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«CROWDING» OUT OF PRIVATE CAPITAL FORMATION BY GOVERNMENT BORROWING IN THE PRESENCE OF INTERGENERATIONAL GIFTS AND BEQUESTS

By W. H. Buiter*

I. *Introduction*

In two recent studies (Buiter and Tobin, 1979, Tobin and Buiter, 1980), James Tobin and I concluded that debt neutrality —the property that the real trajectory of the economic system is invariant under changes in the financing mix, for a given level and composition of real government spending— is a theoretical curiosum. The assumptions required for it to be valid can easily be shown to be contradicted by practical experience. In this paper, I provide a detailed statement of the case against debt neutrality in the context of a model constructed expressly to be as favorable as possible to classical invariance theorems. The model is a generalization of Diamond's version of Samuelson's overlapping generations model (Diamond (1965)), and allows for voluntary intergenerational gifts and bequests. (See Barro (1974), and Buiter (1979)). A comprehensive treatment of the subject can be found in Carmichael (1979). Except for one significant simplification, the treatment of the case of agents with "two-sided intergenerational caring" replicates the original work of Carmichael.

The overlapping generations model used to develop the non-neutrality theories is "classical" in the sense that private actions are derived from explicit optimizing behavior, perfect foresight prevails and all markets are in equilibrium all of the time. All private agents act as price takers. I shall study the behavior of this decentralized, competitive economy when a given government spending program is financed by different combinations of lump-

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sum taxation or current borrowing. Without loss of generality the level of government spending is asssumed to equal zero, which allows us to rephrase the argument in terms of the real effects of alternative debt issue-taxation programs. The restriction to lump-sum taxes is necessary to give the neutrality proposition a chance. Non-lump-sum taxes on labor income, profits, wealth or any other base, will introduce real distortions, impose excess burdens and, except in uninteresting special cases, have real effects.

Private, voluntary intergenerational gifts —from parents to children (bequests) or from children to parents— are essential for the debt neutrality property to prevail. Briefly, the argument for neutrality goes as follows. The stock of real government interest-bearing debt has no effect on private behavior because corresponding to every pound's worth of income on these bonds is a pound's worth of tax payments to finance the bond income. The value of the government bonds on the asset side of private portfolios is the present discounted value of these future income payments. The value of these bonds is therefore exactly matched by the present discounted value of the future tax payments required to service them. Even if we grant that the future payments stream and the future tax payments stream are identical and that both are discounted in the same manner, a shift from tax financing to borrowing could cause non-neutrality because of an intergenerational redistribution of resources. If the bonds are one-period bonds and each individual is supposed to live for two periods, the intergenerational redistribution that can be associated with such issues is immediately apparent. Let an extra Pound's worth of bonds be issued in period t. It is bought by the then young members of generation t. Next period interest and repayment of principal occur. The tax revenue required for the debt servicing could be levied on the then young members of generation $t+1$. In that case, real resources have been redistributed from the young to the old. Consumption and capital formation will be affected. An unfunded social security program will have broadly similar effects. Longer maturity bonds can be incorporated in the analysis without materially altering it. Voluntary intergenerational gifts can remove the real consequences of involuntary intergenerational redistribution through the borrowing-taxation mechanism. Provided the taxes are lump-sum, such private intergenerational transfers will restore the original consumption-investment equilibrium as long as such private actions do not violate the non-negativity constraints on these voluntary intergenerational transfers. If, before the extra Pound's worth of public debt is issued the members of the older generation were all leaving positive bequests to their descendants, the option of redistributing resources from the young to the old through a cut in bequests was already open to the older generation. Their decision not to exercise this option reflects that, at the margin, they receive greater utility from the well-being of their heirs than from their own consumption. The government's attempt to redistribute "gross resources" from the young to the old will in that case be met be increased bequests from the old to the young, leaving the "net resources" available to each generation unchanged. If, on the other hand, the older generations were initially at a "zero bequest corner", i.e., if in order to increase their own life-time resources they would gladly have left their children a negative legacy, had this not been ruled out by law, the involuntary intergenerational redistribution would not have been neutralized by an exactly matching voluntary transfer in the opposite direction.

Within the bounds set by the non-negativity constraints on gifts and bequests, lump-sum redistribution through borrowing or unfunded social security schemes will be neutralized by voluntary intergenerational transfers, if bequest or gift motives are present. Private non-market transactions are required to neutralize public non-market transactions. A formal analysis follows below.

Notation

c_t^1 : consumption while young by a member of generation t

c_t^2 : consumption while old by a member of generation t

K_t : capital stock in existence at the beginning of period t

L_t : size of generation t

k_t : K_t/L_t

B_t : saving by a member of generation t while old (i.e., his bequest to young members of generation $t+1$)

G_t : gift by a young member of generation t to old members of generation $t-1$.

D_t : stock of real one-period government debt in existence at the beginning of period t

$d_t \equiv D_t/L_t$

T_t : lump-sum tax levied on members of generation t while young

$\tau_t \equiv T_t/L_t$

r_t : interest rate on savings carried from period t—1 into period t

n : one-period proportional rate of growth of population

δ : one-period discount rate applied to the utility of one's immediate descendant

ϱ : one-period discount rate applied to the utility of one's immediate forebear

II. *Government Financing in an Overlapping Generations Model without Gifts or Bequests*

Each generation consists of identical households that live for two periods. During the first period of their lives each household works a fixed amount, 1. Income earned in the first period is either consumed or saved. These savings, plus accumulated interest, are the only source of income in the second period of a household's life when it is retired. Households are also identical across generations. Initially there is no government borrowing or lending and no taxation. On the output side, the model has a single commodity that can either be consumed or used as a capital good. Until government bonds are introduced, real capital is the only store of value. The model is "real": there are no money balances. The dual role to be performed by durable output —that of being an input in the production function and of being the only store of value may lead to inefficiencies in a decentralized, competitive economy. (See Diamond (1965), Buiter (1979) and Carmichael (1979)).

II. a. *A Competitive Economy without Government Debt*

In the absence of government borrowing and lending, the utility maximization program faced by a representative household of generation t is given by:

$$\max_{c_t^1,\ c_t^2} u(c_t^1, c_t^2)$$ [1]

subject to

$$c_t^1 + \frac{c_t^2}{1+r_{t+1}} \leq w_t \tag{1}$$

$$c_t^1,\ c_t^2 \geqq 0$$

Equation (1) states that the present discounted value of lifetime consumption cannot exceed that of labor income. Given our assumptions about the utility function, u, the budget constaint will hold with equality and all solutions for c_t^1 and c_t^2 will be interior. Utility is a function of own lifetime consumption only. There is no gift or bequest motive. The model is completed by adding the economy-wide constraints, (2), (3), and (4).

$$w_t = f(k_t) - k_t f'(k_t) \tag{2}$$

$$r_t = f'(k_t) \tag{3}$$

$$w_t - c_t^1 = k_{t+1}(1+n) \tag{4}$$

Output is produced by a well-behaved neoclassical production function which is linear homogeneous in capital and labor. In intensive form it can be written as: $f(k_t)$ with $f(o) = o;\ f'>o;\ f''< o.$ [2] Equation (2) states that the labor market clears and is competitive. The real wage equals the marginal product of labor. Equation (3) states that the capital rental market clears and is competitive with the rental rate (which in the one-commodity model also equals the interest rate) equal to the marginal product of capital. Equation (4) is the economy-wide capital market equilibrium condition. The stock of capital in existence at the beginning of period t, K_t, is equal to the savings of the previous period. Only the young save in this model without bequests, so saving in t—1 is given by $(w_{t-1} - c_{t-1}^1)\ L_{t-1}$. Our conditions on the production function imply $k_t > 0$.

The interior first-order condition for an optimum is:

$$\frac{u_1(c_t^1, c_t^2)}{u_2(c_t^1, c_t^2)} = (1+r_{t+1}) \tag{5}$$

Its interpretation in terms of a tangency between an indifference curve in c_t^1, c_t^2 space and the intertemporal budget constraint is familiar. From the

1. u is assumed to be strictly quasiconcave and increasing in c^1 and c^2. $u_1(0, c^2) = u_2(c^1, 0) = +\infty$, $u_1(\infty, c^2) = u_2(c^1, \infty) = 0$.

2. We also assume $f'(0) = +\infty$; $f'(\infty) = 0$.

first-order condition and the budget constraint, (1), we can solve for c_t^1 (and c_t^2) as a function of w_t and r_{t+1}. Substituting the solution for c_t^1 into the capital market equilibrium condition (4) and using (2) and (3) to substitute for w_t and r_{t+1} we obtain a first-order difference equation in k_t describing the evolution over time of this economy from any arbitrary set of initial conditions.[3]

(6) $$f(k_t)-k_t f'(k_t)-c^1(f(k_t)-k_t f'(k_t),\ f'(k_{t+1})) = k_{t+1}(1+n).$$

This system will be locally stable and converge to a steady state equilibrium

i.f.f. $$\left|\frac{\partial k_{t+1}}{\partial k_t}\right| < 1, \text{ i.e., when}$$

$$\left|\frac{\left(\frac{\partial c^1}{\partial w}-1\right)kf''}{1+n+\frac{\partial c^1}{\partial r}f''}\right| < 1.$$

In what follows we shall assume existence, uniqueness and stability and proceed to analyze steady-state equilibria only.

In steady-state equilibrium, the capital—labor ratio and through that all real stock-stock and stock-flow ratios are constant. We solve for it by setting $k_t = k_{t+1}$ in equations (1) through (5). By substituting the marginal productivity conditions (2) and (3) into (1) and (4) we obtain equations(7) and (8), the stationary private budget constraint and aggregate capital market equilibrium condition.

(7) $$c^1+\frac{c^2}{1+f'(k)} = f(k)-kf'(k)$$

(8) $$f(k)-kf'(k)-c^1 = k(1+n).$$

From (7) and (8) we can solve for the stationary decentralized consumption possibility locus, as in (9a) and (9b).

(9a) $$c^2 = \psi(c^1)$$

3. A solution will exist if c^1 and c^2 are both normal goods and if c^1 does not increase when r_{t+1} increases. It will be unique if the utility function is homothetic. See Carmichael (1979).

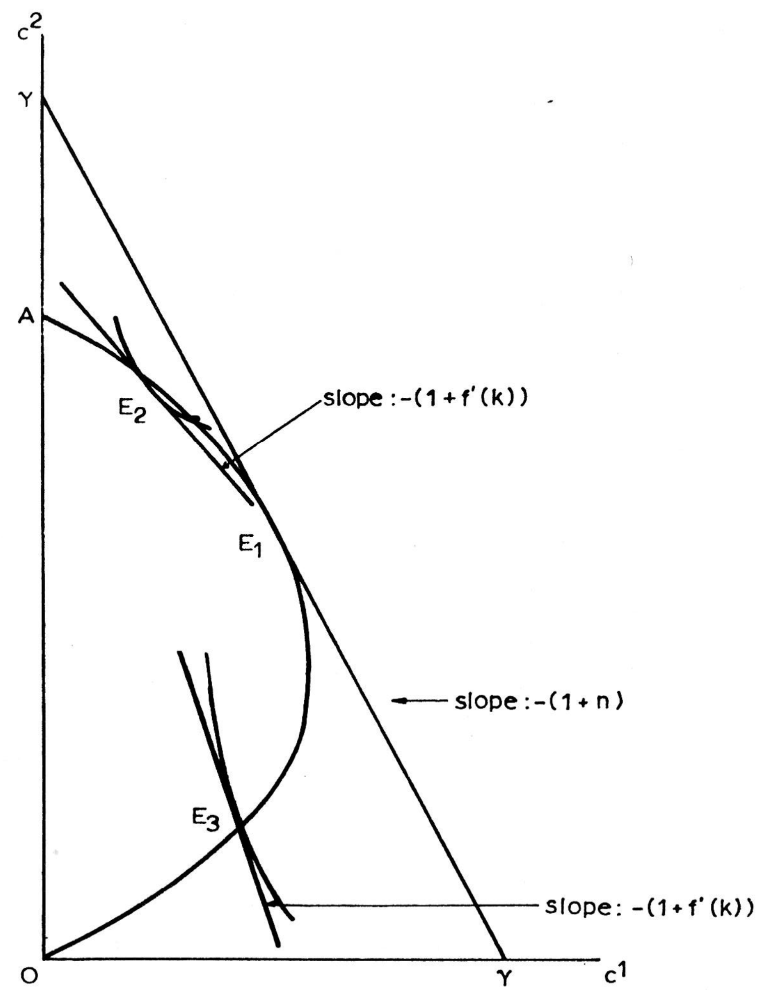


Figure 1

(9b) $$\psi' = -(1+f')\left[[1+\frac{k(n-f')}{1+f'}f''(1+n+kf'')^{-1}]\right].$$

If the production function is Cobb-Douglas, with $f(k) = k^{a}$, $0 < a < 1$,

$$\psi' = \frac{(1+n)\,(1+a^2k^{(a-1)})}{(1-a)ak^{(a-1)}-(1+n)}.$$

The stationary decentralized consumption possibility locus for the Cobb-Douglas case is graphed in Figure 1. At the origin its slope is $\frac{a}{1-a}(1+n)$ k increases monotonically as we move up from 0 towards A. As k approaches infinity (which would be beyond A in the infeasible region) the slope of the consumption possibility locus becomes —1. The locus is strictly concave towards the origin. For large k and more general constant returns production functions than the Cobb-Douglas, $\frac{\partial c^2}{\partial c^1}$ can even become positive again, a case of extreme overaccumulation. With the Cobb-Douglas this is not possible. At the golden rule capital-labor ratio, when $f'(k) = n$, $\psi' = -\frac{\partial c^2}{\partial c^1} = -(1+f') = -(1+n)$.

The steady-state equilibrium of a decentralized competitive economy could be achieved anywhere on this locus. Steady-state equilibria like E_3, corresponding to a capital-labor ratio below the golden rule capital-labor ratio k^*, defined by $f'(k^*) = n$, are possible as are those like E_2 corresponding to a capital-labor ratio in excess of k^*. The golden rule capital-labor ratio k^* could be achieved by a competitive equilibrium at E_1, but this is not more likely than any other point on the locus. A competitive stationary equilibrium satisfies two criteria: it lies on the stationary consumption possibility locus and it has a tangency between an indifference curve and a private budget constraint with slope $-(1+r) = -(1+f'(k))$. The private budget constraint will always cut the stationary consumption possibility locus in the manner indicated at E_3 and E_2. Only at the golden rule (E_1) will the private budget constraint be tangent to the locus.

It is instructive to contrast the private decentralized solution with the solution achieved by an omnipotent social planner. The latter is only subject to the aggregate resource constraint:

$$c_t^1 L_t + c_{t-1}^2 L_{t-1} = L_t f(k_t) - L_t(k_{t+1}(1+n) - k_t) \quad \text{or}$$

(10) $$c_t^1 + \frac{c_{t-1}^2}{1+n} = f(k_t) - k_{t+1}(1+n) + k_t$$

The stationary aggregate resource constraint is

(11) $$c^1 + \frac{c^2}{1+n} = f(k) - nk$$

In order to maximize the stationary per capita amount of resources available for consumption, the social planner selects the golden rule capital-labor ratio k^*. The stationary social consumption possibility locus is the straight line $\gamma\gamma$ with slope $-(1+n)$. By distribution through administrative fiat, any point on this $\gamma\gamma$ locus is available to the social planner. A decentralized competitive equilibrium with a capital-labor ratio below k^*, as at E_3, is not inefficient. During any transition from E_3 to E_1, say, capital deepening has to occur, requiring the sacrifice of consumption during the transition in exchange for a permanently higher consumption path after E_1 has been achieved. A capital-labor ratio in excess of the golden rule is inefficient because it is possible to reduce the capital-labor ratio and thus to have a temporary consumption binge while enjoying a permanently higher path of consumption after k^* has been achieved. This inefficiency is due to capital's dual role as a store of value and a factor of production. In an attempt to shift consumption towards retirement, private agents save by accumulating capital. This depresses the rate of interest. By making available a store of value that has no additional intrinsic use, either as a consumption good or a capital good, government borrowing can alleviate and even eliminate any such inefficiency due to overaccumulation.

II. b. A Competitive Economy with Government Debt

Now consider the case in which the government issues real-valued one-period bonds. Bonds floated during period t are repaid with interest at a rate r_{t+1} in period $t+1$. Government bonds and real capital are perfect substitutes in private sector portfolios. D_t can be negative, in which case the public sector lends to the private sector. Such public sector lending to the private sector consists of public purchases of private sector bonds (which are also perfect substitutes for public bonds), not of real capital. T_t is the total lump-sum tax bill paid by the younger generation in period t. It can be negative in which case it constitutes transfer payments to the young. With government debt and taxes the economy we are considering can be represented as follows:

$$\max_{c_t^1,\, c_t^2} \quad u(c_t^1, c_t^2)$$

subject to $\quad c_t^1, c_t^2 \geqq 0$

(12) $$(w_t - c_t^1 - \tau_t)(1 + r_{t+1}) = c_t^2$$

(13) $$(1 + r_t)\, D_{t-1} = D_t + T_t$$

(14) $$(w_t - c_t^1 - \tau_t)L_t = D_t + K_{t+1}$$

$$w_t = f(k_t) - k_t f'(k_t)$$

$$r_{t+1} = f'(k_{t+1})$$

$$k_t \geqq 0$$

Equation (12) is the modified household budget constraint, allowing for taxes while young. (13) is the government budget constraint. (14) is the modified capital market equilibrium condition. Total saving has to be equal to the total stock of assets consisting of government bonds and real capital. Private life-cycle optimizing behavior yields a consumption function $c_t^1 = c^1(w_t - \tau_t, r_{t+1})$. We again assume $o < c_1^1(.) < 1$ and $c_2^1(.) \leqq 0$.

The complete solution of the model is:

(15) $$c_t^1 = c^1(w_t - \tau_t,\ r_{t+1})$$

(12) $$(w_t - c_t^1 - \tau_t)(1 + r_{t+1}) = c_t^2$$

(16) $$w_t - c_t^1 - \tau_t = d_t + k_{t+1}(1+n)$$

(17) $$(1 + r_t)d_{t-1} = d_t\,(1+n) + \tau_t(1+n)$$

(2) $$w_t = f(k_t) - k f'(k_t)$$

(3) $$r_{t+1} = f'(k_{t+1})$$

$$c_t^1, c_t^2, k_t \geqq 0$$

At each point in time, t, this system of six equations determines the values of c_t^1, c_t^2, w_t, k_{t+1} and two of the three government instruments τ_t, d_t and r_{t+1}, given the value assigned to the remaining government instrument and the values of the predetermined variables r_t, d_{t-1} and k_t. I shall, through the rest of this section on debt neutrality, consider the case in which d_t, the per capita stock of real government debt is kept at a constant value $d_t = d$. In that case (17) simplifies to:

(17′) $$\tau_t = \frac{(r_t - n)d}{1+n}.$$

The model is stable *if* $\left|\frac{\partial k_{t+1}}{\partial k_t}\right| < 1$ in equation (18).

(18) $$f(k_t) - k_t f'(k_t) - c^1\!\left(f(k_t) - k_t f'(k_t) - \frac{(f'(k_t) - n)d}{1+n},\ f'(k_{t+1})\right) - \frac{(1 + f'(k_t))d}{1+n} = k_{t+1}(+n).$$

Stability requires

(19) $$\left|\frac{(c_1^1 - 1)f''\left(k + \frac{d}{1+n}\right)}{1 + n + c_2^1 f''}\right| < 1.$$

Since $k + \frac{d}{1+n}$ is non-negative (because c^2 is non-negative), this stability condition is qualitatively the same as that for the model without government debt, given in (6).

The steady state equilibrium of the model with public debt is given in equations (20) — (23).

(20) $$c^1 = c^1\left(f(k) - kf'(k) - \frac{(f'(k) - n)d}{1+n},\ f'(k)\right)$$

(21) $$\left(f(k)-kf'(k)-\frac{(f'(k)-n)d}{1+n}-c^1\right)(1+f'(k))=c^2$$

(22) $$f(k)-kf'(k)-\left(\frac{1+f'(k)}{1+n}\right)d-c^1=k(1+n)$$

(23) $$\tau=\frac{(f'(k)-n)d}{1+n}.$$

By substituting (20) into (22) we can derive the steady state effect of an increase in the per capita stock of public debt on the capital-labor ratio:

(24) $$\frac{\partial k}{\partial d}=\frac{(1+c_1^1n+(1-c_1^1)f')(1+n)^{-1}}{(c_1^1-1)f''\left(k+\frac{d}{1+n}\right)-[1+n+c_2^1f''].}$$

If the model is stable (equation 19) and if $c_2^1 \leqq 0$ and $0 < c_1^1 < 1$ the denominator of (24) is negative. The numerator is positive. We therefore obtain the familiar result that, comparing steady states, government debt issues reduce the capital-labor ratio, i.e., crowd out real capital. This "crowding out" result also obtains in the short run, as can be checked from equation (18). Given k_t, the effect on k_{t+1} of an increase in d is

(25) $$\frac{\partial k_{t+1}}{\partial d}=\frac{(1+c_1^1n+(1-c_1^1)f')(1+n)^{-1}}{-[1+n+c_2^1f''}.$$

This is negative if $c_2^1 \leqq 0$.

Note that the steady-state value of d can be chosen to be negative or positive. Irrespective of d, if the economy is at the golden rule, no net taxes or transfers are required (23). The growth in the total demand for debt required to keep the per capita stock of debt constant just suffices to repay the debt held by the old generation, plus interest at the rate of population growth. Positive d requires positive τ at capital-labor ratios below the golden rule ratio k^* (at interest rates above n), negative τ in the inefficient region when $f'(k) < n$.

The steady-state effect of government debt issue can be illustrated using a generalization of the stationary competitive consumption possibility locus of Figure 1. The effect of an increase in d on the stationary competitive consumption possibility locus is to shift it up at a rate $-(1+n)$. From (21) and (22) it is

easily seen that, at any given k, $\frac{\partial c^1}{\partial d}=-\frac{(1+f')}{(1+n)}$, while $\frac{\partial c^2}{\partial d}=1+f'$. Thus the rate at which, for any given k, c_2 is traded off for c_1 when d increases is $-(1+n)$. Figure 2 shows the general nature of the shift in the locus while Figure 3 focuses on a particular capital-labor ratio, $\bar{k}$ below the golden rule ratio k^*. A budget line with a common slope $-(1+f'(\bar{k}))$ passes through E_1 and E_2 in Figure 3.

II. c. *Bequests*

With bequests, the utility function, the budget constraint and the capital market equilibrium condition are altered. B_t is the bequest left in the second period of his life by a member of generation t to the members of generation $t+1$. The bequests are received by members of generation $t+1$ at the end of the first period of their lives. The value of the bequest to members of generation $t+1$ at the beginning of their second period is $B_t(1+r_{t+2})$. When the rate of population growth is nonzero bequests shared equally among all descendants. Note that bequests must be non-negative, a useful institutional constraint.

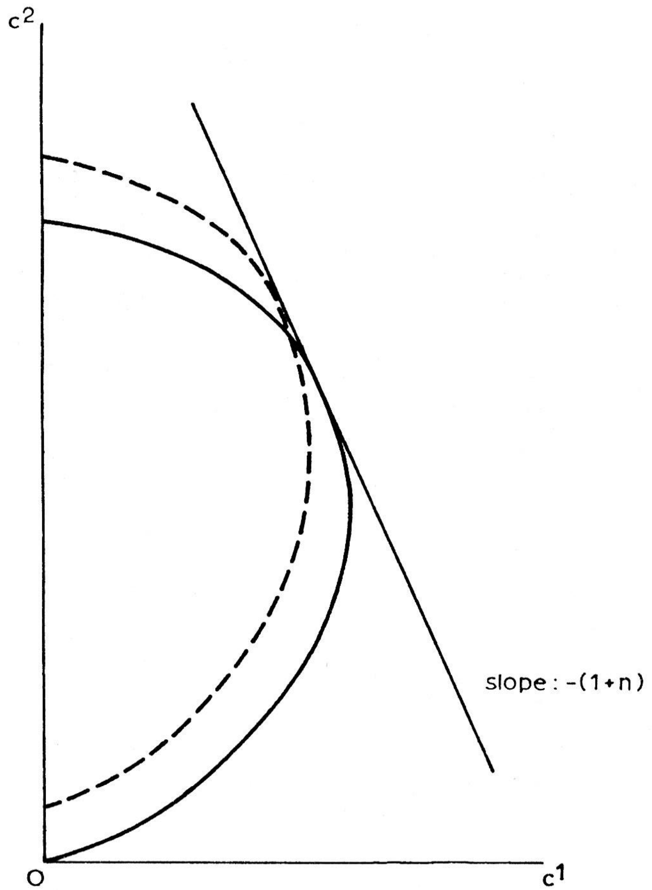
(26) $$B_t \geqq 0 \quad \text{for all } t.$$

The utility function of a member of generation t is $W_t = v(c_t^1, c_t^2, W^*_{t+1})$. The utility of a member of generation t depends on his own life-time consumption, c_t^1, c_t^2 and on the maximum utility level attainable by a member of the next generation. For simplicity I shall consider the additively separable function:

(27) $$W_t = u(c_t^1, c_t^2)+(1+\delta)^{-1}W^*_{t+1}.$$

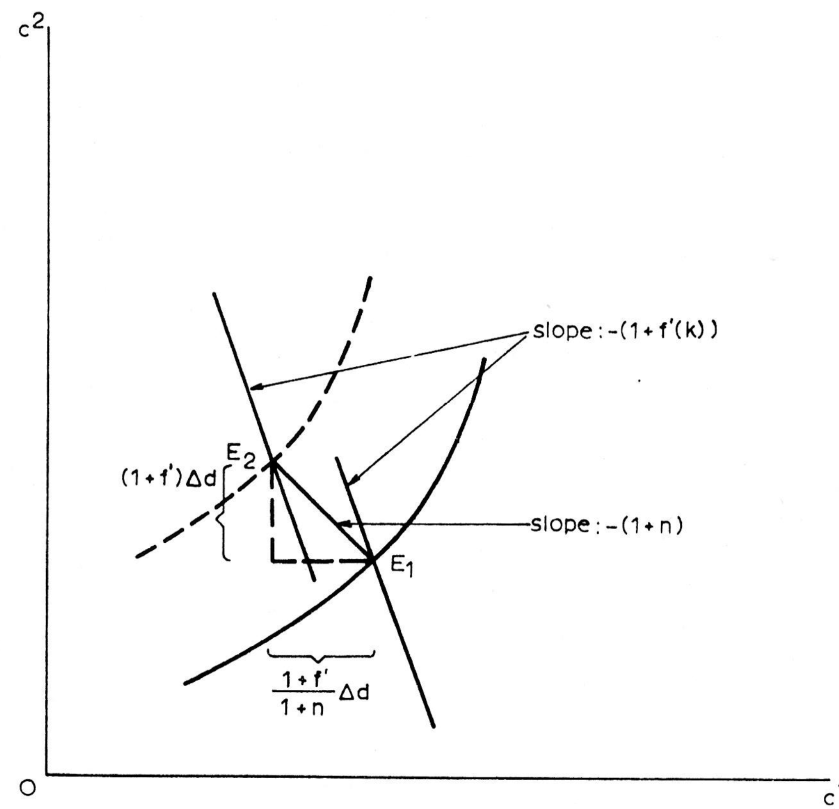
u has all the properties attributed to the utility function of the household without a bequest motive. This ensures interior solutions for c_t^1 and c_t^2 and strict satisfaction of the household budget constraint. δ is the "generational" discount rate; it is not to be confused with the individual's pure rate of time preference. Convergence, i.e., boundedness of W_t requires $\delta > 0$. The optimization problem solved by a representative member of generation t is given in equations (28) and (29). The new economy-wide capital market equilibrium condition is given in equation (30).

(28) $$W_t^* = \max_{c_t^1, c_t^2, B_t} W_t = \max_{c_t^1, c_t^2, B_t} [u(c_t^1, c_t^2)+(1+\delta)^{-1}W^*_{t+1}];\ c_t^1, c_t^2, B_t \geqq 0.$$



Stationary competitive consumption possibility loci with low d (solid line) and with high d (dashed line).

Figure 2



Shift of the stationary competitive consumption possibility locus when d increases, for a given capital-labor ratio $\overline{k} < k^*$.

Figure 3

subject to

(29) $$B_t \leq \frac{B_{t-1}(1+r_{t+1})}{1+n} + (w_t - c_t^1)(1+r_{t+1}) - c_t^2$$

(30) $$w_t - c_t^1 + \frac{B_{t-1}}{1+n} = (1+n)k_{t+1}.$$

The individual's budget constraint now contains the bequest he receives and the bequest he leaves. The capital market equilibrium condition recognizes that now both the young and the old generation can save. As before we have $r_t = f'(k_t)$ and $w_t = f(k_t) - k_t f'(k_t)$

The first order conditions for an optimum are:

(31a) $$u_1(c_t^1, c_t^2) = (1+r_{t+1})\, u_2(c_t^1, c_t^2)$$

(31b) $$u_2(c_t^1, c_t^2) \geq \frac{(1+r_{t+2})u_2(c_{t+1}^1, c_{t+1}^2)}{(1+n)\ (1+\delta)}.$$

If $B_t > 0$, i.e., if there is an interior solution for bequests, (31*b*) holds with equality. If there is a corner solution for bequests, i.e., if $B = 0$ is a binding constraint, (31*b*) holds with strict inequality. The interpretation of these first order conditions is straightforward. (31*a*) says that the discounted marginal utility of consumption in the second period of one's life should equal the marginal utility of consumption in the first period of one's life. (31*b*) states that if bequests are positive, the marginal utility of own consumption should equal the marginal utility of leaving a bequest. A marginal unit of income saved by an old member of generation t yields resources $(1+r_{t+2})$ times larger to generation $t+1$. The marginal utility to a member of generation t of bequests can be expressed as the discounted value of the marginal utility of consumption of a member of generation $t+1$. The appropriate discount rate is the generational discount rate δ. Finally, since it is the utility of a representative member of generation $t+1$ that was assumed to enter into the utility function of generation t, rather than the utility of all $1+n$ descendants, the population growth factor $1+n$ also discounts the marginal utility of consumption of generation $t+1$. If the marginal utility of own consumption exceeds the marginal utility of bequests, here will be a corner solution with $B = 0$.

The steady state equilibrium of the model with bequests is given in equations (32a)-(32d).

(32a) $$u_1(c^1,c^2) = (1+r)\, u_2(c^1,c^2)$$

(32b) $$(1+n)\ (1+\delta) \geq 1+r$$

if $B > 0$, $(1+n)\ (1+\delta) = 1+r$

if $B = 0$ and the zero bequest constraint is binding, $(1+n)(1+\delta) > 1+r$

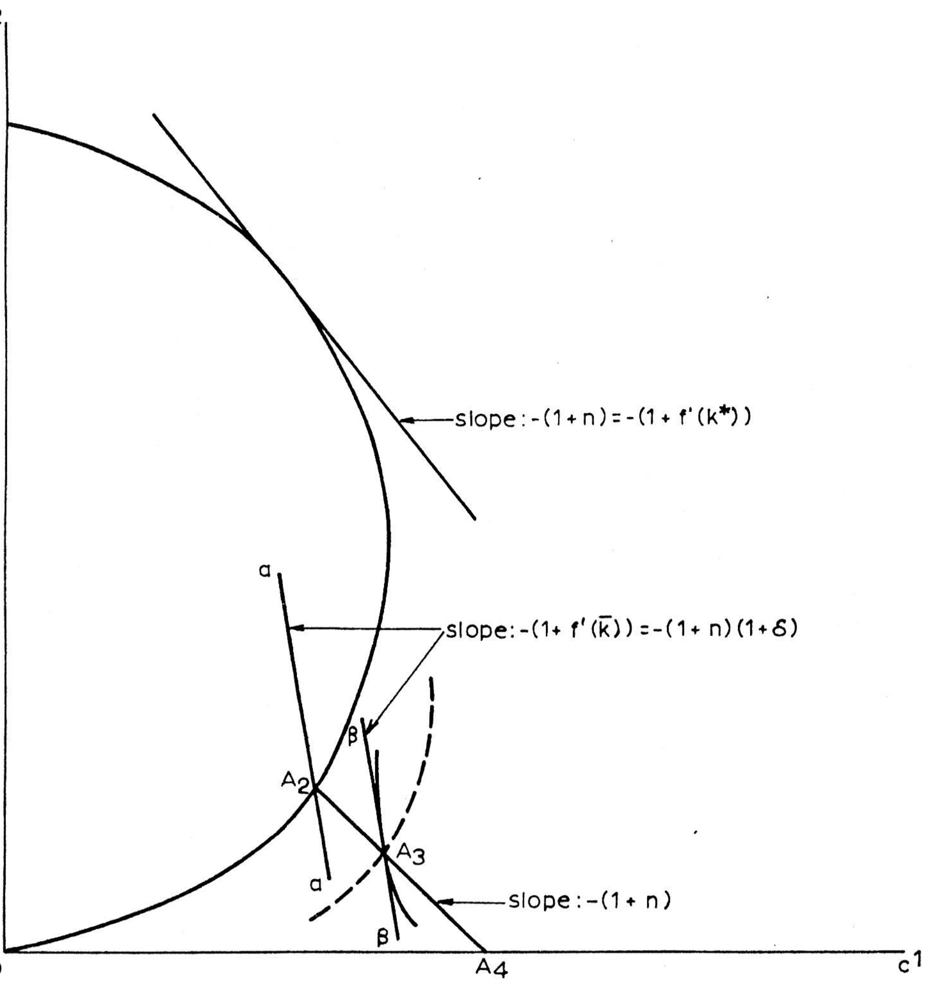
(32c) $$\left(\frac{n-r}{1+n}\right) B = (w-c^1)\ (1+r) - c^2$$

(32*d*) $$(1+n)k = \frac{B}{1+n} + w - c^1$$

$$r = f'(k)$$

$$w = f(k) - kf'(k).$$

The stationary competitive consumption possibility locus with bequests is drawn in Figure 4. OA_2A_1 is the no bequest locus. At capital-labor ratios so high that $(1+n)(1+\delta) > 1+f'(k)$, $B = 0$ and the no-bequest locus is again the relevant one. This critical capital-labor ratio, $\bar{k}$ is at A_2. Since $\delta > 0$, $\bar{k} > k^*$, the golden rule capital-labor ratio. Considering equations (32c) and (32d), we can draw a consumption possibility locus for each value of B. A higher value of B shifts the locus down and to the right at a rate—$(1+n)$. Thus all steady-state equilibria with interior (positive) solutions for bequests lie on the line segment $A_2A_3A_4$. All interior bequest solutions have the same capital-labor ratio, defined by $f'(\bar{k}) = (1+n)(1+\delta)$ which is below the golden rule capital-labor ratio. One such interior solution for bequests is drawn at A_3, where an indifference curve is tangent to a budget constraint with slope $-(1+f'(k))$ on the line segment $A_2A_3A_4$. The stationary consumption possibility locus for the appropriate positive value of B is represented by the dashed curve through A_3. The complete stationary locus with bequests is given by the no-bequest locus above A_2 and the line segment $A_2A_3A_4$. If the stationary competitive equilibrium is on A_1A_2, i.e., if there is a corner solution for bequests, the effect of government lending and borrowing is as in the no-bequest model. If the model is stable, the introduction of government borrowing ($d > 0$) will reduce the equilibrium capital labor-ratio, while the introduction of government lending ($d < 0$) will increase it. However, government borrowing can never reduce the capital-labor ratio below $\bar{k}$. Once k falls to $\bar{k}$, any further increase in government borrowing (which represents an involuntary redistribution of income from the young to the old) will be matched by exactly offsetting bequests, voluntary transfers from the old to the young. This is most easily appreciated if we consider the effect of government lending. Start from an initial equilibrium, without



An interior solution with bequests.

Figure 4

government lending, with positive bequests as at A_3. With bequests, bonds and taxes the private budget constraint (29) and the capital market equilibrium condition (30) are replaced by (29′) and (30′) respectively.

(29′) $$B_t = \frac{B_{t-1}(1+r_{t+1})}{1+n} + (w_t - c_t^1 - \tau_t)\ (1+r_{t+1}) - c_t^2$$

(30′) $$w_t - c_t^1 - \tau_t + \frac{B_{t-1}}{1+n} = d + k_{t+1}(1+n).$$

We also have the budget constraint

(17′) $$(r-n)d = \tau_t(1+n).$$

The stationary constraints are:

(33a) $$\frac{(n-r)}{1+n}(B - d(1+r)) = (w - c^1)\ (1+r) - c^2$$

(33b) $$\frac{1}{1+n}(B - d(1+r)) + w - c^1 = k(1+n).$$

If, with $d = 0$, a stationary solution obtains with $B = B^0 > 0$, a negative value of $d = d^0$ will still permit the same consumption-capital stock equilibrium to obtain as long as $B^0 \geqq |d^0(1+r)|$, i.e., as long as bequests can be reduced by an amount equal, in present value, to the amount of government lending. Then the involuntary government redistribution from the old to the young will be neutralized as regards its effect on the life-time resources of the two generations alive at any one moment, by the reduction in voluntary private redistribution from the old to the young. Given any initial value of bequests, however, there always exists a volume of government lending large enough to put private agents in a zero-bequest corner. Thus, with bequests, the government can always raise the capital-labor ratio above $\bar{k}$. It can never bring it down below $\bar{k}$.

II. d. Gifts from the Young to the Old

With gifts from the young to the old, the utility function is $W_t = v(c_t^1, c_t^2, W^*_{t-1})$. W^*_{t-1} is the maximum level of utility attained by a member o generation $t-1$. We again adopt the additively separable form:

$$W_t = u(c_t^1,\ c_t^2) + (1+\varrho)^{-1} W^*_{t-1}$$

We note that unlike standard time discounting, the utility of a member of the earlier generation is not compounded, but discounted. Convergence requires that the discount rate applied to parents' utility be positive, $\varrho > 0$. Gifts of course cannot be negative. $G_t \geqq 0$. The behavior of the competitive economy with gifts is summarized below

(34) $$W_t^* = \max_{c_t^1,\, c_t^2,\, G_t} [u(c_t^1, c_t^2)+(1+\varrho)^{-1}W_{t-1}^*];\; c_t^1,\, c_t^2,\, G_t \geqq 0.$$

subject to

(35) $$G_{t+1}(1+n)+w_t(1+r_{t+1}) \geqq (c_t^1+G_t)(1+r_{t+1})+c_t^2$$

with

(36) $$w_t - c_t^1 - G_t = k_{t+1}(1+n)$$

and, as before
$$r_t = f'(k_t)$$

$$w_t = f(k_t) - k_t f'(k_t).$$

The private budget constraint allows for gifts handed out and received. The capital market equilibrium condition reflects the fact that resources given by the young to the old, who do not save, are no longer available for capital formation.

The first order conditions of the private optimization problem are:

(37a) $$u_1(c_t^1, c_t^2) = (1+r_{t+1})\, u_2(c_t^1, c_t^2)$$

(37b) $$\frac{u_2(c_{t-1}^1, c_{t-1}^2)}{(1+\varrho)(1+r_{t+1})}(1+n) \leqq u_2(c_t^1, c_t^2).$$

If $G_t > 0$, i.e., if there is an interior solution for gifts, (37b) holds with equality. If there is a corner solution for gifts, i.e., if $G = 0$ is a binding constraint, (37b) holds with strict inequality. Equation (37a) is the condition for the optimal allocation of consumption for a member of generation t between the two periods of his life. (37b) states that if gifts are given from generation t to generation $t-1$, the marginal utility of own consumption should equal the marginal utility of gifts. The marginal utility of gifts is then expressed in terms of the marginal utility of own consumption of a member of generation $t-1$. This marginal utility of own consumption of generation $t-1$ is discounted at the generational discount factor $(1+\varrho)$. Second-period consumption of members of generation $t-1$ takes place one period before second-period consumption of members of generation t, so interest is foregone and further discounting by $(1+r_{t+1})$ is required. Finally, there are more members of generation t than of generation $t-1$. A member of generation t—1 therefore receives $G_t(1+n)$ for G_t given up by a member of generation t. If the marginal utility of own consumption exceeds the marginal utility of gifts, $G_t = 0$. The stationary solution with gifts is given by:

(38a) $$u_1(c^1, c^2) = u_2(c^1, c^2)(1+r)$$

(38b) $$\frac{(1+n)}{1+\varrho} \leqq 1+r.$$

$G > 0$ implies that (38b) holds with equality. A corner solution with $G = 0$ binding implies that (38b) holds with strict inequality. Stationary equilibrium is given by:

(38c) $$G(r-n) = (w-c^1)(1+r)-c^2$$

(38d) $$k(1+n)+G = w-c^1$$

$$r = f'(k)$$

$$w = f(k)-kf'(k)$$

The interesting equation is (38b). Since ϱ is positive, $G > 0$ implies $r < n$. An interior solution for gifts implies that the economy is dynamically inefficient, at a capital-labor ratio $\bar{\bar{k}}$ above the golden rule capital-labor ratio k^*. In models with inifite-lived households with a constant pure rate of time preference Ω, such an inefficiency can never arise. Steady state equilibrium is characterized by $(1+n)(1+\Omega) = 1+r$. With $\Omega > 0$ this implies $r > n$. Earlier consumption is *cet. par.* valued more than later consumption. This is not true when we have a child-parent gift motive. Own earlier consumption may well be valued more than own later consumption. The pure rate of time preference for own consumption, $\Omega(c) = u_1(c,c)/u_2(c,c)-1$, may well be positive. Earlier consumption by parents, however, is *cet. par.* valued less than later consumption by oneself. Parental utility is discounted, even though it "accrues" earlier. Thus child-parent gifts do not make a private decentralized economy with finite-lived agents equivalent to an economy with infinite-lived agents. It also does not rule out the possibility of dynamic inefficiency through overaccumulation. Quite the contrary, if gifts are positive in the steady state, the steady state is necessarily inefficient. An operative gift motive is indeed a reflection of a very strong desire to shift resources away from early consumption towards later consumption.

The effect of gifts on the steady state consumption possibility locus is ndicated in Figure 5. OA_2A_1 is the locus without gifts. For capital-labor ratios below $\bar{\bar{k}}$, defined by $f'(\bar{\bar{k}}) = \frac{1+n}{1+\varrho}$, the locus with gifts is identical with the locus without gifts because the equilibrium solution for G is zero. The stationary capital-labor ratio can never be above $\bar{\bar{k}}$ when there is a gift motive. All solutions with $G > 0$ lie on the line segment $A_4A_3A_2$ with slope $-(1+n)$. Starting at A_2 where $G = 0$ and $k = \bar{\bar{k}}$, an increase in G shifts the stationary consumption possibility locus up and to the left at a rate $-(1+n)$. A typical interior solution for G is drawn at A_3. An indifference curve is tangent to a budget constraint with slope $-(1+f'(k))$ on the line segment $A_4\ A_3\ A_2$. The stationary consumption possibility locus for the appropriate positive value of G is represented by the dashed curve through A_3. The entire stationary consumption possibility locus with gifts is given by the segment of the no-gift locus OA_2 and the straight line $A_4\ A_3\ A_2$.

The effect of government borrowing and lending in the presence of a gift motive is easily analyzed. As long as the economy stays in the range of capital-labor ratios below $\bar{\bar{k}}$, government lending and borrowing will have the same effect as in the model without gifts and bequests. If $k = \bar{\bar{k}}$ initially (with $d = 0$), government lending ($d < 0$) will not have any effect on the steady-state consumption path and capital-labor ratio. Involuntary government redistribution from the old to the young will be neutralized immediately by matching voluntary gifts from the young to the old. Government borrowing, $d > 0$, will also be neutralized by matching reductions in gifts from the young to the old, up to the point that the constraint $G \geqq 0$ becomes binding. The private budget constraint, capital market equilibrium condition and government budget contraint with gifts, borrowing and taxes are

$$G_{t+1}\ (1+n)+(w_t-\tau_t)\ (1+r_{t+1}) = (c_t^1+G_t)\ (1+r_{t+1})+c_t^2$$

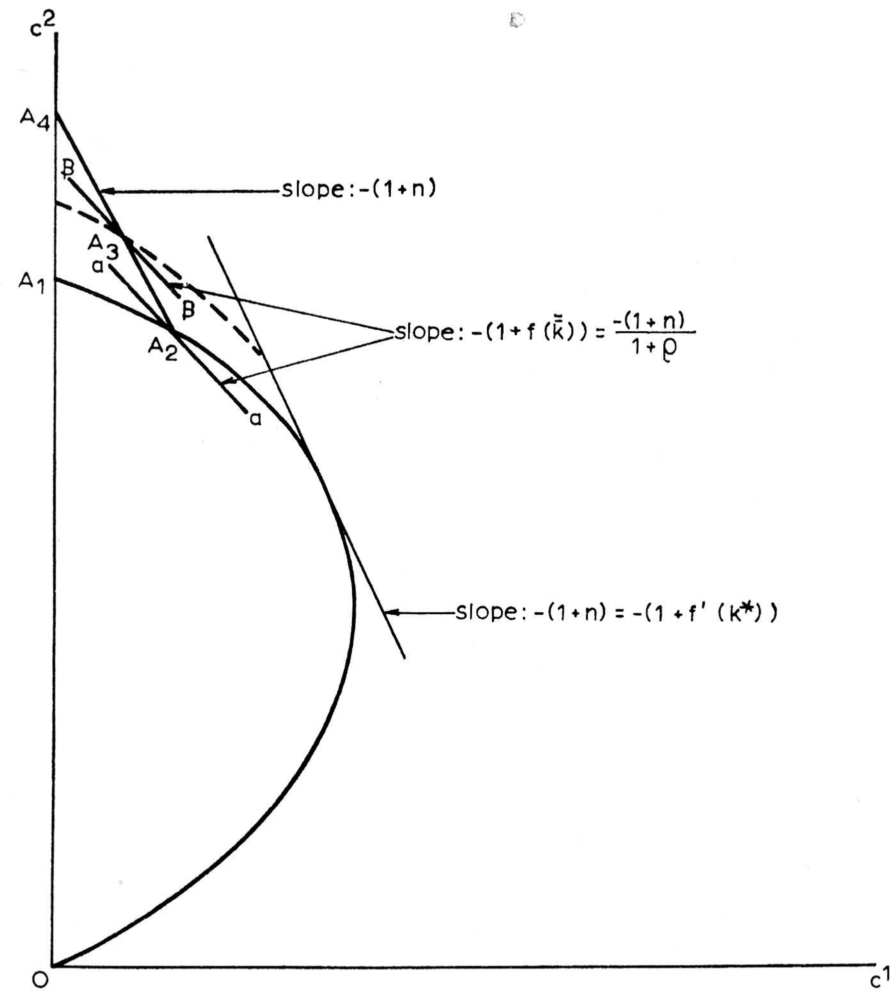
and

$$w_t-\tau_t-c_t^1-G_t = d+k_{t+1}(1+n)$$

$$(r_t-n)\ d = \tau_t\ (1+n)$$

The stationary equations are:

$$(n-r)\,(G+\frac{d(1+r)}{1+n})+(w-c^1)\,(1+r)-c^2 = 0 \qquad \text{(39a)}$$



An interior solution for child-parent gifts.

Figure 5

(39b) $$G+\frac{d(1+r)}{1+n}+k(1+n) = w-c^1$$

Equations (33a) and (39b) show that G and $\frac{d(1+r)}{1+n}$ are "perfect substitutes" as long as the constraint $G \geqq 0$ is not violated. Thus as long as, with $d = 0$, the initial G, (G^0, say) is larger than $d^0 \frac{(1+r)}{1+n}$, where d^0 *is* the size of the real per capita government bond issue, the private sector can and will undo the effects of the government action on consumption and the capital-labor ratio by reducing voluntary gifts from the young to the old. For any initial G, however, there always exists a government borrowing program large enough to make $G \geqq 0$ a binding constraint. Such actions will move the economy from $A_4A_3A_2$ onto A_20, lowering the capital-labor ratio. In view of the inefficiency of the private, decentralized competitive solution with positive gifts, such borrowing will always constitute a Pareto improvement as long as it does not lower k below k^*.

II.e. Gifts and Bequests

I now consider the case of "two-sided caring". Each generation cares about the welfare of its immediate ancestors and its immediate descendants. The utility function is:

$$W_t = v(c_t^1, c_t^2, \overset{*}{W}_{t-1}, \overset{*}{W}_{t+1}).$$

The special case of the additively separable function is again considered:

$$W_t = u(c_t^1, c_t^2)+(1+\delta)^{-1} \overset{*}{W}_{t+1}+(1+\varrho)^{-1} \overset{*}{W}_{t-1}.$$

Convergence now requires not only $\delta > 0$ and $\varrho > 0$ but $\delta\varrho > 1$.

It might be thought that the solution to the problem with both gifts and bequests is in some way a simple combination of the solutions to the cases with just gifts and just bequests. This is not so. With "one-sided caring" (either gift or bequest motives but not both) the private agent's optimization problem is a standard problem in dynamic programming. With a bequest motive, each agent in generation t cares potentially for all his descendants. *Directly* as regards his immediate heir, *indirectly* through the dependence of the utility of his immediate descendant on the utility of generation $t+2$, etc. In the same way, with a gift motive, each agent potentially cares for all his ancestors. In either case utility chains stretch out in one direction only. With both gift and bequest motives, this unidirectional simplicity no longer applies. An agent in generation t cares directly about generations $t-1$ and $t+1$. These generations both care directly about generation t. Generation $t-1$ also cares directly about $t-2$ and generation $t+1$ about $t+2$. Immediately, utility chains can be seen to be running in both directions. These issues were discussed for the first time in Carmichael (1979). A particularly simple solution emerges when the following assumptions are made about the "game" played by a member of generation t with past and future generations:

1. A member of generation t acts competitively in his labor and capital markets, i.e. he takes w_t and r_{t+1} as parametric. He also assumes that all past and future generations have acted or will act competitively in their factor markets.

2. A member of generation t, in formulating his consumption-gift-bequest plan, knows the utility levels and actions of all past generations and correctly anticipates utility levels and actions of all future generations (rational expectations or perfect foresight).

3. A member of generation t plays a *non-cooperative* gift and bequest game with past and future generations. He rationally believes that all past and future generations play the same game. This strategy is *closed-loop* as regards the utility and actions of the two generations with which he overlaps (generations $t-1$ and $t+1$). This means that when evaluating the alternative actions open to him at the beginning of period t, he believes that he can affect the total utility and the actions of his immediate descendants and his immediate forebears.

His strategy is *open-loop* as regards the utility and actions of all other generations ($t-i$, $i \geqq 2$ and $t+j$, $j \geqq 2$). Thus, when evaluating the effects of marginal changes in his actions, he ignores the impact on the actions and utility of generations that are already dead when he is born or that are born after his lifetime.[4]

4. When formulating his closed-loop strategy vis-a-vis generations $t-1$ and $t+1$, he believes that he can alter the behavior of these generations (i.e. their consumption, gift and bequest choices) only by altering the total resources available to them, i.e., only through direct transfers.

4. This is different from Carmichael (1979) who considers the case where an individual only ignores the impact of marginal changes in his actions on generations that are already dead, i.e. Carmichael uses a closed-loop strategy vis-a-vis all later generations. The symmetry imposed in my specification considerably simplifies the analysis.

5. Each generation acts so as to maximize its utility.

6. Each generation views the world and plays the game in the same way as the member of generation t just described.

The resulting equilibrium in this differential game is a Nash equilibrium.

The behavior of this economy can be summarized as follows:

(40) $$W^*_t = \max_{c^1_t, c^2_t, B_t, G_t} W_t = \max_{c^1_t, c^2_t, B_t, G} [u(c^1_t, c^2_t) + (1+\delta)^{-1} W^*_{t+1} + (1+\varrho)^{-1} W^*_{t-1}]$$

subject to:

(41) $$B_t - G_{t+1}(1+n) \leqq \left(\frac{B_{t-1}}{1+n} - G_t\right)(1+r_{t+1}) + (w_t - c^1_t)(1+r_{t+1}) - c^2_t$$

$$B_t,\ G_t,\ c^1_t,\ c^2_t \geqq 0,$$

with economy-wide constraints:

(42) $$w_t - c^1_t + \frac{B_{t+1}}{1+n} - G_t = k_{t+1}(1+n)$$

and

$$r_{t+1} = f'(k_{t+1})$$

$$w_t = f(k_t) - k_t f'(k_t)$$

The first order conditions are:

(43a) $$\frac{\partial W^*_t}{\partial c^1_t} = (1+r_{t+1}) \frac{\partial W^*_t}{\partial c^2_t}$$

(43b) $$\frac{\partial W^*_t}{\partial B_t} \leqq \frac{\partial W^*_t}{\partial c^2_t}$$

If $B_t > 0$, (43b) holds with strict equality. If $B_t = 0$ is a binding constraint, (43b) holds with strict inequality.

(43c) $$\frac{\partial W^*_t}{\partial G_t} \leqq \frac{\partial W^*_t}{\partial c^1_t}$$

If $G_t > 0$, (43c) holds with strict equality. If $G_t = 0$ is a binding constraint, (43c) holds with strict inequality. Using assumptions (1) through (6), the first order conditions and the private budget constraints of current, past and future generations, we can express generation t's marginal utility from bequests in terms of the marginal utility of own consumption of generation $t+1$.

(44) $$\frac{\partial W^*_t}{\partial B_t} = \frac{(1+\varrho)(1+r_{t+2})}{[(1+\varrho)(1+\delta)-1](1+n)} \frac{\partial W^*_{t+1}}{\partial c^2_{t+1}}.$$

Combining (44) and (43b) we get

(45) $$\frac{(1+\varrho)(1+r_{t+2})}{[(1+\varrho)(1+\delta)-1](1+n)} \frac{\partial W^*_{t+1}}{\partial c^2_{t+1}} \leqq \frac{\partial W^*_t}{\partial c^2_t}.$$

If $B_t > 0$, then (45) holds with strict equality. If $B_t = 0$ is a binding constraint, (45) holds with strict inequality.

In an exactly analogous manner we can express generation t's marginal utility from gifts in terms of the marginal utility of own consumption of generation $t-1$.

(46) $$\frac{\partial W^*_t}{\partial G_t} = \frac{(1+n)(1+\delta)}{(1+\varrho)(1+\delta)-1} \frac{\partial W^*_{t-1}}{\partial c^2_{t-1}}$$

Combining (46) and (43c) and using (43a), we obtain:

(47) $$\frac{(1+n)(1+\delta)}{[(1+\varrho)(1+\delta)-1]} \frac{\partial W^*_{t-1}}{\partial c^2_{t-1}} \leqq \frac{\partial W^*_t}{\partial c^2_{t-1}} (1+r_{t+1}).$$

If $G_t > 0$, then (47) holds with strict equality. If $G_t = 0$ is a binding constraint, (47) holds with strict inequality.

The steady state conditions are:

(48a) $$u_1(c^1, c^2) = (1+r)\, u_2(c^1, c^2)$$ [5]

(48b) $$1+r \leqq \left[\frac{[(1+\varrho)(1+\delta)-1}{1+\varrho}\right](1+n)$$

5. From III.43a we have, in the steady state, $\frac{\partial W}{\partial c^1} = (1+r)\frac{\partial W}{\partial c^2}$

$$W = \frac{u(c^1, c^2)}{1-(1+\delta)^{-1}-(1+\varrho)^{-1}}$$

If B > 0 then (48b) holds with equality. If B = 0 is a binding constraint then (48b) holds with strict inequality.

(48c) $$1+r \geqslant \frac{(1+\delta)(1+n)}{(1+\varrho)(1+\delta)-1}$$

If $G > 0$ then (48c) holds with equality. If $G = 0$ is a binding constraint then (48c) holds with strict inequality.

(48d) $$\left(\frac{n-r}{1+n}\right) B+(r-n)\ G = (w-c^1)(1+r)-c^2$$

(48e) $$k(1+n) = \frac{B}{1+n} - G+w-c^1.$$

and

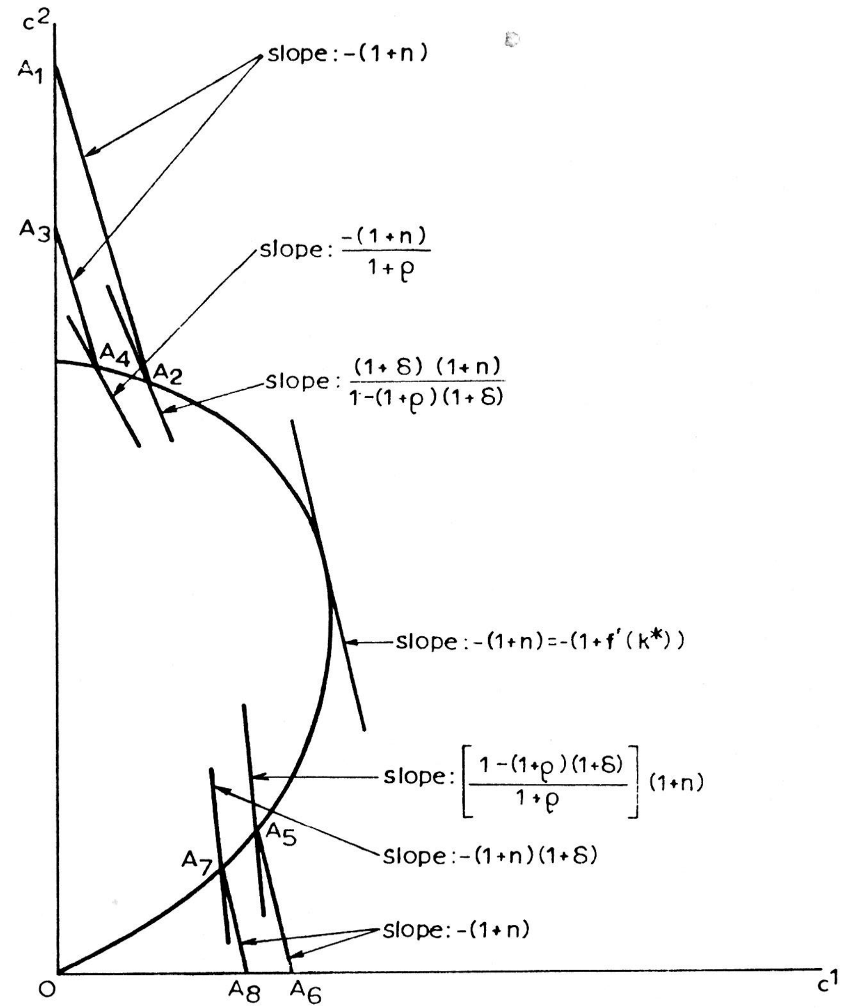
$$r = f'(k)$$

$$w = f(k)-kf'(k)$$

With n, ϱ and δ strictly positive, (48b) and (48c) cannot both hold with equality. This is the commonsense result that there will not be both gifts and bequests in the steady state. Thus, if $B > 0$, then $G = 0$, and if $G > 0$ then $B = 0$. However, it is possible for (48b) and (48c) to both hold with strict inequality, i.e., for both gifts and bequests to be zero.

Note that if (48b) holds with equality, i.e., if there is an interior solution for bequests, we have $r > n$: the capital-labor ratio is below the golden rule capital-labor ratio.[6] Also, if (48c) holds with equality, i.e., if there is an interior solution with gifts, we have $r < n$: the capital-labor ratio is above its golden level. Figure 6 illustrates the stationary consumption possibility locus when there is both a bequest and a gift motive. For capital labor ratios below that defined by $(1+f'(k)) = \frac{(1+\delta)(1+n)}{(1+\varrho)(1+\delta)-1}$ but above that defined by $(1+f'(k)) = \frac{[(1+\varrho)(1+\delta)-1](1+n)}{1+\varrho}$, the stationary consumption possibility locus is the same as it is without gifts and bequests. On the curve segment A_2A_5, there are corner solutions for gifts and bequests: $G = B = 0$. When there is an interior solution for gifts the equilibrium is on the line segment $A_1\ A_2$ with slope$-(1+n)$. A_2 is defined by$-(1+f'(k)) = \frac{(1+\delta)(1+n)}{1-(1+\varrho)(1+\delta)}$.

6. We use the condition $\varrho\delta > 1$, for the stationary utility function to be bounded.



Stationary consumption possibility loci, without gifts and bequests with either gifts or bequests and with both gifts and bequests.

Figure 6

As in the gifts only case, larger positive values of G shift the locus up and to the left at a rate $-(1+n)$. Note that the degree of overaccumulation relative to the golden rule is less when $G > 0$, if there is both a gift and a bequest motive than if there is only a gift motive.[7] The tendency to "oversave", represented by the gift motive is partly, but not completely, neutralized by the presence of a bequest motive. $A_3 A_4$ would be the locus of interior solutions for G if there were only a gift motive. If the bequest motive is operative, i.e., if $B > 0$, all stationary solutions lie on the line segment $A_5 A_6$ with slope $-(1+n)$. A_5 is defined by $(1+f'(k)) = \frac{[(1+\varrho)(1+\delta)-1]}{1+\varrho}(1+n)$. As in the case of bequests only, larger positive values of B shift the locus down and to the right at a rate $-(1+n)$. $A_7 A_8$ would be the locus of interior solutions for B if there were just a bequest motive. The degree of underaccumulation, relative to the golden rule, is less if there is both a bequest and a gift motive than if there is only a gift motive.[8]

The effects of government lending and borrowing on the steady-state capital-labor ratio are a straightforward combination of the effects of such policies when there was either a gift or a bequest motive but not both. Consider an initial equilibrium without government debt: $d = 0$. If the initial equilibrium is in the range of k for which there is a corner solution for both B and G, i.e, on $A_2 A_5$, government borrowing $(d > 0)$ cannot lower k below the value defined by $1+f'(k) = \frac{[(1+\varrho)(1+\delta)-1]}{1+\varrho}(1+n)$ nor can government lending $(d < 0)$ raise k above the value defined by $1+f'(k) = \frac{(1+\delta)(1+n)}{(1+\varrho)(1+\delta)-1}$.

If there is an interior solution for gifts, on $A_1 A_2$, government borrowing will be offset by a reduction in gifts of equal present value, thus leaving c^1, c^2 and k unchanged, unless the increase in d is larger, in present value, than the original value of G. In other words, there is always a positive value of d large enough to make $G \geqq 0$ a binding constraint. If there is an interior solution for bequests, on $A_5 A_6$, government lending $(d < 0)$ will be offset by a reduction in bequests of equal present value which leaves c^1, c^2 and k unchanged. Again, if the value of lending is larger, in present value, than the original bequest, the constraint $B \geqq O$ will become binding.

With gifts, bequests, borrowing and taxes the private and public sector

7. We assume ϱ to be the same in both cases.
8. We assume δ to be the same in both cases.

budget constraints and the capital market equilibrium conditions are:

$$B_t - \frac{B_{t-1}(1+r_{t+1})}{1+n} - G_{t+1}(1+n) + G_t(1+r_{t+1}) - (w_t - c_t^1 - \tau_t)(1+r_{t+1}) \tag{49a}$$

$$c_t^2 = 0$$

$$w_t - c_t^1 - \tau_t + \frac{B_{t-1}}{1+n} - G_t = d + k_{t+1}(1+n). \tag{49b}$$

and

$$(r_t - n)d = \tau_t(1+n).$$

The stationary equations are:

$$(n-r)\left[\frac{B}{1+n} - G - \frac{d(1+r)}{1+n}\right] = (w-c^1)(1+r) - c^2 \tag{50a}$$

$$\frac{B}{1+n} - G - \frac{d(1+r)}{1+n} = k(1+n) - w + c^1 \tag{50b}$$

We know that if $B > 0$, then $G = 0$ and if $G > 0$ then $B = 0$. Thus if $B > 0$, a reduction in d by an amount Δd will be neutralized by a reduction in B by the amount $\Delta d(1+r)$, as long as this does not violate the constraint $B \geqq 0$. If $G > 0$, an increase in d by an amount Δd will be neutralized by a reduction in G by the amount $\frac{\Delta d(1+r)}{1+n}$, as long as this does not violate the constraint $G \geqq 0$.

III. Conclusions

The policy conclusions of this theoretical investigation of debt neutrality are straightforward. While operative intergenerational gift and bequest motives turn finite-lived households into infinite-lived households in a certain sense, there remain essential differences. In particular, if the child-parent gift motive is operative, the decentralized, competitive equilibrium is socially inefficient because it is characterized by a capital-labor ratio above the golden rule. There is therefore a prima facie case for government intervention in the saving-investment process. The second conclusion concerns debt neutrality. Here every neutrality theorem is matched by a non-neutrality theorem. If neither bequests nor gifts are operative, goverment borrowing crowds out capital formation. If child-parent gifts are operative, small increases in government borrowing are neutralized by reductions in gifts. If bequests are operative ,small reductions in government borrowing (or increases in lending) are neutralized by reductions in bequests. There always exists an increase in lending or borrowing that will make $B = 0$, respectively $G = 0$ a binding constraint. There always is a government financial strategy that puts the private sector in a zero gift and zero bequest corner solution, where financial policy will affect the capital-labor ratio. In our simple model, all agents are identical, so either everyone is at a corner or no one is. If instead we visualize a distribution of agents, by δ, by ϱ, and by the other parameters of their utility functions and the constraints they face, increasing government borrowing can be expected to make the zero gift constraint binding for an increasing number of agents. The same analysis can be applied to the study of the effects of the introduction of an unfunded social security retirement scheme in a world where previously only private saving provided for retirement. Subject to the qualifications mentioned earlier, such a scheme will reduce private saving in the short run and the capital-labour ratio in the long run.

Even in this most classical of models, the conclusion emerges inexorably that the way in which the government finances its real spending program can have major consequences for saving and capital formation. Debt neutrality is not a plausible theoretical proposition. Future research should concentrate on empirical assessments of the extent and nature of non-neutrality.

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