1$ ) and the effect of anticipated output on money ( $c_2$ ) if we have two independent restrictions on the reduced form coefficients  $\delta_1$  and  $\delta_2$ . To identify just  $c_1$ , the exclusion restriction  $a_1 = 0$  is sufficient. This is of course the standard identification problem, familiar from partial equilibrium demand and supply analysis, when one does not encounter the additional problem of innovations from one process entering as arguments into another process. If innovations in either process can enter as an argument in the other, the identification problem is compounded. Evidence other than the time series properties of  $\dot{m}_t, Y_t$  and  $X_t$  is required to establish the validity of the restriction that there is no effect of unanticipated output on monetary growth, a restriction required for the identification of the effect of unanticipated money on output.

#### 2.4. Lagged money and output innovations

In empirical applications the output equation given in (1) has been modified to include current and lagged anticipated and unanticipated money as arguments. The rationale for this is inertia in the real output process due to costs of adjustment (capital stock, inventories, quasi-fixed labour etc.), or to lags in the perception of new information [see, e.g., Lucas (1975)].<sup>9</sup> Note that is the presence of such real costs of adjustment the anticipated real rate of return on money balances, which is a function of the anticipated component of the money supply process, should affect the probability density function of real output. The empirical importance of this 'inflation tax' argument would be reflected in the coefficients on current and lagged anticipated future monetary growth. (See section 3.1 below.) Monetary growth can also, in principle, be a function of current and lagged anticipated

<sup>9</sup>But see footnote 6 for a brief discussion of a problem associated with this specification.

and unanticipated output. The monetary reaction function can incorporate *measurement or perception* lags (the time interval between the occurrence of an event and its observation) and *realisation* lags (the time needed to decide upon and realise a control action) [see Deissenberg (1979)]. Eqs. (18) and (19) are a generalisation of (1) and (11), with  $c_t$  output a function of a  $T$ -period distributed lag on anticipated and unanticipated money and money a function of a  $T$ -period distributed lag on anticipated and unanticipated output. It is assumed that the maximal lag is known *a priori*,

$$Y_t = a_1 X_t + \sum_{i=0}^{T-1} b_{1,i} (\dot{m}_{t-i} - \hat{m}_{t-i}) + \sum_{i=0}^{T-1} c_{1,i} \dot{m}_{t-i} + u_{1t}, \quad (18)$$

$$\dot{m}_t = a_2 X_t + \sum_{i=0}^{T-1} b_{2,i} (Y_{t-i} - \hat{Y}_{t-i}) + \sum_{i=0}^{T-1} c_{2,i} Y_{t-i} + u_{2t}. \quad (19)$$

$u_{1t}$  and  $u_{2t}$  are as before.

The presence of the distributed lag terms does alter the conditions for identification. A general discussion, including a detailed analysis of the case where  $T=1$  is given in Buiter (1980b). The main result is that when output is a function of lagged as well as current anticipated and unanticipated monetary growth, the identification of the effect of current anticipated and monetary growth is 'easier' than when only current anticipated and unanticipated monetary growth are included as arguments in the output equation. The identification of the effect of current money growth innovations, however, is 'more difficult' than before. It still requires the *a priori* restriction that current unanticipated output does not affect monetary growth, but this is no longer sufficient for identification. When  $T=1$ , the simplest sufficient condition appears to be that current unanticipated output does not affect monetary growth and that anticipated and unanticipated lagged output have the same effect on monetary growth.

Underlying the analysis of the conditions for identification of the effects of anticipated and unanticipated monetary growth, is the assumption that the maximal orders of the distributed lags are known *a priori*. If anticipated and unanticipated money and output enter their respective equations with distributed lags of unknown order, or if the  $u_{it}$  are not white noise but instead follow a general ARIMA process, identifying and estimating the response of output to money growth and tests of the relevant hypotheses become much more difficult. The maximal orders of the distributed lags must be inferred from the data and the lagged endogenous variables are less informative from the point of view of identification [see Sims (1980) and Wallis (1980)].

The identification of the output effects of unanticipated monetary growth is essential for the empirical confirmation of the new monetary theories of

the business cycle [see, e.g., Lucas (1977)]. These theories require both that anticipated monetary growth has no effect on deviations of actual from capacity output and that monetary innovations account for a significant fraction of the variance of real output. Failure to identify the output effects of unanticipated money would make the new monetary theories of the business cycle untestable.

### 3. Anticipated and unanticipated money in the output equation and the role of monetary policy

In this section of the paper I show that even if an effect of unanticipated output on monetary growth can be ruled out *a priori*, there remain a number of serious problems with the interpretation of the results obtained by Barro et al., especially as regards their implications for the role of monetary policy. The central issue is the observational equivalence between models for which the density function of real output is invariant under alternative deterministic (and known) feedback rules for the money stock and models for which this invariance property does not hold.

The output eq. (18) used by Barro et al. is a restrictive special case which omits at least three potentially important transmission mechanisms for monetary policy. Various not implausible structural macromodels will yield 'semi-reduced form' output equations that include anticipated and unanticipated money in a number of ways not included in eq. (18). A more general output equation is given in (20),

$$Y_t = a_1 X_t + \sum_{i=0}^{T_1} b_{ij} [\dot{m}_{t-i} | I_{t-i-j}] + \sum_{i=0}^{T_1} c_i \dot{m}_{t-i} \\ + \sum_{k=1}^Q \sum_{i=0}^{T_2} d_{ijk} [E(\dot{m}_{t-j+i} | I_{t-j}) - E(\dot{m}_{t-j+i} | I_{t-j-k})] \\ + \sum_{i=0}^{T_3} e_{ij} E(\dot{m}_{t+i} | I_{t-j}) + u_{1t}. \quad (20)$$

#### 3.1. Anticipated future monetary growth

Anticipated future inflation and therefore anticipated future monetary growth will affect the density function of real output in equilibrium models if output is a function of the real stock of capital and money is not superneutral [Tobin (1965) and Fischer (1979)]. This 'Tobin effect' affects actual output and capacity output equally. It therefore concerns the structural or allocative role of monetary policy rather than its stabilization role. There are two ways of allowing for such an effect in empirical work. One is to keep actual output as the dependent variable and include a term

like  $\sum_{i=0}^{T_3} \sum_{j=0}^{S_3} e_{ij} E(\dot{m}_{t+i} | I_{t-j})$  on the right-hand side. The other is to re-specify the output equation with the difference between actual and capacity output as the dependent variable. In that case the measure of capacity output should be constructed in such a way as to allow for the possible presence of a Tobin effect. Existing empirical work [e.g., Barro (1978), Barro and Rush (1980)] which has actual output as the dependent variable and allows for changes in capacity output merely by including a time trend among the regressors precludes the consideration of the Tobin effect. The question of the direction and magnitude of the bias imparted to estimates of  $b_{ij}$  and  $c_i$  by the omission of anticipated future monetary growth as an explanatory variable if  $e_{ij} \neq 0$  for some  $i$  and  $j$  is an 'omitted variables' problem. [See, e.g., Maddala (1979, pp. 155-157) and Schmidt (1976, pp. 39-40).] It depends both on the magnitudes of the  $e_{ij}$  and on the correlation between past anticipated and unanticipated monetary growth and future anticipated monetary growth. In the quarterly model of Barro and Rush (1980), monetary growth depends (among other things) on its own past values at lags one through six. The matrix of regression coefficients from the 'auxiliary' regression of anticipated future monetary growth on past monetary growth is therefore unlikely to be the zero matrix.

#### 3.2. A distributed lag on forecast horizons

In all empirical work except for Fischer (1980), unanticipated monetary growth is represented by a distributed lag on unperceived contemporaneous monetary growth.<sup>10</sup> In (20) this is generalized to prediction errors from forecasts of  $\dot{m}_{t-i}$ ;  $i=0, 1, \dots, T_1$ , made at the beginning of period  $t-i$  and in earlier periods

$$t-i-j, j=1, 2, \dots, S_1 \quad \text{or} \quad \sum_{i=0}^{T_1} \sum_{j=0}^{S_1} b_{ij} (\dot{m}_{t-i} - E(\dot{m}_{t-i} | I_{t-i-j})).$$

Such a reduced form could emerge if the money wage in period  $t$  had to be set in open-loop or unconditional fashion in a period before  $t$  [see, e.g., Fischer (1977), Phelps and Taylor (1977), Taylor (1979)].

With a simple example I will show that models in which  $b_{ij} \neq 0$  for some  $j > 0$  ('Phelps-Fischer-Taylor' or P.F.T. models) can under certain conditions be observationally equivalent to models in which  $b_{ij} = 0$ ,  $j \neq 0$  ('Barro models'). This can be so, regardless of whether  $c_i = 0$ ,  $i=1, \dots, T_1$ , that is regardless of whether or not past anticipated monetary growth affects output. This issue is of some interest because if  $b_{ij} > 0$  for some  $j > 0$ , the second

<sup>10</sup>In practice this is approximated by prediction errors from forecasts, made at the beginning of the unit period of analysis, of monetary growth during that period. The forecasting equations typically have been estimated using observations from the entire sample period, i.e., including data generated in periods beyond the date at which the forecasts are made.

moment of the density function of real output will not be invariant under changes in the deterministic components of monetary feedback rules.

In the example  $T_1 = 0$  and  $S_1 = 1$ ;  $G_t$  denotes real public spending.

$$Y_t = a_{11}G_t + b_{00}(\dot{m}_t - \dot{m}_{t-1}) + b_{01}(\dot{m}_t - \dot{m}_{t-1}) + c_0\dot{m}_t + c_1\dot{m}_{t-1} + u_{1t} \quad (21)$$

$$\dot{m}_t = a_{21}G_t + a_{22}\dot{m}_{t-1} + u_{2t} \quad (22)$$

Eq. (21) is a P.F.T. equation. The reduced form equations of this model are given by (22) and (23)

$$\begin{aligned} Y_t = & (a_{11} + c_0a_{21})G_t + (c_0a_{22} + c_1)\dot{m}_{t-1} + u_{1t} + (c_0 + b_{00} + b_{01})u_{2t} \\ & + b_{01}a_{22}u_{2t-1} + b_{00}a_{21}(G_t - \hat{G}_{t|t}) + b_{01}a_{22}a_{21}(G_{t-1} - \hat{G}_{t-1|t-1}) \\ & + b_{01}a_{21}(G_t - \hat{G}_{t|t-1}). \end{aligned} \quad (23)$$

Now consider a Barro version of eq. (20) with  $T_1 = 1$  and  $S_1 = 0$ ,

$$\begin{aligned} Y_t = & a_{11}G_t + b_{00}(\dot{m}_t - \dot{m}_{t|t}) \\ & + b_{10}(\dot{m}_{t-1} - \dot{m}_{t-1|t-1}) + c_0\dot{m}_t + c_1\dot{m}_{t-1} + u_{1t}. \end{aligned} \quad (24)$$

The reduced form equation for output under the Barro hypothesis is, using (22)

$$\begin{aligned} Y_t = & (a_{11} + c_0a_{21})G_t + (c_0a_{22} + c_1)\dot{m}_{t-1} + u_{1t} + (c_0 + b_{00})u_{2t} \\ & + b_{10}u_{2t-1} + b_{00}a_{21}(G_t - \hat{G}_{t|t}) + b_{10}a_{21}(G_{t-1} - \hat{G}_{t-1|t-1}). \end{aligned} \quad (25)$$

One problem with a money growth equation such as (22) is that by permitting  $\dot{m}_t$  to respond to the current value of the fiscal variable,  $G_t$ , the authorities are given an informational advantage over the private sector, unless  $G_t = \hat{G}_{t|t}$ . A role for stabilization policy then emerges even in the Barro model as can be seen by noting that in eq. (25) the coefficients on  $G_t - \hat{G}_{t|t}$  and  $G_{t-1} - \hat{G}_{t-1|t-1}$  are functions of the policy parameter  $a_{21}$ . To avoid this problem, I shall assume that for all  $t$ ,  $G_t \equiv \hat{G}_{t|t}$ . Almost identical conclusions emerge if it is assumed that  $G_t \neq \hat{G}_{t|t}$  but  $G_{t-1}$  or  $\hat{G}_{t-1}$  is substituted for  $G_t$  in (22).

Leaving aside comparisons of alternative policy regimes (specifically a change in  $a_{22}$ ), we can hope to discriminate between the P.F.T. eq. (21) with  $b_{10} = 0$  and  $b_{01} \neq 0$  and the Barro eq. (24) with  $b_{10} \neq 0$  and  $b_{01} = 0$  only through the presence of the term  $G_t - \hat{G}_{t|t-1}$  in (23), the reduced form P.F.T.

equation. Consider the assumption made by Fischer (1980, p. 235): 'Finally, the exogenous variables *FEDV*, *MIL* and *MINW* were assumed known with perfect foresight'. With  $G_t = \hat{G}_{t|t-1}$  (and  $G_t = \hat{G}_{t|t}$ ) (23) and (25) are observationally equivalent.

Note that the Barro-type exclusion restriction  $a_{11} = 0$ , which permits the identification of  $c_0$  and  $c_1$ , is not sufficient for distinguishing between P.F.T. and Barro models. While the  $a_{11} = 0$  restriction is necessary for the identification of the effects of *past* anticipated monetary growth, it is neither necessary nor sufficient to resolve the observational equivalence of a distributed lag on 'contemporaneous' forecast errors ( $b_{10} \neq 0$  and  $b_{01} = 0$  in our example) and forecast errors from forecasts of monetary growth in a given period, made contemporaneously and at various earlier dates ( $b_{10} = 0$  and  $b_{01} \neq 1$  in our example). This can be seen immediately by setting  $a_{11} = 0$  in (23) and (25). Indeed, even if one were to accept the further hypothesis that anticipated monetary growth does not affect output ( $c_0 = c_1 = 0$ ) the problem of observational equivalence of the Barro and P.F.T. models remains. This can be seen by setting  $a_{11} = c_0 = c_1 = 0$  in (23) and (25), which yields (again assuming that  $G_t = \hat{G}_{t|t}$ ),

$$Y_t = u_{1t} + (b_{00} + b_{01})u_{2t} + b_{01}a_{22}u_{2t-1} + b_{01}a_{21}(G_t - \hat{G}_{t-1|t-1}), \quad (23')$$

$$Y_t = u_{1t} + b_{00}u_{2t} + b_{10}u_{2t-1}. \quad (25')$$

With  $G_t = \hat{G}_{t|t-1}$  eqs. (23') and (25') are observationally equivalent. Yet in (23') the distribution function of  $Y_t$  depends on a known parameter of the monetary feedback rule,  $a_{21}$ , while in (25') the deterministic part of the monetary rule is irrelevant for the behaviour of real output. The only way to discriminate between (23') and (25') is along the lines suggested by Sargent (1976). If a change in the monetary regime (a different value of  $a_{22}$ ) is known to have occurred at some specific point in time, (25') would be invariant under such a change while (23') would be affected.

The foregoing argument does not mean that it is never possible to discriminate between Barro and P.F.T. models in a given policy regime. It does mean that discrimination between the two models requires restrictions on the stochastic processes governing the regressors (other than  $\dot{m}_{t-1}$ ) in the money growth equation —  $G_t$  in our example. If  $G_t$  can be predicted exactly on the basis of information available at the beginning of period  $t-1$ , as we assumed in our example, observational equivalence is inescapable. More generally,  $G_t - \hat{G}_{t|t-1} = \varepsilon_t$ , where  $\varepsilon_t$ , the public spending forecast error, has the property that  $E(\varepsilon_t | I_{t-1}) = 0$ . Eq. (23') becomes (23'') while eq. (25') remains unchanged,

$$Y_t = u_{1t} + (b_{00} + b_{01})u_{2t} + b_{01}a_{22}u_{2t-1} + b_{01}a_{21}\varepsilon_t. \quad (23'')$$

Note that  $a_{21} \neq 0$  is a necessary condition for discriminating between Barro and P.F.T. models.  $a_{11} = 0$  is neither necessary nor sufficient.

Let  $\varepsilon_t$  be given by

$$\varepsilon_t = \pi_1 u_{1t} + \pi_2 u_{2t} + \pi_3 u_{2t-1} + \pi_4 \zeta_4.$$

where  $E(\zeta_i | u_{1t}, u_{2t}, u_{2t-1}) = 0$  and  $\pi_i, i = 1, \dots, 4$ , are constants. Remember that  $u_{2t-1}$  does not belong to  $I_{t-1}$ . Eq. (23'') now becomes

$$Y_t = (1 + b_{01} a_{21} \pi_1) u_{1t} + (b_{00} + b_{01} (1 + a_{21} \pi_2)) u_{2t} \\ + b_{01} (a_{22} + a_{21} \pi_3) u_{2t-1} + b_{01} a_{21} \pi_4 \zeta_4. \quad (23''')$$

If the public spending forecast error  $\varepsilon_t$  can be written as an exact linear combination of  $u_{1t}$ ,  $u_{2t}$  and  $u_{2t-1}$ , i.e., if  $\pi_4 = 0$ , eqs. (25) and (23'') will be observationally equivalent. If  $\pi_4 \neq 0$  (and  $a_{21} \neq 0$ ) finding a non-zero coefficient on  $G_t - \hat{G}_{t|t-1}$  in the reduced form output equation permits the rejection of the Barro hypothesis.

A final problem that arises in tests of the P.F.T. model against the Barro model relates to the output effects of public spending. Even if one is willing to grant Barro's exclusion restriction that anticipated (or actual) public spending (*FEDV*) does not belong in the 'structural' output eqs. (21) and (24), i.e., that  $a_{11} = 0$ , unanticipated public spending (or deviations of *FEDV* from normal) may well belong in the structure. If  $G_t - \hat{G}_{t|t-1}$  enters the structural eqs. (21) and (24) *directly*, we can no longer identify its *indirect* presence through  $\hat{m}_t - \hat{m}_{t-1}$  in the reduced form of (21).

### 3.3. Revisions in forecasts of future money supplies

The third transmission mechanism of monetary policy that is omitted in the work of Barro et al. is represented by the term

$$\sum_{k=1}^Q \sum_{j=0}^{S_2} d_{tjk} [E(\hat{m}_{t-j+i} | I_{t-j}) - E(\hat{m}_{t-j+i} | I_{t-j-k})]$$

in (20). Current output may be a function of *revisions* in forecasts of future monetary growth rates (or of future levels of the money stock).<sup>11</sup> If such a transmission channel is present, alternative deterministic monetary feedback rules relating the current money stock to information available arbitrarily far in the past will alter the stochastic process governing real variables. [See, e.g., Turnovsky (1980), Weiss (1980), McCallum (1980), Buiter (1980a, 1981a, b),

<sup>11</sup>The change from growth rates to levels of the money stock is of no consequence for the point we aim to establish.

Buiter and Eaton (1980).] In this subsection I show that models in which this transmission mechanism is present are observationally equivalent to models from which it is absent. Note that this issue is quite distinct from the presence or absence of effects of past or future anticipated monetary growth on real variables. I shall therefore make the point using a model in which past anticipated monetary growth has no effect on real output and in which Barro's exclusion restriction — fiscal variables do not effect output — holds. The model consists of eqs. (26) and (27)

$$Y_t = \beta(p_t - \hat{p}_{t|t-1}) + u_t^y, \quad \beta > 0, \quad (26)$$

$$m_t - p_t = \alpha_1 y_t - \alpha_2 (\hat{p}_{t+1|t} - p_t) + u_t^m, \quad \alpha_1, \alpha_2 > 0. \quad (27)$$

Eq. (26) is a Sargent–Wallace supply function, eq. (27) a Cagan-style money demand function.  $p_t$  is the log of the general price level.  $u_t^y$  and  $u_t^m$  are white noise disturbance terms. The model contains the two ingredients necessary for the forecast revision mechanism to be operative: (1) current output is a function of expectations of a future endogenous variable ( $p_{t+1|t}$ ), and (2) current output is a function of expectations conditioned at different points in time ( $\hat{p}_{t+1|t}$  and  $\hat{p}_{t+1|t}$ ). The information set  $I_t$  conditioning forecasts made in period  $t$  is assumed to contain the true model and all current and past endogenous variables, exogenous variables and random disturbances.

Assuming stability we can obtain the following expression for real output [see Buiter (1980a)]:

$$y_t = \frac{\beta}{1 + \alpha_1 \beta + \alpha_2} [m_t - \hat{m}_{t|t-1}] + \frac{\beta \alpha_2}{(1 + \alpha_1 \beta + \alpha_2)(1 + \alpha_2)} \\ \times \sum_{i=0}^{\infty} \left( \frac{\alpha_2}{1 + \alpha_2} \right)^i (\hat{m}_{t+1+i|t} - \hat{m}_{t+1+i|t-1}) + \frac{1}{1 + \alpha_1 \beta + \alpha_2} ((1 + \alpha_2) u_t^y - \beta u_t^m). \quad (28)$$

Thus, current output is a function of  $m_t - \hat{m}_{t|t-1}$  and of the revision, between periods  $t-1$  and  $t$ , in the forecasts for all future money supplies.<sup>12</sup>

If we replace  $\hat{p}_{t+1|t}$  by  $\hat{p}_{t+1|t-1}$  in eq. (27), we lose the feature of the model that current output is a function of expectations conditioned on information available at different points in time. The corresponding output equation is

$$y_t = \frac{\beta}{1 + \alpha_1 \beta + \alpha_2} (m_t - \hat{m}_{t|t-1}) + \frac{1}{1 + \alpha_1 \beta + \alpha_2} ((1 + \alpha_2) u_t^y - \beta u_t^m). \quad (29)$$

In (28) deterministic monetary feedback rules  $m_t = f(I_{t-i}), i \geq 0$ , i.e., both

<sup>12</sup>Again we have shifted from rates of change of  $m$  to levels of  $m$ .

instantaneous and lagged feedback rules will affect the density function of output. In (29) only instantaneous monetary feedback rules or automatic stabilizers  $m_t = f(I_t)$  will affect the density function of output. Yet it is quite easily appreciated that (28) and (29) are observationally equivalent. Both  $m_t - \hat{m}_{t|t-1}$  and all terms such as  $\hat{m}_{t+1+i|t} - \hat{m}_{t+1+i|t-1}$  are functions only of the 'news' accruing between periods  $t-1$  and  $t$ . This 'news' consists of  $u_t^y$  and  $u_t^m$  and the innovation in the money supply process.

Let  $m_t$  be governed by any indeterministic covariance-stationary stochastic process. It may be one element in a vector of jointly indeterministic covariance-stationary random variables. It will have the moving average representation

$$m_t = d(L)\varepsilon_t, \quad (30)$$

$$d(L) = \sum_{j=0}^{\infty} d_j L^j,$$

where  $L$  is the lag operator,  $Lx_t \equiv x_{t-1}$  and without loss of generality,  $d_0 = 1$  ( $\varepsilon_t$  is the sequence of one-step ahead linear least squares forecasting errors (innovations) in predicting  $m_t$  as a function of all past information  $I_{t-1}$ , i.e.,  $\varepsilon_t \equiv m_t - E(m_t | I_{t-1})$ ).  $I_{t-1}$  includes  $m_{t-i}$ ,  $i > 1$  and the values, at date  $t-1$  and before, of all other variables that may Granger-cause  $m_t$ . Note that  $\varepsilon_t$  need not be contemporaneously independent of  $u_t^y$  and  $u_t^m$ . From (30)

$$m_t - \hat{m}_{t|t-1} = \varepsilon_t, \quad \text{and} \quad (31a)$$

$$\hat{m}_{t+1+i|t} - \hat{m}_{t+1+i|t-1} = d_{t+1}\varepsilon_t. \quad (31b)$$

Substituting (31a) and (31b) into (28) and (29) yields

$$y_t = \frac{\beta}{1 + \alpha_1\beta + \alpha_2} \left[ 1 + \sum_{i=0}^{\infty} \left( \frac{\alpha_2}{1 + \alpha_2} \right)^{i+1} d_{t+1} \right] \varepsilon_t + \frac{1}{1 + \alpha_1\beta + \alpha_2} ((1 + \alpha_2)u_t^y - \beta u_t^m), \quad (28')$$

$$y_t = \frac{\beta}{1 + \alpha_1\beta + \alpha_2} \varepsilon_t + \frac{1}{1 + \alpha_1\beta + \alpha_2} ((1 + \alpha_2)u_t^y - \beta u_t^m). \quad (29')$$

(28') and (29') are 'almost always' observationally equivalent.<sup>13</sup> This result holds for any covariance-stationary process for  $m_t$ . Specifically, it will hold for a process in the spirit of Barro such as  $m_t = \mu_1 G_t + \mu_2 m_{t-1} + \eta_t$  where  $G_t$  is

<sup>13</sup> Exceptions would be such as the following: (a) the  $d_t$  are chosen in such a way as to make the first term on the right-hand side of (28') identically equal to zero and  $\varepsilon_t$  is not an exact linear combination of  $u_t^y$  and  $u_t^m$ . (b)  $\varepsilon_t$  is an exact linear combination of  $u_t^y$  and  $u_t^m$  and the  $d_t$  are chosen so as to make the entire right-hand side of (28') equal to zero.

(an element in) a covariance-stationary (vector) process and  $\eta_t$  is white noise. The two classes of models can be distinguished only if a change in the stochastic process governing  $m_t$  (i.e., in the  $d_t$ ) is known to have occurred during the sample or prediction interval.

#### 4. Conclusion

The main conclusions are stated in the introduction. In future empirical work on the real effects of anticipated and unanticipated money the specification and estimation of the money supply process will require even greater attention. The crucial issue of whether unanticipated output affects monetary growth will have to be resolved. Since most empirical work uses monetary aggregates that are wider than the monetary base, the interpretation of the money supply process as a policy reaction function seems overly simple. It may be necessary to model the behaviour of the private banking sector whose liabilities constitute most of  $M_1$ ,  $M_2$  or  $M_3$ . Another surprising (or even worrying) feature of past empirical work on the money supply process is that no structural break has been reported in that process at the time of the demise of the Bretton Woods adjustable peg exchange rate regime.

As regards the output equation, a firm distinction needs to be made between the proposition that anticipated (current and past) money has no real effects ( $c_t = 0$  for all  $t$ ) and the proposition that deterministic monetary feedback rules have no real effects and cannot be used for stabilization purposes. Known contingent monetary rules can influence monetary forecast errors [Fischer (1977)] and revisions in money supply forecasts [Turnovsky (1980), Weiss (1980)]. This may give monetary policy a handle on the real economy. Serious identification problems make the empirical resolution of these issues doubtful.

Finally, changes in anticipated future monetary growth rates will, by altering the anticipated real rate of return on money *vis a vis* real assets, alter the composition and (in the long run) the magnitude of 'full information' real output. The importance of this monetary transmission channel can in principle be empirically evaluated in a Barro-type framework.

The empirical work on anticipated and unanticipated money has not so far brought us much closer to an assessment of the stabilization and structural (or allocative) roles of monetary policy.

#### References

- Attfield, C.L.F., D. Demery and N.W. Duck, 1981a, Unanticipated monetary growth, output and the price level: U.K. 1946-77, *European Economic Review* 16, 367-385.  
 Attfield, C.L.F., D. Demery and N.W. Duck, 1981b, A quarterly model of unanticipated monetary growth and output in the U.K. 1963-78, *Journal of Monetary Economics* 8, Nov., 331-350.

- Barro, Robert J., 1977, Unanticipated money growth and unemployment in the United States, *American Economic Review* 67, March, 101-115.
- Barro, Robert J., 1978, Unanticipated money, output and the price level in the United States, *Journal of Political Economy* 86, Aug., 549-581.
- Barro, Robert J., 1979, The equilibrium approach to business cycles, Nov., unpublished.
- Barro, Robert J. and M. Rush, 1980, Unanticipated money and economic activity, in: S. Fischer, ed., *Rational expectations and economic policy* (University of Chicago Press, Chicago, IL) 23-73.
- Buiter, Willem H., 1980a, Monetary, financial and fiscal policies under rational expectations, *IMF Staff Papers* 27, Dec., 785-813.
- Buiter, Willem H., 1980b, Real effects of anticipated and unanticipated money: Some problems of estimation and hypothesis testing, NBER working paper no. 601, Dec.
- Buiter, Willem H., 1981a, The role of economic policy after the new classical macroeconomics, in: D. Currie, R. Nobay and D. Peel, eds., *Macroeconomic analysis* (Croom Helm, London) 233-285.
- Buiter, Willem H., 1981b, The superiority of contingent rules over fixed rules in models with rational expectations, *Economic Journal* 91, Sept. 647-670.
- Buiter, Willem H. and Jonathan Eaton, 1980, Policy decentralization and exchange rate management in interdependent economies, NBER working paper no. 531, Aug.
- Deissenberg, Christopher, 1979, Optimal stabilisation policy with delayed controls and imperfect state measurements, Fakultät für Wirtschafts-wissenschaften und Statistik, Universität Konstanz, Working paper, Series A, no. 133, Nov.
- Fischer, Stanley, 1977, Long-term contracts, rational expectations and the optimal money supply rule, *Journal of Political Economy* 85, Feb., 191-206.
- Fischer, Stanley, 1979, Capital accumulation on the transition path in a monetary optimizing model, *Econometrica* 47, Nov., 1433-1440.
- Fischer, Stanley, 1980, On activist monetary policy with rational expectations, in: S. Fischer, ed., *Rational expectations and economic policy* (University of Chicago Press, Chicago, IL) 211-247.
- Gordon, Robert J., 1979, New evidence that fully anticipated monetary changes influence real output after all, NBER working paper no. 361, June.
- Lucas, R.E., 1975, An equilibrium model of the business cycle, *Journal of Political Economy* 83, Dec., 1113-1144.
- Lucas, R.E., 1977, Understanding business cycles, *Journal of Monetary Economics*, Supplement, Carnegie-Rochester Conference Series, Vol. 5.
- Maddala, G.S., 1979, *Econometrics* (McGraw-Hill, New York).
- McCallum, B.T., 1979, On the observational inequivalence of classical and Keynesian models, *Journal of Political Economy* 87, April, 393-402.
- McCallum, B.T., 1980, Rational expectations and macroeconomic stabilization policy, *Journal of Money, Credit and Banking*, Part 2, Nov., 716-746.
- Phelps, Edmund S. and John B. Taylor, 1977, Stabilising powers of monetary policy under rational expectations, *Journal of Political Economy* 85, Feb., 163-190.
- Sargent, T.J., 1976, The observational equivalent of natural and unnatural rate theories of macroeconomics, *Journal of Political Economy* 84, June, 631-640.
- Schmidt, P., 1976, *Econometrics* (Marcel Dekker, New York).
- Sims, Christopher A., 1980, Macroeconomics and reality, *Econometrica* 48, Jan., 1-48.
- Taylor, John B., 1979, Staggered wage setting in a macro model, *American Economic Review* 69, Papers and Proceedings, May, 108-113.
- Tobin, James, 1965, Money and economic growth, *Econometrica* 33, Oct., 671-684.
- Turnovsky, Stephen J., 1980, The choice of monetary instruments under alternative forms of price expectations, *Manchester School*, March, 39-62.
- Wallis, Kenneth F., 1980, Econometric implications of the rational expectations hypothesis, *Econometrica* 48, Jan., 49-73.
- Weiss, Laurence, 1980, The role for active monetary policy in a rational expectations model, *Journal of Political Economy* 88, April, 221-233.