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Granger-Causality and Policy Effectiveness

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Introduction

It is generally recognized that, if a set of monetary and fiscal policy variables Granger-cause[1] real economic variables, this does not imply that alternative deterministic rules for determining the values of these policy instruments will alter the joint density function of the real variables.[2] It has, however, also been asserted that (letting **X** denote a (suitably restricted) vector of "real" economic aggregates, **g** a list of monetary and fiscal policy variables and E the mathematical expectation operator),

> a model *in general* will have classical policy implications if it satisfies
> $$E(\mathbf{X}_t|\mathbf{X}_{t-1}, \mathbf{X}_{t-2}, \ldots; \mathbf{g}_{t-1}, \mathbf{g}_{t-2}, \ldots) = E(\mathbf{X}_t|\mathbf{X}_{t-1}, \mathbf{X}_{t-2}, \ldots)$$
> so that as a block the aggregate real variables **X** are statistically exogenous with respect to (not caused by, in Granger's sense) the variables in **g**. [Sargent, 1976a, p. 221; my italics.]

Cuddington (1980, p. 539) also argues that

> various classical policy implications emerge from models in which the conditional expectations of all real variables are invariant with respect to government policy instruments. This is equivalent to the statistical hypothesis that the simultaneously determined variables of the economic model are jointly exogenous with respect to all policy variables.

Classical policy implications here means independence of the conditional means of real economic variables of the feedback rules for the monetary and fiscal policy variables, or more loosely that "government manipulations of monetary and fiscal policy variables have no predictable effects on unemployment, output or the interest rate and hence are useless for pursuing countercyclical policy" (Sargent, 1976a, p. 208). In other papers (e.g. Sargent, 1976b), Sargent gives a correct diagnosis of the lack of any necessary connection between Granger-causality and policy effectiveness or neutrality.

In Sargent (1976a) it is also recognized that "it is possible to concoct nonclassical systems that will mimic the classical characteristics thay my tests look for" (p. 222) and an example of such a system is given. The comment about this example that, "while the tests might be fooled by such a structure, that structure itself seems unlikely to me" (Sargent, 1976a, p. 223) reinforces the earlier statement that models in which monetary and fiscal policy variables fail to Granger-cause real economic aggregates will *in general* have classical policy implications.

In this paper it is shown that Granger-causality is unnecessary for policy effectiveness, insufficiency having been established already by Sargent. Warnings about the pitfalls in Granger-causality tests are contained in a number of papers, notably Jacobs, Leamer and Ward (1979), Sims (1977) and Zellner (1979). This paper goes beyond these earlier contributions by demonstrating that no inferences about policy ineffectiveness can be drawn from the results of Granger-causality tests. It can be viewed as an investigation of the statement by Sims that "The fact that policy variables are always in one sense causally prior to the other variables in a model is sometimes assumed to make it likely that they are causally prior (i.e. exogenous) in data used for estimation" (Sims, 1979, pp. 105–106).[3] It therefore addresses an issue very similar to the one that concerned Tobin in his paper "Money and Income: *Post Hoc Ergo Propter Hoc*?" (Tobin, 1970), and can indeed be viewed as a stochastic, rational expectations extension of Tobin's analysis.

Most of the statistical literature on Granger-causality has dealt with the relationship between Granger-causality and econometric exogeneity—independence of the regressors and the current and future values of the disturbances (pre-determinedness), or independence of the regressors and current, past and future values of the disturbances (strict exogeneity). For example, Sims (1972) has shown that failure of $\mathbf{x}$ to Granger-cause $\mathbf{y}$ is a necessary, but not a sufficient, condition for $\mathbf{x}$ to be strictly exogenous. This is not what this paper is concerned with. Indeed, a very similar paper could have been written on the absence of any systematic relationship between (strict) econometric exogeneity and policy effectiveness.

Section I deals with the case of an optimizing controller and a fairly general linear model which permits forward-looking rational expectations. Whether the controller pursues an optimal (possibly time-inconsistent) or a time-consistent (possibly non-optimal) policy, the instruments do not Granger-cause the endogenous variables while the endogenous variables, in general, do Granger-cause the policy instruments. The instruments fail to Granger-cause the endogenous variables even though changing the policy rule may alter any of the conditional and asymptotic moments of the joint distribution function of the endogenous variables.

Section II considers the policy effectiveness information content of Granger-causality tests for two important kinds of *ad hoc* policy rules: automatic stabilizers or instantaneous feedback rules, and *ad hoc* lagged feedback rules.

For important classes of policy rules, Granger-causality tests fail to reveal whether or not "the conditional expectations of all real variables are invariant with respect to government policy instruments". If the value of a vector of current endogenous variables is a function of the expected current or lagged instruments, and if the expected instruments are functions of current or lagged values of the endogenous variables, then the instruments won't Granger-cause the endogenous variables even though changing the policy rule will alter the dependence of the endogenous variables on their own lagged values and on the exogenous variables.

A fortiori, Granger-causality tests tend to be uninformative about the invariance of the second or higher moments of the distributions of real variables with respect to policy instruments. Stabilization policy tends to be concerned primarily with the second moments of such variables as output, employment and inflation.[4] The importance of these straightforward propositions lies in the fact that many interesting policy rules have the property that the instruments are functions (often exact, i.e. non-stochastic) of current and lagged values of the endogenous variables.

Before establishing the second half of the two-way non-implication between Granger-causality and policy effectiveness, formal definitions are given of Granger-causality and "structural invariance".

In a linear Gaussian setting, a vector of random variables **x** fails to Granger-cause another vector **y** if, in a least squares regression of **y** on its own past values and on past values of **x**, the regression coefficients on past values of **x** are all equal to (insignificantly different from) zero.

To permit consideration of the more general definition of Granger-causality proposed, e.g. in Granger (1980), the causality concept just referred to will be called causality *in mean*. Formally:

For any vector $\mathbf{x}_t$, let $\mathbf{X}^t \equiv \{\mathbf{x}_s, s < t\}$.

Definition 1. $\mathbf{x}_t$ is said to Granger-cause $\mathbf{y}_t$ in mean if

$$E(\mathbf{y}_t|\mathbf{Y}^t, \mathbf{X}^t) \not\equiv E(\mathbf{y}_t|\mathbf{Y}^t).$$

$\mathbf{x}_t$ fails to Granger-cause $\mathbf{y}_t$ in mean if (Granger, 1980, p. 337)

$$E(\mathbf{y}_t|\mathbf{Y}^t, \mathbf{X}^t) \equiv E(\mathbf{y}_t|\mathbf{Y}^t).$$

A stronger form of Granger-causality refers to the entire conditional distribution of random variables instead of merely their conditional means. For any vector **x** let $\tilde{F}(\mathbf{x}_t|\mathbf{z}_t)$ denote the conditional distribution function of $\mathbf{x}_t$ given $\mathbf{z}_t$.

Definition 2. $\mathbf{x}_t$ is said to Granger-cause $\mathbf{y}_t$ if

$$\tilde{F}(\mathbf{y}_t|\mathbf{Y}^t, \mathbf{X}^t) \not\equiv \tilde{F}(\mathbf{y}_t|\mathbf{Y}^t).$$

$\mathbf{x}_2$ fails to Granger-cause $\mathbf{y}_t$ if (Granger, 1980, pp. 336–337)

$$\tilde{F}(\mathbf{y}_t|\mathbf{Y}^t, \mathbf{X}^t) \equiv \tilde{F}(\mathbf{y}_t|\mathbf{Y}^t).$$

The concept of invariance or structural invariance, which is the relevant one for the issue of policy effectiveness, is defined in Buiter (1982).

Definition 3. $\mathbf{y}_t$ is said to be structurally invariant *in mean* with respect to $\mathbf{x}_t$ if changes in the deterministic components of the stochastic process governing $\mathbf{x}_t$ do not alter the mean of $\mathbf{y}_t$.[5]

A stronger invariance property is:

Definition 4. $\mathbf{y}_t$ is said to be structurally invariant *in distribution* with respect to $\mathbf{x}_t$ if changes in the deterministic components of the stochastic process governing $\mathbf{x}_t$ do not alter the distribution of $\mathbf{y}_t$.

Note that our concept of ineffectiveness, neutrality or structural invariance relates to a restricted class of changes in the distribution function of the policy instruments. This definition essentially restricts the distribution changes to changes in the deterministic coefficients of the linear feedback rules characterizing the behaviour of the policy instruments. Such alterations in the distribution function of the policy instruments as changes in the variance–covariance matrix of the innovations in the policy instrument processes are ruled out; they would virtually always affect the second and higher movements of the distribution function of real economic variables, but have little bearing on the policy effectiveness debate.

I. Granger-causality and Policy Effectiveness with an Optimizing Controller

As has already been recognized by Sims (1972), policy instruments must always fail to Granger-cause target variables, whether nominal or real, if the controller is optimizing a quadratic objective functional subject to the constraints of a linear model. Sims referred to the case where the information lag in the control process is no more than one period. This result remains valid in rational expectations models. Consider the linear-quadratic optimal control problem of equations (1) and (2):

$$(1) \qquad \min_{\{\mathbf{x}_t\}} E\left\{\sum_{t=1}^{T} (\mathbf{y}_t-\mathbf{a}_t)'\mathbf{K}_t(\mathbf{y}_t-\mathbf{a}_t)|\Omega_1\right\}$$

subject to

$$(2) \qquad \mathbf{y}_t=\mathbf{A}_1\mathbf{y}_{t-1}+\mathbf{B}_1E(\mathbf{y}_{t+1}|\Omega_{t-1})+\mathbf{B}_2E(\mathbf{y}_t|\Omega_{t-1})+\mathbf{C}_1\mathbf{x}_t+\bar{\mathbf{b}}_t+\mathbf{u}_t$$

where $\mathbf{y}_t$ is a vector of state, target or endogenous variables, $\mathbf{a}_t$ is a vector of desired values for $\mathbf{y}_t$ and $\mathbf{K}_t$ is a symmetric positive semi-definite matrix; $\mathbf{x}_t$ is a vector of instruments, $\bar{\mathbf{b}}_t$ a vector of exogenous variables, and $\mathbf{u}_t$ a white noise disturbance vector with $E(\mathbf{u}_t\mathbf{u}_t')=\Sigma_{\mathbf{u}}$; $\mathbf{A}_1$, $\mathbf{B}_1$, $\mathbf{B}_2$ and $\mathbf{C}_1$ are constant matrices. The information set Ω_τ conditioning expectations formed in period τ contains the true structures of the models, and $\mathbf{y}_{\tau-i}$, $\mathbf{x}_{\tau-i}$, $\bar{\mathbf{b}}_{\tau-i}$, $i\geq 0$.[6]

By taking expectations of both sides of (2) conditional on Ω_{t-1}, we can eliminate $E(\mathbf{y}_t|\Omega_{t-1})$. Equation (2) becomes

$$(3) \qquad \mathbf{y}_t=\mathbf{A}\mathbf{y}_{t-1}+\mathbf{B}E(\mathbf{y}_{t+1}|\Omega_{t-1})+\mathbf{C}E(\mathbf{x}_t|\Omega_{t-1})+\mathbf{E}(\mathbf{b}_t|\Omega_{t-1})+\eta_t$$

where

$$(4a) \qquad \mathbf{A}=(\mathbf{I}-\mathbf{B}_2)^{-1}\mathbf{A}_1, \qquad \mathbf{B}=(\mathbf{I}-\mathbf{B}_2)^{-1}\mathbf{B}_1, \qquad \mathbf{C}=(\mathbf{I}-\mathbf{B}_2)^{-1}\mathbf{C}_1$$
$$\mathbf{b}_t=(\mathbf{I}-\mathbf{B}_2)^{-1}\bar{\mathbf{b}}_t$$

and

$$(4b) \qquad \boldsymbol{\eta}_t=\mathbf{C}_1\{\mathbf{x}_t-E(\mathbf{x}_t|\Omega_{t-1})\}+\{\bar{\mathbf{b}}_t-E(\bar{\mathbf{b}}_t|\Omega t-1)\}+\mathbf{u}_t.$$

Note that $\boldsymbol{\eta}_t$ will also be white noise.

Taking $E(\mathbf{y}_{t+1}|\Omega_{t-1})$ in (3) as given when $\mathbf{x}_t$ is set in period t, we can apply dynamic programming as in Chow (1980) to find the *time-consistent* feedback control rule:

$$(5) \qquad \mathbf{x}_t=\mathbf{G}_{1t}E(\mathbf{y}_{t+1}|\Omega_{t-1})+\mathbf{G}_{2t}\mathbf{y}_{t-1}+\mathbf{g}_t.$$

where $\mathbf{G}_{1t}$, $\mathbf{G}_{2t}$ and $\mathbf{g}_t$ are non-stochastic functions of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\{\mathbf{b}_t, \mathbf{a}_t, \mathbf{K}_\tau; \tau\geq t\}$. Note that (5) implies that $\mathbf{x}_t=E(\mathbf{x}_t|\Omega_{t-1})$.

If $\mathbf{a}_t=\mathbf{a}$, $\mathbf{K}_t=\mathbf{k}$ and $\mathbf{b}_t=\mathbf{b}$ for all t, then, under conditions given in Chow (1975, pp. 170–172), $\mathbf{G}_{1t}$, $\mathbf{G}_{2t}$ and $\mathbf{g}_t$ may become time-invariant as $T\to\infty$.[7] The behaviour of the covariance-stationary system is governed by

$$(6) \qquad \mathbf{y}_t=\mathbf{R}_1E(\mathbf{y}_{t+1}|\Omega_{t-1})+\mathbf{R}_2\mathbf{y}_{t-1}+\mathbf{r}+\boldsymbol{\eta}_t$$

where $\mathbf{R}_1=\mathbf{B}+\mathbf{C}\mathbf{G}_1$, $\mathbf{R}_2=\mathbf{A}+\mathbf{C}\mathbf{G}_2$, and $\mathbf{r}=\mathbf{b}+\mathbf{C}\mathbf{g}$.

If (6) is covariance-stationary there exists a representation

(7) $\mathbf{y}_t = \mathbf{Q}\mathbf{y}_{t-1} + \mathbf{q} + \boldsymbol{\eta}_t$

where the roots of $\mathbf{Q}$ all have modulus less than one and

$$\mathbf{Q} = (\mathbf{I} - \mathbf{R}_1\mathbf{Q})^{-1}\mathbf{R}_2$$

$$\mathbf{q} = \{\mathbf{I} - \mathbf{R}_1(\mathbf{Q} + \mathbf{I})\}^{-1}\mathbf{r}.$$

Since $E(\mathbf{y}_{t+1}|\Omega_{t-1}) = \mathbf{Q}^2\mathbf{y}_{t-1} + (\mathbf{Q} + \mathbf{I})\mathbf{q}$, the behaviour of the instrument vector $\mathbf{x}$ under optimal control is governed by

(8) $\mathbf{x}_t = (\mathbf{G}_1\mathbf{Q}^2 + \mathbf{G}_2)\mathbf{y}_{t-1} + \mathbf{G}_1(\mathbf{Q} + \mathbf{I})\mathbf{q} + \mathbf{g}.$

Since $\mathbf{x}_t$ is an exact (non-stochastic) function of $\mathbf{y}_{t-1}$, it is clear that $\tilde{F}(\mathbf{y}_t|\mathbf{Y}^t) \equiv \tilde{F}(\mathbf{y}_t|\mathbf{Y}^t, \mathbf{X}^t)$: when the controller behaves in a time-consistent manner, the instruments will never Granger-cause the state variables. The state variables will, however, in general Granger-cause the instruments, since

$$E(\mathbf{x}_t|\mathbf{X}^t) = (\mathbf{G}_1\mathbf{Q}^2 + \mathbf{G}_2)\mathbf{Q}\mathbf{y}_{t-2} + [\mathbf{G}_1\{\mathbf{Q}(\mathbf{Q} + \mathbf{I}) + \mathbf{I}\} + \mathbf{G}_2]\mathbf{q} + \mathbf{g}$$

and

$$E(\mathbf{x}_t|\mathbf{X}^t, \mathbf{Y}^t) = (\mathbf{G}_1\mathbf{Q}^2 + \mathbf{G}_2)\mathbf{Q}\mathbf{y}_{t-2} + [\mathbf{G}_1\{\mathbf{Q}(\mathbf{Q} + \mathbf{I}) + \mathbf{I}\} + \mathbf{G}_2]\mathbf{q} + \mathbf{g} + (\mathbf{G}_1\mathbf{Q}^2 + \mathbf{G}_2)\boldsymbol{\eta}_{t-1} = \mathbf{x}_t.$$

Thus, unless $\mathbf{G}_1\mathbf{Q}^2 + \mathbf{G}_2 = 0$, the state variables, $\mathbf{y}$, Granger-cause the instruments, $\mathbf{x}$.

If, instead of pursuing *time-consistent* policies, the authorities pursue optimal but possibly time-inconsistent policies, Granger-causality tests continue to be uninformative as regards the presence or absence of policy effectiveness. When optimal policies are pursued, the controller does not take $E(\mathbf{y}_{t+1}|\Omega_{t-1})$ as given when choosing $\mathbf{x}_t$, but allows for the effect of $\mathbf{x}_t$ on $E(\mathbf{y}_{t+1}|\Omega_{t-1})$ through the effect of $\mathbf{x}_t$ on $E(\mathbf{x}_t|\Omega_{t-1})$ (see Buiter, 1981a, b; 1983b). As in the case of *time-consistent* policies, the optimal policy will be a non-stochastic linear feedback rule:

$$\mathbf{x}_t = \mathbf{F}_t\mathbf{y}_{t-1} + \mathbf{f}_t.$$

If the system can be made covariance-stationary by using such a rule (virtually a necessary condition for econometric estimation and hypothesis testing), the instruments will be governed by

(9) $\mathbf{x}_t = \mathbf{F}\mathbf{y}_{t-1} + \mathbf{f}.$

Combining (9) and (3) with $\mathbf{b}_t = \mathbf{b}$, we get

$$\mathbf{y}_t = (\mathbf{A} + \mathbf{C}\mathbf{F})\mathbf{y}_{t-1} + \mathbf{B}E(\mathbf{y}_{t+1}|\Omega_{t-1}) + \mathbf{C}\mathbf{f} + \mathbf{b} + \boldsymbol{\eta}_t.$$

If this is covariance-stationary there exists a representation

(10) $\mathbf{y}_t = \bar{\mathbf{Q}}\mathbf{y}_{t-1} + \bar{\mathbf{q}} + \boldsymbol{\eta}_t$

with

(11a) $\bar{\mathbf{Q}} = (\mathbf{I} - \mathbf{B}\bar{\mathbf{Q}})^{-1}(\mathbf{A} + \mathbf{C}\mathbf{F})$

(11b) $\bar{\mathbf{q}} = \{\mathbf{I} - \mathbf{B}(\bar{\mathbf{Q}} + \mathbf{I})\}^{-1}(\mathbf{C}\mathbf{f} + \mathbf{b})$

Again, $\mathbf{x}_t$ will not Granger-cause $\mathbf{y}_t$ in mean or in distribution while, in general, $\mathbf{y}_t$ will Granger-cause $\mathbf{x}_t$.

Whether we consider (7) and (8) or (9) and (10), the Granger-causality tests reveal nothing about policy effectiveness. It can be shown for the model of equations (2) or (3) that, unless $\mathbf{C}=\mathbf{O}$, the expected instruments $\mathbf{x}_t$ will affect the expectation of $\mathbf{y}_t$. Consider again the model of equation (2) or (3) with, for simplicity, $\mathbf{b}_t=\mathbf{b}$ for all t. Solving for $\mathbf{y}_t$ as a function of current and anticipated future values of $\mathbf{x}_t$, we obtain

$$(12)\qquad \mathbf{y}_t=\Pi_1\mathbf{y}_{t-1}+\Pi_2\mathbf{b}+\sum_{i=0}^{\infty}\Lambda_i E(\mathbf{x}_{t+i}|\Omega_{t-1})+\mathbf{C}_1\{\mathbf{x}_t-E(\mathbf{x}_t|\Omega_{t-1})\}+\mathbf{u}_t$$

where

$$(13a)\qquad \mathbf{\Pi}_1=\mathbf{A}+\mathbf{B}\mathbf{\Pi}_1^2$$

$$(13b)\qquad \mathbf{\Pi}_2=\mathbf{B}(\mathbf{\Pi}_1+\mathbf{I})\mathbf{\Pi}_2+\mathbf{I}$$

$$(13c)\qquad \mathbf{\Lambda}_0=\mathbf{C}+\mathbf{B}\mathbf{\Pi}_1\mathbf{\Lambda}_0$$

$$(13d)\qquad \mathbf{\Lambda}_i=\mathbf{B}(\mathbf{\Pi}_1\mathbf{\Lambda}_i+\mathbf{\Lambda}_{i-1}),\qquad i\geq 1.$$

Provided $\mathbf{C}\neq\mathbf{O}$, i.e. provided $(\mathbf{I}-\mathbf{B}_2)^{-1}\mathbf{C}_1\neq\mathbf{O}$, anticipated constant and future values of $\mathbf{x}$ will affect both the conditional first moment of $\mathbf{y}$, $E(\mathbf{y}_t|\Omega_{t-1})$, and the asymptotic or unconditional first moment.

"Policy ineffectiveness" emerges only if $\mathbf{C}=\mathbf{O}$. If in addition $\mathbf{C}_1\neq\mathbf{0}$ we are in a world where anticipated policy actions do not affect any moment of the distribution function of $\mathbf{y}_t$ but unanticipated policy actions can affect the second and higher moments of this distribution.

Except in some non-cooperative games in which randomized strategies may be optimal, rational expected utility-maximizing or expected loss-minimizing policy-makers will always set $\mathbf{x}_t$ (time-consistently or optimally) as a non-stochastic function of the information available at the time the instrument values have to be assigned. In our set-up $\mathbf{y}_{t-1}$ is a sufficient statistic for all relevant information available to the policy-maker (assuming knowledge of the structure of the model and of $\bar{\mathbf{b}}$ or $\mathbf{b}$).

Note that, if there is a longer information lag in the control process, the results of this section remain valid. Assume, for example that when the controller chooses $\mathbf{x}_t$ he knows only $\mathbf{y}_{t-2}$ (as well as equation (2) or (3)). For simplicity let $\bar{\mathbf{b}}_t=\bar{\mathbf{b}}$. A time-invariant optimal or time-consistent policy rule will now take the form

$$(14)\qquad \mathbf{x}_t=\tilde{\mathbf{F}}\mathbf{y}_{t-2}+\tilde{\mathbf{f}}$$

where $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{f}}$ are non-stochastic.

Substituting (14) into (2) or (3), the behaviour of the system under optimal or time-consistent control will be governed by an equation of the form

$$(15)\qquad \mathbf{y}_t=\mathbf{\Pi}_1\mathbf{y}_{t-1}+\mathbf{\Pi}_2\mathbf{y}_{t-2}+\mathbf{\Pi}_3(\mathbf{C}_1\mathbf{f}+\bar{\mathbf{b}})+\mathbf{u}_t$$

where

$$\mathbf{\Pi}_1=(\mathbf{I}-\mathbf{B}_2)^{-1}\{\mathbf{A}_1+\mathbf{B}_1(\mathbf{\Pi}_1^2+\mathbf{\Pi}_2)\},\qquad \mathbf{\Pi}_2=(\mathbf{I}-\mathbf{B}_2)^{-1}(\mathbf{C}_1\mathbf{F}+\mathbf{B}_1\mathbf{\Pi}_1\mathbf{\Pi}_2)$$

and

$$\mathbf{\Pi}_3 = (\mathbf{I} - \mathbf{B}_2)^{-1}\{\mathbf{I} + \mathbf{B}_1(\mathbf{\Pi}_1 + \mathbf{I})\mathbf{\Pi}_3\}.$$

Clearly, $\mathbf{x}$ will fail to Granger-cause $\mathbf{y}$ in mean and distribution while $\mathbf{y}$, in general, Granger-causes $\mathbf{x}$.

II. Granger-causality and Policy Effectiveness with Ad Hoc Policy Rules

Two kinds of *ad hoc* policy rules will be considered. The first is the instantaneous feedback rule or automatic stabilizer; the second, the lagged feedback rule.

Granger-causality and automatic stabilizers

Granger-causality tests convey no information about the presence or absence of policy effectiveness for a set of instruments if during the sample period the instruments have been used as *automatic stabilizers*, that is, if they were governed by an instantaneous feedback rule relating the current values of the policy instruments to *current* endogenous variables. The discussion of automatic stabilizers is usually restricted to fiscal policy: "features of the tax structure that make tax liabilities respond automatically to current economic conditions" (McCallum and Whitaker, 1979, p. 172). McCallum and Whitaker (1979) rationalize the stabilizing potential of automatic stabilizers through the *decentralized* setting of the control variables that they permit. Aggregative information on GDP and the general price level is assumed to be available only with a lag. Transfer payments such as unemployment benefits are, however, paid out according to fixed rules on a decentralized basis, when claims are made by the individual unemployed workers. This permits an immediate response to changing economic conditions without any need to wait for aggregate output and unemployment data to become available. There are many problems associated with this interpretation. One is that, with efficient capital markets, the *timing* of the unemployment benefits would be of no concern. Any rule for delayed payments of the benefits that leaves the real present value of these benefits unchanged would have the same effect on behaviour. Unspecified capital market imperfections must therefore play a role if automatic stabilizers have a stabilizing potential that is not present with feedback rules, which make current policy instrument values functions only of lagged information. As a satisfactory treatment of these issues would require a full paper in its own right, I shall say no more about them here.

If automatic decentralized fiscal stabilizers exist, so, in principle, do monetary stabilizers. Those in charge of the local unemployment registers could be instructed to perform an open-market purchase of government bonds of a given amount for every worker joining the unemployment register, and an open-market sale for every notified vacancy.[8]

Consider the general rational expectations model of equation (2) (or (3)) with $\bar{\mathbf{b}}_t = \bar{\mathbf{b}}$ for simplicity. The simultaneous feedback rule is given by

$$\mathbf{x}_t = \mathbf{R}\mathbf{y}_t + \boldsymbol{\varepsilon}_t$$

$\boldsymbol{\varepsilon}_t$ represents an *ad hoc* random element in the policy rule. It is a white noise disturbance vector with $E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') = \boldsymbol{\Sigma}_{\varepsilon}$.

With this automatic stabilizer the reduced form solution for $\mathbf{y}_t$ is:

(16a) $\mathbf{y}_t = \mathbf{\Pi}_1\mathbf{y}_{t-1} + \mathbf{\Pi}_2\bar{\mathbf{b}} + \mathbf{\Pi}_3(\mathbf{u}_t + \mathbf{C}_1\boldsymbol{\varepsilon}_t)$

with

(16b) $\mathbf{\Pi}_1 = (\mathbf{I} - \mathbf{C}_1\mathbf{R})^{-1}\{\mathbf{A}_1 + (\mathbf{B}_1\mathbf{\Pi}_1 + \mathbf{B}_2)\mathbf{\Pi}_1\}$

(16c) $\mathbf{\Pi}_2 = (\mathbf{I} - \mathbf{C}_1\mathbf{R})^{-1}\{\mathbf{B}_1(\mathbf{\Pi}_1 + \mathbf{I})\mathbf{\Pi}_2 + \mathbf{B}_2\mathbf{\Pi}_2 + \mathbf{I}\}$

(16d) $\mathbf{\Pi}_3 = (\mathbf{I} - \mathbf{C}_1\mathbf{R})^{-1}$.

Clearly, the instruments do not Granger-cause endogenous variables (in mean or in distribution): $\tilde{\mathbf{F}}(\mathbf{y}_t|\mathbf{Y}^t) \equiv \tilde{\mathbf{F}}(\mathbf{y}_t|\mathbf{Y}^t, \mathbf{X}^t)$. If $\mathbf{\Sigma}_\varepsilon = \mathbf{0}$, i.e. if the automatic stabilizer is non-stochastic, $\mathbf{y}_t$ also fails to Granger-cause $\mathbf{x}_t$; the endogenous variables and the instruments are merely linear transformations of each other. If $\mathbf{\Sigma}_\varepsilon \neq \mathbf{0}$, $\mathbf{y}_t$ will, in general, Granger-cause $\mathbf{x}_t$. For example,

$$E(\mathbf{x}_t|\mathbf{X}^t, \mathbf{Y}^t) = \mathbf{R}\mathbf{\Pi}_1\mathbf{y}_{t-1} + \mathbf{R}\mathbf{\Pi}_2\bar{\mathbf{b}} \neq E(\mathbf{x}_t|\mathbf{X}^t).$$

Note that the conditional and unconditional second moments of $\mathbf{y}$ will be functions of the parameters of the automatic stabilizer rule $\mathbf{R}$, if $\mathbf{C}_1 \neq \mathbf{0}$. The conditional and unconditional first moments of $\mathbf{y}$ will be functions of $\mathbf{R}$ provided $\partial\mathbf{\Pi}_1/\partial\mathbf{R}$ and $\partial\mathbf{\Pi}_2/\partial\mathbf{R}$ are not both equal to zero. Changing the nature of the policy rule to, e.g. a fixed, open-loop rule or to a lagged feedback rule will in general alter the first and higher moments of $\mathbf{y}$. Regardless of its capacity for influencing the distribution function of $\mathbf{y}$, $\mathbf{x}$ will fail to Granger-cause $\mathbf{y}$ if during the sample period it has been used as an automatic stabilizer.

Granger-causality and ad hoc lagged feedback rules

It is obvious that any *ad hoc* non-stochastic lagged feedback rule

$$\mathbf{x}_t = \sum_{i=1}^{N} \mathbf{R}_i\mathbf{y}_{t-i} + \mathbf{r}_t$$

where $\mathbf{r}_t$ is the non-stochastic, open-loop, component of the policy rule, will, when applied to the model of equations (2) or (3), result in the policy instruments failing to Granger-cause the endogenous variables "in distribution". Depending on the precise structure of the model, changes in the $\mathbf{R}_i$ may affect any of the conditional or asymptotic moments of $\mathbf{y}_t$ and changes in the non-stochastic open-loop component $\mathbf{r}_t$ may affect the conditional or asymptotic first moments.

Consider instead the stochastic lagged feedback rule:

(17) $\mathbf{x}_t = \mathbf{R}\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t$

where $\boldsymbol{\varepsilon}_t$ is the vector of innovations in the stochastic process governing the instruments. $\boldsymbol{\varepsilon}_t$ is assumed to be orthogonal to Ω_{t-1}. Substituting (17) into (2) with $\bar{\mathbf{b}}_t = \bar{\mathbf{b}}$, and eliminating $E(\mathbf{y}_t|\Omega_{t-1})$, we obtain

$$\mathbf{y}_t = (\mathbf{I} - \mathbf{B}_2)^{-1}(\mathbf{A}_1 + \mathbf{C}_1\mathbf{R})\mathbf{y}_{t-1} + (\mathbf{I} - \mathbf{B}_2)^{-1}\mathbf{B}_1 E(\mathbf{y}_{t+1}|\Omega_{t-1}) + (\mathbf{I} - \mathbf{B}_2)^{-1}\bar{\mathbf{b}} + \mathbf{C}_1\boldsymbol{\varepsilon}_t + \mathbf{u}_t.$$

Provided the system is covariance-stationary, it will have a first-order representation.

(18) $\mathbf{y}_t = \mathbf{\Pi}_1\mathbf{y}_{t-1} + \mathbf{\Pi}_2\bar{\mathbf{b}} + \mathbf{C}_1\boldsymbol{\varepsilon}_t + \mathbf{u}_t$.

Π_1 and Π_2 are constant matrices to be determined, e.g. through the method of undetermined coefficients.

Note that $\mathbf{x}$ will again fail to Granger-cause $\mathbf{y}$ (in mean and in distribution) while $\mathbf{y}$ will Granger-cause $\mathbf{x}$. This result stands if (17) is replaced by the more general process

$$\mathbf{x}_t = \sum_{i=1}^{N} \mathbf{R}_i \mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t.$$

Finally, consider the class of rational expectations models in which anticipated policy has no effect on the first moments of real variables while their second moments are functions of the parameters of the instrument feedback rules. Such stabilization policy effectiveness comes about through anticipations of future policy actions, which change the informational content of observed endogenous variables such as prices, etc. (Turnovsky, 1980; Weiss, 1980; Buiter, 1980, 1981a, b, 1983a, b; King, 1982).

In such models, the semi-reduced form for the endogenous variables (after substituting out current and past expectations of future endogenous variables) has the following form, using the notation of equation (2):

$$(19) \quad \mathbf{y}_t = \Pi_1 \mathbf{y}_{t-1} + \Pi_2 \bar{\mathbf{b}} + \Pi_3 \mathbf{u}_t + \Lambda_0 \{\mathbf{x}_t - E(\mathbf{x}_t|\Omega_{t-1})\} + \sum_{i=1}^{\infty} \Lambda_i \{E(\mathbf{x}_{t+i}|\Omega_t) - E(\mathbf{x}_{t+i}|\Omega_{t-1})\}.$$

The current value of $\mathbf{y}$ is a function of the revision, between periods $t-1$ and t, in the expectations of all future values of the instruments.

Let the instrument vector $\mathbf{x}_t$ be governed by any feedback rule such as

$$(10) \quad \mathbf{x}_t = \sum_{i=1}^{M} \mathbf{F}_i \mathbf{y}_{t-i} + \sum_{i=1}^{M} \mathbf{G}_i \mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t$$

where $\boldsymbol{\varepsilon}_t$ is a white noise disturbance vector. From (20), $\mathbf{x}_t - E(\mathbf{x}_t|\Omega_{t-1}) = \boldsymbol{\varepsilon}_t$. $E(\mathbf{x}_{t+i}|\Omega_t) - E(\mathbf{x}_{t+i}|\Omega_{t-1})$ for $i > 1$ can only be a function of the new information (the news) that accrued between periods $t-1$ and t, that is, of $\mathbf{u}_t$ and $\boldsymbol{\varepsilon}_t$. We can therefore write the last term of the right-handed side of (19) as $\mathbf{K}_1 \mathbf{u}_t + \mathbf{K}_2 \boldsymbol{\varepsilon}_t$ where $\mathbf{K}_1$ and $\mathbf{K}_2$ are functions of the policy parameters $\mathbf{F}_i$ and $\mathbf{G}_i$. Rewrite (19) as

$$(21) \quad \mathbf{y}_t = \Pi_1 \mathbf{y}_{t-1} + \Pi_2 \bar{\mathbf{b}} + (\Pi_3 + \mathbf{K}_1)\mathbf{u}_t + (\Lambda_0 + \mathbf{K}_2)\boldsymbol{\varepsilon}_t.$$

The potential scope for stabilization policy in these models is apparent now. In certain models (Buiter, 1980, 1981b, 1983a) it is possible to choose the policy parameters in such a way that $\Pi_3 = -\mathbf{K}_1$ and $\Lambda_0 = -\mathbf{K}_2$; all randomness in $\mathbf{y}_t$ can be eliminated. More generally, the conditional and asymptotic second moments of $\mathbf{y}$ will be functions of the parameters of the feedback rule. It is, however, clear from (20) and (21) that $\mathbf{x}$ does not Granger-cause $\mathbf{y}$ in mean or in distribution. Specifically

$$E(\mathbf{y}_t|\mathbf{Y}^t) = \Pi_1 \mathbf{y}_{t-1} + \Pi_2 \bar{\mathbf{b}} = E(\mathbf{y}_t|\mathbf{Y}^t, \mathbf{X}^t)$$

and

$$E[\{\mathbf{y}_t - E(\mathbf{y}_t|\mathbf{Y}^t)\}\{\mathbf{y}_t - E(\mathbf{y}_t|\mathbf{Y}^t)\}'|\mathbf{Y}^t]$$
$$= (\mathbf{\Pi}_3 + \mathbf{\Omega}_1)\mathbf{\Sigma}_{\mathbf{u}}(\mathbf{\Pi}_3 + \mathbf{\Omega}_1)' + (\mathbf{\Lambda}_0 + \mathbf{\Omega}_2)\mathbf{\Sigma}_{\varepsilon}(\mathbf{\Lambda}_0 + \mathbf{\Omega}_2)'$$
$$= E[\{\mathbf{y}_t - E(\mathbf{y}_t|\mathbf{Y}^t, \mathbf{X}^t)\}\{\mathbf{y}_t - E(\mathbf{y}_t|\mathbf{Y}^t, \mathbf{X}^t)\}'|\mathbf{Y}^t, \mathbf{X}^t].^9$$

III. CONCLUSION

The purpose of this paper has been to reject the notion that any inferences about policy effectiveness, structural invariance or the ability of instruments to influence endogenous variables in a systematic way can be drawn from the results of Granger-causality tests. It is not necessary to "concoct" unlikely structures in order to support the proposition that there is two way non-implication between Granger non-causality and structural invariance. Just as Sargent (1976a, p. 222) cautioned against the Type I error of inferring non-invariance from Granger-causality, so this paper warns against the Type II error of inferring structural invariance from Granger non-causality.

It is not diffcult to come up with examples in which failure of instruments to Granger-cause endogenous variables coincides with ineffectiveness of the instruments with respect to the endogenous variables. Such findings are, however, merely accidental, like bagging the vicar at a grouse-shoot. Granger-causality tests are tests of "incremental predictive content" (Schwert, 1979). They are one among a number of statistical exogeneity tests (see Engle, Hendry and Richard, 1983) and play an important role in the estimation and testing of data-coherent econometric models. They are not informative as to the presence or absence of structural invariance in general and policy effectiveness in particular. Tests for policy effectiveness require the presence of changes in the generating process of the policy instruments, either in the sample or in the forecast periods, in order to ascertain whether such changes are associated with changes in the distribution functions of the endogenous variables under consideration.

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NOTES

[1] One set of variables, **x**, is said to Granger-cause another set, **y**, if adding past values of **x** in a regression equation for predicting **y**, which already includes all past values of **y** as regressors, improves the predictive power of the equation in the sense that it reduces the mean squared forecast error (e.g. Hsaio, 1979). A more formal definition is given below.

[2] An example of Sargent (1976a, p. 222) suffices to make this point. Let V_t be the unemployment rate and m_t the logarithm of the nominal money stock

$$V_t = \lambda V_{t-1} + \beta_0\{m_t - E(m_t|I_{t-1})\} + \beta_1\{m_{t-1} - E(m_{t-1}|I_{t-2})\} + u_t$$

$$m_t = \sum_{i=1}^{n} \delta_i m_{t-i} + \varepsilon_t.$$

where ε_t and u_t are Gaussian random disturbances, E is the expectation operator and I_t the information set in period t conditioning expectations formed in period t. It is easily seen that

$$E(V_t | V_{t-1}, V_{t-2}, \ldots; m_{t-1}, m_{t-2}, \ldots) = \lambda V_{t-1} + \beta(m_{t-1} - \sum_{i=1}^{n} \delta_i m_{t-1-i}).$$

Therefore, m helps predict, or Granger-causes, V. However, deterministic feedback rules making m_t a linear function of I_{t-i}, $i > 0$, cannot affect the density function of V_t. Note that, if it is possible to relate m_t to I_t, such *instantaneous* feedback rules will affect the density function of v_t (see Section II).

[3] Sims does not endorse the view he refers to.

[4] An earlier version of this paper (Buiter, 1982) analyses the effect of monetary feedback rules on the second moment of output in a number of small rational expectations models and the inability of Granger-causality tests to detect such effects.

[5] This can apply either to the conditional mean or to the unconditional, asymptotic, mean.

[6] It is assumed that when $\mathbf{x}_t$ is set, the controller knows $\mathbf{y}_{t-1}$ and $\mathbf{b}_t$ but not $\mathbf{u}_t$ or $\mathbf{y}_t$. Generalizations giving the controller more information do not alter the conclusions.

[7] Even if $\mathbf{G}_{1t}$ and $\mathbf{G}_{2t}$ are time-invariant but $\mathbf{g}_t$ changes over time in response to variations in $\mathbf{b}_t$, the system under control may remain covariance-stationary. If the system cannot be made covariance-stationary, most of our statistical and econometric techniques become inapplicable.

[8] I owe this idea to John Flemming.

[9] $\Sigma_{\mathbf{u}} = E(\mathbf{u}_t \mathbf{u}_t')$; $\Sigma_{\varepsilon} = E(\varepsilon_t \varepsilon_t')$; for simplicity $\mathbf{u}$ and ε are assumed to be contemporaneously uncorrelated.

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