NBER

REPRINT NO. 677

Policy Decentralization and Exchange Rate Management in Interdependent Economies

Willem H. Buiter and Jonathan Eaton

NATIONAL BUREAU OF
ECONOMIC RESEARCH, INC.

NATIONAL BUREAU OF ECONOMIC RESEARCH, INC.
1050 MASSACHUSETTS AVENUE
CAMBRIDGE, MA 02138

An earlier version of this article appeared as NBER Working Paper No. 531. This article is reprinted with permission from *Exchange Rate Management Under Uncertainty*, edited by Jagdeep S. Bhandari, pages 31-54. Cambridge, MA: MIT Press, 1985.

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1. Introduction

The demise of Bretton Woods and of the short-lived Smithsonian agreement has raised questions about exchange rate management by monetary authorities acting in isolation from one another. For instance, will individual monetary authorities have an incentive to stabilize the exchange rate? To what extent will monetary actions abroad disrupt domestic monetary policy? What are the gains from coordinating monetary policy?

The problems that arise when different agents pursue independent policies in interdependent economies have been explored by a number of authors. Aoki (1976), Cooper (1969), Hamada (1976), Allen and Kenen (1980), McFadden (1969), Patrick (1973), Kydland (1976), and Pindyck (1976), among others, have written on this issue. Different authors have focused on different aspects of decentralized policy formation. Three distinct types of decentralization and coordination can, in principle, be distinguished.

1. *Target* decentralization occurs when the two authorities have different objectives. Full coordination of targets requires that the authorities adopt a common objective. This does not necessarily imply that authorities then act on the basis of the same information. Furthermore even though authorities have a common set of objectives, they need not play a cooperative game in the formal sense; no binding preplay agreements on the choice of policy responses may have been established.

2. Decentralization of *nonstrategic information* occurs when authorities have access to different information concerning the state of the economy. Full coordination of such information requires that the authorities share this information. Each authority will then form expectations and policy in period t on the basis of the pooled information. This type of coordination need not imply that authorities adopt common objectives or that they formulate policy responses cooperatively.

3. Decentralization of *strategic information* arises whenever policy responses are chosen independently. This type of decentralization could arise even in situations in which authorities shared objectives and informa-

tion about the state of the economy but the optimal cooperative strategy is nonunique. It is analogous to the decentralization problem faced by an American football team that has snapped the ball without having called a play in the huddle. Coordination of strategic information means that authorities make binding, preplay agreements on their choice of policy responses, which they may do even though their targets and information about the state of the economy may differ. The Mundell (1962) assignment problem arises because of decentralization of this form.

The purpose of this chapter is to analyze the optimal design of monetary policy in interdependent economies when both targets and strategic information are decentralized. Previous analytic discussion of policy decentralization has been based on deterministic models. Furthermore, with the exception of Hamada (1976), who considers the problem in a classical, full-employment context, these models incorporate the traditional neo-Keynesian assumption of fixed prices. Studies incorporating stochastic elements have also used a neo-Keynesian framework and rely solely on simulation analysis (Pindyck 1976; Kydland 1976). None of the studies incorporates recent contributions to the theory of aggregate supply and expectations formation associated with the Lucas (1972) supply function.

We consider two monetary authorities pursuing domestic targets in two economies connected by trade both in real goods and in national monies. The model is presented in section 2. Each economy is characterized by a supply function of the Lucas type in which deviations of output from its natural level occur only because of deviations of the domestic price level from the value anticipated in the previous period. The natural level itself is stochastic. Agents in each economy hold domestic money for transactions purposes but may speculate on exchange rate movements by holding domestic or foreign money. We assume that money demands are also stochastic. The two economies are linked by a stochastic purchasing power parity relationship between the two price levels and the exchange rate.

Throughout we assume that the only contemporaneous variable observed by the two monetary authorities and the private sectors is the exchange rate. Incomes and price levels are observed only with a one-period lag. Each monetary authority's problem, then, is to infer from the observed current exchange rate the type of shocks affecting the economy and to set the current money supply to offset these shocks, taking into account the response of the foreign monetary authority to its actions.

In section 3 we assume that each country's monetary authority pursues the objective of stabilizing output around the average, or ex ante, natural level of output. In section 4 we modify the objective to one of stabilizing income around the actual but unobserved natural rate. This second objective is equivalent to minimizing price forecast errors and is more likely to lead to a policy of exchange rate stabilization. In section 5 we introduce exchange rate stabilization as an additional, independent goal.

Throughout we derive the policy rules that obtain if the monetary authorities pursue distinct targets independently. In sections 3 and 4 the optimal policy rules are not affected if authorities instead pursue a common objective cooperatively. This result does not extend to more general models such as the one considered in section 5.

A main purpose of the model is to provide insight into the design of optimal policies for managed floating. Models in which current policy can respond only to past information must implicitly assume that either the exchange rate or the money supply is fixed within the period: exchange rates are either fixed or the float is clean within the period (see Buiter 1979). Our model, however, allows for a contemporaneous money supply response to the current exchange rate.[1] Setting the money supply to fix the exchange rate and ignoring the exchange rate in setting the money supply constitute special cases of our model. Indeed we find that optimal monetary policy can involve exacerbating exchange rate movements.[2]

2. Two-Country Model of Monetary Policy and Exchange Rate Determination

We consider a model in which there are two countries, each with a monetary authority that manipulates the national money supply with the objective of stabilizing income in that country. Although the authorities pursue different objectives with different instruments, they share common information about the state of the world. Each authority establishes its money supply in response to this information, taking the other authority's response function as given.

The deviation of actual output from its long-run normal level in each country is proportional to the percentage deviation of the actual price level in that country from the price level anticipated in the previous period. In the home country we have

$$y_t = \Phi(p_t - p_{t|t-1}) + u_t^y \quad \Phi \geq 0 \tag{1}$$

while abroad

$$y_t^* = \Phi^*(p_t^* - p_{t|t-1}^*) + u_t^{*y} \quad \Phi^* \geq 0. \tag{2}$$

Here y_t denotes the logarithm of home country output and p_t the logarithm of the home country price level; $p_{t|t-1}$ denotes the expectation of p_t based on information available in period $t-1$, and u_t^y denotes a Gaussian white noise disturbance term with variance σ_{uy}^2. Equivalent magnitudes in the foreign country are denoted with an asterisk. In equations (1) and (2) we have normalized the expected logarithms of full employment income to equal zero. The actual or ex post natural levels of outputs, u_t^y and u_t^{*y} obtain when $p_t = p_{t|t-1}$ and $p_t^* = p_{t|t-1}^*$, respectively.

Justification for output supply equations of the form (1) and (2) is provided by Lucas (1972) and Sargent and Wallace (1975). Output equations of the form (1) and (2) would arise also if wage contracts are formed one period in advance. Wages in period t, it is assumed, cannot be modified by information that is not available before period t.[3] Other motivations of (1) and (2) involve imperfect observation of the contemporaneous aggregate price level.

We assume that citizens in each country must hold domestic money for transactions purposes but may speculate by holding foreign money. Overall money demand is thus given by a set of currency substitution money demand functions:

$$m_t - p_t = \alpha y_t - \beta(e_{t+1|t} - e_t) + u_t^m \quad \alpha, \beta \geq 0. \tag{3}$$

$$m_t^* - p_t^* = \alpha^* y_t^* + \beta^*(e_{t+1|t} - e_t) + u_t^{*m} \quad \alpha^*, \beta^* \geq 0. \tag{4}$$

Here m_t denotes the logarithm of nominal home country money balances in period t, e_t the logarithm of the spot price of foreign currency, and $e_{t+1|t}$ the value of e_{t+1} expected to occur in period t. The terms u_t^m and u_t^{*m} represent Gaussian white noise disturbances with variances σ_{um}^2 and $\sigma_{u^*m'}^2$ respectively. The parameter α denotes the income elasticity of demand for real money balances and β the expected exchange rate appreciation elasticity. When $e_{t+1|t}$ is high relative to e_t, foreign money balances are more attractive.[4]

Finally we assume that the domestic and foreign price levels are connected by a stochastic purchasing power parity (PPP) relationship:[5]

$$p_t = e_t + p_t^* + u_t^e \tag{5}$$

where u_t^e represents a Gaussian white noise distribance with variance σ_{ue}^2. Also, for simplicity, contemporaneous values of u_t^y, u_t^{*y}, u_t^m, u_t^{*m}, and u_t^e are assumed to be uncorrelated.

We assume that e_t is the only endogenous or exogenous variable observed contemporaneously. All past endogenous and exogenous variables are also part of the common private and public information set. The information set at time t when m_t is chosen, denoted by Ω_t, is therefore given by:[6]

$$\Omega_t = [e_\tau, p_{\tau-1}, p_{\tau-1}^*, y_{\tau-1}, y_{\tau-1}^*] \quad \tau \leq t. \tag{6}$$

or equivalently by

$$\Omega_t = [e_\tau, y_{\tau-1}^y, u_{\tau-1}^{*y}, u_{\tau-1}^m, u_{\tau-1}^*, u_{\tau-1}^e] \quad \tau \leq t. \tag{6'}$$

In addition both the authorities and the private sector know the true structure of the model, including the first and second moments of the distributions of the random disturbances. Expectations of p_t formed in period $t-1$ are conditional on Ω_{t-1}, and expectations of e_{t+1} formed in period t are conditional on Ω_t. The foreign country has the same information set as the home country.

In period t the monetary authority in the home country chooses m_t to minimize

$$E(V_t|\Omega_t) = \sum_{\tau=t}^{\infty} \delta^{\tau-t} E[(y_\tau - \bar{y}_\tau)^2|\Omega_t] \quad 0 < \delta < 1. \tag{7}$$

The foreign monetary authority chooses m_t^* to minimize

$$E(V_t^*|\Omega_t) = \sum_{\tau=t}^{\infty} \delta^{*\tau-t} E[(y_\tau^* - \bar{y}_\tau^*)^2|\Omega_t] \quad 0 < \delta^* < 1. \tag{8}$$

$\bar{y}_\tau$ and $\bar{y}_\tau^*$ are target real output, at home and abroad, to be specified more precisely below. δ and δ^* are discount factors. Both m_t and m_t^* are chosen under the assumption that m_τ and m_τ^* minimize, respectively, $E(V_\tau|\Omega_\tau)$ and $E(V_\tau^*|\Omega_\tau)$ for all $\tau > t$.

By permitting money supplies to respond to the contemporaneous exchange rate out precluding the possibility of exchange rate contingent money wage contracts, known monetary rules will affect real output. In this model policymakers set money supplies by transfers and taxes. Direct exchange market intervention provides another mode that is consistent with our specification if the other country sterilizes the effect of exchange market intervention on its own money supply by transfers and taxes.

Equations (1) through (5) can be solved by substituting (1), (2), and (5) into (3) and (4) to obtain:

$$\begin{bmatrix} p_t \\ p_t^* \end{bmatrix} = B\begin{bmatrix} m_t + \alpha\phi p_{t|t-1} + \beta e_{t+1|t} + v_t + \beta u_t^e \\ m_t^* + \alpha^*\phi^* p_{t|t-1}^* - \beta^* e_{t+1|t} + v_t^* - \beta^* u_t^e \end{bmatrix}\Delta^{-1} \tag{9}$$

where

$$B \equiv \begin{bmatrix} \pi^* + \beta^* & \beta \\ \beta^* & \pi + \beta \end{bmatrix}$$

$$\pi \equiv 1 + \alpha\phi$$

$$\pi^* \equiv 1 + \alpha^*\phi^*$$

$$v_t \equiv -(\alpha u_t^y + u_t^m)$$

$$v_t^* \equiv -(\alpha^* u_t^{*y} + u_t^{*m})$$

$$\Delta \equiv \pi\pi^* + \beta\pi^* + \beta^*\pi.$$

Therefore, from the PPP relationship,

$$e_t = [\pi^*(m_t + \alpha\phi p_{t|t-1} + \beta e_{t+1|t} + v_t) - \pi(m_t^* + \alpha^*\phi^* p_{t|t-1}^* - \beta^* e_{t+1|t} + v_t^*) - \pi\pi^* u_t^e]\Delta^{-1}. \tag{10}$$

Equations (9) and (10) represent reduced form expressions for the price levels and the exchange rate given past expectations of the current price levels and current expectations of the future exchange rate. We now turn to the design of monetary policy.

3. Optimal Monetary Policy: The Nash Solution

We first consider optimal monetary policy when each monetary authority follows a memoryless Nash strategy; that is, each monetary authority sets its money supply as a function only of the contemporaneous values of the state variables, taking as given the other monetary authority's money supply rule.

In this section, target output is the ex ante natural level of output.[7] We thus set $\bar{y}_\tau = \bar{y}_\tau^* = 0$ for all τ. Given the supply functions, equations (1) and (2), and the objective functions, equations (7) and (8), the authorities minimize

$$\sum_{\tau=t}^{\infty} \delta^{\tau-t} E[[\phi(p_\tau - p_{\tau|\tau-1}) + u_\tau^y]^2|\Omega_t] \tag{11}$$

and

$$\sum_{\tau=t}^{\infty} \beta^{*\tau-t} E[[\phi^*(p_\tau^* - p_{\tau|\tau-1}^*) + u_\tau^{*y}]^2|\Omega_t], \tag{12}$$

respectively.

We assume that each monetary authority sets its money supply as a linear, nonstochastic function of the information set. It is tedious but straightforward to show that the optimal money supply response is in fact of this form. In the appendix we show that the only element of the information set relevant for stabilization is the exchange rate. The monetary authority can achieve the same impact, however, by responding to any combination of current and lagged values of the exchange rate in appropriate proportion. For concreteness we consider only policies involving a response to the current exchange rate.[8] We thus consider policies of the form

$$m_t = ae_t, \tag{13}$$

$$m_t^* = a^* e_t. \tag{14}$$

Policy rules of this form are examples of contemporaneous or instantaneous feedback rules: policy variables respond to current observations of current endogenous variables. This formulation has been applied to stabilization policy by, for example, Poole (1970), Boyer (1978), McCallum and Whitaker (1979), Turnovsky (1982, 1983), Marston (1980), Frenkel and Aizenman (1982), and Canzoneri (1982). This specification is useful here because it allows an analysis of exchange rate management policies other than pure floating and fixed rates.

Since m_t and m_t^* respond only to e_t, all endogenous variables in our model depend, given expectations, only on current disturbances. Taylor (1977) has shown that models such as ours, which incorporate current or past expectations of future endogenous variables, have an infinite number of solutions in which current endogenous variables depend on the entire past history of exogenous variables. There is only one solution in which current variables depend on only a finite number of lagged variables, however. We restrict our analysis to this so-called minimal state solution in which no lagged variables appear. Thus e_t, y_t, y_t^*, p_t, and p_t^* depend linearly only on u_t so that

$$p_{t|t-1} = p^*_{t|t-1} = e_{t+1|t} = 0. \tag{15}$$

Since y_t and y_t^* do not depend on m_τ and m_τ^* for $t \neq \tau$, the authorities' problem reduces to one of choosing a to minimize $E(y_t^2|e_t)$ for the home country and a^* to minimize $E(y_t^{*2}|e_t)$ for the foreign country in each period t.

Each country will optimally choose its monetary policy rule, taking as given the rule of the other country. Considering the home country first, minimization of $E(y_t^2|e_t)$ is equivalent to choosing a money supply rule such that $E(y_t|e_t) = 0$,[9] given the rule followed by the foreign authority: a is chosen to satisfy:

$$E(y_t|e_t) = \Delta^{-1}\Phi[(\pi^* + \beta^*)ae_t + \beta a^*e_t + (\pi^* + \beta^*)E(v_t|e_t) + \beta E(v_t^*|e_t) + \pi^*\beta E(u_t^e|e_t)] + E(u_t^y|e_t) = 0. \tag{16}$$

The foreign country chooses its money supply rule (a^*) such that $E(y_t^*|e_t) = 0$ given the rule followed by the domestic authority (a):

$$E(y_t^*|e_t) = \Delta^{-1}\Phi^*[\beta^* ae_t + (\pi + \beta)a^*e_t + \beta^* E(v_t|e_t) + (\pi + \beta)E(v_t^*|e_t) - \pi\beta^* E(u_t^e|e_t)] + E(u_t^{*y}|e_t) = 0. \tag{17}$$

Equations (16) and (17) can be rewritten as reaction functions as in (18) and (19):

$$ae_t = -\left[\frac{\beta}{\pi^* + \beta^*}a^*e_t + E(v_t|e_t) + \frac{\beta}{\pi^* + \beta^*}E(v_t^*|e_t) + \frac{\pi^*\beta}{\pi^* + \beta^*}E(u_t^e|e_t) + \frac{\Delta\Phi^{-1}}{\pi^* + \beta^*}E(u_t^y|e_t)\right] \tag{18}$$

$$a^*e_t = -\left[\frac{\beta^*}{\pi + \beta}ae_t + \frac{\beta^*}{\pi + \beta}E(v_t|e_t) + E(v_t^*|e_t) - \frac{\pi\beta^*}{\pi + \beta}E(u_t^e|e_t) + \frac{\Delta\Phi^{*-1}}{\pi + \beta}E(u_t^{*y}|e_t)\right]. \tag{19}$$

Note from (18) and (19) that the domestic money supply responds negatively to the money supply abroad. An increase in the foreign money supply causes an appreciation of the exchange rate, creating expectations of depreciation, which reduce the demand for domestic currency. As all disturbances are independently, identically distributed, and there are no other sources of inertia in the model (specifically $m_{t|t-1} = 0$), rational expectations are regressive. To prevent the reduction in demand for domestic currency from raising the domestic price level, an accommodating reduction in domestic money supply must occur. Note also that given the money supply in the foreign country (m_t^*), the optimal domestic money supply in general responds to expectations of all types of shocks, both domestic and foreign and both monetary and real.

Noting that the optimal (least squares) predictor of some variable z_t given e_t is given by

$$E(z_t|e_t) = E(e_t)^{2^{-1}}E(z_te_t)\cdot e_t \tag{20}$$

we obtain

$$E(v_t|e_t) = \Delta\Lambda\pi^*\sigma_v^2 \textstyle\sum^{-1} e_t, \tag{21a}$$

$$E(v_t^*|e_t) = -\Delta\Lambda\pi\sigma_{v^*}^2 \textstyle\sum^{-1} e_t, \tag{21b}$$

$$E(u_t^e|e_t) = -\Delta\Lambda\pi\pi^*\sigma_{ue}^2 \textstyle\sum^{-1} e_t, \tag{21c}$$

$$E(u_t^y|e_t) = -\Delta\Lambda\alpha\pi^*\sigma_{uy}^2 \textstyle\sum^{-1} e_t, \tag{21d}$$

$$E(u_t^{*y}|e_t) = \Delta\Lambda\alpha^*\pi\sigma_{u^*y}^2 \textstyle\sum^{-1} e_t, \tag{21e}$$

$$\textstyle\sum \equiv \pi^{*2}\sigma_v^2 + \pi^2\sigma_{v^*}^2 + (\pi\pi^*)^2\sigma_{ue}^2,$$

$$\Lambda \equiv 1 - \Delta^{-1}(\pi^* a - \pi a^*),$$

$$\sigma_v^2 \equiv E(v_t^2);\ \sigma_{v^*}^2 \equiv E(v_t^{*2});\ \sigma_{ue}^2 \equiv E(u_t^{e2});\ \sigma_{uy}^2 \equiv E(u_t^{y2});\ \sigma_{u^*y}^2 \equiv E(u_t^{*y2}).$$

Assuming the system given by (18) and (19) to be of full rank, the Nash equilibrium solution for a and a^* is given by:

$$a = \beta + \frac{\Phi^{-1}\alpha\pi\pi^*\sigma_{uy}^2 - \pi^*\sigma_v^2}{\pi\pi^*\sigma_{ue}^2 + \Phi^{-1}\alpha\pi^*\sigma_{uy}^2 + \Phi^{*-1}\alpha^*\pi\sigma_{u^*y}^2}, \tag{22}$$

$$a^* = -\beta^* + \frac{-\Phi^{*-1}\alpha^*\pi\pi^*\sigma_{u^*y}^2 + \pi\sigma_{v^*}^2}{\pi\pi^*\sigma_{ue}^2 + \Phi^{-1}\alpha\pi^*\sigma_{uy}^2 + \Phi^{*-1}\alpha^*\pi\sigma_{u^*y}^2}, \tag{23}$$

or, noting that $\sigma_v^2 = \alpha^2\sigma_{uy}^2 + \sigma_{um}^2$ and $\sigma_{v*}^2 = \alpha^{*2}\sigma_{u*y}^2 + \sigma_{u*m}^2$,

$$a = \beta + \frac{\Phi^{-1}\alpha\pi^*\sigma_{uy}^2 - \pi^*\sigma_{um}^2}{\pi\pi^*\sigma_{ue}^2 + \Phi^{-1}\alpha\pi^*\sigma_{uy}^2 + \Phi^{*-1}\alpha^*\pi\sigma_{u*y}^2}, \tag{22'}$$

$$a^* = -\beta^* + \frac{-\Phi^{*-1}\alpha^*\pi\sigma_{u*y}^2 + \pi\sigma_{u*m}^2}{\pi\pi^*\sigma_{ue}^2 + \Phi^{-1}\alpha\pi^*\sigma_{uy}^2 + \Phi^{*-1}\alpha^*\pi\sigma_{u*y}^2}. \tag{23'}$$

β is the absolute value of the elasticity of demand for domestic money with respect to the expected proportional rate of depreciation of the domestic currency and $-\beta^*$ the corresponding elasticity for the foreign currency. In a currency substitution framework they can be viewed as the exchange rate speculation elasticities of home and foreign currency, respectively. The first terms of (22) or (22′) and (23) or (23′) therefore suggest that monetary policy accommodates changes in the demand for money due to unanticipated changes in the exchange rate, thereby neutralizing the effect of unanticipated exchange rate changes on the price level. This policy helps to insulate the economy from real effects of unanticipated exchange rate changes. Remember that since $e_{t|t-1} = 0$, a monetary rule contingent on e_t is a monetary rule contingent on the deviation of the actual exchange rate in period t from the exchange rate for period t anticipated in period $t-1$.

To the extent that monetary policy does accommodate swings in speculative demand, monetary authorities lean with the wind in the foreign exchange market; they expand the money supply when the price of domestic currency is lower than had been expected, and conversely (that is, $a > 0$ and $a^* < 0$).[10] Such a policy will exacerbate movements in the exchange rate, as can be seen from equation (24), the reduced form expression for the exchange rate:

$$e_t = (\pi^* v_t - \pi v_t^* - \pi\pi^* u_t^e)(\Delta - \pi^* a + \pi a^*)^{-1}. \tag{24}$$

The denominator of the second term on the right-hand side of (22′) and (23′) is positive. Thus an increase in the variability of the demand for domestic money (σ_{um}^2) will reduce the degree to which the authorities lean with the wind and may even reverse this policy. An unexpected increase in the demand for domestic money will be associated with an unanticipated appreciation of the home currency. Rather than contracting the money supply as would be optimal if the main sources of uncertainty were foreign, optimal monetary policy will at least in part accommodate the unexpected increase in the demand for money by expanding the money supply.

The variability of foreign money demand has no effect on optimal domestic monetary policy, however. This result may seem surprising since, from (16) and (17), foreign monetary shocks do affect domestic income and, from (18) and (19), domestic monetary policy, given foreign monetary policy, does respond to perceived shocks in the demand for foreign money. If foreign monetary authorities pursue an optimal monetary policy, however, they minimize the effect of their own monetary disturbances. Domestic monetary policy can then ignore such disturbances.

An increase in the variance of domestic output shocks raises the optimal degree to which monetary authorities should lean with the wind. A positive output shock raises the demand for money and appreciates the exchange rate. To offset the effect of a positive income shock, authorities should contract the money supply. Hence when exchange rate variation is caused in large part by instability in the supply of domestic output, monetary authorities should act to augment exchange rate changes.

The variability of foreign output shocks, unlike the variability of foreign monetary shocks, does affect the optimal intervention policy. A positive foreign output shock will tend to depreciate the exchange rate and engender a foreign monetary action that further depreciates the exchange rate. (In contrast a foreign monetary disturbance engenders an offsetting foreign monetary action.) Foreign output shocks thus create exchange rate variability that is unrelated to domestic disturbances. Any response designed to offset the effects of domestic shocks, as perceived through exchange rate variation, on domestic targets will be diminished because the foreign disturbances make the exchange rate a noisier indicator of those domestic disturbances. Thus as σ_{y*}^2 rises, optimal domestic policy is aimed increasingly at accommodating speculative behavior.

For the same reason increased variability in shocks to the purchasing power parity relationship also reduces the extent to which monetary policy can offset the effects of domestic shocks on income. As σ_{ue}^2 rises, policy increasingly should isolate the domestic price level from the effects of exchange rate speculation.

It is illuminating to consider monetary policy in four special cases of the model.

1. No domestic shocks. When $\sigma_{uy}^2 = \sigma_{um}^2 = 0$, there are no domestic

sources of disturbances in the home country. The only shocks it faces are exchange rate disturbances resulting either from the stochastic nature of the purchasing power parity relationship ($\sigma_{u^e}^2 > 0$) or from uncertainty in the rest of the world ($\sigma_{u^{*m}}^2, \sigma_{u^{*y}}^2 > 0$). In this case (22′) reduces to $a = \beta$. (Similarly if there are no sources of disturbances internal to the foreign country, $\sigma_{u^{*y}}^2 = \sigma_{u^{*m}}^2 = 0$ and (23′) reduces to $a^* = -\beta^*$). The money supply rule is entirely accommodating. When the exchange rate depreciates unexpectedly, the money supply expands. In the absence of changes in the money supply, a depreciation of the exchange rate creates expectations of appreciation (since $e_{t+1|t} = 0$). These expectations increase the speculative demand for home country money, which lowers the home country price level and therefore income. To offset this, the monetary authority acts to accommodate exactly the higher money demand with a higher supply. Therefore a country facing temporary shocks largely from abroad through the exchange rate will adopt a monetary rule that exacerbates the exchange rate changes in order to stabilize real income.

2. No domestic shocks and no currency substitution. If there are no domestic shocks and if the demand for domestic currency is inelastic with respect to exchange rate changes (if $\sigma_{u^y}^2 = \sigma_{u^m}^2 = \beta = 0$) then the optimal money supply is independent of the exchange rate ($a = 0$). Thus, except in the improbable event that the various components of (22′) cancel exactly, a policy of free floating is optimal if and only if the demand for money is inelastic and there are no domestic disturbances. Even if the demand for money does not depend on the expected change of the exchange rate (if $\beta = 0$) exchange rate changes signal in part domestic shocks to which the money supply should respond. This result is analogous to Poole's (1970) finding that in the closed economy IS-LM model, the optimal money supply is invariant to the interest rate if and only if the demand for money is interest inelastic and the economy is not subject to a variable demand for money.

3. No real or purchasing power parity shocks. When the only source of uncertainty is in the demand for either currency (when $\sigma_{u^y}^2 = \alpha_{u^{*y}}^2 = \sigma_{u^e}^2 = 0$), then policy makes the supply of money perfectly elastic. The exchange rate is pegged. Turnovsky (1983) also obtains this result for a single small, open economy. This result is analogous to Poole's finding that for a closed economy, a policy of fixing the interest rate is optimal when the only source of disturbances is in the demand for money. Note that if pegging the exchange rate is the optimal policy for one country, it is so for both.

Unless the two countries peg at the same level, however, the model becomes inconsistent.

4. Infinitely elastic currency substitution. If individuals view domestic and foreign currency as perfect substitutes, then $\beta = \beta^* = \infty$, and a policy of pegging minimizes income variability even if the economy is subject to real disturbances. Otherwise the exchange rate becomes indeterminant. This can be seen from equation (10). Setting $\beta = \beta^* = \infty$, this expression reduces to $e_t = e_{t+1|t}$. Any exchange rate that remains constant over time is compatible with equilibrium (see Kareken and Wallace 1981). Again consistency requires that monetary authorities peg to the same exchange rate.

4. Optimal Exchange Rate Management When Minimizing the Price Forecast Error Is the Objective

So far we have assumed that the policy makers' objectives are to minimize output variation around the ex ante expected natural rates ($\bar{y}_\tau = \bar{y}_\tau^* = 0$). One might assume instead that policymakers are concerned with the deviation of income around the ex post actual natural rates ($\bar{y}_\tau = u_\tau^y$, $\bar{y}_\tau^* = u_\tau^{*y}$), which are unobserved contemporaneously. Such an objective is equivalent to minimizing price forecast errors since

$$y_t - u_t^y = \Phi(p_t - p_{t|t-1}) \tag{25}$$

and

$$y_t^* - u_t^{*y} = \Phi^*(p_t^* - p_{t|t-1}^*). \tag{26}$$

The alternative specification of objective functions as

$$\sum_{\tau=t}^{\infty} \delta^{\tau-t} E[(y_\tau - u_\tau)^2|\Omega_t] \quad 0 < \delta < 1 \tag{27}$$

and

$$\sum_{\tau=t}^{\infty} \delta^{*\tau-t} E[(y_\tau^* - u_\tau^*)^2|\Omega_t] \quad 0 < \delta^* < 1 \tag{28}$$

is plausible if one believes that price forecast errors themselves, rather than output fluctuations, are a primary source of inefficiency. If such a specification is adopted, optimal policy rules are derived for the home country by choosing a such that $E(y_t - u_t|\Omega_t) = 0$, given a^*, and for the foreign country by choosing a^* such that $E(y_t^* - u_t^*|\Omega_t) = 0$, given a. This yields

$$a = \beta - (\sigma_{u^m}^2 + \alpha^2\sigma_{u^y}^2)/[(1 + \alpha\Phi)\sigma_{u^e}^2], \tag{29}$$

$$a^* = -\beta^* + (\sigma_{u^{*m}}^2 + \alpha^{*2}\sigma_{u^{*y}}^2)/[(1 + \alpha^*\Omega^*)\sigma_{u^e}^2]. \tag{30}$$

Three basic differences from our previous analysis emerge.

First, supply uncertainty now contributes toward the optimality of a policy of exchange rate stabilization (leaning against the wind in the exchange market) rather than the opposite. The reason is that an unanticipated increase in output will increase money demand, lower the price level, and cause the currency to appreciate. To eliminate the unanticipated price decline, money expansion is now appropriate, dampening the exchange rate change.

Second, the variability of foreign shocks, regardless of whether they are monetary or real in origin, has no effect on optimal domestic monetary policy. The domestic effects of foreign shocks of either type are minimized by optimal foreign monetary policy. In other words each monetary authority acts to offset the effects of local shocks on both itself and the other country.

Third, regardless of the variability of money demand or output supply in either economy, if the purchasing power relationship is nonstochastic, a policy of pegging the exchange rate is optimal.

5. Cooperative Pareto-Optimal Solution

So far we have assumed that each monetary authority acts independently to attain a domestic policy objective, taking the monetary policy of the other country as given. In this section we compare such policies with those that would arise if the two monetary authorities were to cooperate in setting monetary policy to attain a mutual objective. To derive the set of Pareto-optimal policies, we assume that policymakers jointly set monetary policy in period t to minimize an objective of the form

$$w\sum_{\tau=t}^{\infty}\delta^{\tau-t}E[(y_\tau^2|\Omega_\tau)|\Omega_t] + w^*\sum_{\tau=t}^{\infty}\delta^{*\tau-t}E[(y_\tau^{*2}|\Omega_\tau)|\Omega_t] \tag{31}$$

$$w, w^* > 0, 0 < \delta < 1$$

in period t.

As section 4 demonstrated, however, current values of m_t and m_t^* do not affect values of y_τ and y_τ^* for $\tau > t$. Choosing m_t^* to minimize (31) is equivalent to choosing m_t and m_t^* to minimize

$$wE(y_t^2|\Omega_t) + w^*E(y_t^{*2}|\Omega_t)$$

$$= wE[(y_t - E(y_t|\Omega_t))^2|\Omega_t] + w^*E[(y_t^* - E(y_t^*|\Omega_t))^2|\Omega_t] \tag{32}$$
$$+ w[E(y_t|\Omega_t)]^2 + w^*[E(y_t^*|\Omega_t)]^2.$$

From note 9 it follows that the first two terms of the right-hand side of expression (32) are independent of m_t and m_t^*. Minimizing (32) with respect to m_t and m_t^*, then, is equivalent to minimizing

$$wE[(y_t|\Omega_t)]^2 + w^*E[(y_t^*|\Omega_t)]^2. \tag{33}$$

First-order conditions for a minimum are

$$wE\left(y_t\frac{dy_t}{dm_t}|\Omega_t\right) + w^*E\left(y_t^*\frac{dy_t^*}{dm_t}|\Omega_t\right) = 0, \tag{34}$$

$$wE\left(y_t\frac{dy_t}{dm_t^*}|\Omega_t\right) + w^*E\left(y_t^*\frac{dy_t^*}{dm_t^*}|\Omega_t\right) = 0. \tag{35}$$

From equation (A8) in the appendix and since all forward expectations are zero, dy_t/dm_t, dy_t^*/dm_t, dy_t/dm_t^*, and dy_t^*/dm_t^* are nonstochastic. These first-order conditions therefore obtain when m_t and m_t^* satisfy

$$E(y_t|\Omega_t) = E(y_t^*|\Omega_t) = 0. \tag{36}$$

Since (34) and (35) are linear functions of m_t and m_t^*, the values of m_t and m_t^* that satisfy (36) constitute a unique solution. These are exactly the same values of m_t and m_t^* that satisfy the Nash equilibrium. In our model, then, the Nash solution is also the unique Pareto-optimal solution. This result is not surprising since, in our model, each country has one independent instrument, its money supply, and one independent target, the level of its income. In such a context there are no gains from policy coordination.

To show that the equivalence of the Nash and Pareto-optimal solutions does not generalize to systems in which there are more targets than instruments, consider a system in which one or both countries also have exchange rate stabilization as another goal; that is, in period t the home country seeks to minimize

$$E\sum_{\tau=t}^{\infty}\delta^{\tau-t}[E(y_\tau^2 + \omega e_\tau^2|\Omega_\tau)|\Omega_t] \qquad \omega > 0, \quad 0 < \delta < 1 \tag{37}$$

while the foreign country minimizes

$$E\sum_{\tau=t}^{\infty}\delta^{*\tau-t}[E(y_\tau^{*2} + \omega^* e_\tau^2|\Omega_\tau)|\Omega_t] \qquad \omega^* > 0, \quad 0 < \delta^* < 1. \tag{38}$$

First-order conditions for Nash equilibrium values of m_t and m_t^* are given by

$$E(y_t|\Omega_t)\frac{dy_t}{dm_t} + \omega e_t\frac{de_t}{dm_t} = 0, \tag{39}$$

and

$$E(y_t^*|\Omega_t)\frac{dy_t^*}{dm_t^*} + \omega^* e_t\frac{de_t}{dm_t^*} = 0. \tag{40}$$

With weights of w and w^* placed on the home and foreign countries' objective functions, however, first-order conditions for Pareto-optimal values of m_t and m_t^* are

$$wE(y_t|\Omega_t)\frac{dy_t}{dm_t} + w^*E(y_t^*|\Omega_t)\frac{dy_t^*}{dm_t} + (w\omega + w^*\omega^*)\frac{de_t}{dm_t} = 0, \tag{41}$$

$$wE(y_t|\Omega_t)\frac{dy_t}{dm_t^*} + w^*E(y_t^*|\Omega_t)\frac{dy_t^*}{dm_t^*} + (w\omega + w^*\omega^*)\frac{de_t}{dm_t^*} = 0. \tag{42}$$

These are not equivalent to (39) and (40) except when $\omega = \omega^* = 0$. In general, the Nash solution is not Pareto optimal.

If each authority's number of targets does not exceed the number of linearly independent instruments available to that authority, each target responds to each instrument, targets are not inconsistent, and the authorities' instruments are linearly independent of each other, then in a linear world each authority can use its own instruments to obtain an outcome independent of the other authority's rule. One authority could do no better in attaining its own targets if it had access to the other's instruments. When the number of targets that one authority pursues exceeds the number of instruments available to it, it could achieve its targets more precisely if it had access to the other authority's instruments as well.[11] The Pareto-optimal policy rules will depend in this case on the weights assigned to each authority's objective function in calculating global welfare.

In the above example, exchange rate stabilization constitutes a public good that is likely to be pursued inadequately under a decentralized solution. Comparing equation (39) to equation (41) and (40) to (42), note that the weight on the exchange rate effect relative to the home income effect in the first-order conditions for optimality rises from ω to $\omega + (w^*/w)\omega^*$ and from ω^* to $\omega^* + (w/w^*)\omega$ in moving from the decentralized Nash solution to the centralized Pareto-optimal solution.

6. Conclusion

This study of optimal monetary policy or exchange rate management in interdependent economies has abstracted from many real world complications in order to obtain a transparent structure. Nevertheless, a number of results are likely to be robust under further generalizations of the model.

1. Neither a fixed nor a freely floating exchange rate is likely to be optimal. Optimal monetary policy in general requires a finite response of the money supply to the exchange rate.
2. Output stabilizing monetary policy may well require leaning with the wind in the foreign exchange markets, expanding the money supply when the home currency depreciates, thus increasing the volatility of the exchange rate.
3. Depending on the nature of the objective function pursued abroad, optimal foreign monetary policy may allow the domestic monetary authority to establish an optimal intervention rule independent of the variance of some or all foreign disturbances.
4. There are likely to be gains from policy coordination when the number of targets exceeds the number of instruments.

Appendix 2A

Note that for any variable q one has $q_{t|t-i} \equiv E(q_t|\Omega_{t-i})$ and that $E[E(q_t|\Omega_{t-i})|\Omega_{t-i-j}] = E(q_t|\Omega_{t-i-j})$, $i, j \geq 0$. We thus obtain, from (9),

$$\begin{bmatrix} p_{t|t-i} \\ p^*_{t|t-i} \end{bmatrix} = B\begin{bmatrix} m_{t|t-i} + \alpha\Phi p_{t|t-i} + \beta e_{t+1|t-i} \\ m^*_{t|t-i} + \alpha^*\Phi^* p^*_{t|t-i} - \beta^* e_{t+1|t-i} \end{bmatrix}\Delta^{-1}. \tag{A1}$$

Subtracting (A1) for $i = 1$ from (9) yields:

$$\begin{bmatrix} p_t \\ p_t^* \end{bmatrix} - \begin{bmatrix} p_{t|t-1} \\ p_{t|t-1}^* \end{bmatrix}$$
$$= B\begin{bmatrix} m_t - m_{t|t-1} + \beta(e_{t+1|t} - e_{t+1|t-1}) + v_t + \beta u_t^e \\ m_t^* - m_{t|t-1}^* - \beta^*(e_{t+1|t} - e_{t+1|t-1}) + v_t^* - \beta^* u_t^e \end{bmatrix}\Delta^{-1}. \tag{A2}$$

Using

$$e_t = p_t - p_t^* - u_t^e \tag{A3}$$

which, since $u_{t+1|t}^e = u_{t+1|t-1}^e = 0$, implies, for $i > 0$,

$$e_{t+i|t} - e_{t+i|t-1} = p_{t+i|t} - p_{t+i|t-1} - (p_{t+i|t}^* - p_{t+i|t-1}^*) \tag{A4}$$

we may write (A2) as

$$\begin{bmatrix} p_t - p_{t|t-1} \\ p_t^* - p_{t|t-1}^* \end{bmatrix} = BD\begin{bmatrix} p_{t+1|t} - p_{t+1|t-1} \\ p_{t+1|t}^* - p_{t+1|t-1}^* \end{bmatrix}\Delta^{-1}$$
$$+ B\begin{bmatrix} m_t - m_{t|t-1} \\ m_t^* - m_{t|t-1}^* \end{bmatrix}\Delta^{-1} + B\begin{bmatrix} v_t + \beta u_t^e \\ v_t^* - \beta^* u_t^e \end{bmatrix}\Delta^{-1} \tag{A5}$$

where

$$D = \begin{bmatrix} \beta & -\beta \\ -\beta^* & \beta^* \end{bmatrix}.$$

From (A1) and (A4), however, note that

$$\begin{bmatrix} p_{t+i|t} - p_{t+i|t-1} \\ p_{t+i|t}^* - p_{t+i|t-1}^* \end{bmatrix} = ABD\begin{bmatrix} p_{t+i+1|t} - p_{t+i+1|t-1} \\ p_{t+i+1|t}^* - p_{t+i+1|t-1}^* \end{bmatrix}\Delta^{-1}$$
$$+ AB\begin{bmatrix} m_{t+i|t} - m_{t+i|t-1} \\ m_{t+i|t}^* - m_{t+i|t-1}^* \end{bmatrix}\Delta^{-1} \tag{A6}$$

where

$$A \equiv \left\{I - B\begin{bmatrix} \alpha\Phi & 0 \\ 0 & \alpha^*\Phi^* \end{bmatrix}\Delta^{-1}\right\}^{-1}.$$

By repeated forward substitution and assuming stability, we may write (A6) as

$$\begin{bmatrix} p_{t+i|t} - p_{t+i|t-1} \\ p_{t+i|t}^* - p_{t+i|t-1}^* \end{bmatrix} = \sum_{j=i}^{\infty} C^{j-i}AB\begin{bmatrix} m_{t+j|t} - m_{t+j|t-1} \\ m_{t+j|t}^* - m_{t+j|t-1}^* \end{bmatrix} \tag{A7}$$

where $C \equiv ABD$.

Substituting (A7) into (A5) implies that

$$\begin{bmatrix} p_t - p_{t|t-1} \\ p_t^* - p_{t|t-1}^* \end{bmatrix} = BD\sum_{j=1}^{\infty} C^jAB\begin{bmatrix} m_{t+j|t} - m_{t+j|t-1} \\ m_{t+j|t}^* - m_{t+j|t-1}^* \end{bmatrix}\Delta^{-1}$$
$$+ B\begin{bmatrix} m_t - m_{t|t-1} \\ m_t^* - m_{t|t-1}^* \end{bmatrix}\Delta^{-1} + B\begin{bmatrix} v_t + \beta u_t^e \\ v_t^* - \beta^* u_t^e \end{bmatrix}\Delta^{-1}. \tag{A8}$$

We define the following: $u_t \equiv (u_t^y, u_t^{*y}, u_t^m, u_t^{*m}, u_t^e)$, $\tilde{u}_t \equiv u_t - E(u_t|\Omega_t)$, and $\tilde{e}_t \equiv e_t - E(e_t|\Omega_{t-1})$. We may thus define $\tilde{\Omega}_t \equiv (\tilde{e}_t, \tilde{u}_{t-1})$ as the new information available in period t. Since u_{t-i}, $i \geq 2$ is known at period $t-1$, $\tilde{e}_t$ and thus $\tilde{\Omega}_t$ can depend only on $\tilde{u}_{t-1}$ and u_t unless monetary policy is itself random. Revisions of expectations in period t about monetary policy can depend only on information newly available in period t: $\tilde{\Omega}_t$. Restricting ourselves to linear time-invariant nonstochastic policies, we may write

$$m_{t+j|t} - m_{t+j|t-1} = \gamma_t\tilde{e}_t + \gamma'\tilde{u}_{t-1} \qquad j \geq 0 \tag{A9}$$

and

$$m_{t+j|t}^* - m_{t+j|t-1}^* = \gamma_j^*\tilde{e}_t + \gamma_j^{*\prime}\tilde{u}_{t-1} \qquad j \geq 0. \tag{A10}$$

Substituting (A9) and (A10) into (A8) we obtain

$$\begin{bmatrix} p_t - p_{t|t-1} \\ p_t^* - p_{t|t-1}^* \end{bmatrix} = BD\sum_{j=1}^{\infty} C^jAB\begin{bmatrix} \gamma_j\tilde{e}_t + \gamma_j'\tilde{u}_{t-1} \\ \gamma_j^*\tilde{e}_t + \gamma_j^{*\prime}\tilde{u}_{t-1} \end{bmatrix}\Delta^{-1}$$
$$+ B\begin{bmatrix} \gamma_0\tilde{e}_t + \gamma_0'\tilde{u}_{t-1} \\ \gamma_0^*\tilde{e}_t + \gamma_0^{*\prime}\tilde{u}_{t-1} \end{bmatrix}\Delta^{-1} + B\begin{bmatrix} v_t + \beta u_t^e \\ v_t^* + \beta^* u_t^e \end{bmatrix}\Delta^{-1}. \tag{A11}$$

Substituting (A11) into (11) and (12) it is clear that since u_t and u_{t-1}, and therefore $\tilde{e}_t$ and $\tilde{u}_{t-1}$, are orthogonal, policies for which γ_j' and $\gamma_j^{*\prime}$ are nonzero increase the minimum expected loss. Such policies introduce additional randomness in the form of the unobserved (as of last period) component of last period's disturbance into the current period price forecast error. We thus restrict ourselves to monetary policies that do not respond to $\tilde{u}_{t-1}$.

If, in fact, $\gamma_j' = \gamma_j^{*\prime} = 0$, then policy responds only to currently observed components of the current disturbances. Since these can be observed only by e_t, policy can respond only to e_t. We thus restrict ourselves to policies of the form

$$m_t = \sum_{\tau=-\infty}^{t} a_{t-\tau} e_\tau, \tag{A12}$$

$$m_t^* = \sum_{\tau=-\infty}^{t} a_{t-\tau}^* e_\tau. \tag{A13}$$

Substituting (A12) and (A13) into (A8) we obtain

$$\begin{bmatrix} p_t - p_{t|t-1} \\ p_t^* - p_{t|t-1}^* \end{bmatrix} = \Psi \Delta^{-1} e_t + B \begin{bmatrix} v_t + \beta u_t^e \\ v_t^* - \beta^* u_t^e \end{bmatrix} \Delta^{-1} \tag{A14}$$

where

$$\Psi \equiv BD \sum_{j=1}^{\infty} C^j AB\hat{a}_j + B\hat{a}_0,$$

$$\hat{a}_j \equiv \begin{bmatrix} a_j \\ a_j^* \end{bmatrix}; \quad \hat{a}_0 \equiv \begin{bmatrix} a_0 \\ a_0^* \end{bmatrix}.$$

Observe that any given values of Ψ and Ψ^* can be achieved by linear combinations of an infinite number of variations of the underlying policy parameters a_j and a_j^*. For example, a policy rule that sets $\hat{a}_j = 0, j \neq 0$, $\hat{a}_0 = \bar{a}_0$ will have the same effect on the objective functional as one that sets $\hat{a}_j = 0, j \neq 1$, and $\hat{a}_1 = (BDCAB)^{-1} B\bar{a}_0$. In general the government can achieve the same objective by responding only currently to current information (e_t) that it can achieve by responding to this information later. It is interesting to note that even if the governments were to have inferior information to the private sector in the sense that they learn e_t at a later date, they can achieve their objectives equally well. Turnovsky (1980) and Weiss (1982) provide other examples of this phenomenon. (See also Buiter 1980). For convenience we restrict curselves to current response only. We thus assume $a_j = a_j^* = 0, j \neq 0$.

Notes

Buiter would like to acknowledge financial support from the National Science Foundation. Much of Eaton's work on this chapter was done while he was visiting the Graduate Institute of International Studies, Geneva. Earlier versions were presented at the Ninety-second Annual Meeting of the American Economic Association, Atlanta, Georgia, December 1979, at the First Annual Conference of the Society for Economic Dynamics and Control in Cambridge, England, June 1979, and at the University of Warwick Summer Workshop in July 1980. Stanley Black, Marcus Miller, and Douglas Purvis made useful comments.

1. In this respect our model resembles that of Boyer (1978) and Roper and Turnovsky (1980); however, they consider a single open economy characterized by Keynesian unemployment. In a closed economy setting the current response issue has been studied by Woglom (1979) and McCallum and Whitaker (1979). In a recent paper Turnovsky (1983) derives optimal exchange market intervention rules for a small open economy characterized by a Lucas supply function. See also Frenkel and Aizenman (1982).

2. Turnovsky (1982, 1983) also finds a leaning with the wind policy to be optimal in some situations.

3. Contracts that do not allow wages to respond to contemporaneous data might arise because such data are not available symmetrically to workers and employers, leading to problems of moral hazard.

In fact the coefficients ϕ and ϕ^* may depend on the variance of the price level and hence on policy rules themselves. An increase in price level stability may lead to a greater use of long-term contracts embodying fixed nominal wages and prices—for example, raising ϕ and ϕ^*. We do not pursue this line of thought here but assume a short-run supply response independent of policy rules. This assumption is appropriate if, for example, the nature of long-term contracts and their significance in the economy are not affected by price variability over the range of variation relevant to our analysis. Flood and Marion (1982) discuss the implications of allowing output supply elasticities to depend on the variance of the price level. They consider only regimes of fixed and pure floating exchange rates, however. Their analysis, unlike ours, assumes that the aggregate price level is part of current information or that partial or complete wage indexation is feasible.

4. Barro (1978) also assumes that the demand for money responds to expected exchange rate changes in his model of monetary policy in a small open economy.

5. There are a number of reasons why the purchasing power parity relationship may be stochastic without creating possibilities for profitable commodity arbitrage. To illustrate one explanation, consider the standard two-country, two-commodity, two-factor Heckscher-Ohlin-Samuelson trade model appended to include a nontraded good produced in each country. With incomplete specialization and in the absence of transport costs, trade will cause exact purchasing power to hold with respect to traded goods alone and will also result in factor price equalization. The price of nontraded goods will be determined by competition in the nontraded goods sector. Random variation in relative productivities in the two sectors, if not perfectly correlated between the two countries, will cause the relative price of non-traded goods to vary independently in the two countries. The purchasing power parity relationship as applied to all goods, both traded and nontraded, therefore will be subject to random variation. Following this interpretation then, a positive value of u_t^e would imply that productivity in the nontraded goods sector at home is relatively lower, compared to that abroad, than is normally the case. The domestic price level is consequently higher.

6. We assume away all problems of nonuniqueness through extraneous information. The information sets of all agents therefore are limited to variables that appear in the structural model, given expectations—that is, to market fundamentals in the sense of Flood and Garber (1980). See also Taylor (1977).

7. Flood and Marion (1982) show that minimizing the deadweight loss due to wage contracts leads to an objective function somewhere between minimizing variability around the ex ante and ex post natural rates.

8. This case uniquely has the virtue of yielding time-consistent policies. A monetary policy that responds in period t to e_τ, $\tau < t$, does not affect y_t by this response but only y_τ. Since y_τ in period t is a bygone, time-consistent monetary authorities have no reason to stabilize output by expectations of future policy. At time t they may want the private sector to believe that they will respond in some particular way at some future period, but when that period comes they have no incentive to do so. Hence the private sector has no reason to believe them.

9. First note that

$$E(y_t^2|e_t) = E((y_t - E(y_t|e_t))^2|e_t) + [E(y_t|e_t)]^2.$$

In our model y_t and e_t are jointly normally distributed. Therefore, conditional on e_t, y_t is normally distributed with mean given by expression (16) and variance

$$E\{[y_t - E(y_t|e_t)]^2|e_t\} = E(y_t^2) - [E(y_t \cdot e_t)]^2/E(e_t)^2$$

(see Hogg and Craig 1978), pp. 63–65). Since this expression t contains only the unconditional variances and covariance of e_t and y_t, it is independent of the e_t actually observed.

10. See Harris and Purvis (1981) and Eaton and Turnovsky (1982) for a single economy model of exchange rate determination including permanent as well as transitory disturbances. Introducing permanent disturbances is likely to reduce the extent to which a policy of leaning with the wind is optimal since rational expectations consequently are less likely to be regressive. See also Eaton (1982) and Turnovsky (1982) for a discussion of this point.

11. This is also likely to be the case even if the number of targets does not exceed the number of instruments when the coefficients of the system are stochastic. See Brainard (1967).

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