

ECONOMIC POLICY IN SHORT-RUN MODELS AND IN LONG-RUN  
EQUILIBRIUM; A THEORETICAL FRAMEWORK AND  
SOME APPLICATIONS

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Econometric Research Program  
Research Memorandum No. 192  
December 1975

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## ABSTRACT

The state-variable description of dynamic systems is presented in Section I as a general theoretical framework for economic systems. Short-run comparative statics, long-run comparative statics and comparative dynamics can conveniently be performed in this setting.

Section II applies this theoretical framework to the study of government policy in a full employment growth model extension of the IS-LM model. The short-run and long-run effects of changes in public spending under different assumptions about the financing of the budget deficit or surplus are analyzed. Fiscal policy is shown to have real long-run effects even in a full employment model.

Economic Policy in Short-Run Models and in Long-Run Equilibrium;  
a Theoretical Framework and Some Applications\*

Introduction

Considerable attention has been devoted recently to the study of the long-run effects of fiscal and monetary policy<sup>(1)</sup>. The limitations of the textbook Neo-Keynesian models are inherent in their short-run character: many of the parameters of the short-run model are only exogenous in the short run and become endogenous as time passes. Examples are asset stocks, the size of the labor force and the state of expectations about the future. Often the mechanisms for endogenizing these short-run exogenous but long-run endogenous variables are already specified in the short-run model. The investment function gives the current rate of change of the capital stock. The government has to finance its budget deficit or surplus through some combination of borrowing and money creation. Other long-run endogenous variables, such as expectations, commonly lack behavioral equations in the short-run model.

In this paper I shall present a general framework for modeling the short run-long run dichotomy and the transition from the first to the second. Section I proposes the state-variable

\*This paper is based on parts of my Ph.D. Dissertation Temporary Equilibrium and Long-Run Equilibrium, Yale, 1975. The advice and help of James Tobin, Gary Smith and Katsuhito Iwai are gratefully acknowledged. I have also benefitted from discussions with Gregory Chow and from comments by participants in the Research in Progress Seminar at Princeton.

description of a dynamical system as a suitable representation of short-run and long-run models of the economy. Section II illustrates this general approach with a long-run growth model that extends the full employment IS-LM model. This model will be used to analyze the long-run effects of a number of government policy actions. Pure fiscal policy changes - changes in some parameter of public spending which leave the nominal quantity of money or its rate of change unchanged - will be studied in some detail. An interesting conclusion is that even in full employment models there can be long-run expansionary effects of increases in public spending.

The model is sufficiently flexible to permit the analysis of virtually any combination of fiscal and monetary policy changes. Additional policy combinations considered are changes in government spending with money-financed deficits and surpluses and changes in government spending with a "mixed-financing" policy. In all these cases the long-run government spending multiplier is non-zero. In a number of cases it is clearly positive while in others it is ambiguous in sign.

The models I shall be considering in Section II are similar in certain respects to the money and growth models of the late sixties and early seventies<sup>(2)</sup>. It is not surprising that attempts to generalise the simple single-asset growth models by introducing portfolio choice and attempts to extend the IS-LM model to the long run converge to similar, though by no means identical, models.

## Section I

### The State-Variable Description of Dynamic Economic Systems

It is frequently possible to partition the endogenously determined variables of a dynamic economic model into two categories: short-run endogenous variables and short-run exogenous (predetermined) but long-run endogenous variables. The values of the short-run endogenous variables are determined once the contemporaneous values of the short-run exogenous variables and the other parameters of the model are given.

In the terminology of system theory, the short-run endogenous variables are the outputs of the system and the short-run exogenous but long-run endogenous variables are the state variables<sup>(3,4)</sup>. Let  $y(t)$  denote an  $m \times 1$  vector of outputs,  $x(t)$  an  $n \times 1$  vector of state variables and  $u(t)$  a  $k \times 1$  vector of control variables. The state-variable representation of a dynamical system is, in continuous time,

$$(1a) \quad y(t) = F(x(t), u(t)) \quad (\text{output equation})$$

$$(1b) \quad \dot{x}(t) = G(x(t), u(t)) \quad (\text{state equation})$$

and in discrete time<sup>(5)</sup>

$$(1a') \quad Y_t = F(x_t, u_t)$$

$$(1b') \quad x_t = G(x_{t-1}, u_t) \quad (6)$$

Equation 1a, the output equation, is the reduced form of the short-run model. Equation 1b, the state equation, represents a dynamic reduced form: an system of  $n$  first order non-linear differential equations in normal form<sup>(7)</sup>. The linearised version of the model is, in continuous time.

$$(2a) \quad y(t) = Cx(t) + Du(t)$$

$$(2b) \quad \dot{x}(t) = Ax(t) + Bu(t)$$

and in discrete time

$$(2a') \quad y_t = Cx_t + Du_t$$

$$(2b') \quad x_t = Ax_{t-1} + Bu_t$$

where  $A, B, C$  and  $D$  are respectively,  $n \times n$ ,  $n \times k$ ,  $m \times n$  and  $m \times k$  matrices, which, without loss of generality, will all be assumed to be of full rank.  $C = [c_{ij}]$ ,  $D = [d_{ij}]$ ,  $A = [a_{ij}]$ ,  $B = [b_{ij}]$ .

Given the way in which economic theory suggests the structural equations of our models, we are unlikely to start with a system like 1a and 1b or 2a and 2b. What we are likely to start with is

$$(3) \quad H(y, x, \dot{x}, u) = 0 \quad \text{or}$$

$$(3') \quad H(y_t, x_t, x_{t-1}, u_t) = 0$$

a system of  $n+m$  simultaneous non-linear first-order differential or difference equations in implicit form. By linearizing this system we still may be capable of analysing its local stability and controllability properties in the form given by 2a and 2b.

If the total number of endogenous variables is greater than the number of state variables, the dimensionality of the dynamic system we have to analyse will be less than the total number of endogenous variables, which generally constitutes a considerable simplification.

In what follows I shall concentrate on the linear model (2a and 2b or 2a' and 2b'), which may be the linearized version of a non-linear model. For reasons of space only the continuous time case will be considered.

The short-run model can be solved for the impact or short-run multipliers: the effect on the current values of the short-run endogenous variables of changes in the current values of the state variables or the controls. This will be called short-run comparative statics.

From equation 2a

$$y(t) = Cx(t) + Du(t)$$

we get

$$\frac{\partial y_i(t)}{\partial x_j(t)} = c_{ij} \quad \begin{array}{l} i = 1 \dots m \\ j = 1 \dots n. \end{array}$$

and

$$\frac{\partial y_i(t)}{\partial u_j(t)} = d_{ij} \quad \begin{array}{l} i = 1 \dots m \\ j = 1 \dots k. \end{array}$$

A long-run equilibrium model is a short-run model that is invariant over time.  $y(t) = F(x(t), u(t))$  is a long-run equilibrium if both the state variables and the controls are constant over time. Assuming  $u(t)$  to be constant a long-run equilibrium requires the state to be an equilibrium state, ie.

$$(4) \quad 0 = G(x, u)$$

or

$$(5) \quad 0 = Ax + Bu$$

Equations 1a and 4 or 2a and 5 give us the long-run or steady state multipliers. This will be called long-run comparative statics. In the linear case the steady-state effects of changes in the control variables on the state variables are given by

$$\frac{dx_i}{du_j} = \alpha_{ij} \quad \begin{array}{l} i = 1 \dots n \\ j = 1 \dots k \end{array}$$

where  $\alpha_{ij}$  is the  $ij$ th element of  $-A^{-1}B$ . The steady-state effects of changes in the control variables on the output variables are in the linear case given by

$$\frac{dy_i}{du_j} = \beta_{ij} \quad \begin{array}{l} i = 1 \dots m \\ j = 1 \dots k \end{array}$$



where  $\beta_{ij}$  is the  $ij$ th element of  $-CA^{-1}B + D$ .

The "real-time multipliers", giving the effects of changes in the values of the control variables at  $t = t_0$  on the values of the state variables and the outputs at  $t = t_1 > t_0$ , can be found by what will be called comparative dynamics. The state equation is integrated forward from  $t_0$  till  $t_1$  for the original values of the controls and for the new values of the controls at  $t = t_0$ . We can then compare  $x(t_1)$  and the outputs corresponding to  $x(t_1)$  on these two trajectories. The same procedure permits one to derive the "cumulative real-time multipliers", by comparing two trajectories with different values for  $u(t)$  during a time interval  $t^* < t < t^{**}$ . The system  $\dot{x} = Ax + Bu$  possesses the general solution

$$x(t) = e^{At} x(0) + \int_0^t e^{A(t-v)} Bu(v) dv.$$

For a constant  $u$ ,  $x(t)$  will converge to a long-run equilibrium  $x = -A^{-1}Bu$  i.f.f. all the eigenvalues of  $A$  have negative real parts.

Two Examples

The model analyzed in Section II is a sequential temporary (or momentary) equilibrium model based on the full employment version of the IS-LM model. In each period (at each point in time) the outputs: the price level,  $p$ , the interest rate,  $R$ , and the money wage rate,  $\omega$ , take on the values required to clear the commodity market, the bond market and the labor market. The parameters of the temporary equilibrium are (1) the state variables: the percapita stock of money balances,  $\frac{M}{N}$ , the percapita stock of bonds  $\frac{B}{N}$ , the capital-labor ratio,  $\frac{K}{N} = k$ , and the expected rate of inflation,  $x$ , and (2) the control variables: the level of government spending  $g, g'$  or  $v$ , the tax rate,  $\theta$ , and a deficit financing parameter,  $\delta$ , or an open market parameter  $\phi$ . (8)

It is important to realize, however, that the state variable representation of dynamic economic models is not limited to the representation of temporary equilibrium models or market-clearing models. The short-run model represents the economy at a point in time. The situation it describes may be a non-market clearing situation. An example of this would be a Barro and Grossman<sup>(9)</sup> version of the IS-LM model. Neither the labor market nor the commodity market is cleared. The money wage rate and the money price level are parametric in the short run. Households are income-constrained in their consumption spending because there is excess supply in the labor market. Firms are sales-

constrained in the commodity market. Assume for simplicity that the economic system either stays in this excess supply of labor and excess supply of output regime or reaches full equilibrium, but never overshoots into one of the other possible market disequilibrium regimes. In this market disequilibrium model the outputs are the actual production of commodities, the interest rate, and the employment or unemployment of labor. The state variables are not only the asset stocks and expectations but also the money price level and the money wage rate.

No Outputs

It is quite possible for there to be no short-run endogenous variables in the model. The number of endogenous variables may be equal to the number of state variables. The simplest example of this case is the "IS-LM disequilibrium model." Interest rate and output do not instantaneously assume the values required to equate the demand for and supply of money balances and planned saving and investment. In that model  $R$  and  $Y$  become state variables that adjust to the excess demand for money balances and the excess of planned investment over planned saving. The short-run market disequilibrium adjustment processes and the long-run adjustment of stocks (and expectations) have to be analysed simultaneously with consideration of their interdependence.

Elsewhere I have used this analytical framework to investigate whether it is possible to abstract from the slow-moving adjustment processes if one is interested in the quicker market adjustment processes<sup>(10.)</sup> By partitioning the state variables into slow-moving variables ( $x_2$ ) and quickly moving variables ( $x_1$ ) and by partitioning the  $A$  and  $B$  matrices conformably, the complete model can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} U$$

The elements of  $A_{11}$  and  $A_{12}$  are large relative to the elements of  $A_{21}$  and  $A_{22}$ . The issue is whether the behavior of  $x_1$  can be satisfactorily approximated by a system which has  $A_{21}$  and  $A_{22}$  (and  $B_2$ ) replaced by zero matrices.

### Stock Equilibrium and Flow Equilibrium

In our framework a long-run equilibrium is an invariant sequence of short runs, and a long-run equilibrium is fully characterized by the equilibrium state. The economic entities representing the state variables can be a very heterogeneous bunch: expectations, stocks, flows and many others.

When stocks and flows constitute some of the state variables invariance of stocks and flows over time is not quite specific enough. What we shall usually be referring to is invariance of real per-capita stocks and flows (or even, if there is labor-augmenting technical change, real "per-efficiency unit of labor" stocks and flows).

In the literature there has been a tendency to associate long-run equilibria with stock equilibria and short-run equilibria (short-run models representing a temporary or momentary equilibrium) with flow equilibria. This would seem a confusing use of terminology. In a temporary equilibrium model (or market-clearing model), equilibrium in asset markets can be characterized either as a flow equilibrium or as an instantaneous stock equil-

ilibrium<sup>(11)</sup>. It is common to interpret the simple IS-LM model as a mixed stock-flow short-run equilibrium. Sometimes one of the properties of a long-run equilibrium is that stocks are constant. Other long-run equilibrium conditions such as realization (or at least non-revision) of expectations can hardly be characterized as stock equilibrium conditions. The steady states of growing economic systems are not characterised by constant stocks but by constant real stock-flow or stock-stock ratios i.e. by constant real per capita stocks.

## Section II

Government Policy in a Full EmploymentGrowth Model Extension of the IS-LM ModelNOTATION

- M : nominal stock of money balances.
- B : nominal stock of government interest-bearing debt.  
Bonds are fixed nominal face value, variable interest rate claims.
- K : stock of real reproducible capital
- $k \equiv K/N$
- N : size of the labor force.
- $\lambda \equiv \frac{M+B}{NP}$  : real per-capita government debt
- $\Delta \equiv M+B$  : nominal government debt
- $w \equiv \frac{M+B}{NP} + qK$  : real per capita non-human wealth
- q : valuation ratio.
- $n = \dot{N}/N$  .
- p : money price level of commodities.
- R : nominal rate of interest on bonds.
- x : instantaneous expected proportional rate of change of p.
- y : real per capita output.
- s : real per capita saving.
- i : real per capita investment.
- $\theta$  : constant proportional tax rate on factor income and interest income.

- $g$  : real per capita government consumption spending.  
 $g' \equiv g + (1-\theta)\frac{RB}{NP}$  : real per capita government spending on consumption and net-of-tax debt service.  
 $D$  : nominal government deficit.  $D \equiv Np(g + (1-\theta)\frac{RB}{NP} - \theta f(k))$   
 $\ell$  : fraction of wealth held in money balances.  
 $\delta$  : fraction of the current deficit (surplus) financed by creating (withdrawing) money.  $\dot{M} = \delta D$   
 $\phi$  : fraction of total public debt consisting of money balances  
 $\phi \equiv \frac{M}{M+B} = \frac{M}{\Delta}$   
 $v$  : instantaneous proportional rate of growth of  $\Delta$ .

In what follows I shall combine the IS-LM equations with a neoclassical production function and labor market. The supply of labor is strictly proportional to the size of the labor force. A perfectly flexible money wage equilibrates the labor market at each point in time. The money wage will be suppressed in the analysis, but we can always solve for it by finding the money value of the marginal product of labor. A perfectly flexible money price level clears the output market at each point of time. In the short run per capita output is fixed by the historically determined capital-labor ratio. Expansionary fiscal policy, with a constant nominal money stock will raise the interest rate. Unless the LM curve is vertical the price level will also be in-



creased. In the familiar  $p$ - $y$  diagram, the equilibrium price level is determined by the intersection of the vertical "aggregate supply curve" and the downward-sloping "aggregate demand curve", the locus of IS-LM equilibria in  $p$ - $y$  space.

This demand curve will shift to the right when government spending increases, raising  $p$  (and  $R$ ). Increased government spending "crowds out" private investment and private consumption to the extent required to equate total real demand to the fixed real supply. This is but the impact effect, however. Capital accumulation, deficit financing and changing expectations have to be allowed for it if we want to trace the long-run effects.

The effect of government spending on real per capita income in the long run comes from the endogeneity of the capital-labor ratio. There are no idle resources whose underutilization can be eliminated by government spending and induced changes in private sector effective demand. In the long run per capita output in the full employment model is endogenous because the capital-labor ratio can vary and with it output per worker.

#### The Model (12)

The per capita demand for real money balances is given by

$$l(R(1 - \theta), \frac{Y}{W})^w ; l_1 < 0; l_2 > 0; l_2 \frac{Y}{W} < l.$$

The fraction of wealth held in the form of real money balances is a decreasing function of the after-tax real rate of return differential between "other assets" and money and an increasing function of per capita income. The short-run income elasticity of demand for money balances is less than unity. Bonds and existing capital are perfect substitutes in private portfolios. The nominal rate of return on money balances is institutionally fixed at zero. Portfolio balance is given by:

$$\rho(R(1 - \theta), \frac{Y}{w})w = \frac{M}{P}$$

Non-human private wealth is the real value of the stocks of bonds and money and the market value of claims to the stock of capital.

$$w = \frac{M+B}{NP} + qK.$$

$q$  is the valuation ratio, the ratio of the market value of the claims to the stock of capital to the value of the capital stock at current reproduction costs or, equivalently, the ratio of the rate of return on a dollar invested in the production of new capital goods to the rate of return on a dollar invested in existing capital goods. In long-run equilibrium  $q = 1$ . Divergence of  $q$  from 1 encourages investment ( $q > 1$ ) or discourages it ( $q < 1$ ).

The production function is a well-behaved constant returns to scale neoclassical production function in capital and labor.

$$y = f(k); \quad f' > 0; \quad f'' < 0.$$

With static expectations about future changes in  $q$  the valuation ratio can now be expressed as:

$$q = \frac{f'(k)}{R - x}$$

The investment and saving functions are of the stock adjustment type. The rate of investment is an increasing function of  $q$ . The per capita investment function is

$$i = i(f'(k) - R + x); \quad i' > 0; \quad i(0) = nk$$

When the actual and desired capital-labor ratios coincide, firms invest at the rate required to maintain that capital-labor ratio.

The per capita saving function is:

$$s = s(\hat{\mu}f(k) - \frac{(M+B)}{Np} - qk); \quad s' > 0; \quad s(0) = nw = n(\frac{M+B}{Np} + qk)$$

$\hat{\mu}f(k)$  is target wealth, a multiple  $\mu(1-\theta) = \hat{\mu}$  of after-tax factor income. When desired per capita wealth equals actual per capita wealth, households save at the rate required to maintain that level of per capita wealth.

Three simple price expectations mechanisms are considered:

- (a) Static expectations:  $x = \text{some constant (possibly 0)}$
- (b) Short-run or myopic perfect foresight:  $x = \frac{\dot{p}}{p}$
- (c) Adaptive expectations:  $\dot{x} = \beta(\frac{\dot{p}}{p} - x); \beta > 0$

A number of different government policy combinations will be considered. Whatever the policy under consideration, the government budget identity will hold:

$$g + (1 - \theta)\frac{RB}{Np} - \theta f(k) = \frac{D}{Np} = \frac{\dot{M}}{pN} + \frac{\dot{B}}{pN}$$

Policy I has the government varying  $g$ , public consumption expenditure. In Policy II the government varies  $g' \equiv g + (1 - \theta)\frac{RB}{Np}$ : any endogenous change in the real value of after-tax debt service will be offset by a corresponding change in  $g$ .

Thus, for Policy I, we can rewrite the government budget identity as

$$\frac{\dot{M}}{pN} = \delta[g + (1 - \theta)\frac{RB}{Np} - \theta f(k)] \quad \text{and}$$

$$\frac{\dot{B}}{pN} = (1 - \delta)[g + (1 - \theta)\frac{RB}{Np} - \theta f(k)]$$

while for Policy II this can be written as

$$\frac{\dot{M}}{pN} = \delta(g' - \theta f(k)) \quad \text{and}$$

$$\frac{\dot{B}}{Np} = (1 - \delta)(g' - \theta f(k))$$

Pure bond-financed deficits or surpluses are the case in which  $\delta = 0$ . Pure money-financed deficits or surpluses are the case in which  $\delta = 1$ .

We shall also consider a financing rule that requires the use of both bonds and money to finance deficits. The simple case analysed here has a constant money-bond ratio.

$$\begin{aligned} M &= \phi \Delta & 0 < \phi < 1 & \quad (13) \\ B &= (1 - \phi) \Delta \end{aligned}$$

Policy III (the "v policy") has the government fixing  $v$ , the proportional rate of growth of the nominal quantity of public debt,  $\Delta$ , and therefore, since  $\phi$  is constant, of both the nominal quantity of money and the nominal quantity of bonds. From the government budget identity we find that

$$g + (1 - \theta) \frac{RB}{Np} - \theta f(k) = \frac{D}{Np} = v \frac{\Delta}{Np} = v \lambda .$$

With  $v$  and  $\theta$  determined by the policy maker,  $g$  is endogenously determined. Policy IV again makes  $g'$  the parameter of public spending. Given  $\phi$  and  $\theta$ , the common rate of change of both nominal assets,  $v$ , is determined endogenously. For reasons

of space variations of  $g$  or  $\theta$  with  $v$  determined endogenously will not be considered.

Short-run and Long-run Effects of Changes in Government Spending  
with Policies I and II

The model, under Policies I and II is, given in equations 6 - 11.

$$6.I \quad i(f'(k)-R+x) + g + (1-\theta)\frac{RB}{Np} - \theta f(k) - s(\hat{\mu}f(k) - \frac{(M+B)}{Np} - \frac{f'(k)}{R-x}k) - x\frac{(M+B)}{Np} = 0 \quad IS(I)$$

$$6.II \quad i(f'(k)-R+x) + g' - \theta f(k) - s(\hat{\mu}f(k) - \frac{(M+B)}{Np} - \frac{f'(k)}{R-x}k) - x\frac{(M+B)}{Np} = 0 \quad IS(II)$$

$$7 \quad \ell\left(R(1-\theta), \frac{f(k)}{\frac{M+B}{Np} + \frac{f'(k)k}{R-x}}\right) \left(\frac{M+B}{Np} + \frac{f'(k)k}{R-x}\right) = \frac{M}{Np} \quad LM$$

$$8.I \quad \frac{d}{dt}\left(\frac{B}{Np}\right) = (1-\delta)[g + (1-\theta)\frac{RB}{Np} - \theta f(k)] - \left(\frac{\dot{p}}{p} + n\right)\frac{B}{Np}$$

$$8.II \quad \frac{d}{dt}\left(\frac{B}{Np}\right) = (1-\delta)[g' - \theta f(k)] - \left(\frac{\dot{p}}{p} + n\right)\frac{B}{Np}$$

$$9.I \quad \frac{d}{dt}\left(\frac{M}{Np}\right) = \delta[g + (1-\theta)\frac{RB}{Np} - \theta f(k)] - \left(\frac{\dot{p}}{p} + n\right)\frac{M}{Np}$$

$$9.II \quad \frac{d}{dt}\left(\frac{M}{Np}\right) = \delta[g' - \theta f(k)] - \left(\frac{\dot{p}}{p} + n\right)\frac{M}{Np}$$

$$10 \quad \dot{k} = i(f'(k) - R+x) - nk$$

$$11 \quad x = -n \quad \text{or}$$

$$11' \quad x = \frac{\dot{p}}{p} \quad \text{or}$$

$$11'' \quad \dot{x} = \beta\left(\frac{\dot{p}}{p} - x\right)$$

### Bond-financed deficits and surpluses

With purely bond-financed deficits and surpluses  $\delta = 0$ . Static expectations will be defined as  $x = -n$  so that in long-run equilibrium expectations can be realised. Policy II will be dealt with first.

### Long-run comparative statics with Policy II

The steady state values of  $R, x, k, \frac{B}{Np}$  and  $\frac{M}{Np}$  (or, since  $M$  is given exogenously, of  $R, Np, x, k$  and  $B$ ) are found by setting equations 8.II, 9.II, and 10 equal to zero and taking either 11 or 11' or setting 11'' equal to zero. This plus the momentary equilibrium conditions (IS(II) and LM) gives the five steady conditions:

$$12a \quad \ell(R(1-\theta), \frac{1}{\hat{\mu}})\hat{\mu}f(k) = \frac{M}{Np} \quad (\text{LLM})$$

$$12b \quad g' - \theta f(k) = 0 \quad (GT)$$

$$12c \quad \frac{M}{Np} + \frac{B}{Np} + k = \hat{\mu} f(k)$$

$$12d \quad f'(k) = R - x$$

$$12e \quad x = -n \left( = \frac{\dot{p}}{p} \right)$$

12b is the balanced budget condition. When the government has two nominal debt instruments and uses just one of these to finance deficits or surpluses, the only possible steady states are balanced budget steady states. The real value of both nominal assets will grow at the natural rate of growth  $n$  because of a steady proportional rate of deflation.

We can represent both the short-run and the long-run equilibrium in  $R - Np$  space. Figure 1 represents the long-run equilibrium. We can solve 12d for  $k$  as a function of  $R$ :

$$k = h(R); \quad h' = \frac{1}{f''} < 0.$$

Substituting this into 12a and 12b we get

$$13a \quad \varrho(R(1-\theta), \frac{1}{\hat{\mu}}) \hat{\mu} f(h(R)) = \frac{M}{NP} \quad (LLM)$$

$$13b \quad g' - \theta f(h(R)) = 0 \quad (GT)$$



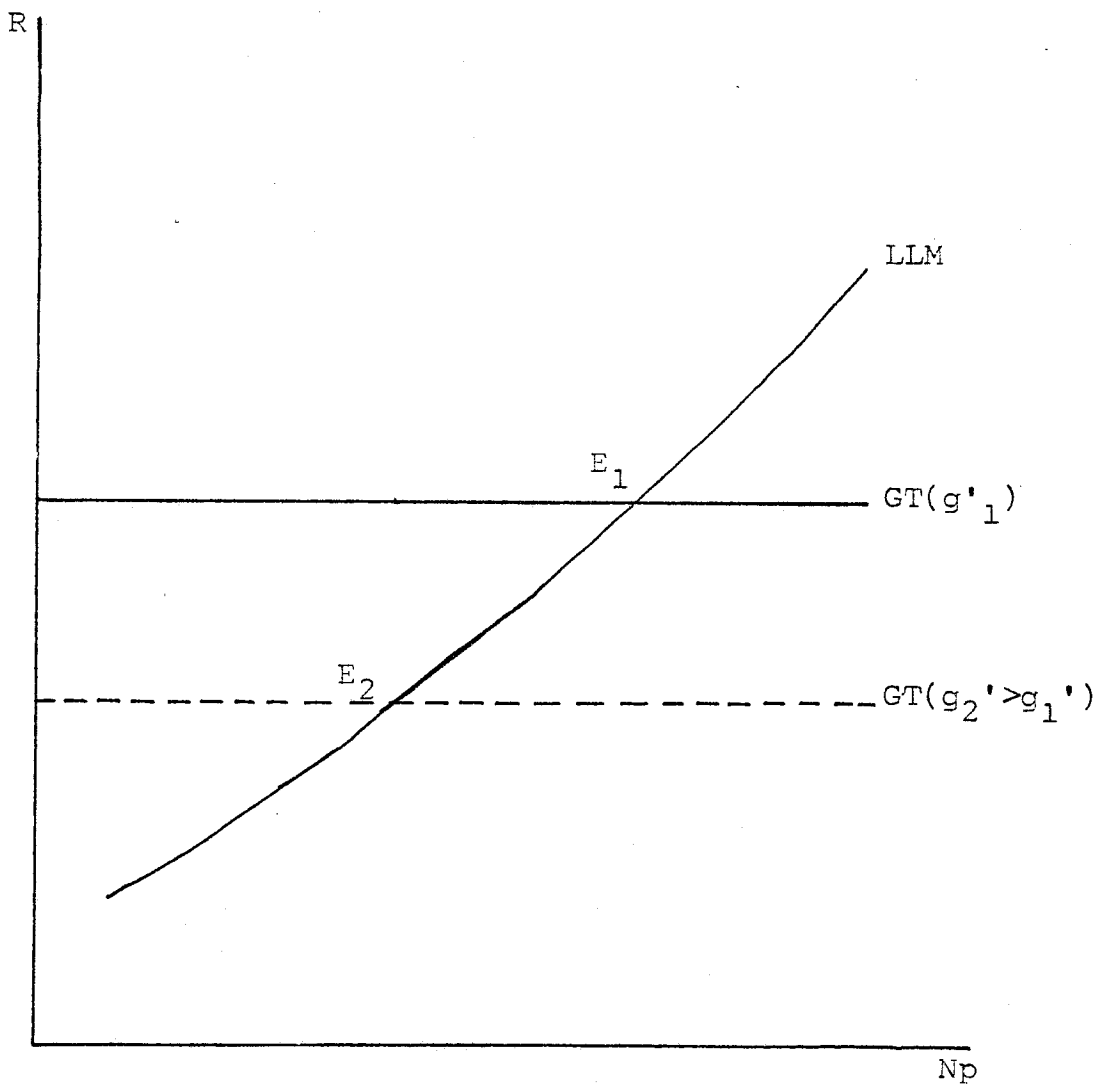


Figure 1

The slope of the LLM curve is

$$\left. \frac{dR}{dNp} \right|_{LLM} = \frac{-Mf''}{(\hat{\mu}f\ell_1(1-\theta)f'' + \ell\hat{\mu}f')(Np)^2} > 0$$

The slope of the GT curve is

$$\left. \frac{dR}{dNp} \right|_{GT} = 0$$

An increase in  $g'$  shifts the GT curve down along the LLM curve in Figure 1, lowering the steady state values of  $R$  and  $Np$ . Algebraically:

$$\frac{dR}{dg'} = \frac{f''}{\theta f'} < 0$$

$$\frac{dNp}{dg'} = \frac{-(\ell_1\hat{\mu}ff''(1-\theta) + \ell\hat{\mu}f')}{\theta f' \frac{M}{(Np)^2}} < 0$$

Across steady states,  $R$  and  $k$  are inversely related. Therefore, even in the full employment model  $\frac{dy}{dg'} > 0$ . The reason here is capital deepening rather than the elimination of unemployment of labor.

Note also that the power of fiscal policy in the long run depends only on the balanced budget condition and the specification of the tax function. As long as government bonds affect

private behavior to any extent (not necessarily through the net worth effect we have postulated), the balanced budget condition is part of the long-run equilibrium conditions and the long-run effectiveness of pure fiscal policy is guaranteed.

The intuitive story behind the long-run comparative statics is the following: an increase in  $g'$  requires, given  $\theta$ , a higher level of real per capita factor income to balance the budget. With full employment this implies a higher capital labor ratio. This in turn requires a lower  $R$  and a higher level of per capita wealth. The LLM curve indicates that both these effects will increase the per capita demand for real money balances. With the nominal stock of money balances fixed by assumption, a lower  $Np$  is required to raise the real value of the stock of money balances. In the new steady state as in the old,  $N$  will be growing at a rate  $n$  and  $p$  will be falling at that same rate:

$$N(t)p(t) = N(0)e^{nt}p(0)e^{-nt} = N(0)p(0).$$
 With  $N(0)$  historically determined, a lower  $N(0)p(0)$  requires a lower  $p(0)$  in a steady state with a higher level of  $g'$ .

Short-run comparative statics with Policy II

Figure 2 shows the impact effect and long-run effect of an increase in  $g'$ . The slope of the IS curve is

$$\left. \frac{dR}{dNp} \right|_{IS} = \frac{-(s'-x) \cdot \frac{(M+B)}{(Np)^2}}{i' + \frac{s'f'(k)k}{(R-x)^2}}$$

When we evaluate this at a long-run equilibrium ( $x = -n$  and  $q = \frac{f'(k)}{R-x} = 1$ ) we get

$$\left. \frac{dR}{dNp} \right|_{IS} = \frac{-(s'+n)(M+B)/(Np)^2}{i' + \frac{s'k}{R+n}} < 0$$

The impact effect of an increase in  $g'$  will be to shift the IS curve up and to the right; algebraically:

$$\frac{\partial R}{\partial g'} = \frac{1}{i' + \frac{s'k}{R+n}} > 0.$$

The slope of the LM curve is

$$\left. \frac{dR}{dNp} \right|_{LM} = \frac{-\frac{\ell_2 f(k)}{NP} \left(1 - \frac{k}{w}\right) - \frac{Mk}{w(Np)^2}}{w\ell_1(1-\theta) + \left(\frac{\ell_2 f(k)}{w} - \ell\right) \frac{f'(k)k}{(R-x)^2}}$$

Evaluated at a long-run equilibrium this becomes

$$\left. \frac{dR}{dNp} \right|_{LM} = \frac{-\frac{\ell_2 f(k)}{Np} \left(1 - \frac{k}{w}\right) - \frac{Mk}{w(Np)^2}}{w\ell_1(1-\theta) + \left(\frac{\ell_2 f(k)}{w} - \ell\right) \frac{k}{R+n}} > 0.$$

The impact effect of an increase in  $g'$ , given  $M, B, k, x, \theta$  and the new higher level of  $g'$  is a new momentary equilibrium, shown in Figure 2 at  $S_{12}$  with higher  $R$  and  $Np$ . (The higher  $Np$  represents,

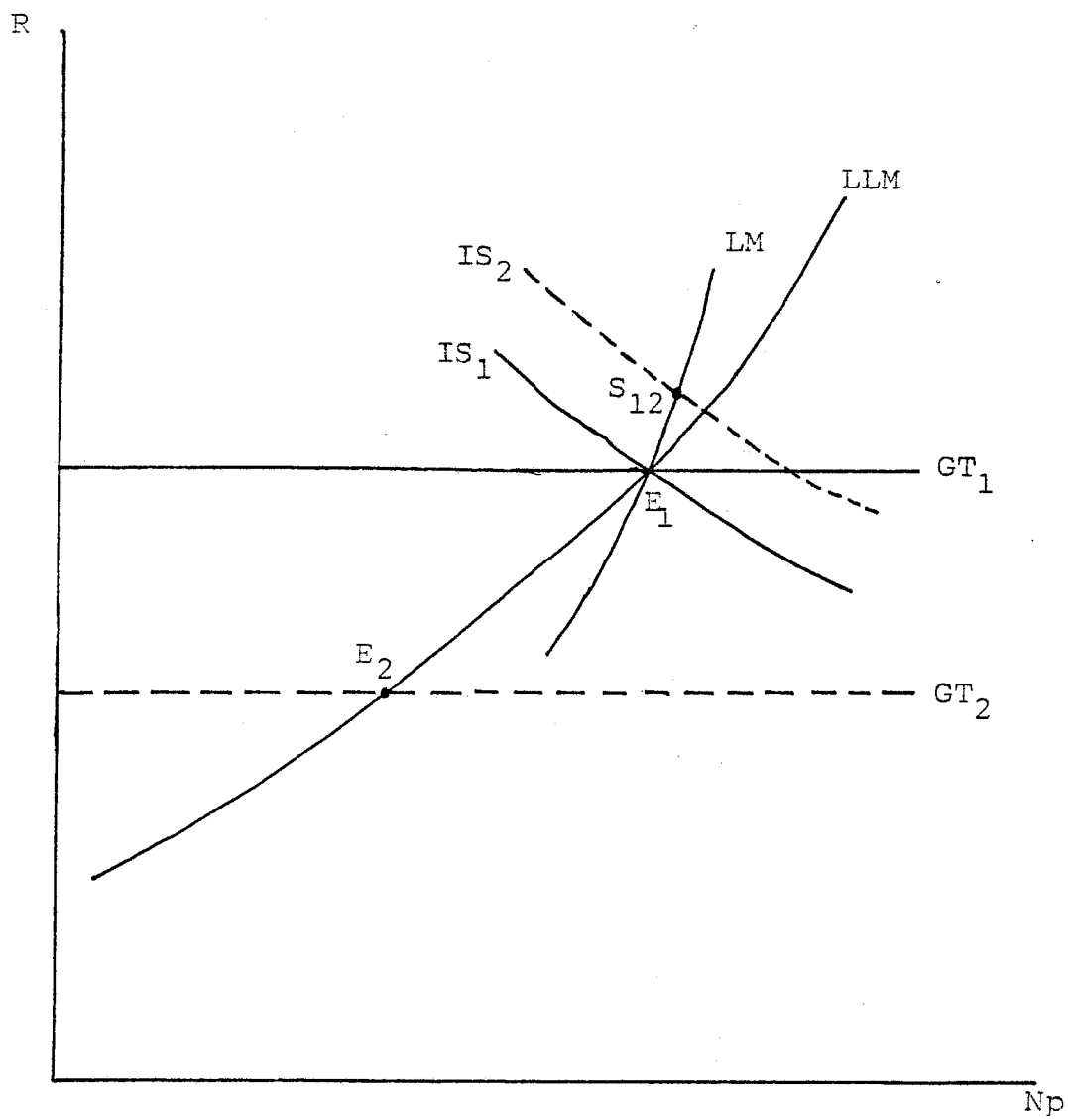


Figure 2

in the short run with  $N$  fixed a higher  $p$ .) Since  $S_{12}$  is not a long-run equilibrium the rates of change of the state variables ( $k$ ,  $\frac{B}{Np}$ ,  $\frac{M}{Np}$  and  $x$ ) at  $S_{12}$  will begin to diverge from their zero steady state rates of change.

Since the government is running a deficit at  $S_{12}$ ,  $\frac{B}{Np}$  will be increasing. The required rate of return on investment has been raised and  $q$  is less than one, causing  $k$  to start falling. Except in the case of static expectations the increase in the price level will be increasing the expected rate of inflation. The new long-run equilibrium, however, is at  $E_2$  with lower  $R$  and lower  $Np$  than at  $E_1$ . The impact effect of an increase in  $g'$  is in the opposite direction from the long-run effect. Prima facie this would seem to suggest instability. For the long-run equilibrium to be stable the impact effects have to be reversed. The economic system has to "back-track" and return beyond its original equilibrium. Nevertheless, under certain circumstances which depend crucially on the expectations mechanism the equilibrium may be stable.

#### Stability:

The mathematics of the local stability conditions is long and tedious. (14) To summarize, the long-run equilibrium is unstable in the case of static expectations and potentially, but not necessarily, stable in the cases of myopic perfect foresight and adaptive expectations. In the "intermediate" case of adaptive expectations, stability requires a certain mini-

imum speed of adjustment of expectations:  $\beta$ .

These results may seem paradoxical. Usually static expectations are considered stabilizing while quick translation of actual price experience into expectations is considered destabilizing. The opposite conclusion here is related to the difference in direction between short-run and long-run effects as shown in Figure 2. The direction of the initial trajectory of the economic system has to be reversed for the long-run equilibrium to be stable. The source of the potential instability is the endogeneity of  $K$  (or  $k$ ) and  $B$ . The new long-run equilibrium,  $E_2$ , requires a higher  $k$  and a lower  $B$  (since  $N_p$  will also be lower, the long-run effect on  $\frac{B}{N_p}$  is ambiguous). Yet at a point like  $S_{12}$  in Figure 2,  $k$  is decreasing and  $B$  is increasing. The decline in  $k$  further accentuates the increase in  $B$  as real tax revenues decline. For the system to be stable in spite of this initial movement away from the new long-run equilibrium 2 conditions must be satisfied. First  $q$  has to be raised above 1 to achieve an above steady state rate of investment. By itself a declining capital-labor ratio, which raises  $f'(k)$ , is favorable to investment but the increasing volume of debt, by raising the rate of interest, works in the wrong direction. What permits the economic system to reduce the required real rate of return by enough to achieve an above steady state rate of investment is inflationary expectations which reduce the real required rate of return corresponding to any given nominal rate. With static expectations

this effect is absent and the initial direction of movement cannot be reversed. The very tight relationship between a smaller  $k$  and an increasing volume of debt through the government financing requirement:  $\frac{\dot{B}}{Np} = g' - \theta f(k)$  is an important source of potential instability. With Policy I this link is a lot looser and stability is more easily obtained:  $\frac{\dot{B}}{Np} = g + (1-\theta)\frac{RB}{Np} - \theta f(k)$ .

Second, ultimately the role of the expected rate of inflation in lowering the real rate of return (and in offsetting the decline in the marginal product of capital if the initial direction of movement has been reversed and  $k$  is increasing) has to be taken over by a lower nominal rate of return, since in the new long-run equilibrium the actual and expected rates of inflation both have to be the same as what they were in the old equilibrium. The expected rate of inflation has three effects on aggregate demand. The first is to increase investment by lowering the real rate of interest. The second is to reduce saving by raising  $q$  and thereby increasing wealth. The third is to reduce real income through expected capital losses on money and bonds. A necessary condition for stability is that the first two effects dominate. This means that if the actual and expected rates of inflation slow down, aggregate demand and the income-related demand for money will weaken. A weakening of aggregate demand leads, in a economy with flexible prices, to a reduction in the price level and the rate of interest, thus permitting the increase



in the capital stock and real income to continue.

Unfortunately, the issue of stability hinges on relatively minor details of specification and on small differences in values of coefficients. Under myopic perfect foresight and adaptive expectations it is possible for the economy to "change direction" after its initial movement away from long-run equilibrium and to converge to the new equilibrium with a lower interest rate, lower stock of nominal debt, higher capital-labor ratio and higher real per capita income. It is also possible for fiscal expansion to set off an unstable spiral of deficits, a rising interest rate and a declining capital-labor ratio.

#### Long-run comparative statics with Policy I

Very briefly I shall now review the effects of pure fiscal policy in a full employment model when  $g$  rather than  $g'$  is the parameter of government spending. The model is given by equations 6.I, 7, 8.I, 9.I, 10 and one of 11, 11' and 11'' with  $\delta = 0$ .

The steady state equations determining the long-run equilibrium values of  $r, k, x, \frac{B}{Np}$  and  $\frac{M}{Np}$  are

$$14a \quad \ell(R(1-\theta), \frac{1}{\hat{\mu}} \hat{\mu} f(k)) = \frac{M}{Np} \quad (\text{LLM})$$

$$14b \quad g + (1-\theta) \frac{RB}{Np} - \theta f(k) = 0 \quad (\text{GT})$$

$$14c \quad \frac{M}{Np} + \frac{B}{Np} + k = \hat{\mu} f(k)$$

$$14d \quad f'(k) = R - x$$

$$14e \quad x = -n \left( = \frac{\dot{p}}{p} \right)$$

The LLM and GT curves can again be used to represent long-run equilibrium in R-Np space

$$15a \quad \ell(R(1-\theta), \frac{1}{\hat{\mu}} \hat{\mu} f(h(R))) = \frac{M}{Np} \quad (\text{LLM})$$

$$15b \quad g + (1-\theta)R(\hat{\mu} f h(R)) - \frac{M}{Np} - h(R) - \theta f(h(R)) = 0 \quad (\text{GT})$$

The slope of the LLM curve is the same as under Policy II

$$\left. \frac{dR}{dNp} \right|_{\text{LLM}} = \frac{-Mf''}{(\hat{\mu} f \ell_1 (1-\theta) f'' + \ell \hat{\mu} f')(Np)^2} > 0$$

The slope of the GT curve is

$$\left. \frac{dR}{dNp} \right|_{\text{GT}} = \frac{-(1-\theta)RM/(Np)^2}{(1-\theta) \left[ \frac{B}{Np} + \frac{R}{f''} (\hat{\mu} f' - 1) \right] - \frac{\theta f'}{f''}}$$

A sufficient but not necessary condition for the GT curve to be downward sloping is  $\hat{\mu} f' < 1$ . Another sufficient condition for the GT curve to be downward sloping is for  $\theta$  to exceed  $(1-\theta)R\hat{\mu}$ . If the GT curve is downward sloping (as drawn in Figure 3), an increase in  $g$  has the long-run effect of lowering  $Np$  and the rate of interest (and thus of raising  $k$  and  $y$ ), as under Policy II.

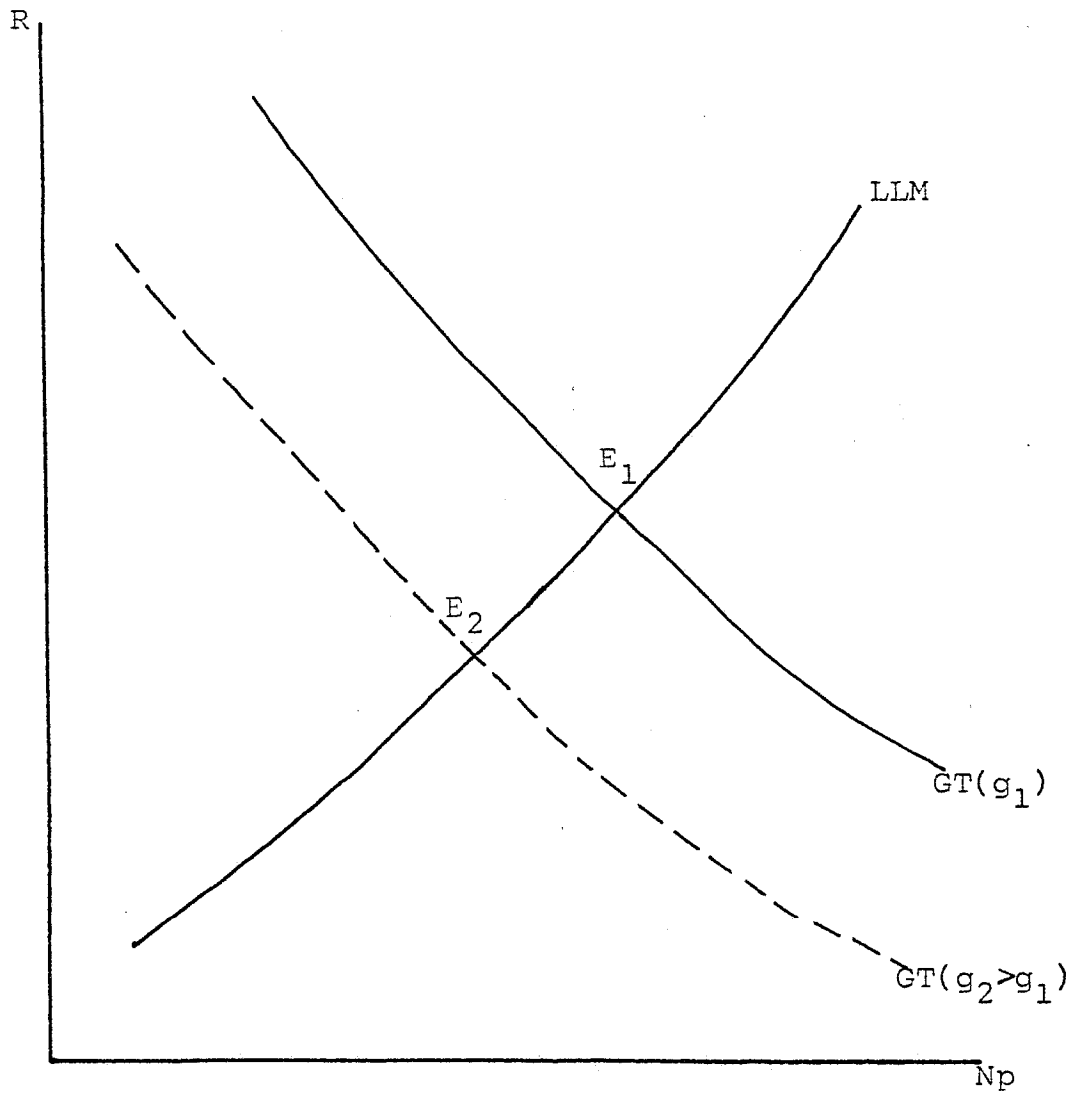


Figure 3

Short-run comparative statics with Policy I

The impact effects of an increase in  $g$  are not quite as unambiguous as those of an increase in  $g'$  because of an ambiguity in the slope of the IS curve. The slope of the LM curve (evaluated at a long-run equilibrium) is as under Policy II.

$$\left. \frac{dR}{dNp} \right|_{LM} = \frac{-\frac{l_2^f}{Np} \frac{(1-k)}{w} - \frac{Mk}{w(Np)^2}}{wl_1(1-\theta) + \left( \frac{l_2^f}{w} - l \right) \frac{k}{R+n}} > 0$$

The slope of the IS curve, evaluated at the long-run equilibrium is

$$\left. \frac{dR}{dNp} \right|_{IS} = \frac{- \left[ (1-\theta) \frac{RB}{(Np)^2} + (s'+n) \frac{(M+B)}{(Np)^2} \right]}{i' + s' \frac{k}{R+n} - (1-\theta) \frac{B}{Np}}$$

If the volume of bonds is large enough, the IS curve may be upward sloping. Through the Pigou effect a higher price level tends to cause excess supply in the commodity market. To bring this market back to equilibrium a higher rate of interest may be required if the value of bond debt is so large that the expansionary effect of the increased debt service associated with a higher  $R$  more than offsets its contractionary effects through reduced investment and increased saving. The impact effect on  $R$  and  $Np$  of an increase in  $g$  depends in a well-known manner on the relative slopes of the IS and LM curves. If the IS curve is downward sloping, both  $R$  and  $Np$  are increased. (Figure 4a) If the IS

curve is upward sloping but cuts the LM curve from above, both  $R$  and  $N_p$  are lowered (Figure 4b). If the IS curve is upward sloping and cuts the LM curve from below, both  $R$  and  $N_p$  are raised (Figure 4c).

In cases (a) and (c) there is again a marked contrast between impact effects and long-run effects. However, in this policy regime, the system is potentially stable even under static expectations because in the new steady state an increase in government spending does not have to be financed exclusively by higher taxes on factor income. Anything that reduces the real value of government debt service (such as a higher  $p$ , a lower  $R$  or a lower  $B$ ) will help to balance the budget.

In conclusion, the theoretical case for the existence of long-run effects of pure fiscal policy changes appears to be very strong, even in a full employment model. In the case of Policy II, the long-run public spending multiplier is equal to the reciprocal of the marginal tax rate -- by virtue of the balanced budget condition -- and the quantitative importance of the long-run multiplier is considerable. Using Modigliani's<sup>(15)</sup> estimate of the U.S. marginal propensity to tax (net of income-related transfers) of about .5, this long-run multiplier would be 2.

The long-run effects of money-financed changes in government spending will not be considered here for reasons of space. The long-run effects of the two mixed financing policies mentioned before will be dealt with briefly.

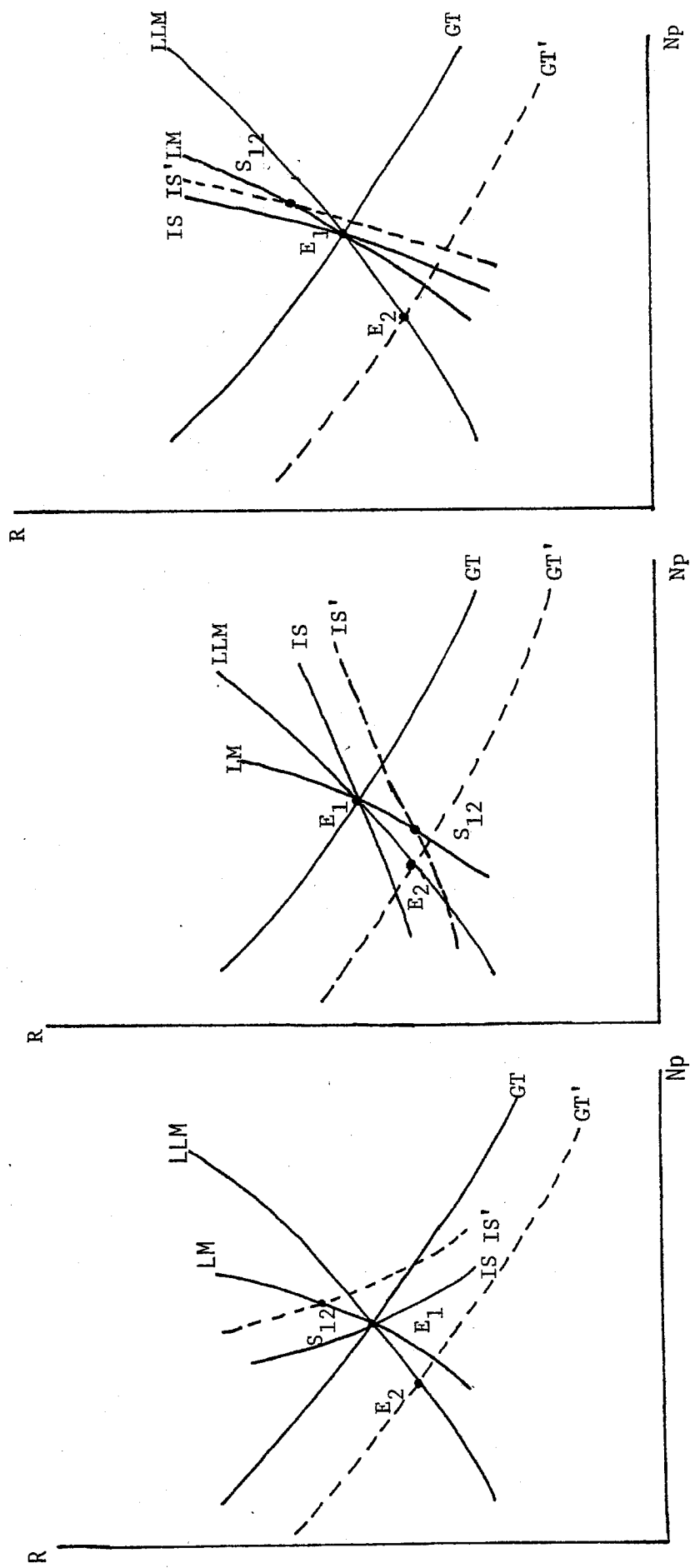


Figure 4

Mixed financing of deficits and surpluses

Policy III (the "v policy")

Under this policy the government determines  $v$ , the proportional rate of growth of the nominal quantity of public debt and therefore, since  $\phi$  is constant, of both the nominal quantity of money and the nominal stock of bonds. With  $\lambda$  denoting the real per capita stock of public debt, the model can be written as follows:

$$16a \quad i(f'(k) - R+x) - s(\hat{\mu}f(k) - \lambda - \frac{f'(k)}{R-x}k) + (v-x)\lambda = 0$$

$$16b \quad \ell(R(1-\theta), \frac{f(k)}{\lambda + \frac{f'(k)}{R-x}k}) (\lambda + \frac{f'(k)}{R-x}k) = \phi\lambda$$

$$16c \quad \dot{k} = i(f'(k) - R+x) - nk$$

$$16d \quad \dot{\lambda} = (v - n - \frac{\dot{p}}{p})\lambda$$

$$\begin{cases} 16e' & x = v - n \\ 16e'' & x = \frac{\dot{p}}{p} \\ 16e''' & \dot{x} = \beta(\frac{\dot{p}}{p} - x). \end{cases}$$

In the static expectations case, the fixed expected proportional rate of change of the price level is assumed to be equal to the steady state proportional rate of change of the price level,  $v-n$ .

The steady state values of  $x, R, k$ , and  $\lambda$  for all three expectations hypotheses are determined by:

$$\begin{aligned}
 17a \quad & x = v - n \\
 17b \quad & f'(k) = R - v + n \\
 17c \quad & \ell(R(1-\theta), \frac{1}{\hat{\mu}}) \hat{\mu} f(k) = \phi \lambda \\
 17d \quad & \hat{\mu} f(k) = \lambda + k.
 \end{aligned}$$

In long-run equilibrium, the expected proportional rate of change of the price level equals the actual proportional rate of change of the price level which is equal to the proportional rate of change of the per capita stock of nominal assets  $v - n$ . Note that, with  $v > 0$ , there will be a steady-state deficit. We can represent the steady state equilibrium in  $\lambda - k$  space by the following two equations:

$$\begin{aligned}
 18a \quad & \ell[(f'(k) + v-n)(1-\theta), \frac{1}{\hat{\mu}}] \hat{\mu} f(k) = \phi \lambda \quad (\text{LLM}) \\
 18b \quad & \hat{\mu} f(k) = \lambda + k \quad (\text{ww})
 \end{aligned}$$

The long-run comparative static effects of an increase in  $v$  (or  $\phi$ ) on  $\lambda$  and  $k$  are shown in Figure 5. Algebraically, the long-run multipliers are given by:

$$\frac{\partial k}{\partial v} = \frac{(1-\theta)w\ell}{\Lambda}$$



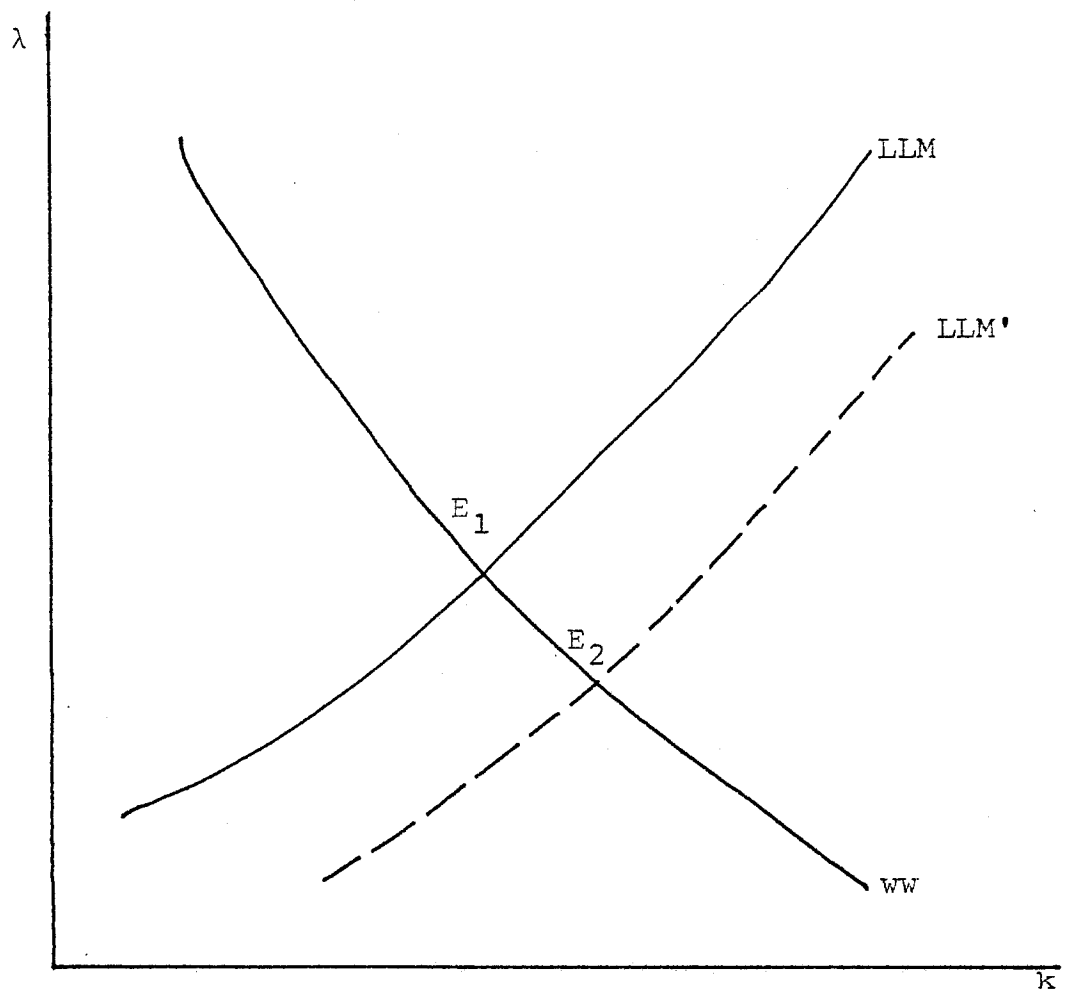


Figure 5

$$\frac{\partial \lambda}{\partial v} = \frac{(1-\theta)w\ell_1(\hat{\mu}f'-1)}{\Lambda}$$

$$\frac{\partial k}{\partial \phi} = -\frac{\lambda}{\Lambda}$$

$$\frac{\partial \lambda}{\partial \phi} = \frac{-\lambda(\hat{\mu}f'-1)}{\Lambda}$$

$$\Lambda = -(1-\theta)w\ell_1 f'' + \hat{\mu}f'(\phi - \ell) - \phi$$

Since  $\phi > \ell$ , ( $\ell w = \phi \lambda$  and  $w > \lambda$ ) the sign of  $\Lambda$  is ambiguous. Again the assumption that  $\hat{\mu}f' < 1$  is sufficient to make  $\Lambda$  negative and the  $wv$  curve downward sloping.

#### "Superneutrality"

If  $\Lambda$  is negative, a higher policy - determined rate of growth of nominal public debt (money and bonds) corresponds to a higher equilibrium capital-labor ratio, as does a higher share of money balances in total nominal public debt. The effect on real per capita holdings of public debt of both these policy changes is negative if an increase in the stock of capital owned by the private sector increases target wealth less than actual wealth. The long-run effect of an increase in  $v$  is therefore very similar to the long-run effect of an increase in  $g'$ .

The reason for the absence of "superneutrality" -- invariance of the the real long-run equilibrium under different proportional rates of change of nominal outside financial claims -- is of course the institutionally fixed nominal rate of return on money

balances. In equilibrium a higher  $v$  corresponds to a higher actual and expected rate of inflation. With an institutionally fixed nominal rate of return on money balances the real rate of return on money balances will be lowered which would cet.par. increase the real rate of returns differential between money and bonds-cum-capital. It is easy to show that if the policy authority were to increase the nominal rate of return on money balances ( $i^M$ ) pari passu with the steady state rate of inflation, no real effects could result from varying  $v$ . Let the policy authority fix  $i_M$  according to the rule  $i^M = x$ . The long-run equilibrium would in that case be characterised by:

$$19a \quad x = v - n$$

$$19b \quad \ell(R-x)(1-\theta), \frac{1}{\hat{\mu}} \hat{\mu} f(k) = \phi \lambda$$

$$19c \quad f'(k) = R - v + n$$

$$19d \quad \hat{\mu} f(k) = \lambda + k$$

The real rate of return differential between money and bonds-cum-capital is

$$[R(1-\theta) - x] - [i^M(1-\theta) - x] = (R-x)(1-\theta)$$

Substituting 19a and 19c into 19b and 19d gives

$$\ell(f'(k)(1-\theta), \frac{1}{\hat{\mu}} \hat{\mu} f(k) = \phi \lambda$$

$$\hat{\mu}f(k) = \lambda + k$$

$$R - x = f'(k)$$

Thus the real variables (capital-labor ratio, real per capita public debt and real rate of interest) are independent of  $v$ . Superneutrality of the steady-state equilibrium with respect to changes in the common proportional rate of growth of all outside financial claims is consistent with any government policy concerning  $i^M$  that has the property  $i_N^M - i_0^M = v_N - v_0$  in long-run equilibrium. (The subscripts N and 0 refer to the new and the old situation respectively.) For example, the government could set the nominal rate of return on money balances equal to some constant  $c$  plus  $x$ , or to  $c + \frac{\dot{p}}{p}$  or to  $v$  itself. By varying  $i^M$  in this manner, the policy authority loses the "wedge" it can drive between the real rate of return on money balances and on alternative assets.

Superneutrality in this model applies to changes in the proportional rate of change of all nominally denominated outside claims. As long as government interest-bearing debt somehow effects private sector behavioral relationships, the economic system will not exhibit superneutrality with respect to changes in the proportional rate of growth of a single nominal asset such as money even if the policy authority were to vary the nominal rate of return on money balances in accordance with the (expected) rate of inflation.

Stability analysis of the  $v$  model is long and tedious. Furthermore, no surprising or easily intuitively interpretable results are forthcoming. Potential stability is the unsurprising property of the model under Policy III and Policy IV.

Policy IV, (the  $g'$  policy)

Under this policy regime the government determines its real per capita expenditure on goods and net-of-tax real debt service and finances any deficits or surpluses it might incur by issuing or retiring its bond and money debt in such a way as to keep  $\phi$  constant. The complete model in this case can be written as:

$$20a \quad i(f'(k) - R+x) - s(\hat{\mu}f(k) - \lambda - \frac{f'(k)}{R-x}k) + g' - \theta f(k) - x\lambda = 0$$

$$20b \quad \ell\left(R(1-\theta), \frac{f(k)}{\lambda + \frac{f'(k)}{R-x}k}\right) \left(\lambda + \frac{f'(k)}{R-x}k\right) = \phi\lambda$$

$$20c \quad \dot{k} = i(f'(k) - R+x) - nk$$

$$20d \quad \dot{\lambda} = g' - \theta f(k) - \left[n + \frac{\dot{p}}{p}\right] \lambda$$

$$20e \quad \left\{ \begin{array}{l} x = (g' - \theta f(k^*)\lambda^{-1} - n \\ 20e' \quad x = \frac{\dot{p}}{p} \\ 20e'' \quad \dot{x} = \beta\left(\frac{\dot{p}}{p} - x\right) \end{array} \right.$$

The steady state equilibrium is characterised by the following equations:

$$21a \quad \ell(R(1-\theta), 1/\hat{\mu})\hat{\mu}f(k) = \phi\lambda$$

$$21b \quad f'(k) = R - (g' - \theta f(k))\lambda^{-1} + n$$

$$21c \quad \hat{\mu}f(k) = k + \lambda$$

and

$$x = \frac{\dot{p}}{p}$$

$$\frac{\dot{p}}{p} = [g' - \theta f(k)]\lambda^{-1} - n$$

Algebraically the LLM curve and the ww curve are given by:

$$22a \quad \ell([\hat{f}'(k) + (g' - \theta \hat{f}(k))\lambda^{-1-n}](1-\theta), 1/\hat{\mu})\hat{\mu}\hat{f}(k) = \phi\lambda \quad (\text{LLM})$$

$$22b \quad \hat{\mu}\hat{f}(k) = k + \lambda. \quad (\text{ww})$$

The long-run multipliers of government spending and open market operations can be derived as before.

Conclusion

In full employment models like the one considered in this paper, the ability of the government to affect the real long-run equilibrium hinges on the government's ability to "drive a wedge" into some private sector behavioral relationship. In the case of the "v policy", different proportional rates of change of nominal debt affect the real rate of return differential between two assets through the policy authority's control over the nominal rate of return on money. The balanced budget condition and the functional form of the tax function determine the long-run government spending multiplier under pure bond financing and pure money financing.

The conclusion that there are long-run affects of pure fiscal policy (and of the various mixed fiscal-monetary policies) is not presented as in any way surprising. If the government can, by fixing prices or rates of return, by changing the availability of a financial claim whose supply is outside the control of the private sector or through its spending and taxing behavior affect the opportunity sets of private economic agents, it will be capable of affecting the trajectory of an economic system with full employment of resources in the long run as well as in the short run.



Footnotes

1. Blinder, A.S. and Solow, R.M. (2).  
Christ, C.F. (7).  
Silber, W.L. (16).  
Ott, D.J. and Ott, A. (14).  
Mundell, R.A. (13).  
Buitter, W.H. (3).
2. Some recent examples are: Foley, D.K. and Sidrauski, M. (8);  
Foley, D.K. and Sidrauski, M. (9); Hadjimichalakis, M.G. (11);  
Stein, J.L. (17).
3. There may be additional short-run endogenous variables  $z(t)$  that are determined by  $z(t) = H(y(t), x(t), u(t))$ , etc.
4. "The state is some compact representation of the past activity of the system complete enough to allow us to predict, on the basis of the inputs, exactly what the outputs will be, and also to update the state itself", Padulo, L and Arbib, M.A. (15). Another useful reference is: Chen, C.T (6).
5. In Section II only continuous time systems will be considered.
6. The outputs could occur in the state equation:

$$\dot{x}(t) = \tilde{G}(x(t), y(t), u(t))$$

Substituting the output equation into the state equation we would get

$$\dot{x}(t) = \tilde{G}(x(t), F(x(t), u(t)), u(t)) = G(x(t), u(t)).$$

7. Higher order systems can always be reduced to first-order systems.
8. When the different versions of the model are studied in detail it will be convenient to use slightly different outputs and state variables as will become clear in Section II.
9. Barro, R.J. and Grossman, H.I. (1).
10. Buitter, W.H. (4).
11. Foley, D.K. (10); see also Buitter, W.H. (5) and (3).
12. The specification of the model is discussed in much greater detail in Buitter, W.H. (3) Chapter IV. See also Tobin, J. and Buitter, W.H. (18).
13.  $\dot{\phi}$  is related to  $\delta$  through the relationship  $\dot{\phi} = (\delta - \phi)\frac{\Delta}{\Delta}$ : when total debt is increasing, the "average" money-bond financing ratio,  $\phi$ , will increase (decrease) when the "marginal" money-bond financing ratio,  $\delta$ , exceeds (falls short of) the average ratio.
14. The stability analysis is obtainable from the author on request.
15. Modigliani, F. (12).

References

1. Barro, R.J. and Grossman, H.I. "A General Disequilibrium Model of Income and Employment", American Economic Review, Vol. 61, No. 1 (March 1971) pp. 82-93.
2. Blinder, A.S. and Solow, R.M. "Does Fiscal Policy Matter", Journal of Public Economics, Vol. 2, No. 4 (November 1973) pp. 319-37.
3. Buiter, W.H. Temporary Equilibrium and Long-run Equilibrium; unpublished Ph.D. thesis, Yale 1975.
4. Buiter, W.H. "Short-run and Long-run Disequilibrium in Dynamic Macromodels", Econometric Research Project Working paper 188, Nov. 1975
5. Buiter, W.H. "On Two Specifications of Asset Equilibrium in Macroeconomic Models; a Note", Econometric Research Project Working Paper 185, Oct. 1975
6. Chen, C.T. Introduction to Linear System Theory, 1970 New York Holt, Rinehart and Winston, Inc.
7. Christ, C.F. "A Simple Macroeconomic Model with a Government Budget Restraint", Journal of Political Economy, Vol. 76, No. 1 (January 1968) pp. 53-67.
8. Foley, D.K. and Sidrauski, M. "Portfolio Choice, Investment and Growth", American Economic Review, Vol. 60, No. 1 (March 1970) pp. 44-64.

9. Foley, D.K. and Sidrauski, M. Monetary and Fiscal Policy in a Growing Economy, London, MacMillan 1971.
10. Foley, D.K. "On Two Specifications of Asset Equilibrium in Macroeconomic Models", Journal of Political Economy, Vol. 83, No. 2 (April 1975) pp. 303-24.
11. Hadjimichalakis, M.G. "Equilibrium and Disequilibrium Growth with Money -- The Tobin Models", Review of Economic Studies Vol. 38(4), No. 116 (October 1971) pp. 457-79.
12. Modigliani, F. "Monetary Policy and Consumption", Consumer Spending and Monetary Policy, The Linkages, Federal Reserve Bank of Boston, Conference Series, No. 5, 1971.
13. Mundell, R.A. "A Fallacy in the Interpretation of Macroeconomic Equilibrium", Journal of Political Economy, Vol. 73 No. 1 (February 1965).
14. Ott, D.J. and Ott, A. "Budget Balance and Equilibrium Income" Journal of Finance, Vol. 20, No. 1 (March 1965).
15. Padulo, L and Arbib, M.A., System Theory, 1974, W.B. Saunders Co., Philadelphia.
16. Silber, W.L. "Fiscal Policy in IS-LM Analysis, A Correction" Journal of Money Credit and Banking, (November 1970).
17. Stein, J.L. "Neoclassical and Keynes-Wicksell Monetary Growth Models", Journal of Money Credit and Banking, Vol. 1, No.2 (May 1969), pp. 164-83.

18. Tobin, J. and Buiter, W.H. "Long-run Effects of Fiscal and Monetary Policy on Aggregate Demand", Forthcoming in Proceedings of the Conference on Monetarism, J. Stein ed. Amsterdam, North Holland.

Appendix AStability analysis for Policy II under pure bond financing and  
pure money financing

For the static expectations and adaptive expectations cases we solve the short-run IS and LM equations for  $R$  and  $N_p$  as functions of  $B$  (or  $M$ ),  $k$  and  $x$ , conditional on the values assumed by  $M$  (or  $B$ ),  $g'$  and  $\theta$  and evaluate these solutions at the long run equilibrium  $(B^*, k^*, x^*)$  or  $(M^*, k^*, x^*)$  with  $x^* = -n$  and  $q = 1$ . The IS/LM solution for  $R$  and  $N_p$  is

$$A.1a \quad R = h^1(B, k, x)$$

$$A.1b \quad N_p = h^2(B, k, x)$$

with bond-financed deficits and surpluses, and

$$A.2a \quad R = h^1(M, k, x)$$

$$A.2b \quad N_p = h^2(M, k, x)$$

with money-financed deficits and surpluses.

The short-run reduced form multipliers can be solved for from:

$$\begin{bmatrix} -(i' + s' \frac{k}{R-x}) & \frac{-s'(M+B)}{(Np)^2} + \frac{x(M+B)}{(Np)^2} \\ w l_1 (1-\theta) + \frac{k}{R-x} (\frac{l_2^f(k)}{w} - l) & \frac{l_2^f(k)}{Np} (1 - \frac{k}{w}) + \frac{Mk}{w(Np)^2} \end{bmatrix} \begin{bmatrix} dR \\ dNp \end{bmatrix}$$

$$= \begin{bmatrix} \frac{(-s'+x)}{Np} dM + \frac{(-s'+x)}{Np} dB + (i'f'' + \theta f' + s' \hat{\mu} f' - s' \frac{k}{R-x} f'' - s') dk \\ - (i' + s' \frac{k}{R-x} - \frac{(M+B)}{Np}) dx \\ \frac{1}{Np} [1 + \frac{l_2^f}{w} - l] dM + (\frac{l_2^f}{Npw} - \frac{l}{Np}) dB + \left( -l_2^f f' + (1 - \frac{k}{R-x} f'') (\frac{l_2^f}{w} - l) \right) dk \\ + \frac{k}{R-x} (\frac{l_2^f}{w} - l) dx \end{bmatrix}$$

$$h_M^1 = \frac{(-s'+x)}{Np} \left[ \frac{l_2^f}{Np} (1 - \frac{k}{w}) + \frac{Mk}{w(Np)^2} \right] + \frac{1}{Np} (1 + \frac{l_2^f}{w} - l) \frac{(s'-x)}{(Np)^2} (M+B)$$

$$h_B^1 = \frac{(-s'+x)}{Np} \left[ \frac{l_2^f}{Np} (1 - \frac{k}{w}) + \frac{Mk}{w(Np)^2} \right] + \left( \frac{l_2^f}{Npw} - \frac{l}{Np} \right) \frac{(s'-x)(M+B)}{(Np)^2} > 0$$

$$h_k^1 = \frac{(-i'f'' + \theta f' + s' \hat{\mu} f' - s' \frac{k}{R-x} f'' - s') (\frac{l_2^f}{Np} (1 - \frac{k}{w}) + \frac{Mk}{w(Np)^2}) +$$

$$+ \frac{(-l_2^f f' + (1 - \frac{k}{R-x} f'') (\frac{l_2^f}{w} - l)) (s'-x)(M+B)}{(Np)^2}$$

$$h_x^1 = \frac{-(i' + \frac{s'k}{R-x} - \frac{(M+B)}{Np}) (\frac{l_2^f}{Np} (1 - \frac{k}{w}) + \frac{Mk}{w(Np)^2}) + \frac{k}{R-x} (\frac{l_2^f}{w} - l) \frac{(s'-x)(M+B)}{(Np)^2}}{\Omega}$$

$$h_x^1 > 0 \text{ if } i' + s' \frac{k}{R-x} - \frac{(M+B)}{Np} > 0$$

$$h_M^2 = \frac{-(i' + s' \frac{k}{R-x}) (1 + \frac{l_2^f}{w} - l) \frac{1}{Np} + \frac{(s'-x)}{Np} (wl_1(1-\theta) + \frac{k}{R-x} (\frac{l_2^f(k)}{w} - l))}{\Omega}$$

$$h_B^2 = \frac{-(i' + s' \frac{k}{R-x}) (\frac{l_2^f}{wNp} - \frac{l}{Np}) + \frac{(s'-x)}{Np} (wl_1(1-\theta) + \frac{k}{R-x} (\frac{l_2^f(k)}{w} - l))}{\Omega}$$

$$h_k^2 = \frac{-(i' + s' \frac{k}{R-x}) (-l_2^{f'} + (1 - \frac{k}{R-x} f'') (\frac{l_2^f}{w} - l))}{\Omega}$$

$$\frac{-(i' f'' + \theta f' + s' \hat{u} f' - s' \frac{k}{R-x} f'' - s') (wl_1(1-\theta) + \frac{k}{R-x} (\frac{l_2^f(k)}{w} - l))}{\Omega}$$

$$h_x^2 = \frac{-(i' + s' \frac{k}{R-x}) \frac{k}{R-x} (\frac{l_2^f}{w} - l) + (i' + s' \frac{k}{R-x} - \frac{(M+B)}{Np}) (wl_1(1-\theta) + \frac{k}{R-x} (\frac{l_2^f(k)}{w} - l))}{\Omega}$$

$$h_x^2 > 0 \text{ only if } i' + s' \frac{k}{R-x} - \frac{(M+B)}{Np} > \frac{k}{R-x} \frac{(\frac{l_2^f}{w} - l) (\frac{M+B}{Np})}{wl_1(1-\theta)}$$



$$\Omega = -(i' + s' \frac{k}{R-x}) \left[ \frac{\ell_2^f(k)}{Np} \left(1 - \frac{k}{w}\right) + \frac{Mk}{w(Np)^2} \right] +$$

$$+ (s' - x) \frac{(M+B)}{(Np)^2} \left[ w \ell_1(1-\theta) + \frac{k}{R-x} \left( \frac{\ell_2^f(k)}{w} - \ell \right) \right] < 0$$

(All these expressions were evaluated at  $x = -n$ )

### Static expectations and bond-financing

With static expectations the state variables are  $B$  and  $k$ .  
Their behavior over time is give by:

$$A.3a \quad \dot{B} = Np(g' - \theta f(k))$$

$$A.3b \quad \dot{k} = i(f'(k) - R - n) - nk$$

Substituting A.1a and A.1b into A.3a and A.3b yields

$$\dot{B} = Np(g' - \theta f(k))$$

$$\dot{k} = i(f'(k) - h^1(B, k) - n) - nk.$$

The linear approximation at the long-run equilibrium  $B^*, k^*$  gives

$$\begin{bmatrix} \dot{B} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} 0 & -Np\theta f'(k) \\ -i' h_B^1 & i'(f''(k) - h_k^1) - n \end{bmatrix} \begin{bmatrix} B - B^* \\ k - k^* \end{bmatrix}$$

Necessary and sufficient conditions for stability are:

$$A.4a \quad i'(f''(k) - h_k^1) - n < 0$$

$$A.4b \quad -i'h_B^1 p\theta f'(k) > 0$$

A.4a may be satisfied but A.4b never is. The long-run equilibrium is unstable under static expectations. Figure A.1 illustrates this instability with the familiar phase diagram in B-k space.

### Static expectations and money financing

Substituting A.2a and A.2b into A.3a and A.3b and linearizing at the long-run equilibrium  $M^*, k^*$  we get:

$$\begin{bmatrix} \dot{M} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} 0 & -pN\theta f'(k) \\ -i'h_M^1 & i'(f''(k) - h_k^1) - n \end{bmatrix} \begin{bmatrix} M - M^* \\ k - k^* \end{bmatrix}$$

Necessary and sufficient stability conditions are

$$A.5a \quad i'(f''(k) - h_k^1) - n < 0$$

$$A.5b \quad -i'h_M^1 pN\theta f'(k) > 0$$

The equilibrium will only be stable if  $h_M^1 < 0$  i.e. if the impact effect of an increase in the stock of money (which will shift both the IS curve and the LM curve to the right) is to lower the rate of interest. For reasons of space, the adaptive expectations

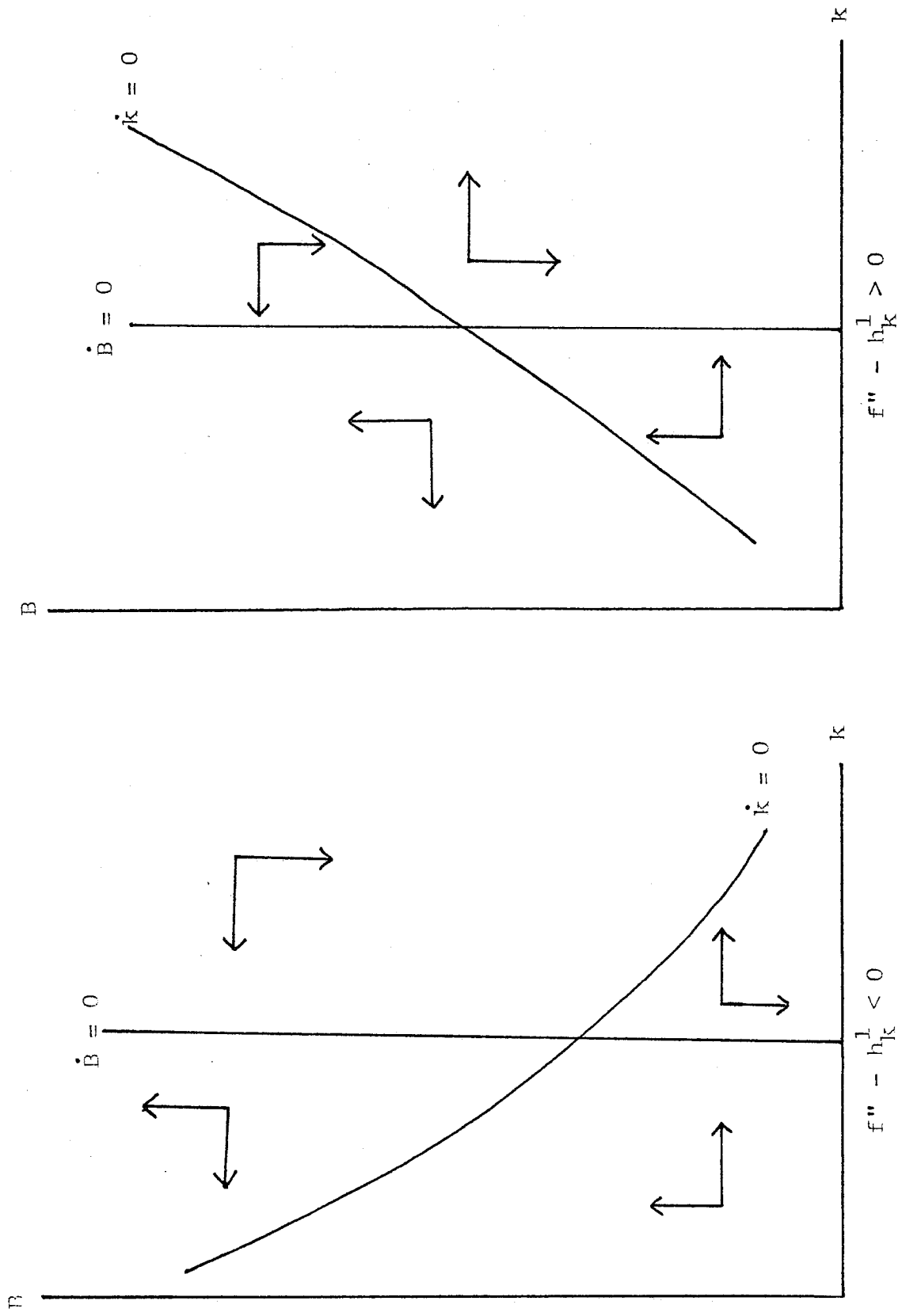


Figure A1

and myopic perfect foresight cases will only be considered for the bond-financing regime.

Adaptive expectations and bond-financing

Substituting A.1a and A.1b into the dynamic equations for the state variables we get:

$$A.6a \quad \dot{B} = Np(g' - \theta f(k))$$

$$A.6b \quad \dot{k} = i(f'(k) - h^1(B, k, x) + x) - nk$$

$$A.6c \quad \dot{x} = \beta \left( \frac{\dot{h}^2(B, k, x)}{h^2(B, k, x)} - x \right)$$

The characteristic equation of the Jacobian matrix obtained by taking the linear approximation of this system at the long-run equilibrium  $(B^*, k^*, x^* = -n)$  will be a cubic which, without loss of generality can be written as:

$$a_0 \gamma^3 + a_1 \gamma^2 + a_2 \gamma + a_3 = 0 \quad (a_0 > 0)$$

Necessary and sufficient conditions for all characteristic roots of this cubic equation to have negative real parts are:

$$a_1 > 0$$

$$a_2 > 0$$

$$a_3 > 0$$

$$a_1 a_2 - a_0 a_3 > 0$$

One of the first two inequalities can be eliminated since it is implied by the remaining three. The linear approximation looks as follows:

$$A.7 \begin{bmatrix} \dot{B} \\ \dot{k} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & -Np\theta f' & 0 \\ -i'h_B^1 & i'(f''-h_k^1)-n & i'(1-h_x^1) \\ z_1 & z_2 & z_3 \end{bmatrix} \begin{bmatrix} B-B^* \\ k-k^* \\ x+n \end{bmatrix}$$

$$z_1 = \frac{-\beta h_k^2 i' h_B^1}{Np - \beta h_x^2}$$

$$z_2 = \frac{Np\beta}{Np - \beta h_x^2} \left[ -h_B^2 \theta f' + \frac{h_k^2}{Np} (i'(f''-h_k^1)-n) \right]$$

$$z_3 = \frac{Np\beta}{Np - \beta h_x^2} \left[ \frac{h_k^2}{Np} (i'(1-h_x^1) - 1) \right]$$

The characteristic equation is:

$$\gamma^3 - \left\{ i'(f'' - h_k^1) - n + \frac{Np\beta}{Np - \beta h_x^2} \left[ \frac{h_k^2}{Np} i'(1-h_x^1) - 1 \right] \right\} \gamma^2$$

$$- \left\{ \left[ i'(f'' - h_k^1) - n \right] \frac{Np\beta}{Np - \beta h_x^2} - i'(1 - h_x^1) \frac{Np\beta}{Np - \beta h_x^2} h_B^2 \theta f' + i' h_B^1 Np \theta f' \right\} \gamma$$

$$- i' h_B^1 Np \theta f' \frac{Np\beta}{Np - \beta h_x^2} = 0$$

The linearized version of the static expectations model is found back as the upper left 2x2 submatrix of the linearized version of the adaptive expectations model. The static expectations model can be regarded as a limiting case of the adaptive expectations model with  $\beta = 0$ .

$a_3 = i' h_B^1 Np \theta f' \frac{Np\beta}{Np - \beta h_x^2} > 0$  is necessary for stability. Since  $h_B^1 > 0$ ,  $a_3 > 0$  i.f.f.  $Np - \beta h_x^2 > 0$ , i.e. only if  $h_x^2 > 0$ : the impact effect of a rise in inflationary expectations on the price level is positive.

$$h_x^2 > 0 \text{ only if } i' + s \frac{k}{R-x} - \frac{(M+B)}{Np} > \frac{\frac{k}{R-x} \left( \frac{l_2^f}{w} - l \right) \frac{M+B}{Np}}{w l_1 (1-\theta)}$$

With  $\beta = 0$  (the static expectations case)  $a_3 = 0$  and the system will not be stable.

### Myopic perfect foresight and bond financing

The complete dynamic system can in this case be written as:

$$A.8a \quad i(f'(k) - R + \frac{\dot{p}}{p}) + g' - \theta f(k) - s(\hat{\mu}f(k) - \frac{(M+B)}{Np}) - \frac{f'(k)}{R - \frac{\dot{p}}{p}}k - \frac{\dot{p}}{p} \frac{(M+B)}{Np} = 0$$

$$A.8b \quad l(R(1-\theta), \frac{f(k)}{\frac{M+B}{Np} + \frac{f'(k)}{R - \frac{\dot{p}}{p}}k}) (\frac{M+B}{Np} + \frac{f'(k)}{R - \frac{\dot{p}}{p}}k) = \frac{M}{p}$$

$$A.8c \quad \dot{B} = Np(g' - \theta f(k))$$

$$A.8d \quad \dot{k} = i(f'(k) - R + \frac{\dot{p}}{p}) - nk.$$

We can solve A.8a and A.8b for  $R$  and  $\frac{\dot{p}}{p}$  as functions of  $B$ ,  $K$  and  $p$ .

$$A.9a \quad R = m(B, k, p)$$

$$A.9b \quad \frac{\dot{p}}{p} = r(B, k, p)$$

The reduced form impact multipliers are solved for from:

$$\begin{bmatrix} -i' - s' \frac{k}{R - \frac{\dot{p}}{p}} & i' + s' \frac{k}{R - \frac{\dot{p}}{p}} - \frac{(M+B)}{Np} \\ w l_1 (1-\theta) + \frac{k}{R - \frac{\dot{p}}{p}} (\frac{l_2 f}{w} - l) & \frac{k}{R - \frac{\dot{p}}{p}} (l - \frac{l_2 f}{w}) \end{bmatrix} \begin{bmatrix} dR \\ \frac{\dot{p}}{p} \end{bmatrix} =$$

$$\left[ \begin{aligned} & -\frac{s'}{Np} dB + (-i'f'' + \theta f' + s' \frac{kf''}{R - \frac{p}{p}} - s') dk + s' \frac{(M+B)}{(Np)^2} dp \\ & \left( \frac{l_2 f'}{wNp} - \frac{l}{Np} \right) dB + \left[ -l_2 f' + \left( \frac{l_2 f'}{w} - l \right) \left( 1 + \frac{k}{R - \frac{p}{p}} f'' \right) \right] dk + \left( \frac{-l_2 f' (M+B) - Mk}{w(Np)^2} \right) dp \end{aligned} \right]$$

The state space representation of the system is given in equations A.10a-c and its linear approximation at the long-run equilibrium  $B^*, k^*, p^*$  in equation A.11.

$$A.10a \quad \dot{p} = pr(B, k, p)$$

$$A.10b \quad \dot{B} = Np(g' - \theta f(k))$$

$$A.10c \quad \dot{k} = i(f'(k) - m(B, k, p) + r(B, k, p)) - nk$$

$$A.11 \quad \begin{bmatrix} \dot{p} \\ \dot{B} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} pr_p & pr_B & pr_k \\ 0 & 0 & -Np\theta f' \\ i'(r_p - m_p) & i'(r_B - m_B) & i'(r_k - m_k + f'') - n \end{bmatrix} \begin{bmatrix} p - p^* \\ B - B^* \\ k - k^* \end{bmatrix}$$

The characteristic equation is:

$$\begin{aligned} & \lambda^3 + [-pr_p - i'(r_k - m_k + f'') + n] \lambda^2 \\ & + p[-i'r_p m_k + i'r_p f'' - r_p n + i'(r_B - m_B) N\theta f' + i'm_p r_k] \lambda \\ & + i'p^2 N\theta f' [r_p m_B - m_p r_B] = 0 \end{aligned}$$



The condition that the coefficient  $a_3$  of the characteristic equation be positive (the condition that the determinant of the Jacobian be negative) is:

$$r_p m_B - m_p r_B > 0$$

This can be rewritten as:

$$i' + s' \frac{k}{R - \frac{p}{p}} - \frac{M+B}{Np} > \left( \frac{\frac{k}{R - \frac{p}{p}} \frac{(M+B)}{p} \left( \frac{\ell_2^f}{w} - \ell \right)}{w \ell_1 (1-\theta)} \right)$$

Comparing this with the necessary and sufficient condition for  $a_3$  to be positive in the adaptive expectations case:

$$i' + s' \frac{k}{R - \frac{p}{p}} - \frac{(M+B)}{Np} - \frac{\left( \frac{k}{R - \frac{p}{p}} \right) \frac{(M+B)}{p} \left( \frac{\ell_2^f}{w} - \ell \right)}{w \ell_1 (1-\theta)}$$

$$> \frac{(-) \left( i' + s' \left( \frac{k}{R - \frac{p}{p}} \right) \left[ \frac{\ell_2^f}{Np} \left( 1 - \frac{k}{w} \right) + \frac{Mk}{w(Np)^2} \right] \right)}{w \ell_1 (1-\theta)}$$

$$+ \frac{\frac{s'(M+B)}{(Np)^2} \left[ w \ell_1 (1-\theta) + \frac{k}{R - \frac{p}{p}} \left( \frac{\ell_2^f}{w} - \ell \right) \right]}{w \ell_1 (1-\theta)}$$

We see that the myopic perfect foresight case can be interpreted as a limiting case of the adaptive expectations case when the speed of adaptation becomes infinite.

The other stability conditions do not provide much economic insight although they are consistent with the a-priori restrictions that were imposed.

Appendix BStability for Policy III

For the case of myopic perfect foresight the model can be written as:

$$B.1a \quad i\left(f'(k) - R + \frac{\dot{p}}{p}\right) - s(\hat{\mu}f(k) - \lambda - \frac{f'(k)}{R - \dot{p}/p}k) + (v - \frac{\dot{p}}{p})\lambda = 0$$

$$B.1b \quad \ell\left(R(1-\theta), \frac{f(k)}{\lambda + \frac{f'(k)}{R - \dot{p}/p}k}\right)\left(\lambda + \frac{f'(k)}{R - \dot{p}/p}k\right) = \phi\lambda$$

$$B.1c \quad \dot{k} = i\left(f'(k) - R + \frac{\dot{p}}{p}\right) - nk$$

$$B.1d \quad \dot{\lambda} = (v - n - \frac{\dot{p}}{p})\lambda$$

The IS and LM equations can be solved for  $R$  and  $\dot{p}/p$  as functions of  $k$  and  $\lambda$ .

$$B.2a \quad R = q^1(k, \lambda)$$

$$B.2b \quad \frac{\dot{p}}{p} = q^2(k, \lambda)$$

Substituting this into equations B.1c and B.1d we get:

$$\dot{k} = i(f'(k) - q^1(k, \lambda) + q^2(k, \lambda)) - nk$$

$$\dot{\lambda} = (v - n - q^2(k, \lambda) \lambda)$$

The linear approximation at the long-run equilibrium  $(k^*, \lambda^*)$  gives:

$$\begin{bmatrix} \dot{k} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} i'(f'' - q_k^1 - q_k^2) - n & i'(q_\lambda^2 - q_\lambda^1) \\ -\lambda q_k^2 & -q_\lambda^2 \end{bmatrix} \begin{bmatrix} k - k^* \\ \lambda - \lambda^* \end{bmatrix}$$

The reduced form multipliers are solved for from:

$$\begin{bmatrix} w l_1 (1-\theta) - \frac{k}{R-v+n} \left( \frac{l_2^f}{w} - l \right) & \frac{k}{R-v+n} \left( l - \frac{l_2^f}{w} \right) \\ \frac{-(i' + s'k)}{R-v+n} & \frac{i' + s'k}{R-v+n} - \lambda \end{bmatrix} \begin{bmatrix} dR \\ d\left(\frac{\dot{p}}{p}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \left[ -l_2^{f'} + \frac{l_2^f}{w} - l + \frac{k f''}{R-v+n} \left( \frac{l_2^f}{w} - l \right) \right] dk + \left[ \frac{l_2^f}{w} - l + \phi \right] d\lambda \\ \left[ \frac{-(i' + s'k)}{R-v+n} f'' + s' \hat{\mu} f' - s' \right] dk + [-s' - n] d\lambda \end{bmatrix}$$

$$q_k^1 = \frac{-(i' + \frac{s'k}{R-v+n} - \lambda)l_2^{f'} + (i' - \lambda(1 + \frac{kf''}{R-v+n}) + s'\hat{\mu}_{R-v+n}^f)(\frac{l_2^{f'}}{w} - l)}{\Omega}$$

$$q_\lambda^1 = \frac{(i' - \lambda)(\frac{l_2^{f'}}{w} - l + \phi) + \frac{s'k}{R-v+n}\phi + \frac{nk}{R-v+n}(l - \frac{l_2^{f'}}{w})}{\Omega}$$

$$q_k^2 = \frac{-wl_1(1-\theta)(f''(i' + s'\frac{k}{R-v+n}) - s'\hat{\mu}^f + s') + \frac{k}{R-v+n}(\frac{l_2^{f'}}{w} - l)s'\hat{\mu}^f}{\Omega}$$

$$- \frac{(i' + s'\frac{k}{R-v+n})l_2^{f'} + i'(\frac{l_2^{f'}}{w} - l)}{\Omega}$$

$$q_\lambda^2 = \frac{-wl_1(1-\theta)(s'+n) - \frac{nk}{R-v+n}(\frac{l_2^{f'}}{w} - l) + i'(\frac{l_2^{f'}}{w} - l + \phi) + \frac{s'k}{R-v+n}\phi}{\Omega}$$

$$\Omega = wl_1(1-\theta)[i' + \frac{s'k}{R-v+n} - \lambda] - \frac{k}{R-v+n}(\frac{l_2^{f'}}{w} - l)\lambda.$$

If it is assumed that

$$i' + \frac{s'k}{R-v+n} - \lambda > \frac{\frac{k}{R-v+n}(\frac{l_2^{f'}}{w} - l)\lambda}{wl_1(1-\theta)}$$

(i.e. that  $\Omega$  is negative)  $q_k^2$  will be negative,  $q_\lambda^1$  will be negative if  $i' - \lambda > 0$ ,  $q_k^1$  will be positive if  $1 + (kf'')/(R+v-n) < 0$  and  $q_k^2$  will be positive if  $f''(i' + (s'k/R-v+n)) - s'\hat{\mu}^f + s' < 0$ .

Without additional information all we can say is that the system is potentially stable. Necessary and sufficient conditions for the local stability of the long-run equilibrium are:

$$i'(f'' - q_{1k}^1 + q_{1k}^2) - n - q_{\lambda}^2 < 0$$

and

$$i'(q_{1k}^1 q_{\lambda}^2 - q_{\lambda}^1 q_{1k}^2) + (n - i'f'')q_{\lambda}^2 > 0.$$

These two stability conditions reduce to long algebraic expressions in terms of the structural parameters of the model that do not appear to have any easy intuitive interpretation. It should be noted that

$$i' + \frac{s'k}{R-v+n} - \lambda > \frac{\frac{k}{R-v+n} \left( \frac{\ell_2^f}{w} - \ell \right) \lambda}{w\ell_1(1-\theta)}$$

( $\Omega < 0$ , is the condition that the impact effect of an increase in the expected rate of inflation on the price level be positive) is not necessary for stability under the mixed financing rule considered here. (See Appendix A) The opposition of short-run and long-run effects found under pure bond-financing does not appear here. For example, a simple set of sufficient conditions for stability with  $\Omega > 0$  is:  $q_{\lambda}^2 > 0$ ;  $q_{1k}^1 > 0$ ;  $q_{\lambda}^1 > 0$ ;  $q_{1k}^2 < 0$  i.e.

$$i' + \frac{s'k}{R-v+n} - \lambda < \frac{\frac{k}{R-v+n} \left( \frac{\ell_2^f}{w} - \ell \right) \lambda}{w\ell_1(1-\theta)}$$

$$i' + \frac{s'k}{R-v+n} - \lambda < \left( \frac{\lambda_2^f}{w} - \lambda \right) \left[ i' - \lambda \left( 1 + \frac{kf''}{R-v+n} \right) + \frac{s'\hat{u}f'k}{R-v+n} \right] \frac{1}{\lambda_2^f}$$

$$i' + \frac{s'k}{R-v+n} - \lambda > \frac{(s'-n)k \left( \lambda - \frac{\lambda_2^f}{w} \right)}{(R-v+n) \left( \frac{\lambda_2^f}{w} - \lambda + \phi \right)}$$

$$-w\lambda_1(1-\theta) \left( f'' \left( i' + \frac{ks'}{R-v+n} \right) - s'\hat{u}f' + s' \right) + \frac{k}{R-v+n} \left( \frac{\lambda_2^f}{w} - \lambda \right) s'\hat{u}f'$$

$$-\lambda_2^f \left( i' + \frac{s'k}{R-v+n} \right) + i' \left( \frac{\lambda_2^f}{w} - \lambda \right) < 0.$$

While these conditions impose rather narrow bounds on the values of the structural parameters of the model that are consistent with stability, they are not inconsistent with the a priori restrictions imposed on the signs and magnitudes of these structural coefficients. As I said before, much more detailed empirical information is required to firmly rule out either stability or instability.