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Chapter 4

LONG-RUN EFFECTS OF FISCAL AND MONETARY POLICY ON AGGREGATE DEMAND

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1. Introduction: The Setting of the Problem

This paper is a theoretical exercise addressed to a rather esoteric and artificial question in the logic of aggregate demand. Does expansionary fiscal policy raise aggregate demand permanently or at best only temporarily? The controversy is reminiscent of the Pigou–Keynes–Lerner controversy on the efficacy of reduction of money wages and prices in expanding aggregate demand, where also much was made of the distinction between short-run impacts and ultimate cumulative effects. The trouble with such discussions, including this one, is that a long run constructed to track the ultimate consequences of anything is a never-never land. For that abstraction we apologize in advance.

A characteristic monetarist proposition is that pure fiscal policy does not matter for aggregate real demand, nominal income, and the price level. The course of aggregate nominal demand, stochastic influences aside, depends solely on the path of the quantity of money somehow defined. Although increases in this monetary aggregate may frequently in practice be associated with budget deficits, the central bank always can break this link and very often does. The fiscal policies alleged not to matter are variations of government expenditure, transfer payments, and taxes while the quantity of money or its path over time remain unchanged.

We have stated this monetarist proposition baldly for the purpose of theoretical discussion. We realize that monetarists, Professor Friedman in particular, usually soften their assertions with qualifying adjectives and adverbs "minor", "almost", etc. After all, no one would wish to have his salvation depend on the literally complete independence of any two variables in a complex interdependent economy. Hedges of this order really do not alter the monetarist message for theory and policy, and they are not intended to. Therefore, let us hope that we can discuss the strong proposition without semantic and textual quarrel about the strength and purity with which it has been asserted.

Non-monetarists have argued on numerous occasions that a necessary condition for the proposition is zero elasticity of demand for money with respect to interest rates, and we have offered against the proposition the theoretical reasons and empirical evidence for believing this elasticity is not zero. In the comparative statics of short-run macro-equilibrium this condition appears as a vertical *LM* curve. When the condition is not met, the analysis indicates that a shift in the *IS* curve, – which could be brought about by an increase in the rate of government expenditures or transfers or by reduction in the flow of tax revenues – will raise aggregate real demand.

The extent to which this expansion evokes an increase in supply depends on how close the economy is to its productive capacity. Perhaps we should stress the point that no one is contending that fiscal policy can increase output when production is supply-constrained. Neither can monetary policy. Moreover, this particular debate is not about the existence or size of the natural rate of unemployment. Logically the natural rate proposition is distinct from the monetarist propositions about fiscal policy; one could accept either one without the other.

Even when output is supply-constrained issues concerning aggregate demand remain. The monetarist proposition then is that expansionary fiscal policy – purged of incidental and extrinsic monetary expansion – does not affect the price level. Basically the assertion is that government cannot change, by its own spending behavior or by measures designed to affect that of taxpayers and other citizens, the income velocity of money.

On its face this is a very surprising assertion. If those present at the conference were to decide to lower our average cash holdings while maintaining our spending, we would all agree that national income velocity would rise, though the change would hardly be detectible by our measuring devices. If the Fortune 100 did likewise, it would be detectible. Why not when the federal government does so– especially considering that the measure of velocity includes its spending in the numerator but excludes its cash from the denominator?

Monetarists argue that their proposition holds whether or not the *LM* curve is vertical. Friedman (1972, pp. 915–917) reaffirmed this view in his rejoinder to Tobin's comments on his "theoretical framework" articles. He says that fiscal effects are "certain to be temporary and likely to be minor", and that our difference of opinion is "mostly, whether one considers only the impact effect of a change or the cumulative effect". He agrees that the impact effect of a rise in government expenditure or reduction in taxes is expansionary; there is a once-for-all shift of "*IS*" and this pulls up income and interest rate along a non-vertical *LM* locus. By labelling this effect on income not only "minor" but "temporary" he seems to be saying that non-monetary financing of the accompanying budget deficits moves *LM* to the left, cancelling the expansionary shift of *IS*. But he does

not say this explicitly, stressing instead that monetary financing of the same fiscal program would be more and longer expansionary than the issue of interest-bearing debt.

Anyway, the issue we wish to discuss is whether and when the impact effects typified by *IS–LM* statics are reversed, modified, or amplified by shifts in those curves. But we have a few more general observations in prelude.

First, how relevant is this issue to the policy controversy which generated the theoretical debate in the first place? The policy controversy concerns such practical matters as the effects of the 1964 tax cut, the anti-inflationary content of the 1968 tax surcharge, the role of the escalation of war spending in escalating inflation in 1966, the importance of budget economy in fighting inflation or accentuating recession in 1974. In cases like these, advocates of fiscal measures were looking for short-term effects on aggregate demand, without committing themselves to changes of expenditures and taxes never to be repeated or reversed. They were certainly not contemplating that the stock of money should remain forever constant while the stocks of other assets grew. When Walter Heller argued that the tax cut of 1964 would increase demand and reduce unemployment he was talking about what would happen in 1965. He was not talking about what would happen in 1970 or 1980 if the tax cut were even then the only change from pre-1964 monetary and fiscal policies. In this context it was no answer to say that years of accumulation of debt in exclusively non-monetary form would be contractionary. It *was* an answer, right or wrong, to say that demand for money is interest-inelastic.

Second, the claim that growth of non-monetary government debt has the same qualitative effects as reduction of money supply depends on a particular view about asset preferences – roughly that non-monetary debt is a closer substitute in portfolios for capital than for money. This is the traditional view, shared by Keynes. In his "Essay on the Principles of Debt Management" Tobin (1971) distinguished between fiscal or flow effects of government budgets and deficits and the monetary or stock effects of the accumulated debt. He pointed out that, for a one-time change of budgetary program, the flow effect is one-shot while the stock effect cumulates. A corollary is that the flow effect is reversed when the budgetary change is reversed, while the stock effect persists. But at the same time Tobin entertained the possibility that the stock effect of non-monetary debt may be expansionary, that such assets are in investors' eyes closer to money than to capital. If so the growth of non-monetary debt would shift the *LM* curve to the right rather than to the left. (The relevant interest rate on the Hicks diagram would be in this case the true Modigliani–Miller cost of equity capital, which would diverge from the rates on government securities.) We mention this here because we shall not pursue the matter in this paper, where we shall acquiesce in

an extreme version of the traditional assumption, namely that government securities and capital are perfect substitutes in portfolios.

Third, there is some tendency to couple the monetarist proposition with a general attack on the use of equilibrium analysis and comparative statics, particularly the *IS–LM* apparatus, in short-run macro-economics. The attack has some justification, because it is generally true that the *incomplete* stock–flow equilibrium determined in such models implies changes in some stocks whose assumed constancy was a condition of the flow equilibrium itself. What is not true is that recognition of the temporary nature of the "equilibrium" invalidates all the propositions of such analysis or validates contrary propositions.

Keynes explicitly restricted his *General Theory* to a time period in which the stock of capital is for practical purposes constant. Yet a Keynesian equilibrium generally involves non-zero net investment, implying changes in capital stock and thus quite possibly in the investment function and other behavioral equations of his model. The same short-run assumption applies to other stocks and flows, including government debt and deficit: the analysis does not apply to a "run" long enough for the flow to make a significant change in the stock. Careful teachers of *IS–LM* and all that have never allowed their students to use the apparatus on questions like "the effects of an increase in government spending financed by printing money" because they knew that the change in money stock was indeterminate and time-dependent. Unfortunately they seldom get around to dynamic models in which the question makes sense.

2. The Plan and Notation of the Paper

In a pioneering paper, Blinder and Solow, inspired by the same questions which concern us, have presented a model of long-run equilibrium similar to the one we shall discuss below.[1] Our reasons for offering another version are two. First, we wish to consider some additional ways of modelling fiscal and monetary policy. Second, we wish to structure the long-run demands for wealth, capital, and money somewhat more definitely and explicitly than Blinder and Solow did, especially in their original article. We shall discuss the differences in some detail in section 3.

[1] Blinder and Solow (1973). A summary, with an important amendment, is given by the same authors in Blinder and Solow (1974, pp. 45–58). The amendment is to include income Y in the investment function; we discuss its significance below in section 3. Blinder and Solow appear to have contributed the first systematic treatment of long-run effects of fiscal policy in an economy without binding labor constraints on output. Of course, growth theory has treated the long-run effects of fiscal and monetary policies in economies with full employment and flexible prices. Even there, fiscal and monetary measures are generally so intertwined that "money-growth" models shed little light on the issues concerning "pure" fiscal and monetary effects.

Like Blinder and Solow, we shall consider a Pigovian stationary state. The advantage of this abstraction is that it allows in a simple manner for adjustments of stocks of capital and other assets. It avoids the possible flow–stock inconsistencies of the short-run equilibrium models. It therefore permits us to consider the monetarist claim that the apparent power of fiscal policy in those models depends wholly on such inconsistencies. Yet the model is artificial in several respects. To be relevant to the issue at hand, the model must permit unemployment even in long-run stationary equilibrium; and this requires the implausible indefinite persistence of wage and price rigidities. We shall also, however, consider the effects of fiscal policy on the price level in a long-run equilibrium with full employment and flexible prices.

The plan of the rest of the paper is as follows: Section 3 discusses briefly the Blinder–Solow contributions. Section 4 discusses the long-run comparative statics and stability properties of pure fiscal measures in economies with unemployed labor. In Model I the instrument of fiscal policy is G', government purchases of goods and services plus debt interest net of taxes on such interest. We trace the effects of once-for-all changes of G', while both the money stock and the proportional tax rate remain constant. As net debt interest changes, government purchases are adjusted dollar for dollar in the opposite direction to hold G' at its policy-determined level.

In Model II, the parameter of fiscal policy is G, government purchases, as it is in Blinder–Solow. This means that the fiscal stimulus varies endogenously as the volume of debt and the interest rate change.

Finally, section 4 analyzes briefly the use of the tax rate θ as an instrument of fiscal policy.

Section 5 takes up, still in the context of long-run unemployment, the effects of changing the quantity of money. Two kinds of monetary change are considered. One is a change in the quantity of money via open market operations, while fiscal instruments, θ and G or G' are held constant. The second is monetary change linked to fiscal policy: budget deficits consequent to a change in G or G' are financed by printing money while the non-monetary public debt is fixed.

Section 6 shifts from the Keynesian world of long-run unemployment to the neo-classical long run of full employment and flexible prices. Once again the questions are how variation of G' affects the long-run equilibrium and whether the equilibrium is stable.

The variables of our models are as follows:

Y = real net national product,
K = capital stock,
N = employment of labor,
$\overline{N}$ = labor force,

D = nominal government interest-bearing debt,
R = nominal rate of return on debt,
$r = R(1 - \theta)$, after-tax rate of return,
G = real government expenditure, not including debt interest,
G' = real government expenditure, including net debt interest,
M = nominal monetary debt of government,
θ = tax rate,
W = real private wealth, $K + \{(D + M)/p\}$,
α = capital share in income,
p = price level,
x = expected rate of inflation.

In the unemployment models of sections 4 and 5, employment N is always less than labor force $\overline{N}$, the price level p is assumed constant, and expected inflation x is zero. In the full employment model of section 6, N is equal to $\overline{N}$, and both p and x are endogenous.

Non-monetary debt is modelled like bills of short maturity, indeed strictly like interest-bearing deposits. It is always valued at par although its yield is market-determined and varies. The only convenient alternative, the one adopted by Blinder and Solow, is to go to the opposite extreme and to assume that all government debts are perpetuities with constant coupons but variable prices. The difference is not consequential for the questions of interest. More realistic specifications, with debts of finite maturity, enmesh dynamic analysis in a morass of complex bookkeeping which is not worth the trouble.

It is assumed throughout that production of Y obeys a constant-returns-to-scale function of capital K and labor N, with the usual neo-classical properties. In equilibrium the marginal product of capital derived from the production function, is equal to the real before-tax rate of return on debt (R in sections 4 and 5, $R - x$ in section 6). As previously stated, we follow – without endorsing – the Keynesian assumption that debt and capital are perfect substitutes in portfolios. At times of disequilibrium the marginal product of capital may differ from the return on debt. Their divergence is the signal and incentive for net investment or disinvestment in capital.

3. The Blinder–Solow Models of Fiscal Effects

Blinder and Solow present first a "long-run" model with a fixed capital stock but variable government debt, and then a model in which both stocks are endogenous. The first has at best expository relevance, since it is unrealistic and

potentially misleading to assume that over a horizon in which wealth and government debt change no capital accumulation can occur. For that reason, we will confine our comments and comparison to their variable-capital model. B will denote the number of bonds and the money value of current debt service. (Bonds are consols with a coupon of one unit of money.) Using our own notation where possible, we can write their model as follows:

$$Y = C[Y + B - T(Y + B), M + (B/R) + K] + I(R, K) + G; \qquad 0 < C_Y < 1, \quad C_W > 0, \tag{1}$$

$$M = L[R, Y, M + (B/R) + K]; \qquad 0 < T' < 1, \tag{2}$$

$$\dot{K} = I(R, K); \qquad I_R < 0, \quad I_K < 0, \tag{3}$$

$$\dot{B} = [G + B - T(Y + B)]R; \qquad L_R < 0, \quad L_Y > 0, \quad 0 < L_W < 1. \tag{4}$$

C and T are, respectively, consumption and tax functions. $Y = F(B, K)$ and $R = H(B, K)$ are the *IS–LM* solutions for income and interest rate, respectively.

The main differences between this model and ours are in the specification and in policy options considered:[2]

(a) We have opted to put more explicit structure on the stationary state demand functions for stocks of wealth and capital. Our short-run saving and investment functions are derived from these demand functions via mechanisms of adjusting actual to desired stocks. In the Blinder–Solow model, in contrast, the long-run desired stocks are implicit in the consumption and investment equations [by setting $C + G = Y$ and $I = 0$ in (1) and (3)].

(b) The investment function (3), in the original Blinder–Solow, is poorly motivated. They argue that the function "is in line with modern investment theory, which envisions an equilibrium demand for capital stock and a disequilibrium demand for investment" [Blinder–Solow (1973, p. 330)]. Yet the omission of Y from the function vitiates this rationalization for a model in which Y is endogenous. The omission is corrected in their second exposition [Blinder–Solow (1974, p. 55)]. The investment function here is $\dot{K} = I(R, Y, K)$, presumably with $I_Y > 0$. However, the stability conditions repeated in the second version are those derived for the model with the misspecified investment equation.

[2] An inessential difference, the modelling of government debt as perpetuities, has already been mentioned.

In their original model, sufficient conditions for stability are

$$F_B > (1 - T')/T' \quad \text{and} \quad I_K + C_W < 0,$$

(F_B is the reduced form impact multiplier of an increase in the number of bonds held by the private sector). It can be shown that in this model stability implies $dB/dG > 0$ and vice versa [if $F_B > (1 - T')/T'$]–the new equilibrium number of bonds after an increase in government spending on goods and services is larger than the old equilibrium number of bonds if, and only if, the equilibrium is stable.

In the model with the amended investment function, two things happen:

First, the stability conditions become very much more stringent–the linearized system is now

$$\begin{bmatrix} \dot{B} \\ \dot{K} \end{bmatrix} = \begin{bmatrix} R(1 - T' - T'F_B) & R(-T'F_K) \\ I_R H_B + I_Y F_B & I_K + H_K I_R + F_K I_Y \end{bmatrix} \begin{bmatrix} (B - B^*) \\ (K - K^*) \end{bmatrix}.$$

$(1 - T')/T' < F_B$ and $F_K < 0$ [$C_W + I_K < 0$] are no longer sufficient for stability, since the determinant condition is no longer necessarily satisfied.

Second, stability of the system now isn't sufficient to guarantee $dB/dG > 0$, or vice versa. If $dB/dG < 0$, the long-run multiplier for bond-financed deficit spending no longer exceeds that for money-financed deficit spending [Blinder–Solow (1973, p. 327)]. It is even possible that dB/dG is negative and so large numerically that it causes

$$\frac{dY}{dG} = \left(1 + (1 - T')\frac{dB}{dG}\right)\Big/T'$$

to become negative. This comparative-static result can obtain whether or not the equilibrium is stable.

In the stable case, this intuitively implausible result would require the government to run more surpluses than deficits along the adjustment path, even though the initial impact of higher government spending may be to create deficits. This problem arises because Blinder and Solow do not restrict, by their consumption function, the long-run equilibrium relationship of wealth to income. It may turn out that, when the public is content with the fixed money stock and has adjusted both capital and wealth to desired magnitudes, there is less, not more, room in portfolios for debt. This possibility is evaded by their original investment function, which implies an equilibrium capital stock independent of Y, but it can occur when desired K rises with Y.

(c) As regards the fiscal policy options considered, Blinder and Solow deal exclusively with our Model II. The possibility that the long-run government

spending multiplier is negative under bond-financed deficits arises only in that regime. In our Model I the multiplier for G' is always positive.

4. Analysis of Effects of Debt-Financed Government Expenditures

Variations of fiscal policy are characterized by changes in government outlays, with the tax rate θ and the money stock M held constant. In Model I the parameter of policy is G'; in Model II, it is G. The former is simpler analytically and in a sense more plausible in examining the long-run consequences of a one-step change of the parameter. Model I assumes that if debt interest increases purchases are curtailed correspondingly.[3] Model II, as in Solow–Blinder, credits a given fiscal policy with expansionary effect just because interest rate increases or deficits raise outlays for debt interest. Some might regard this procedure not as a constant fiscal policy but as an ever more expansionary policy.

4.1. Comparative Statics in the Two Models

Here are the long-run equilibrium equations for Models I and II:

$$W = \mu(Y - RK)(1 - \theta) = \mu(1 - \theta)(1 - \alpha)Y = \hat{\mu}Y. \tag{5}$$

This is the condition for zero private saving. On life cycle principles, wealth is a multiple of disposable labor income. For simplicity, μ and α and thus $\hat{\mu}$, are taken to be constants, but they could made functions of R.

$$\text{(I)} \qquad G' - \theta Y = 0,$$

$$\text{(II)} \qquad G + R(1 - \theta)D - \theta Y = 0. \tag{6}$$

This is the condition for zero government saving, a balanced budget. The tax rate θ applies to wages, capital income, and debt interest.

$$M = L(r, Y/W)W = L(r, 1/\hat{\mu})\hat{\mu}Y; \qquad L_1 < 0, \quad \hat{\mu}L > L_2 > 0. \tag{7}$$

The fraction of wealth held in money is inversely related to the after-tax return on alternative assets and positively related to the income/wealth ratio, which is

[3] An alternative which would be more plausible would to be aim fiscal policy for budget balance at a target income level Y^*, fixing the level of government exhaustive expenditures at G, and letting θ vary with debt interest so as to maintain budget balance at Y^*. We investigated such a model, but the analysis is too messy to report.

constant in equilibrium. Since debt and capital are perfect substitutes, (7) is a complete description of portfolio allocation.

$$K = F(R)Y, \qquad K = (\alpha/R)Y. \tag{8}$$

Technologically, the capital/output ratio is inversely related to the rate of return on capital.

$$W = K + D + M. \tag{9}$$

This is the definition of private wealth given above.

The stationary-state equilibrium of this model has the following properties. At the equilibrium Y the public has the desired amount of wealth W. Some of it is in money, the amount of which is the constant M. The interest rate r is such that the public is willing to hold the fraction M/W of its wealth in monetary form. Some of the public's wealth is in capital K, namely an amount such that the return on capital is R. The rest of the public's wealth is in non-monetary government debt D. These conditions can be met for any Y. But the budget-balance equation for the government is necessary to keep D fixed, and the addition of this requirement determines the equilibrium Y.

The models can be condensed into two relations of Y and R, taking as given G' or G, M, and θ.

$$L(r, 1/\hat{\mu})\hat{\mu}Y = M. \tag{10}$$

This is the long-run "LM" curve, which we shall denote LLM.

$$\text{(I)} \quad G' - \theta Y = \dot{D} = 0,$$

$$\text{(II)} \quad r(\hat{\mu}Y - M - F(R)Y) + G - \theta Y = \dot{D} = 0. \tag{11}$$

This is the long-run budget-balance equation, to be denoted as GT. It also takes into account full adjustment of W and K to Y and R, which with M given implies a value for D. But this D does not meet portfolio preferences unless (10) is also satisfied.

The slope of LLM is given by

$$\left(\frac{\partial R}{\partial Y}\right)_{LLM} = -\frac{M/Y}{L_1\hat{\mu}Y(1-\theta)} > 0. \tag{12}$$

The slope of GT is

$$\text{(I)} \quad \left(\frac{\partial R}{\partial Y}\right)_{GT} = \infty,$$

$$\text{(II)} \quad \left(\frac{\partial R}{\partial Y}\right)_{GT} = \frac{(G - rM)/Y}{(1-\theta)(D - RF'(R)Y)}. \tag{13}$$

For non-negative D, the denominator of the slope for Model II is certainly positive. The numerator is also positive if G exceeds the hypothetical after-tax interest on the money stock, negative otherwise. $(G - rM)/Y$ is equal to $\theta - \hat{\mu}r + \alpha(1 - \theta)$, and is the rate at which an increase of Y raises the budget surplus with R constant, starting from a position of budget balance.

We are interested in the shift of GT for an increase of G' or G.

$$\text{(I)} \quad \left(\frac{\partial Y}{\partial G'}\right)_{GT} = \frac{1}{\theta},$$

$$\text{(II)} \quad \left(\frac{\partial Y}{\partial G}\right)_{GT} = \frac{1}{(G - rM)/Y}. \tag{14}$$

In Model I the long-run "multiplier" is simply the reciprocal of the tax rate. In Model II the multiplier is positive if GT is upward sloping, but GT shifts left if it is downward sloping. With non-negative D, GT always shifts downward;

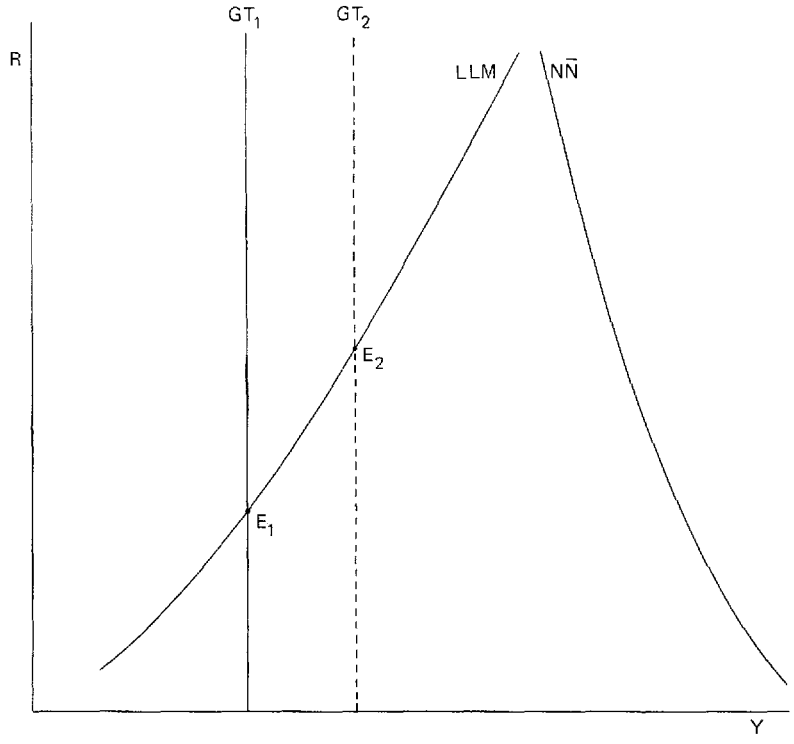

FIGURE 1. Long-run equilibria for Model I.

when G increases, R must decrease in order to lower debt interest and keep the budget balanced.

Model I is pictured in figure 1. To the right of GT, the budget deficit $\dot{D}$ is negative; to the left, it is positive. Above LLM – given M and assuming that W and K are fully adjusted to R and Y – the stock of debt D is too small; the public would like to exchange money for debt. Below LLM, D is too large. As the shift of GT illustrates, an increase of G' leads to higher equilibrium values for both Y and R.

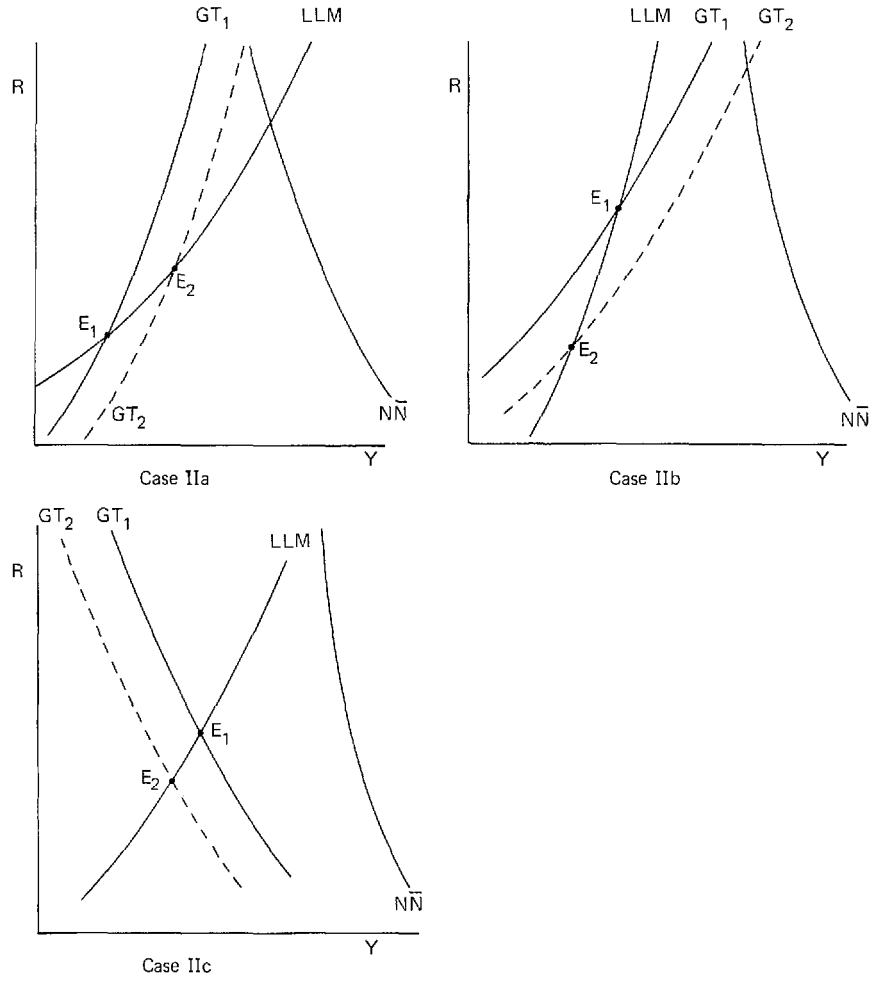

FIGURE 2. Long-run equilibria in Model II.

Model II is pictured in the three panels of figure 2. Case IIa is little different from Model I. In case IIb, the *LLM* curve is the steeper. The comparative statics suggests that fiscal expansion diminishes Y and R, but perverse results of this type make one doubt the stability of the equilibria. In case IIc, GT is negatively sloped, the comparative static result is again "perverse" and there is reason to doubt stability.

These comparative statics apply, of course, only for $N \leqq \overline{N}$. Equilibrium demand for labor depends directly not only on Y but also on R, because higher R means use of labor-intensive technique. The full employment ceiling to Y is shown in figures 1 and 2, as $N\overline{N}$. A demand equilibrium on the far side of $N\overline{N}$ means an inflationary gap. One way it can be eliminated is reduction of the real stock of money by inflation; we discuss this case in section 6.

4.2 Temporary Solutions and Dynamics: Stability in Model I

Our dynamic story is the familiar *IS–LM* tale. We postulate conventional *IS* and *LM* loci for the Keynesian short run, i.e., for given stocks of wealth, debt, capital, and money. The solution of this system determines the momentary values of Y and R. As the stocks change, the solution changes. The question is whether this process leads to the stationary equilibrium.

The short-run *LM* relation is implicit in the equation $L(r,\ Y/W)W = M$, holding W constant. Its short-run slope, $-L_2/\{L_1(1 - \theta)W\}$, is less steep than its long-run slope, $\{-L(r,\ 1/\hat{\mu})\hat{\mu}\}/\{L_1(r,\ 1/\hat{\mu})(1 - \theta)W\}$, on the usual assumption, empirically supported, that the short-run income elasticity of demand for money, $L_2 Y/LW$ is less than unity. It is convenient to put the *LM* curve in explicit form (suppressing θ so long as it is being held constant),

$$R = R(Y,\ W,\ M); \qquad R_1, R_2, > 0; \quad R_3 < 0. \tag{15}$$

The short-run slope is R_1; the long-run slope is $R_1 + \hat{\mu}R_2$.

The short-run *IS* locus is derived from the usual identity that capital accumulation equals the sum of private and public saving. Since asset revaluations have been assumed away, private saving is equal to $\dot{W}$. The identity differs between Models,

$$\begin{aligned} &\text{(I)} \quad \dot{K} + G' = \dot{W} + \theta Y, \\ &\text{(II)} \quad \dot{K} + G = \dot{W} + \theta Y - rD. \end{aligned} \tag{16}$$

We assume investment and saving functions of the stock-adjustment type,

$$\begin{aligned} \dot{K} &= i(F(R)Y - K), \\ \dot{W} &= s(\hat{\mu}Y - W). \end{aligned} \tag{17}$$

The IS curves are combinations of (16) and (17),

$$\begin{aligned} &\text{(I)} \quad Y(s\hat{\mu} + \theta - iF(R)) = G' + sW - iK, \\ &\text{(II)} \quad Y(s\hat{\mu} + \theta - iF(R)) - Dr = G + sW - iK. \end{aligned} \tag{18}$$

The coefficient of Y in (18) is the reciprocal of the conventional multiplier m. We assume throughout that $\hat{\mu}$ exceeds $F(R)$ in the range of relevant values: the desired wealth/income ratio exceeds the desired capital/output ratio. Indeed we shall assume that $1/m$ is positive even if $i > s$. We wish to bypass questions of short-short-run dynamics and instability. Nevertheless, in Model II the IS curve may become upward sloping for high stocks of debt. This can be seen from

$$\begin{aligned} &\text{(I)} \quad \left(\frac{\partial R}{\partial Y}\right)_{IS} = \frac{1/m}{YiF'(r)} < 0, \\ &\text{(II)} \quad \left(\frac{\partial R}{\partial Y}\right)_{IS} = \frac{1/m}{YiF'(R) + D(1-\theta)}. \end{aligned} \tag{19}$$

Consider first the dynamics of Model I. Figure 3 duplicates figure 1 for LLM and GT curves, and in addition shows the short-run LM_1 and IS_1 curves corresponding to the initial equilibrium at E_1, i.e., corresponding to the equilibrium stocks W, K, D, M at E_1 and to the initial value of G'. The short-run loci naturally intersect at E_1. Now suppose that G' is increased in one step, shifting the GT locus and indicating a new long-run equilibrium E_2.

The immediate impact is to shift the IS curve to IS_{12}, producing a short-run solution S_{12}. This is evanescent, of course, because stocks do not remain at their initial E_1 values. The question now is whether the configuration of figure 1 is stable.

At the new equilibrium E_2, with higher Y and R, the public will hold more wealth, but smaller fractions of wealth in money and in capital ($\partial L/\partial R$ and $\mathrm{d}F/\mathrm{d}R$ are both negative). Consequently the volume of debt will be higher, both absolutely and as a proportion of wealth and income. A dynamic path from E_1 to S_{12} to E_2 must involve deficits which achieve this accumulation of debt.

As figure 3 is drawn, IS_{12} is shifted horizontally from E_1 by less than the shift of GT. This means that the short-run multiplier m is less than the long-run multiplier $1/\theta$, and it guarantees that at S_{12} the budget is in deficit. It is conceivable that m exceeds $1/\theta$, if the short-run marginal propensity to invest $iF(R)$ is high. But m would have to exceed $1/\theta$ by some margin to place S_{12} in the budget surplus region to the right of GT_2.

Taking $m < 1/\theta$ as pictured, at S_{12} both W and D are increasing. LM will be shifting up. The growth of wealth also tends to shift IS up (via the sW term). We

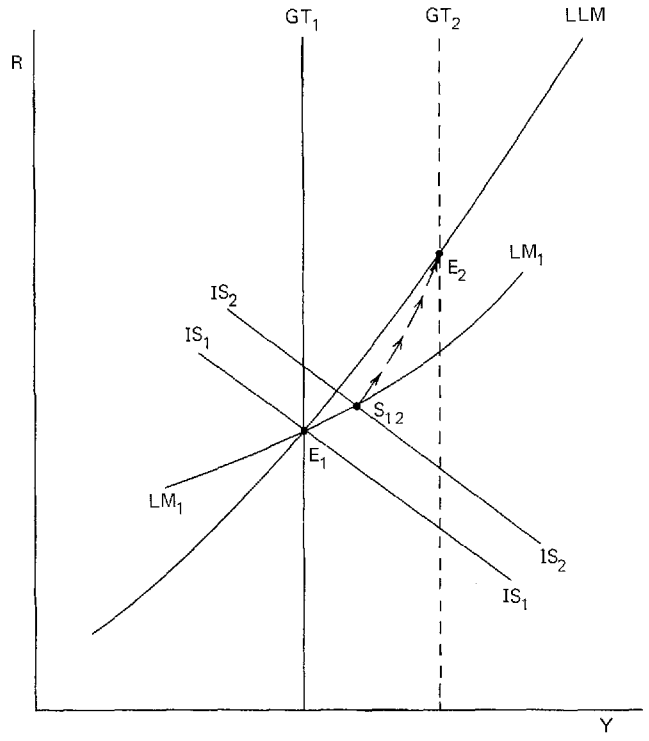

FIGURE 3. Short- and long-run equilibria in Model I.

cannot be sure whether $\dot{K}$ is positive or negative at S_{12}. Compared with E_1, the increase in Y raises the demand for capital but the increase in R lowers it.

A path from S_{12} to E_2 in figure 3 seems plausible. On such a path stocks of assets other than M are increasing and shifting upward both IS and LM. But the dynamics are not easy to display graphically, and the pictured path is not the only possibility. We turn to formal stability analysis.

The system consists of four equations in (K, D, Y, R):

$$\left.\begin{aligned} \dot{K} &= i(F(R)Y - K) \\ \dot{D} &= G' - \theta Y \\ 0 &= Y(s\hat{\mu} + \theta - iF(R)) - sM - sD - (s-i)K - G' \\ 0 &= R - R(Y, M + D + K, M) \end{aligned}\right\} \tag{20}$$

The local stability of the system can be analyzed from the following characteristic quadratic equation in λ:

$$\begin{vmatrix} -i-\lambda & 0 & iF(R) & iYF'(R) \\ 0 & -\lambda & -\theta & 0 \\ -(s-i) & -s & 1/m & -iYF'(R) \\ -R_2 & -R_2 & -R_1 & 1 \end{vmatrix} \tag{21}$$

Here $1/m$ is as before shorthand for the coefficient of Y in the third equation of (20).

Considering (21) as $(a\lambda^2 + b\lambda + c = 0)$, sufficient and necessary conditions for local stability – the real parts of both roots negative – are that both b/a and c/a exceed zero. Both conditions are met on assumptions already made, that the multiplier m is positive, that $\hat{\mu} > F(R)$ (so that there is room in portfolios for assets other than capital), that $F'(R) \leqq 0$, and that R_1 and R_2 are both positive. It turns out that

$$\begin{aligned} a &= (1/m) - R_1 iYF'(R) > 0, \\ c &= \theta is > 0, \\ b &= \theta s + \theta i + is(\hat{\mu} - F(R)) - isF'(R)Y(R_1 + \hat{\mu}R_2). \end{aligned} \tag{22}$$

All the terms of b are positive under the assumptions. Note that, in contrast to the Blinder–Solow conditions, no restriction on the relative sizes of s and i, other than the one required to keep the multiplier positive, is part of the sufficient conditions for stability.

4.3. *Stability in Model II*

The system of four equations is

$$\left.\begin{aligned} \dot{K} &= i(F(R)Y - K) \\ \dot{D} &= G + rD - \theta Y \\ 0 &= Y(1/m) - sM - (s+r)D - (s-i)K - G \\ 0 &= R - R(Y, M + K + D, M) \end{aligned}\right\}. \tag{23}$$

The characteristic equation is

$$\begin{vmatrix} -i-\lambda & 0 & iF(R) & iYF'(R) \\ 0 & r-\lambda & -\theta & D(1-\theta) \\ -(s-i) & -(s+r) & 1/m & -iYF'(R) - D(1-\theta) \\ -R_2 & -R_2 & -R_1 & 1 \end{vmatrix} = 0. \tag{24}$$

Calling this equation $a'\lambda^2 + b'\lambda + c'$,

$$a' = a - R_1 D(1-\theta) = (1/m) - R_1(iYF'(R) + D(1-\theta)), \tag{25}$$

$$b' = b + r(s\hat{\mu} - iF(R)) + rR_1 iYF'(R) - D(1-\theta)(s\hat{\mu}R_2 + (s+i)R_1),$$

$$c' = is(\theta - r\hat{\mu} + rF(R)) - is(R_1 + \hat{\mu}R_2)(D(1-\theta) - rYF'(R)).$$

For interpretation, we recall the slopes of GT and LLM given above in (12) and (13) and restate them in the symbols of this section of the paper,

$$\left(\frac{\partial R}{\partial Y}\right)_{LLM} = R_1 + R_2\hat{\mu}, \tag{12'}$$

$$\left(\frac{\partial R}{\partial Y}\right)_{GT} = \frac{\theta - \hat{\mu}r + rF(R)}{D(1-\theta) - rF'(R)Y}. \tag{13'}$$

Given the non-negativity of the denominator in (13') and the expression for c' in (25),

$$\left(\frac{\partial R}{\partial Y}\right)_{GT} \gtreqless \left(\frac{\partial R}{\partial Y}\right)_{LLM}$$

as

$$(\theta - \hat{\mu}r + F(R)) - (R_1 + \hat{\mu}R_2)(D(1-\theta) - rF'(R)Y) \gtreqless 0,$$

i.e., as $c' \gtreqless 0$. Hence in case IIa, c' is positive, while in cases IIb and IIc, c' is negative.

The value of a' is the effect of G on the short-run solution Y, specifically $\partial Y/\partial G$ calculated from the third equation of (23). The normal expectation is that it is positive; indeed the issue under discussion is whether this positive effect is temporary or not. With positive a', we know that in cases IIb and IIc one of the roots λ is positive. With normal short-run effects, therefore, the stationary equilibrium is unstable in those cases, as previously conjectured.

As for case IIa, with a' positive the equilibrium will be stable if b' is positive, unstable otherwise. Now even with D low enough to make a' positive, b' may be negative. Thus IIa may be either stable or unstable.

A negative value of a' seems at first glance to reverse these stability findings, making IIb and IIc possibly stable. Then one might conclude that fiscal expansion is contractionary both in short run and long run! But this conclusion is illusory. The *IS–LM* solution itself is unstable under usual assumptions about short-run dynamics. This may be seen as follows:

Let the *IS* slope [equation (19), Model II] be $(1/m)/z$. Then $a' = 1/m - R_1 z$. R_1 is the *LM* slope. Here are the possibilities:

	$m > 0$	$m < 0$
$z < 0$	(i) $a' > 0$	(iii) if $a' < 0$ $(1/m)/z > R_1 > 0$
$z > 0$	(ii) if $a' < 0$ $0 < (1/m)/z < R_1$	(iv) $a' < 0$ $(1/m)/z < 0 < R_1$

(i) is the normal case already discussed. In case (ii) an increase in G shifts the IS curve right; it is upward sloping but flatter than LM. In cases (iii) and (iv) an increase in G shifts the IS curve left. In (iii) it is upward sloping but steeper than LM. In (iv) it is downward sloping. The four cases are pictured in figure 4. The short-run dynamics indicated by the arrows follow the usual assumption that R always moves towards LM, while Y moves towards IS if $m > 0$ and away from it if $m < 0$. On this assumption, case (i) is the only stable configuration. (Cases not shown, which are also stable, involve $a' > 0, z < 0$.)

4.4. Variation in Tax Rate

We briefly consider the long-run effects in Model I of a third type of pure fiscal policy: changing the tax rate θ, keeping G' constant and financing temporary deficits by debt issue.

A reduction in θ shifts the vertical GT line of Model I to the right. The long-run multiplier $\partial Y/\partial\theta$ is $-Y/\theta$. But the LLM curve is not invariant to this type of fiscal action. It may shift either way. On the one hand, a tax reduction increases, for given R and Y, the opportunity cost of holding money; this effect moves LLM right. On the other hand, tax reduction increases after-tax human wealth and therefore, other things equal, raises $\hat{\mu}$. This effect moves LLM left. The net effect is uncertain, and so we cannot exclude the possibility that equilibrium R will be lower with lower θ. The stability analysis is the same as in section 4.2.

5. Monetary Policy in the Long Run

Of the several possible meanings of monetary policy in this framework, we shall consider two. The first is a one-shot open market purchase. The second is a combination of fiscal and monetary expansion, a once-for-all increase in government expenditure with deficits financed by printing money instead of issuing interest-bearing debt.

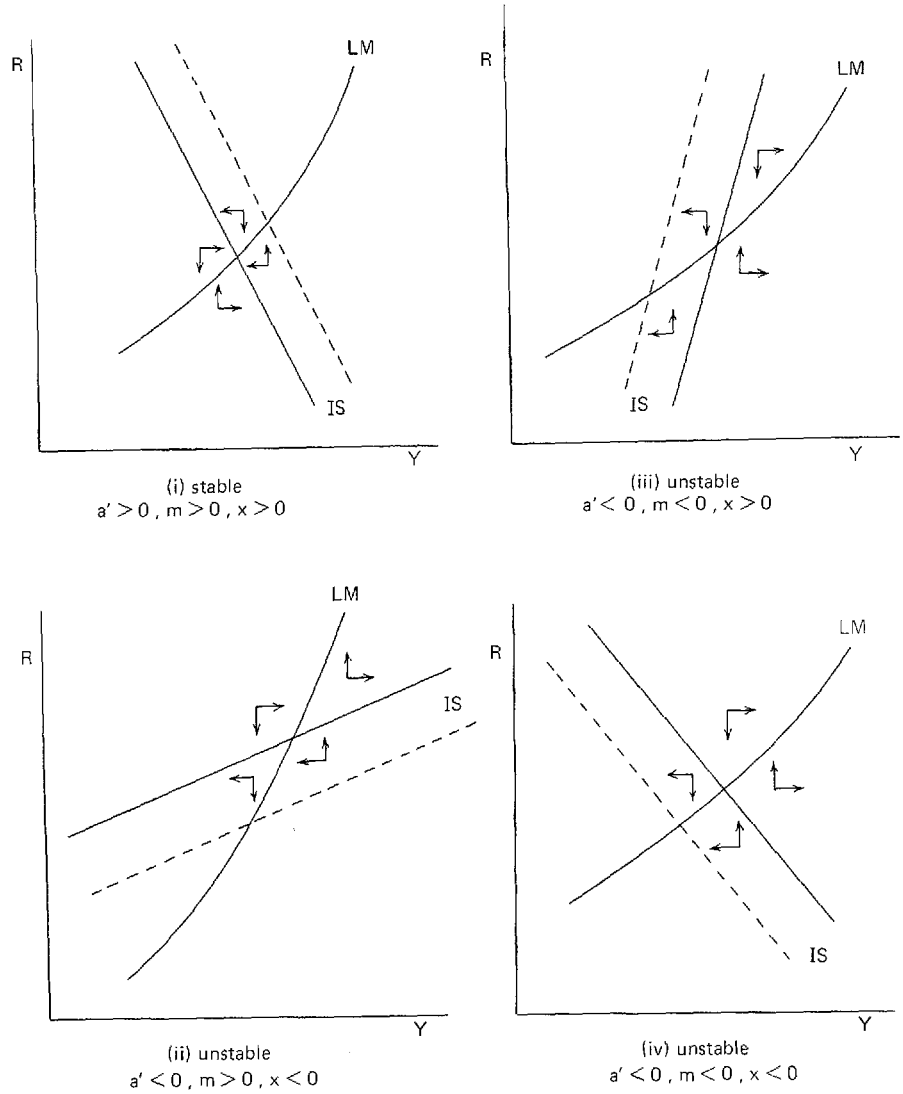

FIGURE 4. Stability or instability of short-run *IS–LM* solution.

5.1. *Open Market Purchase of Bonds*

The story of the open market purchase is pictured in figure 5. The framework is Model I, in which $G' = G + rD$ is the parameter of budget policy. The economy starts in long-run equilibrium E_1, with the associated short-run curves LM_1 and IS_1. The new long-run curve LLM_2 implies a new equilibrium E_2, with unchanged Y and lower R. The new stock equilibrium involves the same wealth $W = \hat{\mu}Y$, but a different portfolio. At E_2 the public will hold more money, and less debt-cum-capital, than at E_1. But they will also hold more capital, because the

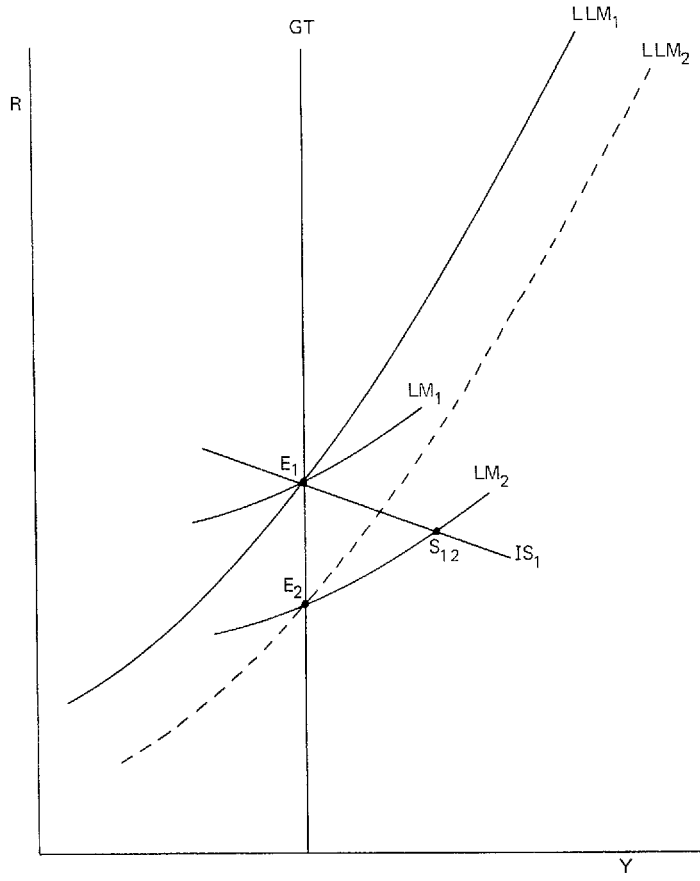

FIGURE 5. Short- and long-run effects of open market purchase in Model I.

desired capital/output ratio $F(R)$ is increased by the reduction in R. Thus ultimately there are two substitutions against debt, one of money in the initial open market operation and one of capital during the process of adjustment.

The shift of short-run LM accomplished by the open market purchase reflects only a money-for-debt substitution. The subsequent capital-for-debt substitution does not affect the LM curves. That is why LM_2 and LLM_2 cross the vertical GT line at the same point. The IS curve is unaffected by the initial operation. (If capital gains on bonds were taken into account, as in Blinder–Solow, IS would shift up because of the increase in wealth.) The first impact of the monetary expansion is the short-run solution S_{12}. But the increase in income is temporary. The government is now running a surplus, and, as we have already seen, the contraction of debt brings income down. Meanwhile the public is also saving more (wealth is below $\hat{\mu}Y$) and investing more [capital is less than $F(R)Y$]. The temporary increase in wealth retards the decline of R, and is reversed once capital and wealth catch up with income, which is on its way down.

In Model II, higher interest rates and debt accumulation have an expansionary fiscal effect because they enlarge the budget deficit. The story for case IIa is essentially the same as for Model I, except that the open market purchase shifts the initial IS curve down. E_2 represents a *smaller* output than E_1. It may seem paradoxical that monetary expansion is, in the long run, contractionary. We do not think the result should be taken seriously, given that it depends on the assumption that monetary expansion entails a fiscal contraction via reduction of debt interest transfers.

In case IIb, E_2 is at higher levels of Y and R than E_1. But there is no way to get there. The open market purchase shifts down the short-run LM curve, increases Y, and decreases R. But from this point the dynamic previously described moves the temporary solution down and to the left.

In case IIc it is quite possible that the dynamics lead to the new equilibrium E_2 with higher Y and lower R.

5.2. *Fiscal Expansion with Money-Financed Deficits*

We turn briefly to the monetary financing of deficits combined with expansionary fiscal policy. The previous discussion of fiscal policy is now altered by holding D constant instead of M. In Model I, the long-run GT locus is unchanged, still vertical. But the LM locus becomes

$$\hat{\mu}Y - L(r, 1/\hat{\mu})\hat{\mu}Y - F(R)Y = D, \tag{26}$$

$$\left(\frac{\partial R}{\partial Y}\right)_{LLM} = \frac{D/Y}{L_1(1-\theta)\hat{\mu}Y + F'(R)Y} < 0. \tag{27}$$

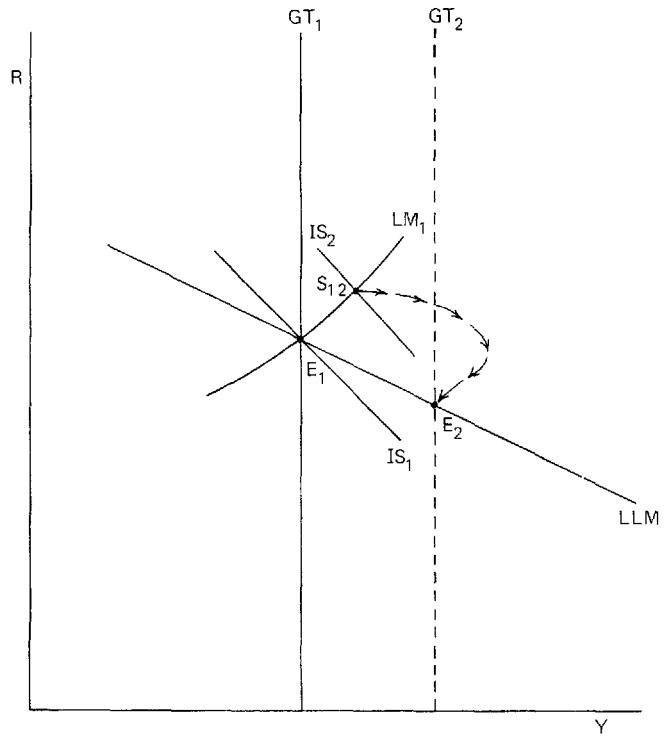

FIGURE 6. Short- and long-run effects of monetized deficit spending in Model I.

Thus we see that the long-run *LLM* locus is negatively sloped. A rightward shift of $\overline{GT}$, as shown in figure 6 is bound to raise equilibrium income Y and lower equilibrium R. The conclusion is not altered in Model II. There the GT locus has an upward slope due the term rD in the budget-balance equation, even though D is constant. But it is still true that a rightward shift in GT raises equilibrium Y and lowers R.

The dynamics are as follows: the short-run LM locus is

$$W - L(R(1 - \theta), Y/W)W - K = D. \tag{28}$$

It is upward sloping, because M does not increase until a budget deficit actually appears,

$$\left(\frac{\partial R}{\partial Y}\right)_{LM} = -\frac{L_2}{L_1(1 - \theta)\ W} > 0. \tag{29}$$

The short-run *IS* curve, defined as before, shifts to the right with the increase of G', but on usual assumptions not as far as the GT line. The temporary solution is at S_{12}. But now there is a budget deficit: M increases, W increases, IS moves up, LM moves down. When the solution crosses GT_2 budget surpluses appear and arrest the expansion.

For formal analysis of the dynamics of this case, it is convenient to express the short-run model in terms of the four variables K, W, Y, and R,

$$\left.\begin{aligned} \dot{K} &= i(F(R)Y - K) \\ \dot{W} &= s(\hat{\mu}Y - W) \\ 0 &= Y(s\hat{\mu} + \theta - iF(R)) - sW + iK - G' \\ 0 &= R - R(K,W,Y) \end{aligned}\right\} . \tag{30}$$

Here the function $R(K,W,Y)$ is implicit in (28), and its partial derivatives are

$$\left.\begin{aligned} R_1 &= \frac{-1}{L_1(1-\theta)W} > 0 \\ R_2 &= \frac{1 - L + L_2(Y/W)}{L_1(1-\theta)W} < 0 \\ R_3 &= \frac{-L_2}{L_1(1-\theta)W} > 0 \end{aligned}\right\} . \tag{31}$$

The characteristic equation is

$$\begin{vmatrix} -i-\lambda & 0 & iF(R) & iF'(R)Y \\ 0 & -s-\lambda & s & 0 \\ i & -s & 1/m & -iF'(R)Y \\ -R_1 & -R_2 & -R_3 & 1 \end{vmatrix} = 0. \tag{32}$$

In the quadratic equation $a\lambda^2 + b\lambda + c$ all three coefficients a, b, c are unambiguously positive, confirming that the comparative static story of figure 6 makes sense.

6. Effects of Fiscal Policy with Full Employment and Flexible Prices

The standard short-run *IS–LM* analysis applies to situations of full employment as well as unemployment, and in this section we extend our long-run analysis to conditions of full employment and flexible prices and wages. In the short run, output is fixed by the full employment labor supply. The standard short-run comparative statics is familiar: Expansionary fiscal policy, with constant nominal

money stock, always raises the interest rate. Unless the demand for money is interest-inelastic and the LM curve vertical, it also raises the velocity of money and the price level. Increased real government expenditure "crowds out" private investment, and possibly also private consumption, to the extent necessary to equate total real demand to fixed real supply. But the short-run analysis does not trace the further effects of these changes in the rates of growth of capital and government debt.

Our long-run full employment model resembles as closely as possible both the long-run models of the previous sections and the standard short-run full employment version of IS–LM analysis. But differences necessarily arise. Once the price level is made endogenous in a long-run model, we must explicitly consider price expectations and distinguish real and nominal rates of return. Moreover, in a long-run full employment model the capital stock is endogenous. Output is not fixed, as it is in the short run, by labor supply; output per worker varies with the capital/output ratio and the real interest rate, as illustrated by the $N\bar{N}$ curves of figures 1 and 2.

Capital and bonds are, as before, perfect substitutes in portfolios. The portfolio choice of money vs. bonds-and-capital depends on the real after-tax rate of return differential between money and bonds. The nominal rate of return on money balances is institutionally fixed at zero. If x is the expected instantaneous proportional rate of change of the price level p, and R the nominal rate of return on bonds, the real rate of return on bonds is $R - x$, and the real after-tax rate of return differential is $R(1 - \theta)$.

Portfolio balance is therefore given by

$$L\left(R(1-\theta),\, Y\Big/\frac{M+D}{P}+K\right)\left(\frac{M+D}{p}+K\right)=\frac{M}{p}. \tag{33}$$

The production function is, as throughout the paper, a well-behaved constant returns to scale neoclassical production function in capital and labor, N,

$$Y = Nf(K/N); \qquad f' > 0, \qquad f'' < 0. \tag{34}$$

Labor is supplied inelastically and is always fully employed. Since we are only considering stationary states we choose units such that $N = 1$.

Rather than specifying the investment function as $I = i[F(R - x)f(K) - K]$, we shall now find it convenient to write it as

$$I = I(f'(K) - (R - x)), \qquad I(0) = 0, \quad I' > 0. \tag{35}$$

This function makes the rate of investment an increasing function of the difference between the rate of return obtainable from investing a dollar in the production of new capital goods and the rate of return on existing assets.

We shall consider 3 simple mechanisms for generating price expectations:

Static expectations: $x(t) = 0,$ (36a)

Myopic perfect foresight: $x(t) = \dot{p}(t)/p(t),$ (36b)

Adaptive expectations: $\dot{x}(t) = \beta[(\dot{p}/p) - x(t)]; \quad \beta > 0.$ (36c)

Government consumption expenditure is fixed in real terms. With bond-financed deficits, the government budget restraint is therefore given by

(I) $\dot{D}/p = G' - \theta f(K),$ (37a)

(II) $\dot{D}/p = G + (1 - \theta)R\,(D/p) - \theta f(K).$ (37b)

For reasons of space we shall consider only Model I.

6.1. The Long-Run Equilibrium

The complete dynamic model is

$$(IS) \qquad I[f'(K) - R + x] + G' - \theta f(K) - s\left[\hat{\mu} f(K) - \left(\frac{M + D}{p}\right) - K\right] - x\left(\frac{M + D}{p}\right) = 0. \tag{38}$$

The last term in (38) represents expected additions, positive or negative, to wealth, due to changes in the real values of nominal stocks of money and debt. It is assumed that current saving from income is adjusted correspondingly.

$$(LM) \qquad L\left[R(1 - \theta), \frac{pf(K)}{M + D + pK}\right]\left[\frac{M + D}{p} + K\right] = \frac{M}{p}, \tag{39}$$

$$\left(\frac{\dot{D}}{p}\right) = G' - \theta f(K) - \frac{\dot{p}}{p}\frac{D}{p}, \tag{40}$$

$$\left(\frac{\dot{M}}{p}\right) = -\frac{\dot{p}}{p}\frac{M}{p}, \tag{41}$$

$$\dot{K} = I[f'(K) - R + x], \tag{42}$$

$$x = 0, \tag{43a}$$

$$x = \frac{\dot{p}}{p}, \tag{43b}$$

$$\dot{x} = \beta\left[\frac{\dot{p}}{p} - x\right]. \tag{43c}$$

In a stationary long-run equilibrium, expectations are realized, momentary equilibrium holds at each point of time, and real stocks and flows remain constant. Since we are considering only fiscal policies with a constant nominal quantity of money, actual and expected rates of inflation must be zero in long-run equilibrium. In summary,

$$\dot{p}/p = x = \dot{x} = \dot{D} = \dot{K} = 0.$$

The choice of expectations function is irrelevant for the comparative static results of the model, although it crucially affects the dynamics. The stationary state equilibrium is completely described by the following four equations, which as it happens are recursive, in K, R, p and D,

$$(GT) \qquad G' = \theta f(K), \tag{44}$$

$$R = f'(K), \tag{45}$$

$$(LLM) \qquad L\left[R(1-\theta), \frac{1}{\hat{\mu}}\right]\hat{\mu}f(K) = \frac{M}{p}, \tag{46}$$

$$\hat{\mu}f(K) = \frac{M+D}{p} + K. \tag{47}$$

Equation (45) gives a stationary state relationship between K and R,

$$K = g(R), \qquad g'(R) = 1/f'' < 0.$$

We can therefore represent the long-run equilibrium in $R - p$ space by the GT and LLM curves,

$$G' = \theta f(g(R)),$$

$$L\left[R(1-\theta), \frac{1}{\hat{\mu}}\right]\hat{\mu}f(g(R)) = \frac{M}{p}.$$

These are illustrated in figure 7.

The slope of the LLM curve is

$$\left(\frac{dR}{dp}\right)_{LLM} = \frac{-Mf''(K)}{[\hat{\mu}f(K)L_1(1-\theta)f''(K) + L\hat{\mu}f'(K)]p^2} > 0.$$

The slope of the GT curve is

$$\left(\frac{dR}{dp}\right)_{GT} = 0.$$

An increase in G' from G_1' to G_2' shifts the GT curve down along the LLM curve

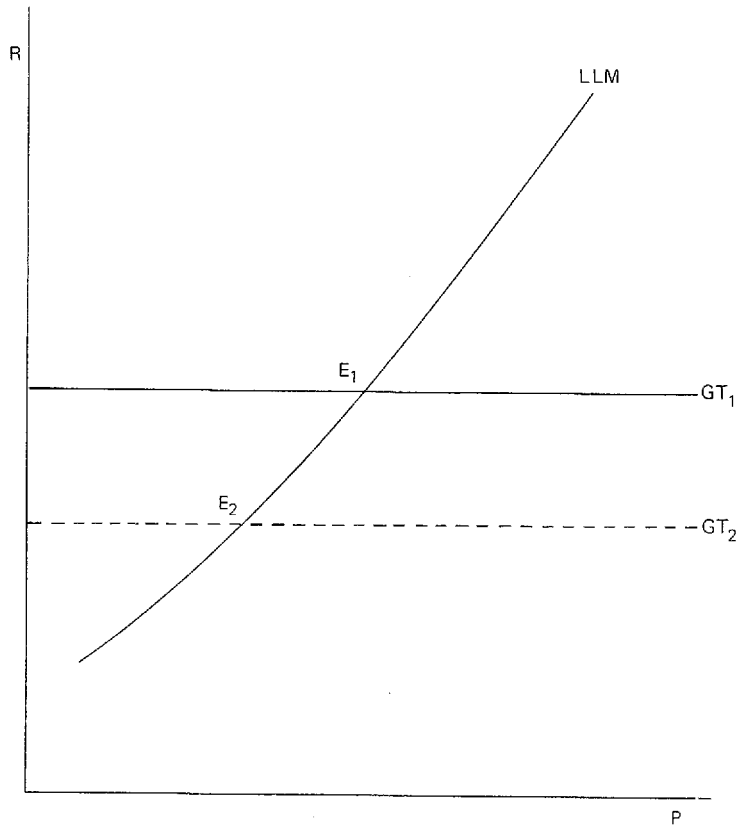

FIGURE 7. Long-run equilibria in the full employment model.

in figure 7, lowering equilibrium R and p. Algebraically,

$$\text{(a)}\ \frac{\partial R}{\partial G'} = \frac{f''}{\theta f'} < 0,$$
$$\text{(b)}\ \frac{\partial p}{\partial G'} = -\frac{L_1 \hat{\mu} f f''(1-\theta) + L\hat{\mu} f'}{\theta f'(K)\,(M/p^2)} < 0. \tag{48}$$

Note that even in the full employment model $\partial Y/\partial G' > 0$; the reason here, however, is capital deepening rather than the elimination of unemployment of labor.

Note also that the effectiveness of fiscal policy here is not at all dependent on the fact that government interest-bearing debt has been counted as part of private sector net worth. Even if the capitalized value of future taxes "required" to service the debt were exactly equal to the value of these bonds – and there are many sound economic reasons for arguing against such a complete offset – the balanced budget condition: $G' = \theta f(K)$ will guarantee the long-run effectiveness of fiscal policy, in a comparative-static sense.

The intuitive story behind equations (16) is simple: An increase in G' requires, given θ, a higher level of income to balance the budget. With full employment and a fixed labor force, this means a larger capital stock. This in turn requires a lower R and a higher stationary state level of wealth. The LLM curve shows that both these effects will increase the demand for real money balances. Since the nominal stock of money is fixed, the price level must be lower to increase the real value of the fixed nominal quantity of money. Prima facie these results would seem to be unstable. As we observed at the outset of this section, the impact effect of an increase in G' is to shift the short-run IS curve to the right and to raise R and p. For the long-run equilibrium to be stable, these impact effects have to be reversed, implying that the economy "overshoots" in the short run. Nevertheless, under certain conditions, which depend crucially on the expectations mechanism and on the precise numerical values of certain coefficients, the model may be stable.

6.2. *The Impact Effects*

Figure 8 shows the impact effect of an increase in government expenditure G'. The slope of the IS curve is

$$\left(\frac{\mathrm{d}R}{\mathrm{d}p}\right)_{IS} = -\frac{(s-x)(M+D)}{I'p^2}.$$

When we are considering the IS curve that goes through the long-run equilibrium, $x = 0$ and

$$\left(\frac{\mathrm{d}R}{\mathrm{d}p}\right)_{IS} = -\frac{s(M+D)}{I'p^2} < 0.$$

The impact effect of an increase in G' will be to shift the IS curve to the right,

$$\left(\frac{\partial R}{\partial G'}\right)_{IS} = \frac{1}{I'} > 0.$$

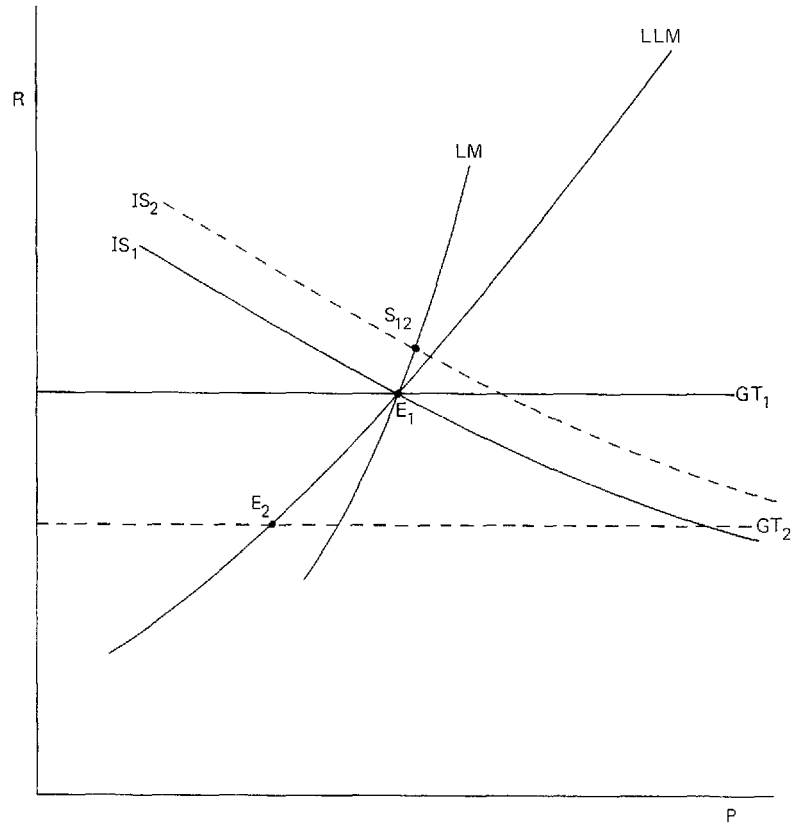

FIGURE 8. Short- and long-run effects of expansionary fiscal policy in the full employment model.

The slope of the LM curve is

$$\left(\frac{\mathrm{d}R}{\mathrm{d}p}\right)_{LM} = \left(-L_2\frac{f(K)}{p}\left[1-\frac{K}{W}\right]-\frac{MK}{Wp^2}\right)\Big/WL_1(1-\theta) > 0.$$

The impact effect of an increase in G' – given M, D, K, x, θ and the new higher level of G' – is a new temporary equilibrium, as shown in figure 8, (before stocks and expectations have had time to change) at S_{12} with higher p and R. At S_{12}, D will be increasing and K will be decreasing. Except in the case of static expectations, $\dot{x}$ will be positive. The new long-run equilibrium, however, is at E_2, with lower R and p than at E_1.

6.3. *Stability*

The tedious mathematics of the local stability conditions for the full employment model are relegated to the appendix. To summarize, the long-run equilibrium is unstable in case of static expectations [(43a)] (6.11a) and potentially but not necessarily stable in cases of myopic perfect foresight [(43b)]. In the intermediate case of adaptive expectations [(43c)], stability requires but is not guaranteed by a finite minimum speed of adaptation.

These results may seem paradoxical. Usually static expectations are considered stabilizing, while quick translation of actual price experience into expectations is considered destabilizing. The opposite conclusion here is related to the difference in direction between short- and long-run effects, as exhibited in figure 8. This means that some over-shooting is necessary for the long-run equilibrium to be stable.

The apparent paradox arises from the endogeneity of K and Y, and the related endogeneity of $\dot{D}$ and D. At a position like S_{12} in figure 8, the Pigou effect is pulling consumption down, and the increase in $R - x$ is unfavorable to investment. On the other hand, the real deficit is positive and D is growing, an effect accentuated as real tax revenues decline along with the capital stock K and real income Y. The latter effect, which is the source of instability, is absent from the short-run story. The expected rate of inflation itself has two effects on short-run aggregate demand along the adjustment path. One is to raise investment by lowering the real rate of interest, and the other is to increase saving to make up for expected real capital losses on money and debt. A necessary, but not sufficient, condition for stability is that the investment effect is the stronger. This means that if the rate of inflation slows down and if the slowdown is translated fairly promptly into expectations, aggregate demand and the income-related demand for money will weaken. A weakening of aggregate demand leads, in an economy with flexible prices, to a reduction in the price level and the real interest rate and to an increase in capital stock and real income. Unfortunately, the issue of stability turns on relatively minor details of specification and on small differences in values of coefficients. It is possible that fiscal expansion sets off an unstable spiral of inflation, deficits, rising interest rates, and dwindling capital stock and output. It is also possible that is the start of an oscillation that converges to an equilibrium with lower price level, lower interest rate, higher capital stock and income.

7. Conclusion

Nothing in the analysis of this paper supports the claim that expansionary fiscal effects on aggregate demand are only transitory. To investigate the question, the

main section of this paper focussed on the pure logic of aggregate demand. Supply constraints were assumed away – there is always labor available to produce the output demanded. In this situation, an increase in government expenditure either leads to a new long-run equilibrium with higher real income or, in unstable cases, to an explosive increase in income and interest rate. We do not stress the latter possibility, since it depends on built-in fiscal expansion via debt interest payments and since the economy would sooner or later hit a full employment ceiling.

Interest-inelasticity of demand for money seems to be crucial for the strong monetarist proposition after all. A fiscally driven expansion could, of course, occur but vanish if the short-run *LM* curve is not vertical while the long-run *LLM* curve is vertical. We are not aware that this argument has been made.

In future work on this subject, not motivated by any particular propositions, monetarist or otherwise, we would embed the moving short-run equilibria in a growth model rather than a stationary state. (The growth model is more confining. A stationary-state equilibrium just requires zero changes of stocks, and there are lots of configurations of stocks consistent with that condition. An equilibrium growth path requires that flows stand in the same relation to each other as the corresponding stocks.) Moreover, we would find more congenial a model which allows debt and capital to be imperfect substitutes with distinct rates of return. Clearly a more satisfactory model would also recognize that even when the economy is not at full employment nominally denominated stocks change in real value from price movements as well as from fiscal and monetary policies.

In section 6 we considered the long-run effects of fiscal expansion in an economy with full employment and flexible prices. Here there is a striking difference between impact and ultimate effects. The new long-run equilibrium, after a permanent increase of government expenditure, has larger real income and capital stock but lower price level and interest rate. But such equilibria are stable, if at all, only if price expectations adapt fairly quickly to price experience.

Finally, we observe again that it is disturbing that the qualitative properties of models – the signs of important system-wide multipliers, the stability of equilibria – can turn on relatively small changes of specification or on small differences in values of coefficients. We do not feel entitled to use the "correspondence principle" assumption of stability to derive restrictions on structural equations and parameters. There is no divine guarantee that the economic system is stable.

Appendix: Stability in the Full Employment Model

We solve the short-run *LM* and *IS* equations for R and P as functions of D, K and x, given M, G' and θ and evaluate these solutions at the long-run equilibrium $(D^*, K^*, 0)$,

$$R = h^1(D, K, x; M, G', \theta), \tag{A-1a}$$

$$P = h^2(D, K, x; M, G', \theta). \tag{A-1b}$$

The reduced-form impact multipliers are solved for from

$$\begin{bmatrix} -I' & \dfrac{-s(M+D)}{p^2} \\ WL_1(1-\theta) & \dfrac{L_2 f(K)}{p}\left(1-\dfrac{K}{W}\right)+\dfrac{MK}{Wp^2} \end{bmatrix} \begin{bmatrix} \mathrm{d}R \\ \mathrm{d}P \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{-s}{p}\mathrm{d}D + ((\theta + s\hat{\mu})f'(K) - I'f''(K) - s)\mathrm{d}K - \left(I' - \dfrac{(M+D)}{p}\right)\mathrm{d}x \\ \left(\dfrac{L_2 f(K)}{pW} - \dfrac{L}{p}\right)\mathrm{d}D + \left(-L_2 f'(K) + \dfrac{L_2 f(K)}{W} - L\right)\mathrm{d}K \end{bmatrix}$$

The signs of these derivatives are not all determined, given the a priori restrictions we have imposed so far,

$h^1_D > 0$,
h^1_K is undetermined; if $(\theta + s\hat{\mu})f' - I'f'' - s < 0$, $h^1_K > 0$,
h^2_D is undetermined,
h^2_K is undetermined; if $(\theta + s\hat{\mu})f' - I'f'' - s > 0$, $h^2_K < 0$,
h^1_x is undetermined; $h^1_x > 0$ if and only if $I' - (M + D)/p > 0$,
h^2_x is undetermined; $h^2_x > 0$ if and only if $I' - (M + D)/p > 0$.

We now consider stability for the three mechanisms for generating expectations:

Static expectations: $x = 0$

The dynamic equations are

$$\dot{D} = p[G' - \theta f(K)], \tag{A-2a}$$

$$\dot{K} = I[f'(K) - R]. \tag{A-2b}$$

Substituting (A-1a) and (A-1b) into these equations, and taking the linear approximation at the long-run equilibrium (D^*, K^*) we get

$$\begin{bmatrix} \dot{D} \\ \dot{K} \end{bmatrix} = \begin{bmatrix} 0 & -p\theta f' \\ -I'h^1_D & I'[f'' - h^1_K] \end{bmatrix} \begin{bmatrix} D - D^* \\ K - K^* \end{bmatrix}. \tag{A-3}$$

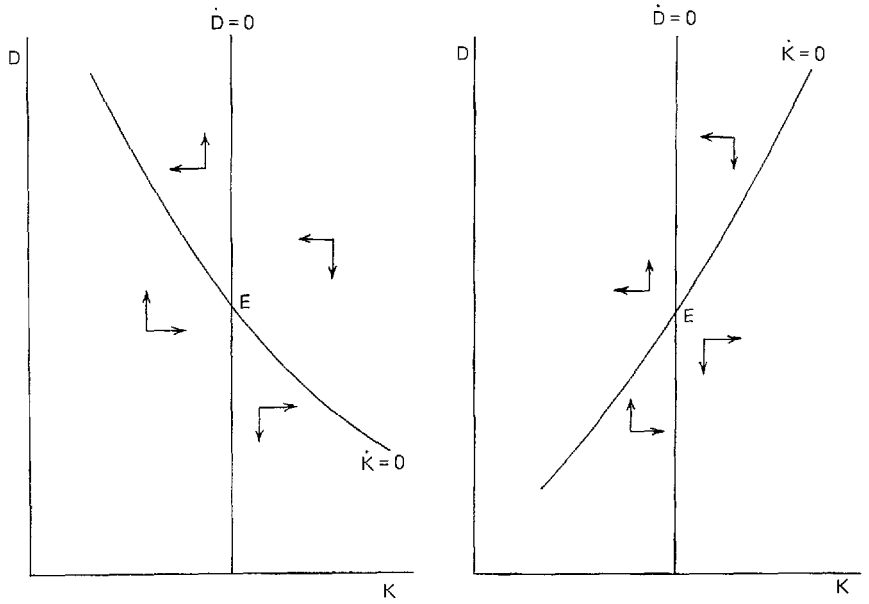

FIGURE 9. Stability in the full employment model under static expectations

Necessary and sufficient conditions for stability are

$$I'(f'' - h_K^1) < 0, \tag{A-4a}$$

$$-I'h_p^1 p\theta f' > 0. \tag{A-4b}$$

(A-4a) may be satisfied, (A-4b) never is; the long-run equilibrium is unstable under static expectations. Figure 9 illustrates this instability with the familiar phase diagram in D, K space.

Myopic perfect foresight: $\dot{p}/p = x$

The complete dynamic system can in this case be written as

$$I\left[f'(K) - R + \frac{\dot{p}}{p}\right] + G' - \theta f(K)$$

$$- s\left[\hat{\mu}f(K) - \frac{M + D}{p} - K\right] - \frac{\dot{p}}{p}\frac{M + D}{p} = 0, \tag{A-5a}$$

$$L\left[R(1-\theta), \frac{pf(K)}{M+D+pK}\right]\left[\frac{M+D}{p}+K\right] = \frac{M}{p}, \tag{A-5b}$$

$$\dot{D} = p[G' - \theta f(K)], \tag{A-5c}$$

$$\dot{K} = I\left[f'(K) - R + \frac{\dot{p}}{p}\right]. \tag{A-5d}$$

We solve (A-5b) for R as a function of p, K and D, given M and θ,

$$R = R(p, D, K), \tag{A-5b'}$$

$$R_p = \left(\frac{-L_2 f(K)}{p}\left[1 - \frac{K}{W}\right] - \frac{MK}{Wp^2}\right)\Big/ WL_1(1-\theta) > 0, \tag{A-6a}$$

$$R_D = \left(\frac{L_2 f(K)}{pW} - \frac{L}{p}\right)\Big/ WL_1(1-\theta) > 0, \tag{A-6b}$$

$$R_K = \left(-L_2 f'(K) + \frac{L_2 f(K)}{W} - L\right)\Big/ WL_1(1-\theta) > 0. \tag{A-6c}$$

Substituting (A-5b) into (A-5a), (A-5c) and (A-5d) and linearizing at the equilibrium (K^*, D^*, p^*) gives

$$\begin{bmatrix}\dot{p}\\ \dot{D}\\ \dot{K}\end{bmatrix} = \begin{bmatrix} \dfrac{\left[I'R_p + s\dfrac{(M+D)}{p^2}\right]p}{I' - \dfrac{M+D}{p}} & \dfrac{\left[-\dfrac{s}{p} + I'R_D\right]p}{I' - \dfrac{M+D}{p}} & \dfrac{[-I'f''(K) + I'R_K + \theta f'(K) + s\hat{\mu}f'(K) - s]p}{I' - \dfrac{M+D}{p}} \\ 0 & 0 & -p\theta f'(K) \\ \dfrac{I'\dfrac{M+D}{p}\left[R_p + \dfrac{s}{p}\right]}{I' - \dfrac{M+D}{p}} & \dfrac{\dfrac{I'}{p}[R_D(M+D) - s]}{I' - \dfrac{M+D}{p}} & \dfrac{I'\left[-f''(K)\dfrac{M+D}{p} + R_K\dfrac{M+D}{p} + \theta f'(K) + s\hat{\mu}f'(K) - s\right]}{I' - \dfrac{M+D}{p}} \end{bmatrix}\begin{bmatrix} p - p^* \\ D - D^* \\ K - K^* \end{bmatrix} \tag{A-7}$$

The characteristic equation of the coefficient matrix can be written as

$$a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0; \qquad a_0 > 0.$$

Necessary and sufficient for all characteristic roots to have negative real parts are

$$\begin{aligned} a_1 &> 0, \\ a_2 &> 0, \\ a_3 &> 0, \\ a_1a_2 - a_0a_3 &> 0, \end{aligned}$$

One of the first two inequalities can be eliminated since it is implied by the remaining three.

The characteristic equation of the system is

$$\lambda^3 + \left(\frac{-I'\left[\frac{(M+D)}{p}(R_K - f'') + f'(\theta + s\hat{\mu}) - s\right] - \left[I'R_P + s\frac{(M+D)}{p^2}\right]p}{I' - \frac{M+D}{p}} \right)\lambda^2$$

$$+ I'\left(\frac{\theta f'[R_D(M+D) - s] + R_P[f'(\theta + s\hat{\mu}) - s]p + \frac{s}{p}(M+D)(f'' - R_K)}{I' - \frac{M+D}{p}} \right)\lambda$$

$$+ \frac{s\theta I'f'[pR_P + (M+D)R_D]}{I' - \frac{M+D}{p}} = 0.$$

A detailed analysis of the necessary and sufficient conditions for stability would require a lot of space for rather little additional insight; some brief remarks will suffice:

$a_3 > 0$, requires $I' - (M + D)/p > 0$: the effect of an increase in the expected rate of inflation is to create excess demand in the goods market. As we shall see this condition is also necessary for stability in the adaptive expectations case.

$a_1, a_2 > 0$, these two conditions set rather strict bounds on the permissible values of s; it has to be large enough to make the numerator of a_1 positive, but small enough to make the numerator of a_2 positive.

$a_1a_2 - a_0a_3 > 0$, no great intuitive insight can be obtained from this condition; it does not contradict any a priori sign restrictions on the coefficients; stability however becomes a rather detailed empirical question.

Adaptive expectations: $\dot{x} = \beta\left(\frac{\dot{p}}{p} - x\right)$

This case too, will turn out to be at least potentially stable. Substituting (A-la) and (A-lb) into the dynamic equations, we get

$$\dot{D} = p[G' - \theta f(K)],$$
$$\dot{K} = I[f'(K) - h^1(D, K, x) + x],$$
$$\dot{x} = \left(\beta \frac{\dot{h}^2(D, K, x)}{h^2(D, K, x)} - x\right).$$

A linear approximation at the equilibrium $(D^*, K^*, 0)$ gives

$$\begin{bmatrix} \dot{D} \\ \dot{K} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & -p\theta f' & 0 \\ -I'h^1_D & I'(f'' - h^1_K) & I'(1 - h^1_x) \\ \dfrac{-\beta h^2_K I' h^1_D}{p - \beta h^2_x} & \dfrac{p\beta}{p - \beta h^2_x}\left[-h^2_D \theta f' + \dfrac{h^2_K}{p} I'(f'' - h^1_K)\right] & \dfrac{p\beta}{p - \beta h^2_x}\left[\dfrac{h^2_K}{p} I'(1 - h^1_x) - 1\right] \end{bmatrix} \begin{bmatrix} D - D^* \\ K - K^* \\ x - 0 \end{bmatrix} \tag{A–8}$$

The characteristic equation is

$$\lambda^3 - \left\{I'(f'' - h^1_K) + \frac{p\beta}{p - \beta h^2_x}\left[\frac{h^2_K}{p} I'(1 - h^1_x) - 1\right]\right\} \lambda^2$$
$$- \left\{I'(f'' - h^1_K)\frac{p\beta}{p - \beta h^2_x} - I'(1 - h^1_x)\frac{p\beta}{p - \beta h^2_x} h^2_D \theta f' + I' h^1_D p\theta f'\right\} \lambda$$
$$- I' h^1_D p\theta f' \frac{p\beta}{p - \beta h^2_x} = 0.$$

The linearized version of the static expectations model (A-3) is found back as the UPPER LEFT 2×2 submatrix in the linearized version of the adaptive expectations model (A-8). The static expectations model can therefore be regarded as a limiting case of the adaptive expectations model, when $\beta = 0$ [and

terms like h_x^1 and h_x^2 are irrelevant because $x \equiv 0$],

$$a_3 = -I'h_D^1 p\theta f' \frac{p\beta}{p - \beta h_x^2} > 0$$

is necessary for stability. Since $h_D^1 > 0$, $a_3 > 0$ iff $p = \beta h_x^2 < 0$, i.e., only if $h_x^2 > 0$; the impact effect of a rise in inflationary expectations is to increase the price level. $h_x^2 > 0$ iff $I' - [(M + D)/p] > 0$ which is the condition we derived for myopic perfect foresight. It is clear that with $\beta = 0$ [the static expectations case] $a_3 = 0$ and the system won't be stable.

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