This book is dedicated to my sister
Hens Buiter
in love and admiration

Macroeconomic theory and stabilization policy

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Chapter 8

Staggered wage setting with real wage relativities: variations on a theme of Taylor

Written with Ian Jewitt

I Introduction

In a number of influential recent papers, Taylor [1979a,b; 1980a,b] has analysed the behaviour of an economy characterized by staggered overlapping wage contracts and rational expectations. His model has the 'Keynesian' feature that the second moment of the distribution function of real output is not invariant under changes in the deterministic (and known) components of monetary policy rules. The reason for this is the inertia in the money wage process induced by the staggered multi-period contracts and the assumption that the wage payments due in any given period under the contract are not 'indexed', that is not made contingent on the actually realized values of such nominal variables as the general price level. We retain the crucial assumption of multi-period (staggered) non-contingent contracts but wish to examine the consequences of altering Taylor's assumption that wage bargainers are influenced by relative money wages rather than relative real wages. In Taylor's model money wage contracts are negotiated without reference to past, current and expected future prices. Our suggested modification that wage bargainers are influenced by relative real wages, which we consider somewhat more plausible, has some interesting implications for the empirical estimation of models with staggered wage contracts (see especially Taylor [1980b]).

Section II presents our general *N*-period overlapping staggered real wage model or relative real wage model (RRW) and contrasts it with Taylor's relative money wage model (RMW). One can distinguish three influences on the contract wage. These are the average price level expected to prevail over the contract, demand effects and the response of bargainers to relative (real or nominal) wages. Taylor's model contains the demand effect and the relative nominal wage effect. Section III analyses staggered real and money wage contracts without relative wage effects. One conclusion is that Taylor's RMW model is observationally equivalent to a staggered real wage model without relative wage effects. Section IV considers some empirical implications of the RRW model.

II A comparison of the general solution of Taylor's relative money wage model and the relative real wage model

Taylor's RMW model is presented in equations (1)–(4)

$$x_t = \sum_{s=1}^{N-1} d_{-s}\, x_{t-s} + \sum_{s=1}^{N-1} d_s\, \hat{x}_{t-1|t+s} + \frac{\gamma}{N}\sum_{s=0}^{N-1} \hat{y}_{t-1|t+s} + \varepsilon_t; \qquad \gamma > 0 \quad (1)$$

$$y_t + p_t = m_t + v_t \quad (2)$$

$$p_t = \frac{1}{N}\sum_{i=0}^{N-1} x_{t-i} \quad (3)$$

$$m_t = (1-\beta)p_t \quad (4)$$

The logarithm of the money contract wage negotiated in period t is denoted by x_t. Wage contracts last N periods. A constant fraction $\frac{1}{N}$ of all firms and all workers settle in any given period. The contract wage is constant over the duration of the contract. We shall assume in this section, along with Taylor [1980a], that the weights on future and past contract wages are symmetric, linearly declining in s and sum to unity, i.e.

$$d_s = d_{-s} = b_s = [N(N-1)]^{-1}(N-s) \qquad s = 1, \ldots, N-1 \quad (5)$$

y_t, the log of real output, is also a measure of excess demand in the labour market, since the level of full employment output is assumed to be constant throughout. p_t and m_t are the logs of the price level and the nominal money stock respectively. The terms ε_t and v_t are white noise disturbances with zero means and a constant contemporaneous variance-covariance matrix. For any variable, z, say, $\hat{z}_{t-1|t+s}$ is the mathematical expectation of z_{t+s} conditional on the information available in period $t-1$. Equation (2) is a very simple aggregate demand equation.[1] Equation (3) specifies the current price level, p_t, as a proportional mark-up on the average of the contract wages in effect during period t. Equation (4) is an instantaneous monetary policy response function.[2]

In Taylor's own words, the two key assumptions of the RMW model are '(1) wage contracts are staggered, that is, not all wage decisions in the economy are made at the same time, and (2) when making wage decisions, firms (and unions) look at the wage rates which are set at other firms and which will be in effect during their contract period' (Taylor [1980a, p. 2]).

It is important to note that the second assumption refers to current multi-period *money* wage contract decisions that are made with reference to all other *money* wage contracts that will be in effect during the periods covered by the current contract. While currently contracting firms and unions may well be interested in their wages relative to those of firms and unions contracting at earlier and later dates, rational behaviour would seem to require that the relative real values of these contracts and not the relative money wages *per se* should be the proper focus of concern. It can therefore be argued that Taylor's RMW model does not isolate the implications of having multi-period non-contingent (i.e., open-loop) money wage contracts[3] from those of having a form of money illusion.

The RRW model modifies the wage setting process in the following way. The contract money wage of firms and unions settling in period t, x_t, is set to achieve a given expected (target) real wage over the duration of the contract. This expected target real wage depends on expected average excess demand during the contract interval and on the real wages that are expected to be achieved by other firms and unions whose contracts overlap with the period t contract. With N-period contracts, the current contract money wage is therefore directly dependent on current expectations of the price level during the current and the following N-1 periods. Indirectly, because of the dependence of the current contract wage on the expected real value of the contract wages with which it overlaps, the current contract wage will depend on current expectations of the price level during the next $2N$-2 periods.

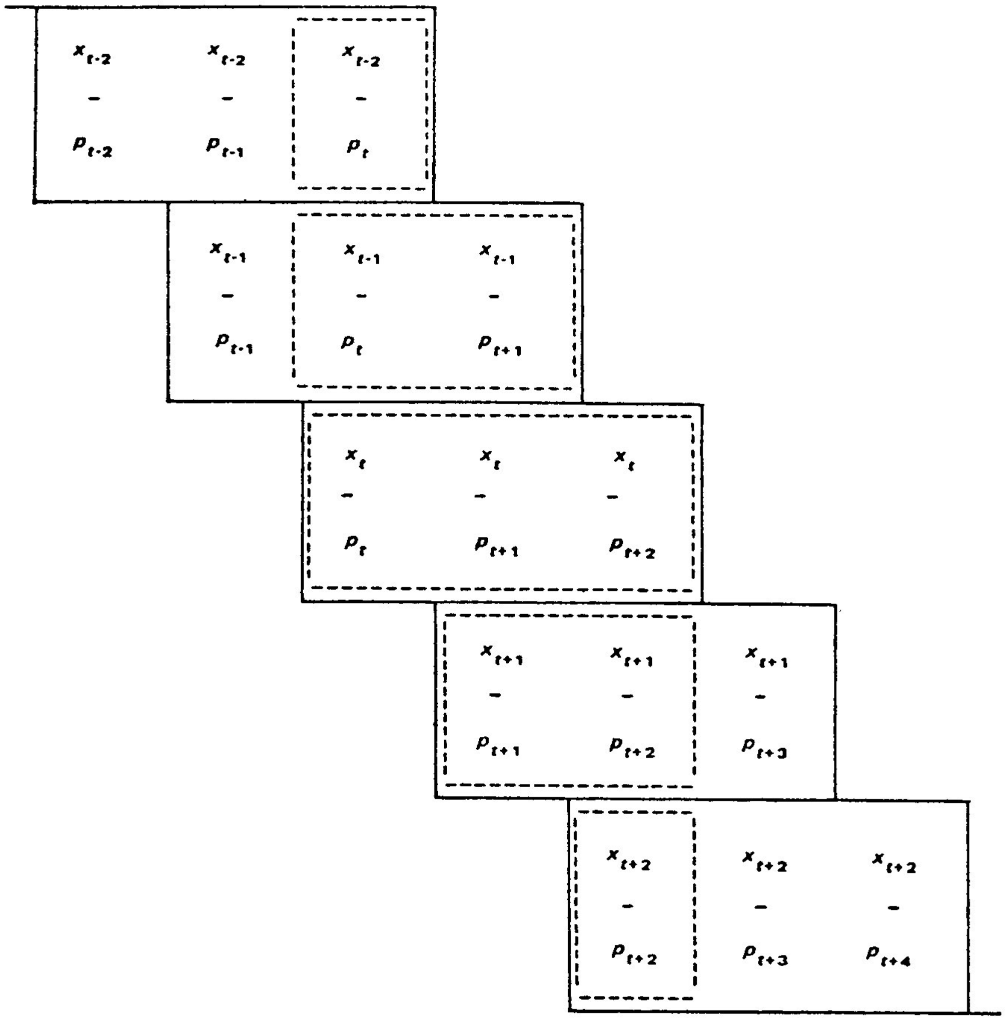
In general there are three kinds of arguments in the structural equation for the current contract wage. First, the price expectation effect, which reflects the influence of the average price level expected over the life of the contract. This effect is absent in the RMW model. Second, the relative wage effect, which can be in terms of either real or money wages. Finally, the demand effect, which represents the influence of the average level of excess demand in the labour market expected over the life of the contract.

Formally, the RRW model retains equations (2)–(5) but replaces (1) by

$$\begin{aligned} x_t - \frac{1}{N}\sum_{s=0}^{N-1}\hat{p}_{t-1|t+s} &= \sum_{s=1}^{N-1} d_{-s}\left(x_{t-s} - \frac{1}{N}\sum_{j=0}^{N-1}\hat{p}_{t-1|t-s+j}\right) \\ &+ \sum_{s=1}^{N-1} d_s\left(\hat{x}_{t-1|t+s} - \frac{1}{N}\sum_{j=0}^{N-1}\hat{p}_{t-1|t+s+j}\right) \\ &+ \frac{\gamma}{N}\sum_{s=0}^{N-1}\hat{y}_{t-1|t+s} + \varepsilon_t \end{aligned} \quad (6)$$

Note that in (6) we specify the expected real contract wage represented by the money contract wage that is expected to be negotiated in period

Table 1 Real wage perceptions of firms and unions contracting during period t in the RRW model (solid rectangles) and the RMW model (dashed rectangles).



$t + s$ by $\hat{x}_{t-1|t+s} - \frac{1}{N}\sum_{j=0}^{N-1} \hat{p}_{t-1|t+s+j}$. This is the expectation of the average real wage negotiated by firms and unions contracting in period $t + s$ over the entire duration of that contract, i.e., from period $t + s$ to period $t + s + N - 1$. One interpretation of the Taylor RMW model is that firms and unions negotiate not on the basis of the actual and expected values of contracts established by other firms and unions but on the basis only of those real wages earned by other workers whilst the currently negotiated

contract is in force. According to this interpretation, (6) should be replaced by

$$x_t - \frac{1}{N}\sum_{s=0}^{N-1} \hat{p}_{t-1|t+s} = \sum_{s=1}^{N-1} d_{-s}\left(x_{t-s} - \frac{1}{N-s}\sum_{j=s}^{N-1} \hat{p}_{t-1|t-s+j}\right) + \sum_{s=1}^{N-1} d_s\left(\hat{x}_{t-1|t+s} - \frac{1}{N-s}\sum_{j=0}^{N-1-s} \hat{p}_{t-1|t+s+j}\right) + \frac{\gamma}{N}\sum_{s=0}^{N-1} \hat{y}_{t-1|t+s} + \varepsilon_t \tag{7}$$

The two possible interpretations of relative real wages are illustrated in Table 1. In the model of equation (6) firms and unions contracting in period t interpret the real value of the contract formed in period $t + 2$ as $\hat{x}_{t-1|t+2} - \frac{1}{3}(\hat{p}_{t-1|t+2} + \hat{p}_{t-1|t+3} + \hat{p}_{t-1|t+4})$. The model of (7), on the other hand, would substitute $\hat{x}_{t-1|t+2} - \hat{p}_{t-1|t+2}$ for this expression. It can be shown (and is apparent from Table 1) that

$$\frac{1}{N}\sum_{s=0}^{N-1} \hat{p}_{t-1|t+s} = \sum_{s=1}^{N-1} b_s(N-s)^{-1}\sum_{j=s}^{N-1} \hat{p}_{t-1|t-s+j} + \sum_{s=1}^{N-1} b_s(N-s)^{-1}\sum_{j=0}^{N-1-s} \hat{p}_{t-1|t+s+j} \tag{8}$$[4]

Therefore, if (5) holds, equation (7) is identically equal to equation (1), the relative money wage equation of the RMW model.

Taylor argues that firms and unions negotiating the current contract wage look at the contract wages which are set at other firms and will be in effect during their contract period. If overlapping contract wages matter because of what they imply for the real standards of living achieved by other workers and the real wage costs paid by other firms, the RRW model is clearly the appropriate one.

The solution method for the RRW model is similar to that for the RMW model (see Taylor [1980a]), although the algebra is somewhat more involved. From (3) we see that $\hat{p}_{t-1|t+k} = \frac{1}{N}\sum_{i=0}^{N-1} \hat{x}_{t-1|t+k+i}$ and from (2), (3) and (4) that $\hat{y}_{t-1|t+s} = -\frac{\beta}{N}\sum_{i=0}^{N-1} \hat{x}_{t-1|t+s-i}$. Substitute these two expressions into (6) to obtain:

$$x_t - \frac{1}{N^2}\sum_{j=0}^{N-1}\sum_{i=0}^{N-1} \hat{x}_{t-1|t+j-i} = \sum_{s=1}^{N-1} b_s\left(x_{t-s} - \frac{1}{N^2}\sum_{j=0}^{N-1}\sum_{i=0}^{N-1} \hat{x}_{t-1|t-s+j-i}\right) + \sum_{s=1}^{N-1} b_s\left(\hat{x}_{t-1|t+s} - \frac{1}{N^2}\sum_{j=0}^{N-1}\sum_{i=0}^{N-1} \hat{x}_{t-1|t+s+j-i}\right) - \frac{\gamma\beta}{N^2}\sum_{s=0}^{N-1}\sum_{i=0}^{N-1} \hat{x}_{t-1|t+s-i} + \varepsilon_t \tag{9}$$

Taking expectations of (9) as of $t-1$ and rearranging yields:

$$\hat{x}_{t-1|t} = \sum_{s=1}^{N-1} b_s \hat{x}_{t-1|t-s} + \sum_{s=1}^{N-1} b_s \hat{x}_{t-1|t+s} + \frac{(1-\gamma\beta)}{N^2} \sum_{s=0}^{N-1} \sum_{i=0}^{N-1} \hat{x}_{t-1|t+s-i} - \frac{1}{N^2} \sum_{s=1}^{N-1} b_s \sum_{j=0}^{N-1} \sum_{i=0}^{N-1} \hat{x}_{t-1|t-s+j-i} - \frac{1}{N^2} \sum_{s=1}^{N-1} b_s \sum_{j=0}^{N-1} \sum_{i=0}^{N-1} \hat{x}_{t-1|t+s+j-i} \tag{10}$$

Like Taylor we use the following identity:

$$\frac{1}{N^2} \sum_{s=0}^{N-1} \sum_{i=0}^{N-1} \hat{x}_{t-1|t+s-i} \equiv \left(\frac{N-1}{N}\right) \sum_{s=1}^{N-1} b_s \hat{x}_{t-1|t-s} + \left(\frac{N-1}{N}\right) \sum_{s=1}^{N-1} b_s \hat{x}_{t-1|t+s} \tag{11}$$

Equation (10) becomes:

$$\begin{aligned} \hat{x}_{t-1|t}\left(1-\left(\frac{1-\beta\gamma}{N}\right)\right) = {} & \left[1+(1-\beta\gamma)\left(\frac{N-1}{N}\right)-\frac{1}{N}\right] \sum_{s=1}^{N-1} b_s \hat{x}_{t-1|t-s} \\ & + \left[1+(1-\beta\gamma)\left(\frac{N-1}{N}\right)-\frac{1}{N}\right] \sum_{s=1}^{N-1} b_s \hat{x}_{t-1|t+s} \\ & - \left(\frac{N-1}{N}\right) \sum_{s=1}^{N-1} \sum_{j=1}^{N-1} b_s b_j \hat{x}_{t-1|t-s+j} \\ & - \left(\frac{N-1}{N}\right) \sum_{s=1}^{N-1} \sum_{j=1}^{N-1} b_s b_j \hat{x}_{t-1|t+s-j} \\ & - \left(\frac{N-1}{N}\right) \sum_{s=1}^{N-1} \sum_{j=1}^{N-1} b_s b_j \hat{x}_{t-1|t-s-j} \\ & - \left(\frac{N-1}{N}\right) \sum_{s=1}^{N-1} \sum_{j=1}^{N-1} b_s b_j \hat{x}_{t-1|t+s+j} \end{aligned} \tag{12}$$

Equation (12) can be written as:

$$\sum_{s=1}^{2N-2} B_{-s} \hat{x}_{t-1|t-s} + B_0 \hat{x}_{t-1|t} + \sum_{s=1}^{2N-2} B_s \hat{x}_{t-1|t+s} = 0 \tag{13}$$

where, as we show in the appendix, the B_i are given by:

$$B_0 = (3N^2 - N - 1 + 3N\beta\gamma)/3N^2 \tag{14a}$$

$$B_s = B_{-s} = -\frac{N-1}{N}\left((2-\beta\gamma)b_s - \sum_{i=1}^{N-1-s} b_i\, b_{s+i} - \sum_{i=1}^{s-1} b_i\, b_{s-i}\right), \quad s = 1, \ldots, N-1 \tag{14b}$$

$$B_s = B_{-s} = \frac{N-1}{N}\left(\sum_{i=s-N+1}^{N-1} b_i\, b_{s-i}\right) \qquad s = N, \ldots, 2N-2 \tag{14c}$$

Using well-known formulae, the summations in (14b) and (14c) yield

third-order polynomials in s. When the summations run from a larger to a smaller number in (14b) we take the terms to vanish.

The following points are worth noting. First, B_0 is always positive for positive $\beta\gamma$. This can be contrasted with Taylor's model. Second, in (14c) B_s is always positive and independent of $\beta\gamma$. Third, in (14b) B_s is a function of $\beta\gamma$; since the terms in the summation signs are all non-negative these B_s will all change sign for some sufficient large value of $\beta\gamma$, which will in general be different for different s.

Equation (13) can now be solved using the method presented in Taylor [1980a]. Let $L^s x_t \equiv x_{t-s}$ and define the polynomial $B(L) = \sum_{s=-(2N-2)}^{2N-2} B_s L^s$, where B_0 and $B_s = B_{-s}$, $s = 1, 2, \ldots, 2N-2$, are as in (14a–c). Equation (13) can now be rewritten as:

$$B(L)\hat{x}_{t-1|t} = 0 \tag{15}$$

Because of its symmetry $B(L)$ can be factored as in (16):

$$B(L) = \lambda A(L)\, A(L^{-1}) \tag{16}$$

where λ is a normalization constant and $A(L)$ is a polynomial of order $2N-2$

$$A(L) = \sum_{s=0}^{2N-2} \alpha_s L^s \tag{17}$$

with $\alpha_0 = 1$.

A unique rational expectations solution to (15) is obtained by choosing the polynomial $A(L)$ that corresponds to the unstable roots of $B(L)$. With this choice of $A(L)$ we can divide (15) by $\lambda A(L^{-1})$. This yields:

$$A(L)\hat{x}_{t-1|t} = 0 \tag{18}$$

A rational expectations reduced form stochastic difference equation for x_t, the contract wage, is therefore given by:

$$A(L)x_t = \varepsilon_t \tag{19}$$

The α_i, $i = 0, \ldots, 2N-2$ are obtained by solving the $2N-1$ equations

$$B_s = \lambda \sum_{u=0}^{2N-2-s} \alpha_u \alpha_{u+s} \qquad s = 0, 1, 2, \ldots, 2N-2. \tag{20}$$

It is instructive to compare the behaviour of our RRW model with that of Taylor's RMW model, for identical values of all parameters, including contract length. This will serve to bring out the separate contributions of money illusion and 'nominal inertia' due to overlapping, staggered non-contingent money wage contracts.

The reduced form solution for the contract wage in the N-period RRW model (equations (14a–c), (19) and (20)) differs dramatically from the corresponding solution to Taylor's RMW model. The analogue to (13) in the RMW model is:

$$\sum_{s=1}^{N-1} \bar{B}_{-s}\, \hat{x}_{t-1|t-s} + \bar{B}_0\, \hat{x}_{t-1|t} + \sum_{s=1}^{N-1} \bar{B}_s\, \hat{x}_{t-1|t+s} = 0 \tag{21}$$

where

$$\bar{B}_0 = \frac{-(N + \beta\gamma)}{N - (N-1)\beta\gamma} \tag{22a}$$

$$\bar{B}_s = \bar{B}_{-s} = b_s, \qquad s = 1, \ldots, N-1 \tag{22b}$$

Using the same approach as in equations (15)–(20), we get:

$$\bar{A}(L)x_t = \varepsilon_t \tag{23}$$

Here $\bar{A}(L)$ is an $N-1$ degree polynomial.

$$\bar{A}(L) = \sum_{s=0}^{N-1} \bar{\alpha}_s L^s \tag{24}$$

The $\bar{\alpha}_s$, $s = 1, \ldots, N-1$ are found by solving the N equations

$$\bar{B}_s = \lambda \sum_{u=0}^{N-1-s} \bar{\alpha}_u \bar{\alpha}_{u+s} \qquad s = 0, 1, 2, \ldots, N-1 \tag{25}$$

where the $\bar{B}_s$, are defined in (22a,b).

Thus with N-period contracts, the RRW model yields a $2N$-2 order stochastic difference equation in the contract wage while Taylor's RMW model yields an $N-1$ order stochastic difference equation in the contract wage. In Taylor's RMW model as $\beta\gamma$ increases, the coefficients on all lagged values of the contract wage decline monotonically to -1. The coefficients change sign together at $\beta\gamma = N/N-1$.[5] This change of sign is due to two effects working in opposite directions. If $\beta\gamma$ is low then a higher value of x_t will be associated with higher money wage settlements in all subsequent periods, through the relative (money) wage effect. If policy is more restrictive, $\beta\gamma > N/N-1$, the demand effect dominates the relative wage effect and a higher value of x_t leads through expectations of monetary contraction and consequent excess supply of labour to lower wage settlements in subsequent periods. The above pattern can be contrasted with the RRW model. In this model the coefficients on lagged contract wages x_{t-s} tend to -1 as $\beta\gamma \rightarrow +\infty$ for $1 \leqslant s \leqslant N-1$ and to 0 for $M \leqslant s \leqslant 2N-2$.[6] We expect the coefficients on $x_{t-2N+2}, \ldots, x_{t-N}$ to be negative for all finite positive values of $\beta\gamma$[7] and the coefficients on $x_{t-N+1}, \ldots, x_{t-1}$ to be positive for small values of $\beta\gamma$. Table 2 shows how the lag weights vary with $\beta\gamma$ for $N = 2, 3$ in the RMW model and for $N = 2$ in the RRW model.

Table 2 Examples of the contract wage equation in the RMW and RRW models

The RMW model, $N = 2$										
$\beta\gamma$ =	0	.1	.2	.3	.4	.5	.6	1.0	2.0	$+\infty$
$-\bar{\alpha}_1$ =	1.0	.64	.52	.44	.38	.33	.29	.17	0	−1
The RMW model, $N = 3$										
$-\bar{\alpha}_1$ =	.73	.46	.38	.31	.27	.23	.19	.09	−.07	−1
$-\bar{\alpha}_2$ =	.27	.19	.16	.14	.11	.10	.09	.04	−.03	−1
The RRW model, $N = 2$										
$-\alpha_1$ =	2	.81	.63	.52	.44	.38	.33	.19	0	−1
$-\alpha_2$ =	−1	−.27	−.21	−.18	−.16	−.15	−.13	−.11	−.07	0
Money wage model without relative wage effects, $N = 2$										
$-\alpha_1$ =	0	−.024	−.046	−.065	−.084	−.101	−.116	−.172	−.537	−1
Money wage model without relative wage effects, $N = 3$										
$-\alpha_1$ =	0	−.022	−.041	−.058	−.077	−.092	−.106	−.159	−.533	−1
$-\alpha_2$ =	0	−.011	−.021	−.030	−.040	−.048	−.056	−.086	−.363	−1

III Staggered wage contracts without relative wage effects

In this section we examine the money and real wage models when Taylor's second key assumption is removed. That is, when the bargaining process does not take into account relative wages. This corresponds to the special case of the wage equations in (1) and (6) when $d_s = d_{-s} = 0$. It enables us to separate the consequences of having relative wage effects from those having staggered, overlapping contracts *per se*.

The contract determination equation in the money wage model becomes

$$x_t = \frac{\gamma}{N} \sum_{s=0}^{N-1} \hat{y}_{t-1|t+s} + \varepsilon_t \tag{26}$$

Substituting, in for expected demand and taking expectations of both sides gives

$$\hat{x}_{t-1|t} = -\frac{\beta\gamma}{N} \sum_{s=0}^{N-1} \hat{p}_{t-1|\,t+s} \tag{27}$$

Substituting for expected prices and using (11) we get

$$-\frac{\hat{x}_{t-1|t}}{\beta\gamma} = \frac{N-1}{N} \sum_{s=1}^{N-1} b_s\, \hat{x}_{t-1|t-s} + \frac{\hat{x}_{t-1|t}}{N} + \frac{N-1}{N} \sum_{s=1}^{N-1} b_s\, \hat{x}_{t-1|t+s}$$

or rearranging

$$-\left(\frac{N + \beta\gamma}{\beta\gamma(N-1)}\right)\hat{x}_{t-1|t} = \sum_{s=1}^{N-1} b_s\, \hat{x}_{t-1|t-s} + \sum_{s=1}^{N-1} b_s\, \hat{x}_{t-1|t+s} \tag{28}$$

We can solve equation (28) exactly as before. Table 2 presents the lag weights on the resulting stochastic difference equation for $N = 2, 3$ and various values of $\beta\gamma$.

In the money wage model without relative wage effects, less accommodating policy (that is, larger values of $\beta\gamma$) *increases* the dependence of current wages on past wages, as reflected in the larger absolute values of the (negative) lag coefficients. The behaviour of Taylor's RMW model is similar to that of the money wage model without relative wage effects, when the demand effect dominates, that is for large values of $\beta\gamma$. For small values of $\beta\gamma$ the relative wage effect dominates the demand effect in Taylor's model and the dependence of current wages on past wages *diminishes* as $\beta\gamma$ increases from zero to $N(N-1)^{-1}$. This behaviour can be seen clearly in Table 2.

In the real wage model we can similarly disentangle the demand effect from the relative real wage effect and the price expectation effect. In equation (6) we set $d_s = d_{-s} = 0$, which gives

$$x_t - \frac{1}{N}\sum_{s=0}^{N-1} \hat{p}_{t-1|t+s} = \frac{\gamma}{N}\sum_{s=0}^{N-1} \hat{y}_{t-1|t+s} + \varepsilon_t. \tag{29}$$

Substituting for expected demand we have

$$x_t - \frac{1}{N}\sum_{s=0}^{N-1} \hat{p}_{t-1|t+s} = -\frac{\beta\gamma}{N}\sum_{s=0}^{N-1} \hat{p}_{t-1|t+s} + \varepsilon_t. \tag{30}$$

Now substituting money wages for prices and taking expectations of both sides gives

$$\hat{x}_{t-1|t} = \frac{1-\beta\gamma}{N^2}\sum_{s=0}^{N-1}\sum_{i=0}^{N-1} \hat{x}_{t-1|t+s-i},$$

which upon applying (11) and rearranging gives

$$\frac{N+(N-1)^{-1}N\beta\gamma}{N-N\beta\gamma}\,\hat{x}_{t-1|t} = \sum_{s=1}^{N-1} b_s\,\hat{x}_{t-1|t-s} + \sum_{s=1}^{N-1} b_s\,\hat{x}_{t-1|t+s} \tag{31}$$

Comparing this with the equivalent expression in the RMW model (21, 22a, 22b) we can see that the equations are identical when $\beta\gamma$ in this model takes a value $(N-1)N^{-1}$ times that in the RMW model. This means that policy is $N(N-1)^{-1}$ times more effective in this model than it is in the RMW model, but that otherwise they are identical.

The only way to distinguish empirically between the money wage model with relative wage effects and the real wage model without relative wage effects is by knowing $\beta\gamma$ and N *a priori*. Otherwise the two methods are observationally equivalent.

It has already been shown (Phelps and Taylor [1977]; Fischer [1977]) that in order to obtain the conclusion that known monetary policy rules

affect real output, it is sufficient to have multi-period non-contingent money wage (or price) contracts. It is neither essential to have relative wage effects nor for contracts to be staggered. Different policy rules will have different effects whenever the money stock can be made to respond to new information before private agents can revize wage contracts. To include such features as relative wage effects and staggered contracts may of course be desirable for its own sake because it captures an essential feature of reality.

IV Some empirical implications of the relative real wage model

With the aggregate demand equation (2), the price equation (3) and the policy reaction function as our maintained hypothesis, the RRW model and the RMW model have some directly testable implications even if the contract wage cannot be observed. From the price level equation (3) we see that, if $N = 2$, the price equation of the RMW model is the ARIMA (1, 1) process

$$p_t = -\bar{\alpha}_1 p_{t-1} + \tfrac{1}{2}\varepsilon_t + \tfrac{1}{2}\varepsilon_{t-1}$$
$$\text{with } -1 \leqslant -\bar{\alpha}_1 \leqslant 1. \tag{32a}$$

With $N = 2$, the price equation of the RRW model is the ARIMA (2, 2) process

$$p_t = -\alpha_1 p_{t-1} - \alpha_2 p_{t-2} + \tfrac{1}{3}\varepsilon_t + \tfrac{1}{3}\varepsilon_{t-1} + \tfrac{1}{3}\varepsilon_{t-2}$$
$$\text{with } -1 \leqslant -\alpha_1 \leqslant 2 \text{ and } -1 \leqslant -\alpha_2 \leqslant 0. \tag{32b}$$

The price equation for the RMW model with $N = 3$ is also an ARIMA (2, 2) process:

$$p_t = -\bar{\alpha}_1 p_{t-1} - \bar{\alpha}_2 p_{t-2} + \tfrac{1}{3}\varepsilon_t + \tfrac{1}{3}\varepsilon_{t-1} + \tfrac{1}{3}\varepsilon_{t-2}$$
$$\text{with } .732 \geqslant -\bar{\alpha}_1 \geqslant -1;\ .268 \geqslant -\bar{\alpha}_2 \geqslant -1;\ |\bar{\alpha}_1| > |\bar{\alpha}_2|.$$
$$\operatorname{sign}(-\bar{\alpha}_1) = \operatorname{sign}(-\bar{\alpha}_2). \tag{32c}$$

Given *a priori* knowledge of N, we can test the RRW hypothesis that the price data follow an ARIMA $(2N-2, 2N-2)$ process against the RMW hypothesis test that the price data follow an ARIMA $(N-1, N-1)$ process. It is much less straightforward to include in these tests the inequality constraints on the α_i and $\bar{\alpha}_i$ coefficients shown for $N = 2$ in (32b) and (32c). Alternatively, without prior knowledge of N, we can select the best-fitting ARIMA (i, i) process $(i \geqslant 1)$ for the price data. If $i = 1$ and the other restrictions of (32a) are not rejected by the data, the RMW model is consistent with the data and the RRW model is not. If $i > 1$, non-nested hypothesis tests are in general required to discriminate between the RRW and the RMW model on the basis of inequality constraints such as those given in (32b) and (32c). However, finding a positive coefficient for the longest lag on p_t is always inconsistent with the RRW model and finding coefficients with different signs is always inconsistent with the RMW model. Taylor, in his empirical work on the RMW model (see specifically Taylor [1980b], but also Taylor [1979b]) estimates the $N = 2$ version of the model using annual data, which yields the ARIMA (1, 1) process of equation (32a). This equation could never have been generated by the RRW model which always yields at least an ARIMA (2, 2) process for the general price level. In the most direct test of the RMW model (Taylor [1980b]) with $N = 2$, it is found necessary to let the random disturbances ε_t and v_t follow a first order MA process. It is, in principle, always undesirable to have to attribute systematic explanatory power to the disturbance terms of a model. In this case, however, one might go further and argue that equation (32a) with an MA (1) process for ε_t may well be a misspecification that should be tested against (32b), the RRW model with $N = 2$, or against (32c), the RMW model with $N = 3$, both with *i.i.d.* ε_t. However, the RMW model with $N = 3$ is implausible on *a priori* grounds as three year contracts are not found in most of the countries in Taylor's sample.

A problem with such a test, as with other tests of the RMW or RRW hypotheses, is that reduced form equations such as (32a, b and c) reflect not only a particular model of wage and price determination but also the remaining equations of the model such as the rudimentary aggregate demand equation (2) and the simple instantaneous policy feedback rule (4). If these relations were respecified to include, say, lagged output, wage or price terms, the order of the autoregression in p found in equations (32a, b and c) would inevitably be altered. There may well be observational equivalence of reduced forms for a model embodying either the RMW or the RRW hypotheses if the specification of other equations in the model is altered even in quite minor and not implausible ways.[8] If the maintained hypothesis of equations (2) and (4) is deemed unacceptably restrictive, time-series analyses using aggregate data will never permit a conclusive test of the various wage hypotheses against each other.

V Conclusion

The RRW model we developed as an alternative to Taylor's RMW model differs from the latter in one important respect. Taylor's RMW model views the money wage decision of firms and unions contracting in a given period as influenced by the *money* wage rates set (or expected to be set) by other firms and unions that will be in effect during their own contract period. The RRW model views current wage bargainers as attempting to achieve a real wage target over the life of their contract that is influenced by the real wages achieved or expected to be achieved by those other

wage bargainers with whose contract periods there is some degree of overlap.

Quite significant differences in behaviour are exhibited in otherwise identical RRW and RMW models. With N-period contracts, the RRW model yields a $2N$-2 order stochastic difference equation for the contract wage and an ARIMA ($2N$-2, $2N$-2) process for the general price level. The corresponding RMW model yields an N-1 order stochastic difference equation for the contract wage and an ARIMA (N-1, N-1) process for the general price level. With the RMW model the coefficients on lagged contract wages in the contract wage equation always have a common sign and decline both in the order of the lag and with the degree of non-accommodation of monetary policy. The lag coefficients in the RRW model will generally have mixed signs. It was also shown that Taylor's RMW model is observationally equivalent to a real wage model without relative wage effects.

What is perhaps the major qualitative conclusion of Taylor's RMW model is not affected, however. This is that rational expectations combined with nominal inertia due, e.g., to overlapping, staggered, non-contingent money wage contracts, leave scope for known contingent monetary policy rules to influence such real variables as the variance of output.

Appendix

In this appendix we show that equation (13), with the B_j as given in (14a,b,c), is equivalent to equation (12). Except for rearrangement we only have to determine the coefficients of $\hat{x}_{t-1|t+r}$, $\hat{x}_{t-1|t-r}$, for $r = 0, \ldots, 2N - 2$, in the bilinear forms on the right-hand side of equation (12). There are four such bilinear forms but by symmetry we need only consider two cases. The problem, then, is to find K_r and M_r such that (A1) and (A2) hold:

$$\left(\sum_{s=1}^{N-1}\sum_{j=1}^{N-1} b_s b_j \hat{x}_{t-1|t-s+j} + \sum_{s=1}^{N-1}\sum_{j=1}^{N-1} b_s b_j \hat{x}_{t-1|t+s-j} =\right) \tag{A1}$$

$$2\sum_{s=1}^{N-1}\sum_{j=1}^{N-1} b_s b_j \hat{x}_{t-1|t-s+j} = \sum_{r=1}^{N-2} K_r \hat{x}_{t-1|t+r} + K_o \hat{x}_{t-1|t} + \sum_{r=1}^{N-2} K_r \hat{x}_{t-1|t-r}$$

$$\sum_{s=1}^{N-1}\sum_{j=1}^{N-1} b_s b_j \hat{x}_{t-1|t+s+j} = \sum_{r=2}^{2N-2} M_r \hat{x}_{t-1|t+r}. \tag{A2}$$

Consider (A1) first. In this equation we have

$$j = s + r \tag{A3}$$

The fact that j takes values only between 1 and $N - 1$ means that, for $r \geq 0$, s is constrained to taking the values between 1 and $N - 1 - r$. For each s in this domain we have a corresponding j (given by (A3)) and a value for $b_j b_s$; the sum of all these (multiplied by 2) is the coefficient of $\hat{x}_{t-1|t+r}$. Hence

$$K_{-r} = K_r = \sum_{i=1}^{N-1-r} b_i b_{r+i}, \qquad r = 0, \ldots, N-2 \tag{A4}$$

Because of the linearity of b_k in k we can easily carry out the summation in (A4). Substituting in for b_i and b_{r+i} (A4) becomes:

$$K_{-r} = K_r = \sum_{i=1}^{N-1-r} \frac{(N-i)(N-r-i)}{[N(N-1)]^2}$$

$$= \sum_{i=1}^{N-1-r} \frac{N(N-r) - (2N-r)i + i^2}{[N(N-1)]^2} \tag{A4$'$}$$

Now using the following identities:

$$\sum_{i=1}^{N-1-r} N(N-r) = (N-1-r)N(N-r)$$

$$\sum_{i=1}^{N-1-r} (r-2N)i = \tfrac{1}{2}(N-1-r)(N-r)(r-2N)$$

$$\sum_{i=1}^{N-1-r} i^2 = \tfrac{1}{6}(N-1-r)(N-r)(2N-2r-1)$$

we have

$$K_{-r} = K_r = \frac{(N-1-r)(N-r)(2N+r-1)}{6[N(N-1)]^2} \qquad r = 0, \ldots, N-2. \tag{A5}$$

And in particular

$$K_0 = \frac{2N-1}{6N\,(N-1)} \tag{A6}$$

Now we deal with (A2). Proceeding as before, M_r is the sum of the series $b_s b_j$ for which

$$s + j = r, \tag{A7}$$

and both s and j lie between 1 and $N-1$. Hence,

$$1 \leqslant j \leqslant N-1 \qquad \text{and}$$
$$1 \leqslant s = r - j \leqslant N - 1 \Rightarrow r - N + 1 \leqslant j \leqslant r - 1$$

The upper and lower bounds on j depend on the value of r. There are two cases

$2 \leqslant r \leqslant N-1$, in which case j lies between 1 and $r-1$ (I)

$2N-2 \geqslant r \geqslant N$, in which case j lies between $r-N+1$ and $N-1$ (II)

Therefore,

$$M_{-r} = M_r = \sum_{j=1}^{r-1} b_j b_{r-j} \qquad r = 2, \ldots, N-1 \tag{A8}$$

$$M_{-r} = M_r = \sum_{j=r-N+1}^{N-1} b_j b_{r-j} \qquad r = N, \ldots, 2N-2 \tag{A9}$$

We can now write equation (12) as

$$\frac{N-1}{N}\sum_{s=N}^{2N-2} M_{-s}\hat{x}_{t-1|t-s} - \frac{N-1}{N}\sum_{s=1}^{N-1}((2-\beta\gamma)b_s + K_{-s} + M_{-s})\hat{x}_{t-1|t-s}$$

$$+ \left(1 - \frac{(1-\beta\gamma)}{N} + \frac{N-1}{N}K_0\right)\hat{x}_{t-1|t}$$

$$- \frac{N-1}{N}\sum_{s=1}^{N-1}\left(((2-\beta\gamma)b_s + K_s + M_s)\hat{x}_{t-1|t+s}\right.$$

$$+ \frac{N-1}{N}\sum_{s=N}^{2N-2} M_s\hat{x}_{t-1|t+s} = 0 \tag{A10}$$

Notes

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1 The simplest interpretation of (2) is that of the quantity theory equation of exchange with the (logarithm of the) income velocity of circulation represented by a white noise disturbance term v_t.

2 In Taylor's RMW model, as in the RRW model specified below, output would be affected by lagged monetary feedback as well (e.g., $m_t = \delta p_{t-1}$). Unless the policy rule is 'symmetric in time' (e.g., $m_t = \delta p_{t-1} + (1-\beta)p_t + \delta p_{t-1|t+1}$) the simple algebraic structure of the RMW model and the RRW model is lost because the polynomial equation in the expected contract wage is no longer symmetric.

3 The non-contingent nature of these contracts means that the N-period wage contract negotiated in period t does not make the wages paid under the contract in periods $t+1, \ldots, t+N-1$ contingent on information (on future contract wages, future prices and future excess demands) that may become available during the life of the contract. If a multi-period contract made the money wages paid over the life of the contract contingent on future information, it would be equivalent to a sequence of single-period contracts.

4 Substituting for b_s in equation (8) gives

$$(N-1)\sum_{s=0}^{N-1}\hat{p}_{t-1|t+s} = \sum_{s=1}^{N-1}\left(\sum_{j=s}^{N-1}\hat{p}_{t-1|t-s+j} + \sum_{j=0}^{N-1-s}\hat{p}_{t-1|t+s+j}\right)$$

Noticing that

$$\sum_{j=s}^{N-1}\hat{p}_{t-1|t-s+j} = \sum_{j=0}^{N-1-s}\hat{p}_{t-1|t+j}$$

$$\sum_{j=0}^{N-1-s}\hat{p}_{t-1|t+s+j} = \sum_{j=s}^{N-1}\hat{p}_{t-1|t+j}$$

and

$$\sum_{s=1}^{N-1}\sum_{j=0}^{N-1-s}\hat{p}_{t-1|t+j} = \sum_{s=0}^{N-2}(N-1-s)\hat{p}_{t-1|t+s}$$

$$\sum_{s=1}^{N-1}\sum_{j=s}^{N-1}\hat{p}_{t-1|t+j} = \sum_{s=1}^{N-1}s\hat{p}_{t-1|t+s}$$

the identity is easily established.

5 From (22a) $\bar{B}_0$ goes to infinity as $\beta\gamma$ goes to $\dfrac{N}{N-1}$. Dividing (21) by $\bar{B}_0$ and taking the limit as $\beta\gamma \to \dfrac{N}{N-1}$ yields $\hat{x}_{t-1|t} = 0$. Therefore $x_t = \varepsilon_t$ is a solution. Again from (22a) $\bar{B}_0$ goes to $(N-1)^{-1}$ as $\beta\gamma$ goes to $+\infty$. By direct substitution it can be checked that $a_s = 1$, $s = 1, \ldots, N-1$, satisfies (25). (Note that $\lambda = [(N-1)N]^{-1}$.)

6 From equations (14a,b,c) we get $\lim\limits_{\beta\gamma\to\infty}\left(\dfrac{B_s}{B_0}\right) = \dfrac{N-s}{N}$ for $1 \leqslant s \leqslant N-1$ and $\lim\limits_{\beta\gamma\to\infty}\left(\dfrac{B_s}{B_0}\right) = 0$ for $N \leqslant s \leqslant 2N-2$. Dividing (20) by B_0 we get

$$\frac{B_s}{B_0} = \frac{\sum_{u=0}^{2N-2-s}\alpha_u\alpha_{u+s}}{\sum_{i=0}^{2N-2}\alpha_i^2}, \quad s = 1, \ldots, 2N-2.$$

Taking the limit as $\beta\gamma \to \infty$ this reduces to

$$\frac{N-s}{N} = \frac{\sum_{u=0}^{2N-2-s}\alpha_u\alpha_{u+s}}{\sum_{i=0}^{2N-2}\alpha_i^2}, \quad s = 1, \ldots, N-1$$

$$0 = \frac{\sum_{u=0}^{2N-2-s}\alpha_u\alpha_{u+s}}{\sum_{i=0}^{2N-2}\alpha_i^2}, \quad s = N, \ldots, 2N-2.$$

These two sets of equations are satisfied by

$$\alpha_i = \begin{cases} 1, & i = 1, \ldots, N-1 \\ 0 & i = N, \ldots, 2N-2 \end{cases}$$

7 It is obvious that the coefficient on $\hat{x}_{t-2N+2}$ is always negative for finite positive $\beta\gamma$.

8 We would like to thank an anonymous referee for bringing this to our attention.

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