

THE LONG-RUN EFFECTS OF FISCAL POLICY

Willem H. Buiter\*

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\*Assistant Professor of Economics and  
International Affairs, Woodrow Wilson  
School of Public and International Affairs,  
Princeton University

Econometric Research Program  
PRINCETON UNIVERSITY  
207 Dickinson Hall  
Princeton, New Jersey

ABSTRACT

The impact effects and long-run effects of fiscal policy are compared in a small macroeconomic model with endogenous price level and price expectations. The hypothesis that public spending crowds out private spending is rejected in the short run as well as in the long run.

## THE LONG-RUN EFFECTS OF FISCAL POLICY\*

### Introduction.

Keynes's model and its heterogeneous progeny like the IS/LM model,<sup>1</sup> the Patinkin model<sup>2</sup> and the Metzler model<sup>3</sup> all were short-run models. In a short-run model of the economy some of the variables that are endogenous in the long run are treated as predetermined. They fall into three categories:

First, the stocks of all assets, both real and financial. In the simple macroeconomic model I shall be considering the relevant stocks are the nominal quantity of government fiat money, the nominal quantity of government interest-bearing debt, the stock of real reproducible capital and the size of the labor force (the "fund" of labor services).

Second, expectations about the future. An example from Keynes are the expectations about the future flow of returns to investment that are reflected

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\*This paper is based on Chapters 3 and 4 of my Ph.D. dissertation, Temporary Equilibrium and Long-Run Equilibrium, Yale 1975. Special thanks are due to James Tobin, Gary Smith and Katsuhito Iwai.

<sup>1</sup>Hicks, J.R. "Mr. Keynes and the Classics." Econometrica, Vol. 5, no. 2 (April 1937), pp. 147-59.

<sup>2</sup>Patinkin, D. Money, Interest and Prices, 2nd ed. New York: Harper and Row, 1965.

<sup>3</sup>Metzler, L.A. "Wealth, Saving and the Rate of Interest." Journal of Political Economy, Vol. 59, no. 2 (April 1951), pp. 93-116.

in the marginal efficiency of capital schedule.<sup>4</sup> In the model below, only price expectations will be treated explicitly.

Third, in some of the simple Neo-Keynesian models such as the Hicksian IS/LM model and the Patinkin model with unemployed labor, the money price level and the money wage rate are assumed to be fixed in the short run.<sup>5</sup>

The parameters of government policy are usually treated as exogenous both in the short run and in the long run. Recent work attempting to endogenize policy behavior, while promising, has not yet established any firm operational results.<sup>6</sup> In the model considered below the government controls consist of the level of government spending, the marginal income tax rate and the shares of bonds and money in the financing of deficits or surpluses. The nominal rate of return on money balances is assumed to be fixed at zero, but could be made into an additional policy instrument.

Short-run analysis is the study of the impact effects or first-round effects of changes in the government control variables (or in any of the short-run parameters) on the short-run endogenous variables: the rate of interest and the level of output in the neo-Keynesian models with unemployment; the rate of interest and the price level in the neo-Keynesian full-employment models.

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<sup>4</sup> Keynes, J.M. The General Theory of Employment Interest and Money. Macmillan and Co., Ltd., 1936, ch. 11.

<sup>5</sup> Keynes certainly did not assume rigid prices. See Keynes, J.M. op. cit., ch. 19.

<sup>6</sup> See e.g. W.D. Nordhaus, "The Political Business Cycle," The Review of Economic Studies, Vol. 42(2), April 1975, pp. 169-90, and Rees, A.; Kaufman, H.; Eldersveld, S.J., and Freidel, F., "The Effects of Economic Conditions on Congressional Elections, 1946-58," Review of Economics and Statistics, 44 (1962), pp. 458-463.

Goldfeld, S.M. and Blinder, A.S. "Some Implications of Endogenous Stabilization Policy." Brookings Papers on Economic Activity, 3, 1972, pp. 585-641.

Long-run analysis explicitly allows for the changes in asset stocks, expectations and other short-run parameters that occur as time passes. While it is possible in principle to analyze the complete dynamic behavior of an economic system from arbitrary initial conditions, analytical tractability usually compels one to concentrate on steady-state behavior. The long-run comparative statics (across steady states) can then be supplemented by an analysis of the stability and controllability properties of the model.

The original motivation for investigating the long-run effects of governmental policy -- especially fiscal policy -- was a desire to test the proposition that the effects of pure fiscal policy changes, defined as changes in some parameter(s) of the government spending or tax functions that do not affect the nominal quantity of money are "certain to be temporary and likely to be minor";<sup>7</sup> the expansionary or inflationary effect of, say, an increase in government spending is essentially a first-round effect that will be reversed in the long run unless accompanied by an increase in the quantity of money. In this paper I shall analyze the short-run and long-run effects of such "pure" fiscal policy changes. I do not present such policies as realistic descriptions of government behavior. They simply provide a framework for testing at the theoretical level the "null hypothesis" of 100% crowding-out of private spending by public spending. The analysis suggests that the rather alarmist views on crowding-out that have become fashionable again in policy-making circles are unwarranted.

The usefulness of the model extends beyond the analysis of the long-run effects of pure fiscal policy. Pure monetary policy (of the money-rain variety) and the entire spectrum of mixed fiscal-monetary policy actions can be analyzed with the same analytical framework.

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<sup>7</sup>Friedman, M. "Comment on the Critics." Journal of Political Economy, Vol. 80, no. 5 (Sept/Oct. 1972), pp. 906-50.

A seminal paper by Blinder and Solow<sup>8</sup> and the first part of a paper by Tobin and Buiter<sup>9</sup> analyze the question of the long-run effects of fiscal and monetary policy in the context of a model without binding supply constraints in the long run as well as in the short run. Real output is always determined by effective demand. Money wages and prices are not considered explicitly.

In order for Keynesian involuntary unemployment to exist in long-run stationary equilibrium we have to accept the indefinite persistence of wage and price rigidities. The "Keynesian" aspect of these models coexists very uncomfortably with their focus on the ultimate long-run equilibrium of the stationary state. These models could perhaps not unfairly be characterized as: Keynesian wine in Pigovian bottles.

In what follows I shall endogenize the price level and price expectations and allow for capacity constraints. The question analyzed is the following: will the impact effects of an increase in government spending -- which all parties concerned recognize as being expansionary or inflationary -- be "undone" in the long run as changes in stocks, expectations, etc. modify the temporary equilibrium solution ("shift the LM and IS curves around")?

The principal result is that Blinder's and Solow's conclusion that pure fiscal policy matters for real variables in the long run is not contingent on what might be considered to be an unwarranted extension of a temporary (dis)equilibrium model to the long run of the stationary state.

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<sup>8</sup>Blinder, A.S. and Solow, R.M. "Does Fiscal Policy Matter?" Journal of Public Economics, Vol. 2, no. 4 (Nov. 1973), pp. 319-37.

<sup>9</sup>Tobin, J. and Buiter, W. "Long-Run Effects of Fiscal and Monetary Policy on Aggregate Demand." Forthcoming in Proceedings of the Conference on Monetarism, ed. J. Stein, Amsterdam: North Holland.

Tobin and Buiter<sup>10</sup> and Buiter<sup>11</sup> have endogenized the price level by superimposing the IS/LM framework on a neoclassical production function and assuming full employment. In this paper the price level will be endogenized and a capacity constraint introduced into the standard IS/LM model by adding a "Phillips curve" price equation to the temporary equilibrium model.

### The Model.

#### Notation

Y	real factor income
T	total real taxes
G	real government consumption spending
S	real private saving
K	stock of real reproducible capital
M	nominal stock of money balances
M <sup>d</sup>	demand for nominal money balances
D	nominal stock of bonds
R	nominal rate of interest on government bonds
p	money price level of commodities
q	valuation ratio
x	instantaneous proportional expected rate of change of p
θ	marginal income tax rate
W	total real private sector non-human wealth

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<sup>10</sup>Tobin and Buiter, Op. cit.

<sup>11</sup>Buiter, W. Temporary Equilibrium and Long-Run Equilibrium. Unpublished Ph.D. thesis, Yale University, 1975, pp. 186-202.

The long run considered will be the long run of the stationary state. None of the essential results are changed if labor force growth at a constant exponential rate and labor augmenting technical change are allowed for.

The short-run equilibrium equations of the model will resemble as closely as possible the standard short-run version of IS/LM analysis. Of course, once the price level is made endogenous we must explicitly consider price expectations and distinguish between real and nominal rates of return.

#### The Financial Assets

Bonds and existing capital are assumed to be perfect substitutes in private portfolios. This is the standard Keynesian assumption. Bonds are fixed nominal market value, variable interest rate claims, rather like savings accounts. Money is government-issued fiat money. The portfolio choice of money versus bonds-cum-capital depends on the real after-tax rate of return differential between money and bonds. The nominal rate of return on money balances is institutionally fixed at zero. If  $x$  is the expected instantaneous proportional rate of change of the price level,  $p$ , and  $R$  the nominal rate of return on bonds, the real after-tax rate of return differential is  $R(1-\theta)$ . (Capital gains due to inflation are not taxed.) The demand for real money balances is specified as follows:

$$\frac{M^d}{p} = L(R(1-\theta), \frac{Y}{W})W \quad L_1 < 0 ; L_2 > 0 ; \frac{L_2 Y}{W} < L .$$

The fraction of wealth held in the form of real money balances is inversely related to the after-tax rate of return differential between money and alternative assets (the usual gross-substitutes assumption) and positively related to the income-wealth ratio. The income-wealth ratio can be considered as a



proxy for the ratio of human to non-human wealth.<sup>12</sup> The short-run income elasticity of demand for real money balances is less than unity.

Private sector non-human wealth is the sum of the real value of the stocks of bonds and money and the market value of existing capital.  $q$  is the valuation ratio, the ratio of the market value of the stock of capital to the value of the stock of capital at current reproduction costs, or equivalently, the ratio of the rate of return on a dollar invested in the production of new capital goods to the rate of return on a dollar spent on existing capital assets.

$$\text{Thus, } W \equiv \frac{M+D}{p} + qK .$$

In long-run equilibrium  $q = 1$ . Outside of long-run equilibrium  $q$  can vary, providing the inducement to invest ( $q > 1$ ) or to decumulate ( $q < 1$ ).

Asset market equilibrium is given by:

$$L(R(1-\theta), \frac{Y}{\frac{M+D}{p} + qK}) (\frac{M+D}{p} + qK) = \frac{M}{p} \quad (1)$$

### The Production Function

The (implicit) production function is a well-behaved constant returns to scale neoclassical production function in capital and labor. Labor services are supplied inelastically and are proportional to the size of the constant labor force.

### The Investment and Saving Functions

The investment and saving functions are of the stock adjustment type. The rate of investment is an increasing function of the gap between target capital stock and actual capital stock.

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<sup>12</sup>I am indebted to Milton Friedman for pointing this out during a seminar.

$$I = I(H(R-x)Y-K) \quad I' > 0; \quad H' < 0; \quad I(0) = 0 .$$

$H(R-x)$  is the technologically determined optimal capital-output ratio.

I assume that  $H(R-x)Y = qK$  <sup>13</sup>

Private saving is an increasing function of the gap between target wealth and actual wealth. Target wealth,  $\hat{W}$ , is a multiple of after-tax factor income.

$$\hat{W} = \mu(1-\theta)Y ; \quad [\mu(1-\theta) = \hat{\mu}] .$$

The saving function will be specified as:

$$S = S(\hat{\mu}Y - W) \quad S' > 0; \quad S(0) = 0. <sup>14</sup>$$

### Expectations

Adaptive expectations determine the expected rate of change of prices:

$$\dot{x} = \beta \left( \frac{\dot{p}}{p} - x \right) \quad \beta \geq 0 . \quad (2)$$

This includes static expectations ( $x(t) = 0$ ) and myopic perfect foresight ( $x = \frac{\dot{p}}{p}$ ) as special cases.

### Asset Dynamics

Changes over time in the stocks of capital, money and bonds are given by

<sup>13</sup> For the market value of the stock of capital,  $qK$  to coincide exactly with the firm's target stock of capital,  $H(R-x)Y$ , we either require a Cobb-Douglas production function or static expectations as regards the future earnings of capital. See also Tobin and Buiter, op. cit. and Buiter, op. cit. pp. 162-163.

<sup>14</sup> I assume  $\hat{\mu} > H(R-x)$ ; there is room in the portfolio for assets other than capital.

$$\dot{K} = I . \quad (3)$$

and

$$\frac{\dot{M} + \dot{D}}{P} = \frac{G + RD}{P} - T . \quad (4)$$

(4) is the government budget identity.

### Price Dynamics

The behavior of prices over time is governed by the following Phillips-curve-type equation:

$$\frac{\dot{P}}{P} = \Psi(Y, K) + x \quad \Psi_Y > 0; \quad \Psi_K < 0; \quad \left( \Psi_{YY} > 0; \quad \Psi_{KK} > 0; \quad \Psi_{YK} < 0 \right)^{15} \quad (5)$$

The proportional rate of change of prices is an increasing function of the difference between current effective demand and full employment capacity. When there is neither slack nor tightness ( $\Psi(Y, K) = 0$ ) the actual and expected rates of change of the price level coincide. There is no "dynamic money illusion"; the coefficient of  $x$  in (5) is unity.

### Government Policy

The tax function is a proportional tax on factor-income and interest income  $T = \Theta \left( Y + \frac{RD}{P} \right)$ .

Let  $\delta$  denote the share of money in the financing of the current budget deficit.

<sup>15</sup>Such a price-Phillips curve could, e.g., be derived through a fixed proportional mark-up of price on money wage, from the following wage-Phillips curve:  $\frac{\dot{w}}{w} = \alpha(g(Y, K) - N) + x \quad \alpha > 0$ .  $g(Y, K)$  is the demand for labor, derived from effective demand  $Y = C + I + G$  and a well behaved neoclassical production function.  $N$  is the size of the labor force;  $w$  is the money wage.

$$\frac{\dot{M}}{p} = \delta \left[ G + (1-\theta) \frac{RD}{p} - \theta Y \right]$$

$$\frac{\dot{D}}{p} = (1-\delta) \left[ G + (1-\theta) \frac{RD}{p} - \theta Y \right]$$

Pure fiscal policy is the case where  $\delta = 0$ . Most of the analysis will center on the case where  $G' \equiv G + (1-\theta) \frac{RD}{p}$  is the government spending parameter.  $G'$  is government purchases of goods and services plus real net-of-tax debt payments. As net debt service changes, through endogenous changes in  $R$ ,  $D$  or  $p$ , government purchases of goods and services are adjusted so as to keep  $G'$  at its fixed level. With  $\delta = 0$ , the government budget identity becomes

$$\frac{\dot{D}}{p} = G + (1-\theta) \frac{RD}{p} - \theta Y \quad (\text{Policy I})$$

or

$$\frac{\dot{D}}{p} = G' - \theta Y \quad (\text{Policy II})$$

The complete model is given by the following set of equations.

$$\left\{ \begin{array}{l} I(H(R-x)Y-K) + G + (1-\theta) \frac{RD}{p} - \theta Y - S(\hat{u}Y - \frac{M+D}{p}) - H(R-x)Y - x \frac{M+D}{p} = 0 \quad (6-I) \\ \text{or} \\ I(H(R-x)Y-K) + G' - \theta Y - S(\hat{u}Y - \frac{M+D}{p}) - H(R-x)Y - x \frac{M+D}{p} = 0 \quad (6-II) \end{array} \right.$$

IS curve (Policy I)<sup>16</sup>

IS curve (Policy II)

$$L(R(1-\theta), \frac{Y}{\frac{M+D}{p} + H(R-x)Y}) \left( \frac{M+D}{p} + H(R-x)Y \right) = \frac{M}{p} \quad \text{LM curve} \quad (7)$$

<sup>16</sup>The last term in (6-I) and (6-II) represents expected additions, positive or negative, to wealth due to changes in the real values of nominal stocks of money and bonds. The IS curve represents equilibrium in the commodity market

$$\left\{ \begin{array}{l} \frac{d}{dt} \left( \frac{D}{P} \right) = G + \frac{RD}{P} - \theta \left( Y + \frac{RD}{P} \right) - \frac{\dot{P}}{P} \frac{D}{P} \\ \frac{d}{dt} \left( \frac{D}{P} \right) = G' - \theta Y - \frac{\dot{P}}{P} \frac{D}{P} \end{array} \right. \quad \begin{array}{l} \text{6(I) (Policy I)} \\ \text{6(II) (Policy II)} \end{array}$$

$$\frac{d}{dt} \left( \frac{M}{P} \right) = - \frac{\dot{P}}{P} \frac{M}{P} \quad (9)$$

$$\dot{K} = I(H(r-x)Y-K) \quad (10)$$

$$\frac{\dot{P}}{P} = \psi(Y,K) + x \quad (11)$$

$$\dot{x} = \left( \frac{\dot{P}}{P} - x \right) \quad (12)$$

### Long-Run Equilibrium

In long-run equilibrium, momentary (IS/LM) equilibrium holds at each point of time, real stocks and flows remain constant and expectations are no longer revised.

Since we are considering policy regimes that hold the nominal quantity of money constant, actual and expected rates of inflation must be zero in long-run equilibrium, as can be checked by setting  $\frac{d}{dt} \left( \frac{D}{P} \right)$ ,  $\frac{d}{dt} \left( \frac{M}{P} \right)$ ,  $\dot{K}$  and  $\dot{x}$  equal to zero.

In summary, in the stationary state,  $\frac{\dot{P}}{P} = x = \dot{x} = \dot{D} = \dot{K} = 0$ .

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$Y = C + I + G$ .  $C$  denotes private consumption demand. When income is aggregated across the private and the public sectors, the transfer terms will cancel each other out but the capital gains term remains.

$$C = \left( Y + \frac{RD}{P} \right) (1-\theta) - x \left( \frac{M+D}{P} \right) - S.$$

Substituting this into the commodity market equilibrium condition yields 6(I) or 6(II).

The stationary state is described by the following five equations in the five unknowns  $R$ ,  $Y$ ,  $p$ ,  $K$  and  $D$ :

$$L(R(1-\theta), \frac{1}{\lambda})\hat{\mu}Y = \frac{M}{p} \quad (13)$$

$$H(R)Y = K \quad (14)$$

$$\Psi(Y, K) = 0 \quad (15)$$

$$\hat{\mu}Y = \frac{M+D}{p} + K \quad (16)$$

$$\left\{ \begin{array}{l} G + (1-\theta)\frac{RD}{p} - \theta Y = 0 \quad (\text{Policy I}) \\ G' - \theta Y = 0 \quad (\text{Policy II}) \end{array} \right. \quad (17-I)$$

$$\left\{ \begin{array}{l} G + (1-\theta)\frac{RD}{p} - \theta Y = 0 \quad (\text{Policy I}) \\ G' - \theta Y = 0 \quad (\text{Policy II}) \end{array} \right. \quad (17-II)$$

Considering Policy II first, there is a steady-state relationship between  $G'$  and  $Y$ :  $Y = \frac{1}{\theta}G'$ . The long-run government spending multiplier  $\frac{\partial Y}{\partial G'}$  is therefore the reciprocal of the marginal income tax rate,  $\frac{1}{\theta}$ .

This result that holds by virtue of the balanced budget condition (17-II), therefore carries over from the simple fixed-price IS/LM model.<sup>17</sup> A higher level of government spending on commodities and net debt service has to be offset by a higher level of tax revenues in long-run equilibrium. This requires a higher level of real factor income.

<sup>17</sup> See Blinder and Solow, *op. cit.* and Christ, C.F., "A Simple Macroeconomic Model with a Government Budget Restraint," Journal of Political Economy, Vol. 76, no. 1 (Jan. 1960), pp. 53-67.

We can solve (15) for  $K$  as a function of  $Y$ ,  $K = h(Y)$ ,  $h' = -\psi_Y/\psi_K > 0$ . In  $R$ - $Y$  space the long-run equilibrium can therefore be represented by the following two equations:

$$H(R)Y = h(Y) \tag{18} \text{ (AA curve)}$$

$$G' - \theta Y = 0 \tag{19} \text{ (BB curve)}$$

This is represented in Diagram 1. The old equilibrium is  $E_1$ , the new equilibrium is  $E_2$ . An increase in  $G'$  will raise  $Y$  through the balanced budget condition and hence  $K$  through the zero rate of inflation condition. In  $R$ - $Y$  space the  $BB$  curve shifts to the right. If the  $AA$  curve is downward sloping, ( $H(R) < h'(Y)$ ),  $R$  will be lowered. With lower  $R$  and higher  $Y$  the only way in which the portfolio balance condition can be satisfied if the nominal stock of money is constant is through a lower price level. If the  $AA$  curve is upward sloping ( $H(R) > h'(Y)$ ),  $R$  will be raised and the effect on  $p$  will be ambiguous.

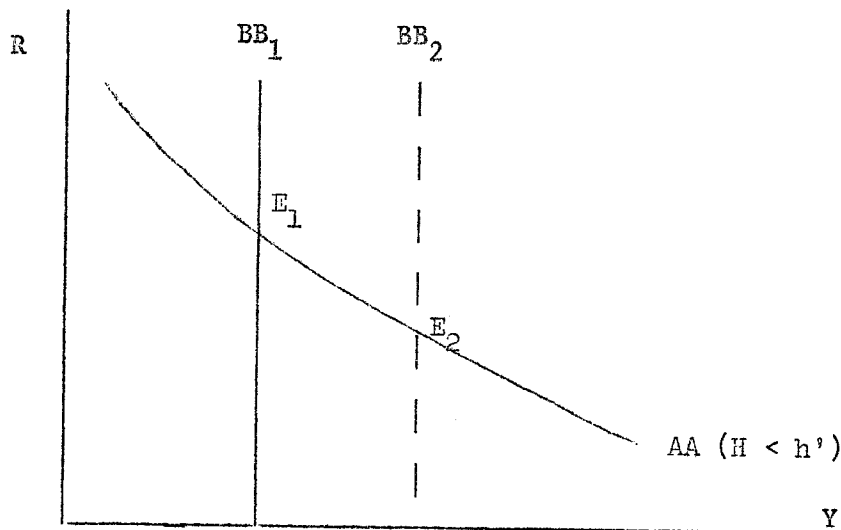


Diagram 1

The effect on R of an increase in  $G^v$  has an intuitive interpretation. If the amount by which the capital stock has to increase to offset the inflationary effect of a higher level of Y ( $-\frac{\Psi_Y}{\Psi_K} \Delta Y$ ) exceeds the increase in the stock of capital that would be optimal at the original level of R ( $H\Delta Y$ ), the marginal product of capital, which in long-run equilibrium is linked directly to the rate of interest, will be lowered.

For Policy II we again can solve (15) for K as a function of Y:  $K = h(Y)$ . The representation of the long-run equilibrium in R-Y space is given by equations (20) and (21).

$$H(R)Y = h(Y) \tag{20} \text{ (AA curve)}$$

$$G + (1-\theta)R \left[ (1-L(R(1-\theta), \frac{1}{\mu})) \hat{u}Y - h(Y) \right] - \theta Y = 0 \tag{21} \text{ (BB curve)}$$

Not only the slope of the AA curve is ambiguous in this case, but the slope of the BB curve and the direction of its shift when G is changed are also ambiguous. The long-run government spending multiplier,  $\frac{\partial Y}{\partial G}$ , while clearly non-zero, cannot be determined without further quantitative information about the structural parameters of the model. The higher level of public spending has to be financed somehow, but a higher volume of tax receipts from a higher level of real income is not the only possibility. Anything that reduces the real value of government debt service, such as a higher price level, a lower rate of interest or a smaller volume of nominal debt will serve the purpose of balancing the budget. <sup>18</sup>

<sup>18</sup>The long-run multiplier is:  $\frac{\Psi_K H' Y M / p^3}{\Psi_Y [-WL_1 (1-\theta)^2 \frac{RM}{p^3} - H' Y (1-\theta) \frac{RM}{p^3} + (1-\theta) \frac{MD}{p^4}] + \Psi_K [(L-1) \hat{u} H' Y (1-\theta) \frac{RM}{p^3} -$

$$\frac{HWL_1 (1-\theta)^2 \frac{RM}{p^3} + H(1-\theta) \frac{MD}{p^4} + \frac{\theta H' Y M}{p^3}]$$

(cont'd)



Impact Effects

We can solve the IS and LM curves for Y and R as functions of D, K, p, x, G (or G') and  $\theta$ , given M. For reasons of space only policy II will be considered here.

$$Y = V(D, K, p, x, G', \theta; M) \quad (22a)$$

$$R = U(D, K, p, x, G', \theta; M) \quad (22b)$$

The impact multipliers (the effect on the short-run equilibrium values of R and Y of changes in D, K, p, x, G' and  $\theta$ ) can be solved for from:

$$\begin{bmatrix} (I' + S')H - \theta - S' \hat{\mu} & (I' + S')YH' \\ L_2 + (L - \frac{L_2 Y}{W})H & WL_1(1-\theta) + (L - \frac{L_2 Y}{W})H'Y \end{bmatrix} \begin{bmatrix} dy \\ dR \end{bmatrix}$$

$$\begin{bmatrix} (\frac{S'+x}{p})dD + I'd_K + (S'\frac{(M+D)}{p^2} - x\frac{(M+D)}{p^2})dp + ((I'+S')YH' + \frac{(M+D)}{p})dx \\ - dG' + (Y-S'\mu Y)d\theta \\ (\frac{L_2 Y}{Wp} - \frac{L}{p})dD - (\frac{(M+D)L_2 Y + MK}{p^2 W})dp + (L - \frac{L_2 Y}{W})H'Ydx + WL_1 R d\theta \end{bmatrix}$$

The numerator is positive as is the first term of the denominator. In the second term,  $\Psi_K \theta H' \frac{YM}{p^3}$  is positive, while  $\Psi_K (L-1) \hat{\mu} H' Y (1-\theta) \frac{RM}{p^3}$ ,  $\Psi_K (-) H W L_1 (1-\theta)^2 \frac{RM}{p^3}$  and  $\Psi_K H (1-\theta) \frac{MD}{p^4}$  are negative.

The impact effect of an increase in  $G'$  on  $Y$  is

$$V_{G'} = \frac{-(WL_1(1-\theta) + (L - \frac{L_2Y}{W})H'Y)}{[(I'+S')H - \theta - S'\hat{\mu}][WL_1(1-\theta) + (L - \frac{L_2Y}{W})H'Y] - [(I'+S')YH'[L_2 + (L - \frac{L_2Y}{W})H]}$$

The impact effect of an increase in  $G'$  on  $R$  is

$$U_{G'} = \frac{L_2 + (L - \frac{L_2Y}{W})H}{[(I'+S')H - \theta - S'\hat{\mu}][WL_1(1-\theta) + (L - \frac{L_2Y}{W})H'Y] - (I'+S')YH'[L_2 + (L - \frac{L_2Y}{W})H]} .$$

The remaining impact multipliers can be found without difficulty. The signs of these impact effects are all as expected and can be interpreted in terms of shifts of the short-run IS and LM curves.

The slope of the IS curve is

$$\left. \frac{dR}{dY} \right|_{IS} = \frac{-(I'+S')H - \theta - S'\hat{\mu}}{(I'+S')YH'}$$

$(-(I'+S')H - \theta - S'\hat{\mu})^{-1}$  is the simple Keynesian government spending multiplier.

If this is positive, as empirical evidence overwhelmingly supports, the IS curve will be downward sloping.

The LM curve is upward sloping,

$$\left. \frac{dR}{dY} \right|_{LM} = \frac{-(L_2 + (L - \frac{L_2Y}{W})H)}{WL_1(1-\theta) + (L - \frac{L_2Y}{W})H'Y} > 0$$

The impact effect of an increase in  $G'$  will, under these conditions, be to shift the IS curve to the right, raising  $R$  and  $Y$ .

Diagram 2 represents both the short-run and the long-run effects of an increase in  $G'$  from an initial position of long-run equilibrium at  $E_1$ .<sup>19</sup> The new long-run equilibrium is  $E_2$ . The temporary equilibrium after  $G'$  has changed, but before  $D, K, p$  and  $x$  have begun to change, is given by  $S_{12}$ . As drawn, the impact effect on  $Y$  is smaller than the long-run effect.  $V_{G'}$  could equally well exceed  $\frac{1}{\theta}$  however.

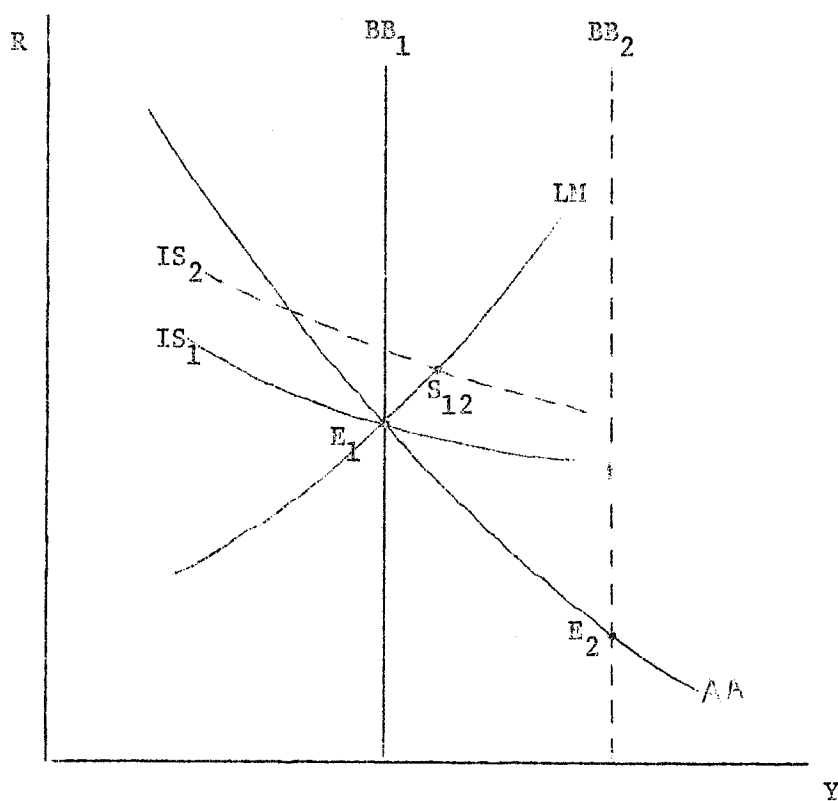


Diagram 2

At  $S_{12}$ , with  $K$  still at the old long-run equilibrium level but with a higher level of  $Y$ , inflation will just be starting (equation (11)). This will generate inflationary expectations (equation (12)).  $R$  and  $Y$  are both higher, so the

<sup>19</sup>For concreteness it is again assumed that the  $AA$  curve is downward sloping.

rate of investment can be either positive or negative. (At  $E_2$   $K$  is higher than at  $E_1$ ; if  $\dot{K}$  is negative initially, this process will have to be reversed somewhere in the adjustment process for the long-run equilibria to be stable.) If  $S_{12}$  is to the left of the  $BB_2$  curve, the government will be running a deficit, which will increase  $D$ . If the long-run output multiplier is less than the short-run multiplier, the government will be running a surplus at  $S_{12}$ .

The dynamic model is obviously too large for meaningful qualitative stability analysis. Only a few general remarks will therefore be made.

### Stability

We can substitute (22a) and (22b) into the dynamic equations for the state variables ( $D, K, p$  and  $x$ ). This gives

$$\dot{D} = p(G' - \theta V(D, K, p, x, G', \theta))$$

$$\dot{K} = I(H(U(D, K, p, x, G', \theta) - x)V(D, K, p, x, G', \theta) - K)$$

$$\dot{p} = p(\Psi(V(D, K, p, x, G', \theta), K) - x)$$

$$\dot{x} = \beta\Psi(V(D, K, p, x, G', \theta), K)$$

The linear approximation of this dynamical system at the long-run equilibrium

$$(D^*, K^*, p^*, x^* = 0, G'^*, \theta^*) \text{ is}$$

$$+ \begin{bmatrix} \dot{D} \\ \dot{K} \\ \dot{p} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} -p\theta V_D & -p\theta V_K & -p\theta V_p & -p\theta V_x \\ I' [H' Y U_D + H V_D] & I' [H' Y U_K + H V_K - 1] & I' [H' Y U_p + H V_p] & I' [H' Y (U_x - 1) + H V_x] \\ p\psi_Y V_D & p(\psi_Y V_K + \psi_K) & p\psi_Y V_p & p(\psi_Y V_x + 1) \\ \beta\psi_Y V_D & \beta(\psi_Y V_K + \psi_K) & \beta\psi_Y V_p & \beta\psi_Y V_x \end{bmatrix} \begin{bmatrix} D - D^* \\ K - K^* \\ p - p^* \\ x - 0 \end{bmatrix}$$

$$+ \begin{bmatrix} -p[\theta V_{G'} - 1] & -p[\theta V_\theta - Y] \\ I' [Y H' U_{G'} + H V_{G'}] & I' [Y H' U_\theta + H V_\theta] \\ p\psi_Y V_{G'} & p\psi_Y V_\theta \\ \beta\psi_Y V_{G'} & \beta\psi_Y V_\theta \end{bmatrix} \begin{bmatrix} G' - G'^* \\ \theta - \theta^* \end{bmatrix} \quad (23)$$

or, more concisely,

$$\dot{Z} = a(Z - Z^*) + b(G - G^*) \quad (24)$$

Z is the vector of state variables (D, K, p, x)'; G is the vector of control variables (G', θ)'. a and b are their respective coefficient matrices. This system will be locally stable if all characteristic roots of the a matrix have negative real parts.<sup>20</sup>

<sup>20</sup>Let  $a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$ ;  $a_0 > 0$  be the characteristic equation of the a matrix. Necessary and sufficient conditions for all roots of this equation to have negative real parts are:  $a_1 > 0$ ;  $a_1 a_2 - a_0 a_3 > 0$ ;  $a_1 a_2 a_3 - a_1^2 a_4 - a_0 a_3^2 > 0$ ;  $a_4 > 0$ .

Without further quantitative information on the structural coefficients we cannot rule out either stability or instability. Qualitative information alone in larger (i.e. more than two-dimensional) models rarely if ever permits one to decide that a model is Q-stable (qualitatively stable). P-stability (potential stability) is all that can be established.<sup>21</sup>

### Controllability

Controllability of a dynamic system refers to the ability of the policy authority to transfer any initial state to any target state in specified finite time through the choice of an appropriate trajectory for its controls.<sup>22</sup>

This would seem to be a more relevant characterization of the options actually open to the policy authority than the more common notion of stability. There is some interest attached to the question as to whether an economic system, after a perturbation of its equilibrium, will ultimately return to the equilibrium or continue to diverge from it with the policy authority rigidly adhering to whatever fixed value it initially assigns to its controls, but the implied inflexibility of policy if the system were to prove unstable appears unduly restrictive.

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<sup>21</sup>The system  $\dot{x} = Ax$  is said to be Q-stable (qualitatively stable) if the solution to  $\dot{x} = Ax$  converges to the equilibrium position, given only the sign pattern of A. If the system converges to the equilibrium for some but not necessarily all numerical values of the coefficients of A consistent with the sign pattern of A, A is said to be P-stable (potentially stable). See Allingham, M.G. and Morishima, M., "Qualitative Economics and Comparative Statics," Theory of Demand, Real and Monetary, M. Morishima et.al., Oxford: Clarendon Press, 1973, pp. 3-69.

<sup>22</sup>See e.g. Padulo, L. and Arbib, M.A., Systems Theory, Philadelphia: W.B. Saunders, 1974; Athans, M. and Falb, P.L., Optimal Control, New York: McGraw Hill, 1966; and Preston, A.J., "A Dynamic Generalization of Tinbergen's Theory of Policy," Review of Economic Studies, Vol. 41(1), no. 125 (January 1974), pp. 65-74.

Since we are considering the linearized version of a highly non-linear system, the controllability properties of the model considered here will be strictly local. Local controllability (in a neighborhood of the equilibrium) refers to the ability of the policy to steer the economy back to the equilibrium, after a perturbation of that equilibrium, through an appropriate choice of time paths for its controls. The importance of this concept for policy-oriented economic models is clearly considerable. In our model controllability answers the question as to whether after an increase in  $G^1$  the policy authority can choose a path for its controls ( $G^1$  and  $\theta$ ) that will bring the economy to the new long-run equilibrium corresponding to the new value of  $G^1$ .

A well-known theorem of control theory states<sup>23</sup> that the non-linear system

$$\dot{x} = f(x,u) \tag{25}$$

(where  $x$  is an  $N \times 1$  vector of state variables and  $u$  is a  $K \times 1$  vector of control variables) with  $f(0,0) = 0$  is locally controllable near the origin if the rank of the matrix  $[A; AB; A^2B, \dots; A^{N-1}B]$  is  $N$ .

$$A = \frac{\partial f}{\partial x} (0,0) \quad (\text{the Jacobian matrix evaluated at the origin})$$

and

$$B = \frac{\partial f}{\partial u} (0,0)$$

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<sup>23</sup> See e.g. Athans and Falb, Op. cit.

Thus the dynamic model will be locally controllable if the rank of the  $4 \times 8$  matrix  $[b; ab; a^2b; a^3b]$  is four. Again this is virtually impossible to establish without more precise quantitative information on the coefficients of the  $a$  and  $b$  matrices. It should be noted however that we have limited the number of controls available to the policy authority perhaps excessively. A somewhat richer tax function, such as  $T = \alpha + \theta(Y + \frac{RD}{p})$ , where  $\alpha$  represents net lump-sum taxes and transfers and authority to set the nominal rate of interest on money balances would provide the policy maker with two additional controls. With both the  $a$  matrix and the  $b$  matrix of full rank (4) the controllability condition will be trivially satisfied. The model may be locally controllable (near an equilibrium) however, even if the number of linearly independent state variables exceeds the number of linearly independent controls.

### Conclusion

When the price level is endogenized and a capacity constraint introduced through a price-Phillips curve there remains a non-zero long-run effect on real output of changes in government spending. Only in the case in which the government-spending parameter is  $G'$  is this effect unambiguous and in the same (expansionary) direction as the short-run effect. The theoretical case for the existence of long-run effects of pure fiscal policy appears to be strong.



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