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GENERATIONAL ACCOUNTS, AGGREGATE SAVING AND INTERGENERATIONAL DISTRIBUTION

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I would like to thank Vito Tanzi and Sheetal K. Chand for enabling me to spend 5 productive and pleasant weeks as a Visiting Scholar in the Fiscal Analysis Division of the Fiscal Affairs Department at the IMF in the summer of 1994. Helpful comments on earlier versions of this paper were received during seminars at the IMF, at the University of Cambridge and at the LSE, especially from Beth Allen, David Newbery, Hashem Pesaran, Alan Marin, Jay Sri Dutta, Ralph Turvey and Tony Venables. This paper is part of NBER's research program in Public Economics. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

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ABSTRACT

Are generational accounts informative about the effect of the budget on the intergenerational distribution of resources and (when augmented with generation-specific propensities to consume out of life-time resources) on aggregate consumption and saving? The paper makes three points. First, the usefulness of generational accounts lives or dies with the strict life-cycle model of household consumption. Voluntary intergenerational gifts or liquidity constraints may therefore adversely affect or even destroy their informativeness. Second, even when the life-cycle model holds, generational accounts only measure the effect of the budget on the lifetime consumption of *private* goods and services. They ignore the intergenerational (re-)distribution associated with the government's provision of public goods and services. Third, generational accounting ignores the effect of the budget on before-tax and before-transfer quantities and prices, including before-tax and -transfer distribution of life-time resources across generations and intertemporal relative prices. That is, it does not handle *incidence* or *general equilibrium repercussions very well.* Although useful, generational accounts should therefore carry the label "handle with great care."

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1 Introduction

In a number of influential papers, Larry Kotlikoff, Alan Auerbach and Jagadeesh Gokhale have extolled the virtues of *generational accounts*, both as the correct way of measuring how the government budget affects intergenerational distribution and as an essential input into the analysis of the effect of the budget on

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saving ([3][4][5]; see also [2][15][16]). In addition to the empirical implementations of this methodology by various subsets of the three aforementioned authors, a not inconsiderable research effort has been undertaken in recent years by a number of national governments and by multilateral organizations such as the IMF and the World Bank, to implement the methodology empirically.

The definition of generational accounts is straightforward. Their empirical implementation makes quite heavy demands on data gathering capacity and involves some quite subtle conceptual problems, many of which are treated extensively in e.g. [3].Generational accounts

> ... are accounts-one for each generation-that tally up, in present value, the amount of receipts less payments the government can expect to collect from each generation over its remaining life span." ([3], p.2).

Strong claims are made by its proponents concerning the merits of the approach:

> "Generational accounting measures directly the amount current and future generations can, under existing public policies, be expected to pay over time in net taxes (taxes paid less transfer payments received) to the government. This type of analysis is essential if we really want to know the burden we are imposing on future generations. It is also critical for understanding how economic policy directly affects national saving and collaterally influences investment, interest rates and growth" ([16], p.22)

and

> "The generational accounts can also be used to assess the effects on national saving of programs to redistribute more or less to current generations."([3] p.3)
>
> "Using recent generation-specific estimates of the propensity to consume out of lifetime resources.....,one can consider the effect on national consumption and national saving of such policy changes." ([3] pp. 3-4)

This paper asks and answers two questions. The first is: what do generational accounts tell us about the way the government budget affects intergenerational distribution? The second is: what do generational accounts (augmented with generation-specific propensities to consume out of life-time resources) tell us about the effect of the budget on saving?

The answer to the first question can be summarized as follows. First, the usefulness of generational accounts as a summary of the budget's impact on the intergenerational distribution of *private* consumption lives or dies with the validity of the life-cycle model as a characterization of private consumption behavior. Second, the generational accounts, as currently constructed, have nothing to say about the intergenerational distribution of public consumption.

The answer to the second question is that the usefulness of generational accounts as a tool for evaluating the impact of budgetary policy on saving is only as high as the degree of validity of the life-cycle model.

The life-cycle model is characterized by the following assumptions. (1) Finite (possibly uncertain) individual lifetimes. (2) No operative Ricardian [1] intergenerational gift motive. (3) Complete markets [2], permitting meaningful present discounted value calculations for streams of future taxes and transfer payments For our purposes this can be weakened to the condition that financial markets are sufficiently rich to ensure that the *timing* of government taxes and transfer payments over the life cycle of an individual does not matter, but only the present value of these taxes and transfers over the life cycle, when they are discounted at the government rate of interest [3]. One of the points this paper makes is the obvious but important one, that generational accounts are uninformative as regards the budget's impact on intergenerational distribution and saving behavior when consumers' decision horizons are *longer* than those characteristic of the life-cycle model (when there is an operative Ricardian bequest motive) or when decision horizon's are *shorter* than those postulated by the life-cycle model, because of the appropriate kind of capital market imperfections.

Even when the strict life-cycle model holds (as is assumed to be the case in Sections 4 and 5 of the paper), great caution should be exercised in interpreting the generational accounts. What *prima-facie* they appear to tell us may be misleading and at worst quite incorrect. There are two reasons for this.

First, the generational accounts ignore the intergenerational distribution aspects of the government's provision of public goods and services. [4]

Second, while generational accounting allows, in principle, for the general equilibrium repercussions of budgetary policy changes on the tax bases and transfer, benefit or subsidy bases, it ignores all changes in before-tax [5] incomes (resources) and relative prices. It therefore fails to record changes in the determinants of the intergenerational distribution of private consumption that do not "pass through" the budget and thus are not recorded in the generational accounts, even when they are caused by the budget. Pre-tax, pre-transfer and pre-subsidy factor incomes and rates of return will in general change as a result of budgetary policy changes. Changes in the pre-tax, pre-transfer and pre-subsidy equilibrium configuration of prices and quantities is what the study of (tax) incidence and of the general equilibrium repercussions of alternative budgetary actions or rules is all about. Such general equilibrium responses of pre-tax, pre-transfer and pre-subsidy factor incomes and rates of return (which are not recorded in the budget or the generational accounts) may reinforce, counteract or even reverse not merely the impact effects of budgetary policy changes (conditional on the original equilibrium configuration of prices and quantities), but even the total effect recorded in the budget (including the general equilibrium repercussions on taxes, transfers and subsidies).

Recapitulating, consider the case where the strict life-cycle model of household consumption holds. Intergenerational distribution should be about the intergenerational distribution of (unobservable) lifetime utility. Translating this into observables, it should be about the intergenerational distribution of the lifetime consumption of private and public goods and services. Generational accounts ignore the intergenerational distribution of consumption of *public* goods and services and provide but a very partial, and potentially misleading picture of the (effect of the budget on the) intergenerational distribution of consumption of *private* goods and services.

The outline of the rest of the paper is as follows. In Section 2, I present a small familiar macroeconomic model to guide the consideration of the issues associated with the construction and use of generational accounts. In Section 3, I show how the presence of a Ricardian intergenerational gift motive may rob the generational accounts of their informativeness as regards the effect of the budget both on intergenerational distribution and on saving.

In Section 4, I discuss the appropriateness of the "Fiscal Balance Rule", a normative rule for generational taxation proposed by Kotlikoff. In Section 5 issues to do with *incidence* and the general equilibrium repercussions of the budget on the after-tax intergenerational distribution of private resources. In Section 6 capital market imperfections are discussed. Intra-generational heterogeneity plays an important role when flawed capital markets are considered. Section 7 concludes.

While this paper does not pretend to contain new theoretical insights, it is, as far as I know, the first comprehensive, critical analytical evaluation of the important theoretical construct and empirical tool of generational accounting.

2 A Simple Two-Period Overlapping Generations Model.

To motivate the discussion that follows, I illustrate how the generational accounts are constructed in a very simple two-period overlapping generations model of a competitive closed economy. While abstract, the economy exhibits some real world features in its tax structure. These serve to restate some of the valid points made by Kotlikoff and others about the construction of the generational accounts in a world where not all taxes and transfers are lump-sum and generation-specific.

2.1 The household accounts.

Each household lives for 2 periods. All households within a generation are identical. Consider a representative household born in period t. She works during the first period of her life for a before-tax real wage w_t and is retired during the second period. Labor supply is exogenous and scaled to unity. She accumulates assets during period t for retirement in period $t+1$ in the form of non-interest bearing government money, $H_{t,t+1}$, one-period nominally denominated government interest-bearing debt $B_{t,t+1}$,with a nominal interest rate $i_{t,t+1}$, one-period index-linked government interest-bearing debt, $D_{t,t+1}$ with a real rate of interest $r_{t,t+1}$, nominally denominated government perpetuities, $B^L_{t,t+1}$ with a constant coupon γ , and a period t price P^L_t , and real reproducible capital $K_{t,t+1}$. The before-tax marginal product of capital in period t (which equals the capital rental rate [6]) is denoted ρ_t . The generation t household consumes private goods, c^1_t while young and c^2_t while old. She may also derive utility from two public consumption goods provided free of charge by the government. The quantities of the two public consumption goods provided in period t by the government are G^o_t and G^y_t respectively. For simplicity we model a pure public consumption good, that is a good that is completely non-rival and completely non-excludable. Households can leave non-negative bequests to the next generation; $\ell_{t,t+1}$ denotes the real bequest left by a member of generation t to generation $t+1$. The bequest is made by generation t during the second period of her life and becomes available to generation $t+1$ during its youth. If there is more than one descendant, bequests are split equally among them. Each member of generation t has $1+n$, descendants. The population growth rate, n, is exogenous. The size of generation t is denoted N_t . Reproduction is through parthenogenesis. When young, households of generation t pay a lump-sum tax (which can be negative) whose real value is τ^1_t. When old a lump-sum tax with real value τ^2_t is paid. Labor income tax is paid at a constant proportional rate τ^w_t by the young in period t. There also is a tax at a constant proportional rate τ^A_t on all asset income. The asset income tax taxes *nominal* interest. Capital gains are not taxed. The general price level in period t is denoted P_t .

The budget constraints while young and while old for a representative member of generation t are given in equations 1 and 2.

$$P_t\left[w_t\left(1-\tau^w_t\right)-\tau^1_t+\frac{\ell_{t-1,t}}{1+n}-c^1_t\right]=P_tK_{t,t+1}+M_{t,t+1}+B_{t,t+1}+P_tD_{t,t+1}+P^L_tB^L_{t,t+1} \tag{1}$$

$$\begin{aligned}&P_{t+1}\left(c^2_t+\ell_{t,t+1}+\tau^2_t\right)\\ =&\left(1-\tau^A_{t+1}\right)\left(\rho_{t+1}P_{t+1}K_{t,t+1}+r_{t,t+1}P_{t+1}D_{t,t+1}+i_{t,t+1}B_{t,t+1}+\gamma B^L_{t,t+1}\right)\\ &+P_{t+1}K_{t,t+1}+M_{t,t+1}+B_{t,t+1}+P_{t+1}D_{t,t+1}+P^L_{t+1}B^L_{t,t+1}\end{aligned} \tag{2}$$

From equations 1 and 2 it follows that the nominal value at period t prices of net taxes paid by a representative member of generation t while young is $P_t\left(\tau_t^w w_t + \tau_t^1\right)$. In addition to these involuntary transfers to the government, a member of generation t transfers while young $M_{t,t+1} + B_{t,t+1} + P_t D_{t,t+1} + P_t^L B_{t,t+1}^L$ to the government through the voluntary acquisition of government liabilities, both interest-bearing and non-interest bearing. The nominal value (at period $t+1$ prices) of *conventionally measured* net taxes paid by the representative member of generation t while old are

$$\tau_{t+1}^A\left(\rho_{t+1}P_{t+1}K_{t,t+1} + i_{t,t+1}B_{t,t+1} + r_{t,t+1}P_{t+1}D_{t,t+1} + \gamma B_{t,t+1}^L\right) + P_{t+1}\tau_t^2.$$

In addition, the old member of generation t receives from the government, in period $t+1$ the gross return (interest plus principal) on the government debt it acquired the previous period, that is, $M_{t,t+1} + (1+i_{t,t+1})B_{t,t+1} + (1+r_{t,t+1})P_{t+1}D_{t,t+1} + (\gamma + P_{t+1}^L)B_{t,t+1}^L$

By combining and rearranging equations 1 and 2 we obtain the following present value relationship for a representative member of generation t .

$$P_t c_t^1 + \frac{P_{t+1}c_t^2}{1+i_{t,t+1}} = P_t w_t + P_t\frac{\ell_{t-1,t}}{1+n} - \frac{P_{t+1}\ell_{t,t+1}}{1+i_{t,t+1}} - \bar{T}_{t,t} - \Psi_{t,t+1} \tag{3}$$

where

$$\bar{T}_{t,t} \equiv \left\{\begin{array}{l} P_t\tau_t^1 + \frac{P_{t+1}\tau_t^2}{1+i_{t,t+1}} + P_t\tau_t^w w_t \\ +\frac{\tau_{t+1}^A}{1+i_{t,t+1}}i_{t,t+1}\left(B_{t,t+1} + P_t D_{t,t+1} + P_t^L B_{t,t+1}^L + P_t K_{t,t+1}\right) \\ +\frac{i_{t,t+1}}{1+i_{t,t+1}}M_{t,t+1} \\ +\Phi_{t,t+1} \end{array}\right\} \tag{4}$$

$$\begin{aligned} \Phi_{t,t+1} \equiv & \left\{\left[1 + i_{t,t+1}\left(1-\tau_{t+1}^A\right)\right] - \tfrac{1}{P_t^L}\left[P_{t+1}^L + \gamma\left(1-\tau_{t+1}^A\right)\right]\right\}\tfrac{P_t^L}{1+i_{t,t+1}}B_{t,t+1}^L \\ & + \left\{\left[1 + i_{t,t+1}\left(1-\tau_{t+1}^A\right)\right] - \tfrac{P_{t+1}}{P_t}\left[1 + r_{r,t+1}\left(1-\tau_{t+1}^A\right)\right]\right\}\tfrac{P_t D_{t,t+1}}{1+i_{t,t+1}} \\ & + \left\{\left[1 + i_{t,t+1}\left(1-\tau_{t+1}^A\right)\right] - \tfrac{P_{t+1}}{P_t}\left[1 + \rho_{t+1}\left(1-\tau_{t+1}^A\right)\right]\right\}\tfrac{P_t}{1+i_{t,t+1}}K_{t,t+1} \end{aligned} \tag{5}$$

and

$$\Psi_{t-1,t} \equiv \left\{\frac{P_t}{P_{t-1}}\left[1 + \rho_{t-1,t}\right] - \left[1 + i_{t-1,t}\right]\right\}P_{t-1}K_t \tag{6}$$

$$= \tau_t^A\left\{\frac{P_t}{P_{t-1}}\rho_{t-1,t} - i_{t-1,t}\right\}P_{t-1}K_t \tag{7}$$

The interpretation of the terms on the R.H.S. of equations 3 and 4 is straightforward. The L.H.S. of equation 3 is the present value (discounting to period the end of period t (or the beginning or period $t+1$) at before-tax interest rates) of lifetime private consumption of a representative member of generation t . The R.H.S. of equation 3 is the present value of all resources available to a member of generation t for financing her lifetime consumption of private goods. Note that consumption of public goods and services figures nowhere in equation 3. The first term on the R.H.S. is the wage income of a member of generation t. The negative of the next two terms, $\frac{P_{t+1}\ell_{t,t+1}}{1+i_{t,t+1}} - P_t\frac{\ell_{t-1,t}}{1+n}$, could be called the *private generational account* of a representative member of generation t, that is the present value of bequests made when old minus the value of bequests received when young. The fourth term on the R.H.S. of equation 3 , $\bar{T}_{t,t}$, is the present value of the taxes net of transfers paid by a representative member of generation t to the government during its lifetime.

Considering $\bar{T}_{t,t}$ in more detail, the first two terms on the R.H.S. of equation 4, $P_t\tau_t^1 + \frac{P_{t+1}\tau_t^2}{1+i_{t,t+1}}$, is the value of net lump-sum taxes paid by generation t when young plus the present discounted value of net lump-sum taxes paid by generation t when old. Note that here, (and throughout the generational accounts computation of the next subsection), discounting is at pre-tax rates of interest, even though all interest income is assumed to be taxed (see [3], page 5). Efficient asset pricing and perfect foresight imply, for instance, that the price of the long-dated government debt instrument equals the present discounted value of all future after-tax coupon payments, discounted using the appropriate after-tax market discount factors, as shown in equation 8.

$$P_t^L = \gamma \sum_{j=1}^{\infty} \prod_{k=1}^{j} \left(\frac{1}{1 + i_{t+k}(1 - \tau_{t+k}^A)} \right) (1 - \tau_{t+j}^A) \tag{8}$$

The period t bond price therefore capitalizes the asset tax rate actually paid by the generation t household in period $t+1$ and all future anticipated tax rates to be paid by future bondholders.

The third term on the R.H.S. of equation 4, $P_t\tau_t^w w_t$, is the value of labor income taxes when young.

The fourth term on the R.H.S. of equation 4 ,

$$\frac{\tau_{t+1}^A}{1+i_{t,t+1}} i_{t,t+1} \left(B_{t,t+1} + P_t D_{t,t+1} + P_t^L B_{t,t+1}^L + P_t K_{t,t+1} \right)$$

represents the present value of conventionally measured [7] taxes on asset income earned by generation t as asset owners when old ,

$$\frac{\tau_{t+1}^A}{1+i_{t,t+1}} \left[i_{t,t+1} B_{t,t+1} + r_{t,t+1} P_{t+1} D_{t,t+1} + \gamma B_{t,t+1}^L + \rho_{t+1} P_{t+1} K_{t,t+1} \right] \tag{9}$$

plus the value of bonds purchased from the government while young minus the present value of the gross (before-tax) income earned on these bonds,

$$B_{t,t+1} + P_t D_{t,t+1} + P_t^L B_{t,t+1} \tag{10}$$

$$-\frac{1}{1+i_{t,t+1}}\left[(1+i_{t,t+1})B_{t,t+1}+P_{t+1}(1+r_{t,t+1})D_{t,t+1}+(\gamma+P^L_{t+1})B^L_{t,t+1}\right]$$

if equalization of after-tax rates of return on non-monetary assets is assumed.

The fifth term on the right-hand side of equation 4, $\frac{i_{t,t+1}}{1+i_{t,t+1}}M_{t,t+1}$, can be interpreted, again following [3] as the present value of the seigniorage paid by generation t to the government over its lifetime. Starting off its life with zero money holdings, the generation t household accumulates $M_{t,t+1}$ worth of money balances during the first period of its life. In the second and last period of its life, it again runs its money balances down to zero. The present value of the net acquisitions of money balances by generation t over its lifetime is therefore $M_{t,t+1}-\frac{M_{t,t+1}}{1+i_{t,t+1}}=\frac{i_{t,t+1}}{1+i_{t,t+1}}M_{t,t+1}$.

Note that

$$\sum_{k=t}^{\infty}\Delta_{k-1,t}(M_{k+1}-M_k)\equiv -M_t+\sum_{k=t}^{\infty}\Delta_{k,t}i_{k+1}M_{k+1} \tag{11}$$

where $\Delta_{k,t-1}$ is the pre-tax nominal discount factor

$$\Delta_{m,n}\equiv\prod_{j=n+1}^{m}\frac{1}{1+i_{j-1,j}} \text{ for } m\geq n+1$$
$$\equiv 1 \text{ for } m=n$$

The initial stock of base money plus the present value of the government's current and future issues of base money equal the present value of the imputed interest cost foregone by the government because of its ability to issue non-interest-bearing monetary liabilities (see [7]).[8]

The sixth term on the R.H.S. of equation 4 reflects differences in after-tax rates of return on non-monetary assets. In equation 5 the term $1+i_{t,t+1}\left(1-\tau^A_{t+1}\right)-\frac{1}{P^L_t}\left[P^L_{t+1}+\gamma\left(1-\tau^A_{t+1}\right)\right]$ measures the excess of the after-tax rate of return on nominally denominated short government debt over the after-tax rate of return on government perpetuities. The term $1+i_{t,t+1}\left(1-\tau^A_{t+1}\right)-\frac{P_{t+1}}{P_t}\left[1+r_{r,t+1}\left(1-\tau^A_{t+1}\right)\right]$ measures the excess of the after-tax rate of return on short nominally denominated government debt over the after-tax rate of return on short index-linked government debt. The term $1+i_{t,t+1}\left(1-\tau^A_{t+1}\right)-\frac{P_{t+1}}{P_t}\left[1+\rho_{t+1}\left(1-\tau^A_{t+1}\right)\right]$ measures the excess of the after-tax rate of return on short nominally denominated government debt over the after-tax rate of return on real capital. If all (pecuniary [9]) after-tax rates of return are equalized, the term $\Phi_{t,t+1}$ equals zero. In their theoretical and empirical approaches to the Generational Accounts, Auerbach, Gokhale and Kotlikoff make an "arbitrage" assumption like this [10]. In view of the large literature documenting an equity premium puzzle, this is not, for empirical purposes, an innocuous assumption. Since the results of this Section go through even if $\Phi_{t,t+1}=0$, I shall assume that this condition holds. This is expressed in equations 12,13 and 14.

$$\frac{P_{t+1}^L + \gamma\left(1-\tau_{t+1}^A\right)}{P_t^L} = 1 + i_{t,t+1}\left(1-\tau_{t+1}^A\right) \tag{12}$$

$$\left[1+\rho_{t+1}\left(1-\tau_{t+1}^A\right)\right]\frac{P_{t+1}}{P_t} = 1 + i_{t,t+1}\left(1-\tau_{t+1}^A\right) \tag{13}$$

$$\frac{P_{t+1}}{P_t}\left[1+r_{t,t+1}\left(1-\tau_{t+1}^A\right)\right] = 1 + i_{t,t+1}\left(1-\tau_{t+1}^A\right) \tag{14}$$

Even when after-tax rates of return on non-monetary assets are equalized, the term $\Psi_{t-1,t}$ will not vanish unless the *before-tax* rate of return on the private asset that is not also a government liability, that is (ownership claims on) the stock of physical capital, equals the before-tax rate of interest. The interpretation of $\Psi_{t-1,t}$ is clear: when the after-tax rate of return on capital equals the after-tax interest rate, it is the difference between the actual tax paid on capital income (in a cash-flow sense) and the tax imputed to it according to the generational accounting conventions: $\tau_t^A P_t \rho_{t-1,t} K_t - \tau_t^A i_{t-1,t} P_{t-1} K_t$.

2.2 Generational Accounts.

In general, the generational account (perhaps better characterized as the *public generational account*) for any generation, say the one born in period k , is defined by Auerbach, Gokhale and Kotlikoff as the present value of net *remaining* lifetime payments to the government by the generation born in period k , discounted back to the beginning of period t (or the end of period $t-1$). It is denoted $T_{k,t-1}$. In our model k ranges from $t-1$ to ∞ and there are but two generations co-existing in any period t. $\bar{T}_{k,t-1}$ is the generational account for a representative member of generation k . For any generation k we have

$$T_{k,j} \equiv N_k \bar{T}_{k,j} \tag{15}$$

$\bar{T}_{t,t-1}$, the generational account of a representative member of generation t,discounted back to the beginning of period t, is given by

$$\bar{T}_{t,t-1} \equiv \left(\frac{1}{1+i_{t-1,t}}\right)\bar{T}_{t,t} \tag{16}$$

where $\bar{T}_{t,t}$ is defined in equation 4.

The generational accounts for the generations other than t are now derived easily. For generation $t-1$, (the old in period t), we have

$$T_{t-1,t-1} \equiv \left(\frac{N_{t-1}}{1+i_{t-1,t}}\right)\left\{\begin{array}{c} P_t\tau_{t-1}^2 \\ +\tau_t^A i_{t-1,t}\left(B_{t-1,t}+P_{t-1}D_{t-1,t}+P_{t-1}^L B_{t-1,t}^L + P_{t-1}K_{t-1,t}\right) \\ -\left[M_{t-1,t}+(1+i_{t-1,t})\left(B_{t-1,t}+P_{t-1}D_{t-1,t}+P_{t-1}^L B_{t-1,t}^L\right)\right] \end{array}\right\} \tag{17}$$

For all generations, $k \geq t$, we have

$$T_{k,t-1} \equiv \Delta_{k,t-1} T_{k,k} \tag{18}$$

Where $T_{k,k}$ is defined from equations 4 and 15 .

2.3 The government account.

The government's single-period budget identity for period t is given in equation 19

$$\begin{array}{c} M_{t+1} - M_t + B_{t+1} - B_t + P_t(D_{t+1} - D_t) + P_t^L(B_{t+1}^L - B_t^L) \equiv \\ P_t(G_t^o + G_t^y) + i_{t-1,t}B_t + P_t r_{t-1,t} D_t + \gamma B_t^L - \tau_t^1 N_t - \tau_{t-1}^2 N_{t-1} \\ -P_t \tau_t^w w_t N_t - \tau_t^A(i_{t-1,t}B_t + P_t r_{t-1,t} D_t + \gamma B_t^L + P_t \rho_t K_t) \end{array} \tag{19}$$

M_t is the aggregate stock of government money at the beginning of period t , B_t is the aggregate stock of short nominally denominated government interest-bearing debt at the beginning of period t , D_t is the stock of short index-linked government interest-bearing debt at the beginning of period t, B_t^L is the aggregate stock of government perpetuities at the beginning of period t and K_t is the aggregate stock of private capital at the beginning of period t .

We note that

$$\begin{array}{rcl} M_t & = & N_{t-1} M_{t-1,t} \\ K_t & = & N_{t-1} K_{t-1,t} \\ B_t & = & N_{t-1} B_{t-1,t} \\ D_t & = & N_{t-1} D_{t-1,t} \\ B_t^L & = & N_{t-1} B_{t-1,t}^L \end{array}$$

Rearranging the government budget identity 19 we obtain

$$\begin{array}{l} B_t + P_{t-1} D_t + P_{t-1}^L B_t^L \\ \equiv \dfrac{1}{1 + i_{t-1,t}} \left\{ \begin{array}{c} B_{t+1} + P_t D_{t+1} + P_t^L B_{t+1}^L + \tau_t^1 N_t + \tau_{t-1}^2 N_{t-1} + P_t \tau_t^w w_t N_t \\ + \tau_t^A i_{t-1,t}[B_t + P_{t-1} D_t + P_{t-1}^L B_t^L + P_{t-1} K_t] \\ -P_t(G_t^o + G_t^y) + M_{t+1} - M_t \\ + \Phi_{t-1,t} + \Psi_{t-1,t} \end{array} \right\} \end{array} \tag{20}$$

In what follows we assume again that $\Phi_{t-1,t} = 0$. Solving 20 recursively forward, we obtain the familiar government present value budget constraint or government solvency constraint given in equation 21, provided the terminal condition given in equation 22 is satisfied.

$$B_t + P_{t-1} D_t + P_{t-1}^L B_t^L \equiv \sum_{k=t}^{\infty} \Delta_{k,t-1} \left\{ \begin{array}{c} \tau_k^1 N_k + \tau_{k-1}^2 N_{k-1} + P_k \tau_k^w w_k N_k \\ + \tau_k^A i_{k-1,k}[B_k + P_{k-1} D_k + P_{k-1}^L B_k^L + P_{k-1} K_k] \\ -P_k(G_k^o + G_k^y) + M_{k+1} - M_k + \Psi_{k,k+1} \end{array} \right\} \tag{21}$$

$$\lim_{k \to \infty} \Delta_{k,t-1}(B_{k+1} + P_k D_{k+1} + P_k^L B_{k+1}^L) = 0 \tag{22}$$

The government solvency constraint given in equation 22 is the no-Ponzi-finance condition stating that, in the long run, the debt cannot grow faster than the rate of interest, i.e. that, if there is a positive stock of debt outstanding, the government ultimately will have to run primary surpluses or resort to seigniorage. As shown in [9], this no Ponzi finance condition can only be derived from acceptable primitives if constraints are imposed on the government's ability to use lump-sum taxes and transfers.

It is easily checked that equation 21 can be rewritten using the generational accounts of all generations currently alive and yet to be born as in equation 23.

$$B_t + P_{t-1}D_t + P_{t-1}^L B_t^L + \sum_{k=t}^{\infty} \Delta_{k,t-1} P_k (G_k^o + G_k^y) = \sum_{k=t-1}^{\infty} T_{k,t-1} \tag{23}$$

Equation 23 states that the value of the government's net outstanding financial liabilities plus the present value of its future consumption program must be covered by the sum of the generational accounts of all existing and future generations. Expressions like 21 and 23 can be extended in a straightforward manner to include public sector capital formation, the sale and purchase of existing financial or real assets by the public sector and government production of intermediate goods and services (intermediate public goods).

2.4 Private consumption behavior.

The decision problem of a competitive, and "policy taking" representative household born in period t is as follows. Taking as given market prices, $P_t, P_{t+1}, w_t, P_t^L, P_{t+1}^L, \rho_{t+1}, r_{t,t}$ and government policy instrument values, τ_t^1, τ_t^2, τ_t^w, $\tau_{t+1}^A, \gamma, G_t^y$ and G_{t+1}^o *and taking as given the bequest received from the previous generation,* $\ell_{t-1,t}$ choose $\{c_t^i, c_t^2, M_{t,t+1}, B_{t,t+1}, D_{t,t+1}, B_{t,t+1}^L, K_{t,t+1}, \ell_{t,t+1}\}$ to maximize the utility functional given in equation 24[11]

$$\begin{aligned} W_t = \tfrac{1}{1-\eta}\left(c_t^1\right)^{1-\eta} + \tfrac{\alpha_1}{1-\eta}\left(c_t^2\right)^{1-\eta} + \tfrac{\alpha_2}{1-\eta}\left(\tfrac{M_{t+1}}{P_{t+1}}\right)^{1-\eta} \\ +\alpha_3 \ln(1+G_t^y) + \alpha_4 \ln(1+G_{t+1}^o) + \delta W_{t+1}^* \end{aligned} \tag{24}$$

$$\eta \geq 0; \alpha_1 > 0; \alpha_j \geq 0, j = 2,3,4; 0 \leq \delta < 1$$

subject to the private budget constraints 1 and 2 and the weak inequalities

$$c_t^i \geq 0, c_t^2 \geq 0, M_{t,t+1} \geq 0, K_{t,t+1} \geq 0, \text{ and } \ell_{t,t+1} \geq 0 \tag{25}$$

W_{t+1}^* is the maximized value of the utility of the generation born in period $t+1$.

Because the utility function satisfies the Inada conditions for its first three arguments, the nonnegativity constraints in consumption while young, consumption when old and money balances will always be satisfied as long as there are positive lifetime resources and finite intertemporal terms of trade. Through appropriate Inada conditions on the production function, the equilibrium capital stock will also always be strictly positive, so we shall not have to worry about the non-negativity constraint on consumer holdings of equity. That leaves just one non-trivial non-negativity constraint, $\ell_{t,t+1} \geq 0$, bequests to the next generation cannot be negative.[12].

The way the bequest motive is introduced is consistent with debt neutrality: generation t cares not about the act of giving or about the consumption levels achieved by the next generation, but by the level of utility achieved by the next generation. All generations have the same utility functionals. Cardinal utility doesn't phase us. For simplicity only one-sided intergenerational caring is considered. Children do not care about their parents and do not make transfers to them. More is required, however, to generate the potential for equilibria with debt neutrality. Specifically, I have ruled out *strategic* bequests by assuming something akin to "intergenerational open-loop Nash behavior" (See [8]). Each member of generation t takes as given when she chooses $c_t^i, c_t^2, M_{t,t+1}, B_{t,t+1}, D_{t,t+1}, B_{t+1}^L, K_{t,t+1}$,and $\ell_{t,t+1}$, the bequest she gets from generation $t-1$, that is $\frac{\ell_{t-1,t}}{1+n}$ [13] .

We assume in what follows that the after-tax rates of return on non-monetary assets are equalized. The household is therefore indifferent about the shares of short bonds, long bonds and equity in her portfolio. The equilibrium shares of these three assets will be determined by their supplies. The interesting first-order conditions are the following

$$\frac{1}{\alpha_1}\left(\frac{c_t^1}{c_t^2}\right)^{-\eta} = 1 + (1-\tau_{t+1}^A)r_{t,t+1} \tag{26}$$

$$\left(\frac{\frac{M_{t+1}}{P_{t+1}}}{c_t^2}\right)^{-\eta} = \frac{\alpha_1}{\alpha_2} i_{t,t+1}\left(1-\tau_{t+1}^A\right) \tag{27}$$

$$\alpha_1\left(c_t^2\right)^{-\eta} \geq \left(1 + (1-\tau_{t+1}^A)r_{t,t+1}\right)\left(\frac{\delta}{1+n}\right)\alpha_1\left(c_{t+1}^2\right)^{-\eta} \tag{28}$$

If there is an interior solution for bequests, $\ell_{t,t+1} > 0$, then equation 28 holds with equality. If equation 28 holds with strict inequality, then $\ell_{t,t+1} = 0$, (the bequest constraint is binding).

Equation 26 equates the marginal rate of intertemporal substitution in consumption over the life of generation t to the after-tax real rate of interest. Equation 27 equates the marginal utility of the services yielded by money when old to its opportunity cost, the marginal utility of consumption when old[14]. Equation 28 says that if one is leaving a positive bequest to the next generation, the marginal utility of one's own consumption when old should just be equal to the marginal utility of bequests. When a member of generation t reduces her period $t+1$ consumption, her utility goes down by $\alpha_1 \left(c_t^2\right)^{-\eta}$ .The resource unit thus saved and made available to generation $t+1$ will, during the next period, have grown to $1+(1-\tau_{t+1}^A)r_{t,t+1}$ units. Since population grows, per capita resources available to a member of generation $t+1$ go up by $\frac{1+(1-\tau_{t+1}^A)r_{t,t+1}}{1+n}$ .Assuming generation $t+1$ is optimizing, its utility will increase by $\frac{1+(1-\tau_{t+1}^A)r_{t,t+1}}{1+n}\alpha_1 \left(c_{t+1}^2\right)^{-\eta}$. Since $\delta < 1$ (each generation is assumed to discount the utility achieved by the next generation (to attach less weight to it than to its own utility)), generation t values its bequest at the margin as $\left(1+(1-\tau_{t+1}^A)r_{t,t+1}\right)\left(\frac{\delta}{1+n}\right)\alpha_1 \left(c_{t+1}^2\right)^{-\eta}$.[15] . If the marginal utility of own consumption exceeds the marginal utility of bequests at $\ell_{t,t+1} = 0$, the nonnegativity constraint on bequests is actually binding (see also [6]).

2.5 Production and factor market equilibrium.

A homogeneous durable commodity, that can be used as a private consumption good, a private capital good or a public consumption good is produced by competitive profit-maximizing firms using a production function with constant returns to scale in capital and labor. The production function is twice continuously differentiable, with positive but diminishing marginal products and satisfies the Inada conditions. Let Y_t denote real output in period t. Then

$$Y_t = F(K_t, N_t) = N_t f\left(\frac{K_t}{N_t}\right) \equiv N_t f(k_t)$$

$$f' > 0; f'' < 0; f(0) = 0; \lim_{k\to 0} f'(k) = \infty; \lim_{k\to\infty} f'(k) = 0 \tag{29}$$

The marginal product of labor equals the before (income)-tax real wage and the before-tax capital rental rate equals the marginal product of capital.

$$w_t = f(k_t) - k_t f'(k_t) \tag{30}$$

$$\rho_t = f'(k_t) \tag{31}$$

3 Debt neutrality, generational accounts, intergenerational redistribution and saving behavior.

Consider an initial or reference equilibrium, whose prices and quantities are denoted by a single star. Let $\ell_{t,t+1}^* > 0$, that is, generation t is planning to leave a strictly positive bequest to generation $t+1$ .Now consider a balanced-budget, lump-sum redistribution from generation t to generation $t+1$., that is, $\Delta\tau_t^2 = -(1+n)\Delta\tau_{t+1}^1 > 0$. As long as the value of bequests planned in the original equilibrium by generation t is not less that the increase in the tax on generation t ($\ell_{t,t+1}^* \geq \Delta\tau_t^2$) generation t will reduce its bequest to generation $t+1$ by the exact amount of the increase in the tax it pays, that is, in the new equilibrium, whose prices and quantities are denoted by double stars, $\ell_{t,t+1}^* - \ell_{t,t+1}^{**} = \Delta\tau_t^2 = -(1+n)\Delta\tau_{t+1}^1$. No other real or nominal equilibrium price or quantity will change. Private intergenerational redistribution exactly offsets lump-sum intergenerational redistribution through the government budget.

Under the policy experiment just considered, the (public) generational account of generation t , $T_{t,t-1}$ increases by $\left(\frac{1}{1+i_{t-1,t}}\right)\left(\frac{P_{t+1}\Delta\tau_t^2}{1+i_{t,t+1}}\right)N_t$ and the (public) generational account of generation $t+1$, $T_{t+1,t-1}$ falls by the same amount. The (public) generational accounts of all other future generations are unchanged. However, the private generational account of generation t , $-\left(\frac{1}{1+i_{t-1,t}}\right)\left(P_t\frac{\ell_{t-1,t}}{1+n} - \frac{P_{t+1}\ell_{t,t+1}}{1+i_{t,t+1}}\right)N_t$, falls by the same amount, $\left(\frac{1}{1+i_{t-1,t}}\right)\left(\frac{P_{t+1}\Delta\tau_t^2}{1+i_{t,t+1}}\right)N_t$, as its public generational account increases through a reduction in $\ell_{t,t+1}$ and the private generational account of generation $t+1$, $-\left(\frac{1}{1+i_{t-1,t}}\right)\left(\frac{1}{1+i_{t,t+1}}\right)\left(P_{t+1}\frac{\ell_{t,t+1}}{1+n} - \frac{P_{t+2}\ell_{t+1,t+2}}{1+i_{t+1,t+2}}\right)N_{t+1}$, increases by the same amount as its public generational account decreases through that same reduction in $\ell_{t,t+1}$. Thus the total lifetime resources available to each generation are invariant under lump-sum redistributions that do alter the public generational accounts. This suggests the following two (obvious) propositions.

Proposition 1 *When the conditions for debt neutrality or Ricardian equivalence are satisfied, the generational accounts are uninformative about the effect of the budget on the intergenerational distribution of resources.*

Proposition 2 *When the conditions for debt neutrality or Ricardian equivalence are satisfied, the generational accounts are uninformative about the effect of the budget on saving.*

Note that even if Ricardian equivalence only characterizes a subset of each generation, or if intergenerational transfers through the government budget are only partly offset by changes in private intergenerational transfers, applications of the generational accounts that do not allow for such "partial Ricardian offsets" will lead to exaggerated estimates of the effect of the budget on intergenerational distribution and on national saving.

4 Government consumption, intergenerational distribution and the "Fiscal Balance Rule".

The proponents of generational accounting do not ignore the "resource exhaustion" aspect of government consumption, but do not attach (intergenerational) distributional significance to the size and composition of the government consumption program. In Kotlikoff ([15]) government consumption occupies a central place in the construction of the prescriptive or normative "Fiscal Balance Rule", but only because and to the extent that the present value of the "exhaustive" public spending program determines the share of total resources that is not available for private absorption.

The potential importance of the issue is recognized in [3]. Atkinson and Stiglitz ([1]), surveying quite a sizable literature on the subject (see especially [17]) suggest that two classes of public (consumption) spending be recognized.

> The first consists of goods where particular beneficiaries can (in theory) be identified-"allocable expenditures"-or of broadly publicly provided private goods (e.g. highways and education). The second group consists of "public goods" that cannot be directly allocated to particular individuals (e.g. defence). For allocable goods, the procedure adopted by Musgrave *et. al.* is similar for that for taxes. For example, unemployment insurance benefits are allocated according to receipts from that source...., education expenditure is allocated to the families of students, ... The second group of public goods are simply allocated on three assumptions:(1) in proportion to total income, (2) in proportion to taxes, and (3) equally to all persons.

Without wishing to minimize the problems involved in getting to any widely accepted allocation of benefits for public consumption goods and public intermediate inputs, the right way forward is surely not to ignore the issue and thus implicitly to allocate all households equal (zero) benefit from public goods.

4.1 The fiscal balance rule.

Having made the point about Ricardian equivalence, we assume in what follows that the life-cycle model applies, that is that $\delta = 0$ (there is no intergenerational gift motive). Money and the three government interest-bearing debt instruments were only introduced to highlight some of the important conceptual and practical aspects (all of them found in the work of Kotlikoff et. al.) of the construction of generational accounts. They are not required to make the remaining points we wish to cover in this paper and they are therefore omitted in the remainder of this paper ($\alpha_2 = 0$). For the remainder of this section, distortionary taxes are also omitted ($\tau^w = \tau^A = 0$).

By adding the uses and sources of funds identities of the private and public sectors together we obtain the intertemporal real resource constraint of the economy as a whole, given in equation 32. Our simplifying assumption that the resource cost of both types of government consumption is equal and constant permits us to obtain the aggregate resource cost of all public consumption, denoted G as

$$G_t = G_t^y + G_t^o$$

$$K_t + \sum_{k=t}^{\infty} \prod_{j=t}^{k} \left(\frac{1}{1+\rho_j}\right) N_k w_k = \sum_{k=t}^{\infty} \prod_{j=t}^{k} \left(\frac{1}{1+\rho_j}\right) \left[N_k \left(c_k^1 + \frac{1}{1+n} c_{k-1}^2\right) + G_k\right] \tag{32}$$

Using the first-order condition for private consumption 26 and the private sector life-time budget constraint 3 the economy-wide intertemporal resource constraint 32 can be rewritten as in equation 33.

$$\begin{aligned} K_t(1+r_{t-1,t}) - \tfrac{\alpha_1(1+r_{t-1,t})}{1+\alpha_1}\left(N_{t-1}w_{t-1} - T_{t-1,t-1}\right) + \textstyle\sum_{k=t-1}^{\infty} \prod_{j=t}^{k} \left(\tfrac{1}{1+r_{j-1,j}}\right) T_{k,k} \\ = \textstyle\sum_{k=t-1}^{\infty} \prod_{j=t}^{k} \left(\tfrac{1}{1+r_{j-1,j}}\right) G_k \end{aligned} \tag{33}$$

Note that $\frac{\alpha_1(1+r_{t-1,t})}{1+\alpha_1}\left(N_{t-1}w_{t-1} - T_{t-1,t-1}\right) = N_{t-1}c_{t-1}^2$, the consumption of the generation that is currently old. What equation 33 says is that the government's current and future consumption plan must be financed, in present value terms, either out of the generational accounts of the young and of all future generations or out of the excess of the existing capital stock over the consumption of the generation that is currently old.

We now define the following notation. The long real rate of interest in period t, r_t^L, is defined by

$$\frac{r_t^L}{1+r_t^L} \equiv \left(\sum_{k=t-1}^{\infty} \prod_{j=t}^{k} \left(\frac{1}{1+r_{j-1,j}}\right)\right)^{-1}$$

Permanent government consumption, G_t^p, is that constant real value of government consumption that has the same present value as the actual planned or expected future path of government consumption, that is,

$$G_t^p \equiv \left(\frac{r_t^L}{1+r_t^L}\right) \sum_{k=t-1}^{\infty} \prod_{j=t}^{k} \left(\frac{1}{1+r_{j-1,j}}\right) G_k$$

Kotlikoff's fiscal balance rule is to choose, for all generations born in period t and later, that constant value of the generational accounts, T_t^p, say, that would satisfy equation 33 [16]. That is,

$$T_t^p \equiv G_t^p - \left(\frac{r_t^L}{1+r_t^L}\right)\left(K_t(1+r_{t-1,t}) - N_{t-1}c_{t-1}^2\right) \tag{34}$$

In the presence of population growth and productivity growth, the fiscal balance rule given in equation 34 could be replaced with 35. E_t is the level of labor-augmenting productivity, which grows in period t at the proportional growth rate ϵ_t. Capital per unit of efficiency labor in period t is denoted $\tilde{K}_t \equiv \frac{K_t}{N_t E_t}$ and consumption when old per member of generation $t-1$, measured in efficiency units is denoted $\tilde{c}_{t-1}^2 \equiv \frac{c_{t-1}^2}{E_{t-1}}$. The excess of the long-run real interest rate over the long-run growth rate of efficiency labor is denote $\tilde{r}_t^L$. It is defined by [17]

$$\left(\sum_{k=t-1}^{\infty}\prod_{j=t}^{k}\left(\frac{(1+n)(1+\epsilon_j)}{1+r_{j-1,j}}\right)\right)^{-1} \equiv \frac{\tilde{r}_t^L}{1+\tilde{r}_t^L}$$

$\tilde{G}_t^p$, is permanent government spending per unit of efficiency labor, that is

$$\tilde{G}_t^p \equiv \left(\frac{\tilde{r}_t^L}{1+\tilde{r}_t^L}\right)\sum_{k=t-1}^{\infty}\prod_{j=t}^{k}\left(\frac{(1+n)(1+\epsilon_j)}{1+r_{j-1,j}}\right)\frac{G_k}{N_k E_k}$$

The modified fiscal balance rule is to chose, for all generations born in period t and later, that constant value of the generational account per member of each generation (measured in efficiency units), denoted $\tilde{T}_t^p$ that satisfies equation 33 , that is

$$\tilde{T}_t^p \equiv \tilde{G}_t^p - \left(\frac{\tilde{r}_t^L}{1+\tilde{r}_t^L}\right)\left(\tilde{K}_t(1+\tilde{r}_{t-1,t}) - \frac{1}{(1+n)(1+\epsilon_t)}\tilde{c}_{t-1}^2\right) \tag{35}$$

Equations 34 or 35 are, I believe, a fair representation of Kotlikoff's fiscal balance rule, in and out of steady state.

While the fiscal balance rule is (trivially) consistent with government solvency in steady state, it is not clear *a-priori*, as noted by Kotlikoff [15], whether adherence to this rule by the government outside the steady state would ensure solvency, let alone ensure convergence to a steady state corresponding to the fiscal balance rule under constant values of all exogenous variables. Even if solvency and convergence to a steady state were ensured, however, we would have established only the feasibility of the rule, not the fairness, desirability or optimality according to some reasonable social welfare function.

4.2 The fiscal balance rule as a normative guide for intergenerational tax and transfer policy.

As a normative rule for intergenerational distribution through the budget, the fiscal balance rule has at least three serious weaknesses.

First, the existing generations (in our model just generation $t-1$) are treated differently from all future generations (generations t and beyond). While all future generations have the same generational account, T_t^p, there is nothing to guarantee that all future generations as a group would have the same generational account as those currently alive. The remaining generational account of the old, $T_{t-1,t-1}$, defined in equation 17 plus the net taxes paid by generation $t-1$ while young, need not add up to T_t^p.

Second, total government consumption [18] varies over time. Even if all generations alive during a given period benefit equally from the government consumption program that period, the value to a generation of the government consumption provided over its lifetime may vary from generation to generation. In the fiscal balance rule, no utility is derived from government consumption, which figures only because it is "exhaustive spending", which pre-empts or uses up real resources and thus reduces the size of the social pie.

Third, even if total real government consumption (or consumption per capita or consumption per capita in efficiency units) is constant over time, its composition can vary over time in a way that affects differentially the utility derived from the public consumption program by different generations [19].

Not all generations alive during a period benefit equally from government consumption during that period. Old age pensioners do not benefit equally from free rock concerts as do teenagers. Education (which has a consumption component as well as a human capital formation component) likewise befits the young more than the old. Expenditure on law and order bestows benefits that are increasing with one's level of non-human wealth. With a constant present value of life-time resources, the life cycle model implies that the old will own most of the non-human wealth in a community. We do indeed see that graying local communities in the USA vote to reduce education expenditure and to increase the police budget. The benefits derived from national defense are also likely to be increasing in one's level of non-human resources[20]. Benches in public parks are most appreciated by the still mobile old and by parents of very young children. Teenagers and young adults prefer soccer pitches and ice rinks. Health expenditures benefit the most the very young and the very old. Free condoms benefit primarily the sexually active age groups.

Clearly, the distribution across generations of benefits from public consumption will depend on its composition. In this paper this issue is dramatized by assuming that one form of public consumption, G_t^y benefits only the young in period t while the second public consumption good, G_t^o, benefits only the old in period t . It is therefore possible in our model, while keeping total government consumption constant over time to completely deprive a particular generation (or indeed every other generation) from the benefits of public consumption. For generation t, for instance, this would involving setting $G_t^y = G_{t+1}^o = 0$ and $G_t^o = G_{t+1}^y = \bar{G} > 0$.

In practice, both the magnitude and the composition of public consumption vary over time, with potentially important distributional consequences. The fiscal balance rule is only intergenerationally distributionally neutral if it is permanent aggregate government consumption that enters into private utility functions. Unless government consumption is actually constant over time, in which case permanent and actual current consumption coincide, it is hard to see how a generation could benefit from public consumption occurring outside its own life span unless an operative chain of bequest motives links all generations.

Conceptually, there is no special problem valuing public consumption. By analogy with the *compensating variation* of standard consumer theory, we can define the value to a member of generation t of the government's consumption program during her lifetime $\left\{G_t^y, G_t^o, G_{t+1}^y, G_{t+1}^o\right\}$ as the value of the smallest lump-sum transfer payment that would have to be made to this household in order to just compensate it for the loss of the government consumption program (holding constant everything else that is assumed parametric to the individual consumer). Alternatively, we could define, by analogy with the *equivalent variation* of standard consumer theory, the value to a member of generation t of the government's consumption program to be the largest lump-sum tax the household would be willing to pay in order not to forego the government's consumption program. These compensating or equivalent lump-sums would then be summed over all members of a generation, discounted properly and subtracted from the conventional generational accounts of each generation, thus providing us with the true generational accounts.

Considering the "compensating variation" in some more detail, let c_t^{1*} and c_t^{2*} be arg max $\left\{u(c_t^1, c_t^2, \hat{G}_t^y, \hat{G}_{t+1}^o)\right\}$ subject to $c_t^1 + \frac{c_t^2}{1+r_{t,t+1}} = w_t - \tau_t^1 - \frac{\tau_t^2}{1+r_{t,t+1}}$ when the government consumption program directly affecting generation t is $\{\hat{G}_t^y, \hat{G}_{t+1}^o\}$. Given any other government consumption program $\{\check{G}_t^y, \check{G}_{t+1}^o\}$, the minimal cost of a private consumption bundle that just compensates the household for the change in the public consumption bundle is given by $c_t^{1**} + \frac{c_t^{2**}}{1+r_{t,t+1}}$ where c_t^{1**} and c_t^{2**} are arg min $\left\{c_t^1 + \frac{c_t^2}{1+r_{t,t+1}}\right\}$ subject to $u(c_t^1, c_t^2, \check{G}_t^y, \check{G}_{t+1}^o) = u(c_t^{1*}, c_t^{2*}, \hat{G}_t^y, \hat{G}_{t+1}^o)$. For the special case of the logarithmic utility function (equation 24 with $\eta = 1$), c_t^{1**} and c_t^{2**} can be solved from the following two equations:

$$\frac{c_t^{2**}}{c_t^{1**}} = \alpha_1(1 + r_{t,t+1})$$

and

$$\begin{aligned}\ln c_t^{1**} + \alpha_1 \ln c_t^{2**} + \alpha_3 \ln(1 - \check{G}_t^y) + \alpha_4 \ln(1 - \check{G}_{t+1}^o) \\ = \ln\left[\left(\tfrac{1}{1+\alpha_1}\right)\left(w_t - \tau_t^1 - \tfrac{\tau_t^2}{1+r_{t,t+1}}\right)\right] \\ +\alpha_1 \ln\left[\left(\tfrac{\alpha_1}{1+\alpha_1}\right)(1 + r_{t,t+1})\left(w_t - \tau_t^1 - \tfrac{\tau_t^2}{1+r_{t,t+1}}\right)\right] \\ +\alpha_3 \ln(1 - \check{G}_t^y) + \alpha_4 \ln(1 - \check{G}_{t+1}^o)\end{aligned}$$

The "compensating variation" is $c_t^{1**} + \frac{c_t^{2**}}{1+r_{t,t+1}} - \left(c_t^{1*} + \frac{c_t^{2*}}{1+r_{t,t+1}}\right)$.

In practice, of course, the quantification of the welfare consequences of public consumption is likely to be an extremely complicated job. If we throw in the towel, however, we are ignoring the welfare implication of real resources amounting, in most industrial countries, to between 20 and thirty percent of GDP, hardly small beer.

We can summarize this Subsection with the following proposition:

Proposition 3 . *The generational accounts do not recognize the intergenerational distributional implications of the government consumption program. This limits their usefulness as a guide to budgetary policy that is intergenerationally neutral or fair.*

5 Incidence and other unpleasant general equilibrium repercussions.

The distinction between impact or direct effects and second-round or indirect effects has been the bread and butter of macroeconomics. The "paradox of thrift" is a staple Keynesian example. Diamond's illustration of the possible perverse effect of an increase in the capital income tax rate on the after-tax rate of return to capital ([10] , see also [1]) is a Neoclassical staple. Diamond's example is also an excellent illustration of possible pitfalls in interpreting the generational accounts.

We restrict ourselves again to the non-monetary version of the model without a bequest motive. There is no productivity growth. The only taxes (transfers) considered are a lump-sum tax on the young, a lump-sum tax on the old and a proportional tax, at a rate τ_t^K on the rental income from capital. The government budget is balanced, there is no public debt and no public consumption spending. Interest on debt is not taxed. Since the after-tax rate of return on capital equals the real interest rate we have

$$r_{t,t+1} = (1 - \tau_{t+1}^K)\rho_{t+1} \tag{36}$$

The model can be summarized in the following two equations:

$$\begin{aligned}
&\left(\frac{\alpha_1^{\frac{1}{\eta}} \left(1+f'(k_{t+1})(1-\tau_{t+1}^K)\right)^{\frac{1-\eta}{\eta}}}{1+\alpha_1^{\frac{1}{\eta}} \left(1+f'(k_{t+1})(1-\tau_{t+1}^K)\right)^{\frac{1-\eta}{\eta}}} \right) (f(k_t) - k_t f'(k_t) - \tau_t^1) \\
&+ \left(\frac{1}{1+\alpha_1^{\frac{1}{\eta}} \left(1+f'(k_{t+1})(1-\tau_{t+1}^K)\right)^{\frac{1-\eta}{\eta}}} \right) \left(\frac{\tau_t^2}{1+f'(k_{t+1})(1-\tau_{t+1}^K)} \right) \\
&= (1+n)k_{t+1}
\end{aligned} \tag{37}$$

$$\tau_t^1 + \frac{\tau_{t-1}^2}{1+n} + \tau_t^K f'(k_t) k_t = 0 \tag{38}$$

The generational account of generation $t-1$ is given in equation 63, that of generation t in equation 64. All future generations are like generation t , with the appropriate discounting to the beginning of period t .

$$T_{t-1,t-1} = N_{t-1}\left(\frac{\tau_{t-1}^2 + \tau_t^K f'(k_t)k_t(1+n)}{1+f'(k_t)(1-\tau_t^K)}\right) \tag{39}$$

$$T_{t,t-1} = \left(\frac{N_t}{1+f'(k_t)(1-\tau_t^K)}\right)\left(\tau_t^1 + \frac{\tau_t^2 + \tau_{t+1}^K f'(k_{t+1})k_{t+1}(1+n)}{1+f'(k_{t+1})(1-\tau_{t+1}^K)}\right) \tag{40}$$

Note that the generational accounts given in equations 63 and 64 are somewhat awkward to derive for this case, in which the rental income from capital is taxed but interest income is not. The lifetime budget constraint for generation t in this case is

$$c_t^1 + \frac{c_t^2}{1+r_{t,t+1}} = w_t - \left(\tau_t^1 + \frac{\tau_t^2}{1+r_{t,t+1}}\right) \tag{41}$$

with the interest rate and the rental rate linked through equation 36

To get equation 41 into the right shape to generate 23 , that the outstanding public debt and the present value of the government's spending program must be financed out of the generational accounts of all generations currently alive and yet to be born, we have to rearrange 41 as in 42 :

$$c_t^1 + \frac{c_t^2}{1+r_{t,t+1}} = \left\{w_t + \left(\frac{\rho_{t+1}-r_{t,t+1}}{1+r_{t,t+1}}\right)\frac{K_{t+1}}{N_t}\right\} - \left\{\tau_t^1 + \frac{\tau_t^2}{1+r_{t,t+1}} + \frac{\tau_{t+1}^K \rho_{t+1}\frac{K_{t+1}}{N_t}}{1+r_{t,t+1}}\right\} \tag{42}$$

The terms within the second set of curly brackets on the R.H.S. of equation 42 are the generational account of generation t (discounted to the beginning of period $t+1$), which is what we need to get to equation 23 .The before-tax and transfer resources of the household (the terms in the first set of curly brackets on the R.H.S. of 42) now include the non-intuitive term $\left(\frac{\rho_{t+1}-r_{t,t+1}}{1+r_{t,t+1}}\right)\frac{K_{t+1}}{N_t}$ (which of course, from 36, equals $\frac{\tau_{t+1}^K\rho_{t+1}\frac{K_{t+1}}{N_t}}{1+r_{t,t+1}}$) . Such terms involving differences in before-tax rates of return are unavoidable whenever arbitrage (which equates after-tax rates of return) does not ensure equalization of before-tax rates of return, only one of which can be used for discounting. If we were instead to discount using the before-tax return factor on capital, $1+\rho$, equation 42 can be replaced by the friendlier-looking expression 43:

$$c_t^1 + \frac{c_t^2}{1+\rho_{t,t+1}} = w_t - \left\{\tau_t^1 + \frac{\tau_t^1}{1+\rho_{t,t+1}} + \frac{\tau_{t+1}^K\rho_{t+1}\frac{K_{t+1}}{N_t}}{1+\rho_{t,t+1}}\right\} \tag{43}$$

However, if there were any positive amount of public debt outstanding, the term

$$-\tau^K_{t+1}\rho_{t,t+1}\frac{D_{t+1}}{N_t} = (r_{t,t+1} - \rho_{t,t+1})\frac{D_{t+1}}{N_t}$$

would appear on the R.H.S. of equation 43, bringing back the awkwardness.

In all three examples, we consider the effect of an increase in the capital rental tax rate τ^K on equilibrium prices (pre-and post-tax) and quantities and on the generational accounts. Since the exercise is for illustrative purposes only, we shall make some further simplifying assumptions: these are that $\eta = 1$ (logarithmic preferences over private goods) and that the increase in τ^K is evaluated at $\tau^K = 0$.

Example 1: *Nothing registers in the generational accounts, but potentially significant changes occur in the pre-and post-tax distribution of life-time resources among generations.*

In the first example it is assumed that $\tau^1_t = 0$. From the government budget identity this implies that the capital income tax is refunded to the old (who are paying the tax) through lump-sum taxes, that is, $\tau^2_t = -\tau^K_{t+1}f'(k_{t+1})k_{t+1}(1+n)$.

The impact effect of an increase in τ^K_{t+1} on k_{t+1} is given by

$$\frac{dk_{t+1}}{d\tau^K_{t+1}} = -\frac{f'(k_{t+1})k_{t+1}}{(1+\alpha_1)(1+f'(k_{t+1}))} \quad (44)$$

As expected, an increase in the capital income tax rate refunded to the tax payers as a lump-sum benefit, reduces the saving of the young in period $t+1$ (and of all future generations) for standard life-cycle reasons. This reduces the equilibrium capital-labor ratio in the short run. Given a diminishing marginal product of capital, this will raise the before-tax capital rental rate. The effect on the after-tax capital rental rate is obtained from equations 36 and 44.

$$\frac{dr_{t,t+1}}{d\tau^K_{t+1}} = \frac{d[\rho_{t+1}(1-\tau^K_{t+1})]}{d\tau^K_{t+1}} = -f'(k_{t+1})\left(1 + \frac{f''(k_{t+1})k_{t+1}}{(1+\alpha_1)(1+f'(k_{t+1}))}\right) \quad (45)$$

It is clear from 45 that the after-tax capital-rental rate will increase with the tax rate on capital rental income if -

$$\frac{-f''(k_{t+1})k_{t+1}}{(1+\alpha_1)(1+f'(k_{t+1}))} > 1 \quad (46)$$

This will occur for a low enough elasticity of substitution between labor and capital[21]. The local stability condition for the model is

$$\frac{-\alpha_1 k_t f''(k_t)}{(1+\alpha_1)(1+n)} < 1 \quad (47)$$

Thus, even if the model is dynamically efficient ($f'(k) > n$), both 46 and 47 can be satisfied. If the model is dynamically inefficient, there is a larger set of parameter values for which both 46 and 47 are satisfied.

Considering the effect of an increase in τ^K on the steady-state capital-labor ratio $\bar{k}$ and the steady state after-tax rental rate, $\bar{\rho}$ we find that

$$\frac{d\bar{k}}{d\tau^K} = \left(\frac{-1}{1+\alpha_1}\right)\left(\frac{1+n}{1+f'(\bar{k})}\right)\left[\frac{f'(\bar{k})\bar{k}}{\left(\frac{\alpha_1}{1+\alpha_1}\right)\bar{k}f''(\bar{k})+1+n}\right] \tag{48}$$

and

$$\begin{aligned}\frac{d\bar{r}}{d\tau^K} &= \frac{d[\bar{\rho}(1-\tau^K)]}{d\tau^K} \\ &= \frac{-f'(\bar{k})\left[f''(\bar{k})\bar{k}\left(\alpha_1[1+f'(\bar{k})]+1+n\right)+(1+\alpha_1)[1+f'(\bar{k})](1+n)\right]}{(1+\alpha_1)[1+f'(\bar{k})]\left[\left(\frac{\alpha_1}{1+\alpha_1}\right)\bar{k}f''(\bar{k})+1+n\right]}\end{aligned} \tag{49}$$

Assuming local stability, the denominators of 48 and 49 are positive. Thus, in the long-run, an increase in the capital income tax rate refunded as lump-sum transfers to the old paying the tax reduces the capital-labor ratio. Again, a low enough value of the elasticity of substitution between labor and capital could result in the after-tax rate of return to capital rising.

The generational accounts for this economy will show zero for all generations, before and after the increase in the capital income tax. If the economy is dynamically efficient, the increase in the capital income tax rate from an initial value of zero will cause efficiency losses as well as welfare losses for at least one generation. Assume the increase in the tax rate is unanticipated and starts in period $t+1$. The old in period $t+1$ (members of generation t), are not affected at all. They suffer a capital levy on period t saving, compared to the scenario without a tax rate increase. Since the capital levy is unanticipated, there are no distortions involved. They get back the resources lost through the capital levy as a lump-sum transfer. Next period, and forever after, the capital-labor ratio will be less that it would otherwise have been. Real wages are lower for each generation, starting with the one born in period $t+1$. If the after-tax rate of return is lower that it would have been without the tax rate increase, all generations starting with those born in period $t+1$ are worse off: their new intertemporal budget constraint lies strictly inside the old one. If the after-tax rate of return is higher, it is possible that some, but not all, generations starting with $t+1$ are better off. If the initial equilibrium is Pareto-inefficient, the reduction in the capital-labor ratio is a good thing and with the introduction of the capital income tax we are in a second-best world with two distortions. This case is left as an exercise to the reader.

The key point is that fiscal policy, including its intergenerational distributional consequences, here works entirely outside the generational accounts, which show zero throughout. Yet fiscal policy certainly influences the life-time private consumption opportunities of the various generations, even for a zero government consumption program, by changing the post-tax labor income and the intertemporal terms of trade faced by successive generations.

Example 2. *Changes in equilibrium factor returns reinforce the intergenerational redistribution recorded in the generational accounts.*

The second example is a straight lump-sum redistribution from the old to the young (a reverse unfunded social security retirement scheme). We now assume $\tau_t^K = 0$, and $\tau_t^1 = -\frac{\tau_{t-1}^2}{1+n}$. The introduction of the scheme in period $t+1$ is unexpected and permanent. We evaluate the increase in τ^2 from an initial value $\tau^2 = 0$.

The generational accounts of equations t and $t+1$ from period $t+1$ (discounted to the end of period t) on are given below in equations 50 and 51.

$$T_{t,t} = N_t \frac{\tau_t^2}{1 + f'(k_{t+1})} \tag{50}$$

$$T_{t+1,t} = \frac{N_{t+1}}{1 + f'(k_{t+1})} \left(-\frac{\tau_t^2}{1+n} + \frac{\tau_{t+1}^2}{1 + f'(k_{t+2})} \right) \tag{51}$$

The short-run effect of an increase in τ_t^2 on the capital-labor ratio is given in equation 52 , the long-run effect in equation 53.

$$\frac{dk_{t+1}}{d\tau_t^2} = \left(\frac{1}{1+\alpha_1}\right)\left(\frac{1}{1+n}\right)\left(\frac{1}{1+f'(k_{t+1})}\right) > 0 \tag{52}$$

$$\frac{d\bar{k}}{d\tau^2} = \left(\frac{1}{1+\alpha_1}\right)\left(\frac{1}{1+f'(\bar{k})} + \left(\frac{\alpha_1}{1+n}\right)\right)\left[\frac{1}{\left(\frac{\alpha_1}{1+\alpha_1}\right)\bar{k} + f''(\bar{k}) + 1 + n}\right] \tag{53}$$

In period t, the young receive a larger transfer and anticipate a higher tax when old. This raises their saving and thus next period's capital stock. In the long run, this effect is reinforced because each generation, in addition to saving part of the increase in period 1 transfer income, now also reduces its consumption while young in anticipation of the higher taxes when old. The saving rate of the young therefore rises unambiguously in the short run and in the long-run; the capital stock in period $t+1$ and the long run capital-labor ratio increase.

It is clear that the old in period $t+1$ are unambiguously worse off. At given factor prices, the increase in τ^2 will raise the permanent income of all generations beginning with $t+1$ if the economy is dynamically efficient ($f'(k) > n$), since in that case the higher taxes paid in old age are discounted at a rate in excess of the biological rate of return. The young in period $t+1$ will enjoy a higher wage because of the larger capital stock inherited by the economy. The interest rate they face will be lower than in the counterfactual scenario without the increased redistribution to the young. If their tastes are skewed sufficiently towards early consumption (if α_1 is small enough), generation $t+1$ is unambiguously better off. All subsequent generations in addition have a higher wage than they would have had without the tax-transfer scheme. With sufficiently low α_1 , they will all be better off.

From equations 50 and 51 it is clear that the generational accounts register the effects on the intergenerational distribution of life-time resources of (1) the direct redistribution at given factor prices and (2) the effect of the endogenous response of the interest rate (which declines) on the present value of lifetime taxes net of transfers. This second effect is of course absent when we consider an infinitesimal change in τ^2 from an initial value of 0. The positive effect of the lump-sum intergenerational redistribution on the equilibrium wage rate is not captured by the generational accounts. In this example the intergenerational distributional consequences of the general equilibrium repercussions on real wages and interest rates from lump-sum tax changes reinforce (are in the same direction as) the distributional effects of the initiating lump-sum tax-transfer change itself. The next example shows that this need not be the case.

Example 3. *The intergenerational redistribution recorded in the generational accounts is counteracted by changes in equilibrium factor incomes.*

Consider the case of an increase in the constant proportional tax rate τ_t^A on all asset income [22]. All revenues from the asset income tax are paid as a lump-sum transfer payment to the young. We also We assume$\tau_t^2 = 0$ and $\tau_t^1 = -\tau_t^A f'(k_t)k_t$ for all t. The model can be reduced to the first-order non-linear difference equation in equation 54.

$$\left(\frac{\alpha_1^{\frac{1}{\eta}}\left(1+f'(k_{t+1})(1-\tau_{t+1}^A)\right)^{\frac{1-\eta}{\eta}}}{1+\alpha_1^{\frac{1}{\eta}}\left(1+f'(k_{t+1})(1-\tau_{t+1}^A)\right)^{\frac{1-\eta}{\eta}}}\right)\left(f(k_t) - k_t f'(k_t) + \tau_t^A f'(k_t)k_t\right) = (1+n)k_{t+1} \tag{54}$$

Consider an unanticipated, permanent increase in the asset tax rate, starting in period t. The effect on k_{t+1} comes through two channels. The first is the intergenerational redistribution effect, given in 55; Since the asset tax imposed in period t was unanticipated, it is effectively a lump-sum tax on the old in period t; since the tax is redistributed to the young in period t, their saving will increase.

$$\frac{\partial k_{t+1}}{\partial \tau_t^A} = \Omega \Xi f'(k_t)k_t > 0 \tag{55}$$

where

$$\Omega = \left\{1 + n - \frac{\alpha_1^{\frac{1}{\eta}}\left(\frac{1-\eta}{\eta}\right)\left(1+f'(k_{t+1})(1-\tau_{t+1}^A)\right)^{\frac{1}{\eta}-2}(1-\tau_{t+1}^A)f''(k_{t+1})[w_t+\tau_t^A f'(k_t)k_t]}{\left[1+\alpha_1^{\frac{1}{\eta}}\left(1+f'(k_{t+1})(1-\tau_{t+1}^A)\right)^{\frac{1-\eta}{\eta}}\right]^2}\right\}^{-1} \tag{56}$$

> 0 if the model is locally stable.

and

$$\Xi = \frac{\alpha_1^{\frac{1}{\eta}} \left(1 + f'(k_{t+1})(1 - \tau_{t+1}^A)\right)^{\frac{1-\eta}{\eta}}}{1 + \alpha_1^{\frac{1}{\eta}} \left(1 + f'(k_{t+1})(1 - \tau_{t+1}^A)\right)^{\frac{1-\eta}{\eta}}} > 0 \tag{57}$$

The second channel is the intertemporal substitution channel. It will increase saving by the young in period t if and only if $\eta > 1$. It is given in equation 58.

$$\frac{\partial k_{t+1}}{\partial \tau_{t+1}^A} = \Omega \Theta f'(k_{t+1})[w_t + \tau_t^A f'(k_t) k_t] < 0 \tag{58}$$

where

$$\Theta = -\frac{\alpha_1^{\frac{1}{\eta}} \left(\frac{1-\eta}{\eta}\right) \left(1 + f'(k_{t+1})(1 - \tau_{t+1}^A)\right)^{\frac{1}{\eta}-2}}{\left[1 + \alpha_1^{\frac{1}{\eta}} \left(1 + f'(k_{t+1})(1 - \tau_{t+1}^A)\right)^{\frac{1-\eta}{\eta}}\right]^2} < 0 \text{ if } \eta > 1 \tag{59}$$

The short-run effect of an increase in both τ_t^A and τ_{t+1}^A on k_{t+1} is the sum of the effects given in 55 and 58, given in 60. It will be negative provided η is sufficiently larger than 1 .

$$\frac{\partial k_{t+1}}{\partial \tau_t^A} + \frac{\partial k_{t+1}}{\partial \tau_{t+1}^A} = \Omega \left(\Xi f'(k) k + \Theta f'(k)[w + \tau^A f'(k) k]\right) \tag{60}$$

The long-run effect of the asset income tax rate increase, with the proceeds transferred lump-sum to the young are given in equation 61.

$$\frac{d\bar{k}}{d\tau^A} = \hat{\Omega} \left(\Xi f'(k) k + \Theta f'(k)[w + \tau^A f'(k) k]\right) \tag{61}$$

where

$$\begin{aligned} \hat{\Omega} &= \Omega + \Xi[k_t f''(k_t)(1 - \tau_t^A) - \tau_t^A f'(k_t)] \\ &> 0 \text{ i.f.f. the model is locally stable} \end{aligned} \tag{62}$$

As shown in equation 60 , a necessary condition for the impact effect on the capital-labor ratio (and therefore on the period $t+1$ wage) to be negative is that the intertemporal substitution effect be negative, which will be the case i.f.f. the intertemporal elasticity of substitution η has a value larger than 1. In addition the negative intertemporal substitution effect has to dominate the positive intergenerational redistribution effect. The long-run effect on the capital-labor ratio and on the real wage can also be negative under very similar conditions (61). We now consider the distributional consequences when these conditions are satisfied.

The old in period t are obviously worse off. The young in period t face a predetermined real wage and receive the lump-sum transfer payment. If the after-tax rate of return to saving they face $(1 + r_{t,t+1}(1 - \tau_{t+1}^A))$ declines as a result of the increase in τ_t^A and τ_{t+1}^A , they too may be worse off, despite the receipt of the transfer payment, if they are sufficiently patient (have a high enough value of α_1). All later generations still receive the lump-sum payment while young, but now have a lower real wage than they would have had without the increase in the asset income tax rate. It is possible that their disposable income while young actually falls (for a high enough intertemporal elasticity of substitution). If disposable income while young falls for generations later than t, then a lower after-tax rate of interest is sufficient for them to be worse off. Even if the after-tax rate of return increases, they will be worse of if they are sufficiently impatient (small values for α_1).

For generations t and beyond, the changes in the generational accounts at constant factor prices may therefore be offset by endogenous changes in factor returns.

The generational accounts for the old and the young in period t are given below. All generations born after period t have generational accounts like the one for generation t.

$$T_{t-1,t-1} = N_{t-1}\left(\frac{\tau_t^A f'(k_t)k_t(1+n)}{1+f'(k_t)}\right) \tag{63}$$

$$T_{t,t-1} = \left(\frac{N_t}{1+f'(k_t)}\right)\left[-\tau_t^A f'(k_t)k_t + \left(\frac{1+n}{1+f'(k_{t+1})}\right)\tau_{t+1}^A f'(k_{t+1})k_{t+1}\right] \tag{64}$$

The generational accounts are not an accurate measure of how exogenous (or counterfactual, in the policy experiment sense) budgetary rules, actions or shocks affect the intergenerational distribution of private resources at given prices, quantities and rates of return; that is they are not an accurate gauge of what we shall call the *direct* effects of the budget. This is because, as shown in examples 1 and 2, the generational accounts register the effects on tax payments and transfer receipts of changes in the tax and transfer bases brought about by the general equilibrium repercussions of the budgetary policy changes. That is, the generational accounts register part of what we shall call the indirect effects of the budget on the intergenerational distribution of private resources considers. The indirect effects are all the general equilibrium changes in after-tax and transfer factor incomes, prices and rates of return due the changes in the budget, both the endogenous changes in the tax bases registered in the national accounts and the endogenous changes in the before-tax distribution of resources across generations ignored by the generational accounts.

This does not mean, of course, that the data that go into the construction of generational accounts are not worth collecting or that they cannot yield important insights into intergenerational distributional consequences of alternative budgetary policies. It does mean that without the aid of a numerically calibrated behavioral model of the economy, it is impossible to extract and interpret the message in the generational accounts.

Proposition 4 *Generational accounts do not measure the direct effect of the budget on the intergenerational distribution of private resources. Neither do they measure the total (direct plus indirect) effect. Instead they measure the sum of the direct effect and a part of the indirect effect—the endogenous response of tax and transfer bases to changes in budgetary policy. The generational accounts can only be interpreted with the help of a fully articulated sequential general equilibrium model of the economy.*

6 Is a tax cut tomorrow as good as a government bond today?

Consider a world without distortionary taxes and transfers and hold constant the government's consumption program. The life cycle model holds and there is no uncertainty. Under these conditions, an individual's generational account or *lifetime net payment* (LNP) to the government

> ...is a sufficient statistic for the government's treatment of individuals; any intertemporal equilibrium will be unaffected by changes in the timing of lifetime net payments to the government that leave individual LNPs unchanged ([15]).

Uncertainty does not change the validity of this statement as long as markets are complete and the proper state-contingent prices are used to price uncertain future government taxes and transfers. These taxes and transfers may be uncertain either because the tax and transfer rules, although deterministic, make realized taxes and transfers functions of random variables such as the future tax base, or because government behavior itself is random (adds noise to the system). If government behavior itself is stochastic, complete markets must include markets to buy and sell goods and services contingent the realization of the additional states of nature created by the government's random behavior.

There are two distinct approaches to the micro foundations of credit rationing and liquidity constraints. The first stresses asymmetric or private information. When exogenous risk characteristics of the borrower are known costlessly to the borrower but unobservable (or observable only at a cost) to the lender, *adverse selection* problems arise, and credit rationing may result. When the riskiness of the project for which funds are borrowed is a function of actions by the borrower that are unobservable (or observable only at a cost) to the lender, *moral hazard* problems arise with credit rationing again as a possible consequence. The second approach emphasizes the importance of the fact that few contracts involve the simultaneous and final exchange of objects of equal value. This creates a key role for third-party enforcement of contracts whenever the net benefits from abiding by the contract vary over the life of the contract and become negative for one party to the contract before they do so for the other party or parties. The two approaches are complementary rather than mutually exclusive.

Since most of the literature has emphasized the asymmetric information motivation for credit rationing, I will elaborate here on the contract enforcement argument.

In their seminal work on sovereign borrowing, default and credit rationing, Eaton and Gersovitz [1981a, b] pointed out that sovereign debtors can, by definition, not bind themselves with contracts that are enforceable by some outside party. Only contracts that are time-consistent (in the sense that it is in the perceived self-interest of all parties to abide by the contract at each point in time during the duration of the contract) will therefore be concluded. Within national economies, the owners of human capital are in a position rather similar to that of a sovereign borrower. Ever since the abolition of slavery, of indentured labor and of the debtor's prison, it has not been possible to attach (to offer a creditor a legally enforceable lien on) future labor income. Gavin Wright [1995] contains a fascinating historical perspective on the "free labor" movement in the USA since the eighteenth century and its even earlier roots in English common law. This historical process has brought about a state of affairs in which third-party enforcement of long-term individual employment contracts is impossible in most of the industrial world. It is almost universally true that the worker cannot legally bind himself; sometimes the same freedom (or inability to secure third-party enforcement) is accorded employers. The prospect of repeated future interaction between employer and worker and the private value of a reputation for reliability may make long-term employment contracts self-enforcing under certain circumstances.

From the point of view of the informativeness of the generational accounts, what matters is that this *inalienability* of labor income makes it very poor collateral for future borrowing. In the most extreme case considered below, it is impossible to borrow against the security of anticipated future labor income at all. This in not because the labor income in uncertain and imperfectly observable by the creditor. The reason is instead that there is no mechanism (such as a legally binding contract enforced through the courts) through which a borrower can commit herself to earmark future labor income for the servicing of a debt they may wish to incur. The example below is extreme, but as long as of a stream of riskless future labor income, discounted at the riskless rate, represents less effective collateral than a government bond of equal value, its conclusions are not changed qualitatively.

The example below works because of the key assumption that it is future labor income *net of taxes and transfer payments* that is inalienable and therefore unsuitable as collateral for consumption loans. In other words, future receipts of government transfer payments are not collateralizable because it is impossible to write a legally binding contract earmarking future welfare checks for the servicing and repayments of consumption loans. This assumption is extreme, and is only made because the polar case it represents is extremely transparent. The key point is that it is clearly much more realistic than the opposite polar case considered in [14], [19] and [15].They assume that a credit-constrained borrower can increase her consumption by the same amount through additional borrowing when she is unexpectedly designated the beneficiary of a future government tax cut or transfer payment worth \$100 one period from now, as she would if she unexpectedly discovered government bonds under the mattress worth $\$100/(1+i)$ where i is the appropriate risk-free discount rate. Note again that this argument holds in a world with complete certainty. It relies on the *institutional* assumption that certain kinds of contracts can and others cannot be enforced through third parties.[23]

I now consider a very simple example, described in a few words in Hayashi [14], p. 117, of how aggregate consumption is affected by the government reducing taxes on each of the young of generation t by an amount ξ in period t and raising taxes by an amount $\xi(1+r_{t,t+1})$ per capita on the same generation when old in period $t+1$.

Each household of generation t maximizes $\ln c_t^1 + \alpha_1 \ln c_t^2$ subject to the following constraints

$$c_t^1 \leq w_t - \tau_t^1 \tag{65}$$

$$c_t^2 \leq e w_{t+1} - \tau_t^2 + (1+r_{t,t+1})(w_t - \tau_t^1 - c_t^1) \tag{66}$$

In this example, households work in both periods of their life. The exogenous labor endowments are 1 when young and e when old. The inalienability of after-tax labor income means that the young may face a borrowing constraint: non-human wealth cannot become negative (equation 65). The riskless lending rate for households and the riskless lending and borrowing rate for the government is r. Note that there is no uncertainty in the model. There is just the certainty of default on (repudiation of) consumption loans to young households, which causes this market to dry up completely.

If the borrowing constraint is non-binding, the consumption decisions of generation t are as given in equations 67 and 68.

$$c_t^1 = \left(\frac{1}{1+\alpha_1}\right)\left(w_t - \tau_t^1 + \frac{e w_{t+1} - \tau_t^2}{1+r_{t,t+1}}\right) \tag{67}$$

$$c_t^2 = \left(\frac{\alpha_1(1+r_{t,t+1})}{1+\alpha_1}\right)\left(w_t - \tau_t^1 + \frac{e w_{t+1} - \tau_t^2}{1+r_{t,t+1}}\right) \tag{68}$$

If the borrowing constraint is binding ($\frac{1}{c_t^1} > \frac{\alpha_1}{c_t^2}(1+r_{t,t+1})$), the consumption decisions of the household are as given in equations 69 and 70.

$$c_t^1 = w_t - \tau_t^1 \tag{69}$$

$$c_t^2 = ew_{t+1} - \tau_t^2 \tag{70}$$

It now becomes essential to introduce some within-generation heterogeneity. The reason is simple. If no member of generation t faces a borrowing constraint, the combination of a tax cut of ξ in period t and a tax increase of $\xi(1 + r_{t,t+1})$ in period $t+1$ for the same persons, will obviously not affect anyone's consumption. If every member of generation t faces a borrowing constraint, on the other hand, the policy experiment of a tax cut financed by borrowing is obviously infeasible: there is no one to buy the additional debt the government wants to issue in order to finance the tax cut (the old in period t won't be around in period $t+1$).

We therefore assume that there is a constant number N^H of households each period who have a high value, $e^H$of the old age labor endowment and that a constant number N^L has a low old age labor endowment e^L. We furthermore assume that $e^L < 1 < e^H$ and that e^L is sufficiently far below and e^H sufficiently far above 1 (the labor endowment when young), that the households endowed with e^L are never faced with a borrowing constraint while those endowed with e^H always are.[24]

This means that aggregate consumption at time t, denoted C_t , is given by

$$\begin{aligned} C_t = N^L\left(\tfrac{1}{1+\alpha_1}\right)\left(w_t - \tau_t^1 + \tfrac{e^L w_{t+1} - \tau_t^2}{1+r_{t,t+1}}\right) + N^H(w_t - c_t^1) \\ +N^L\left(\tfrac{\alpha_1(1+r_{t-1,t})}{1+\alpha_1}\right)\left(w_{t-1} - \tau_{t-1}^1 + \tfrac{e^L w_t - \tau_{t-1}^2}{1+r_{t-1,t}}\right) + N^H(e^H w_t - \tau_{t-1}^2) \end{aligned} \tag{71}$$

At given interest rates and wage rates, the effect of the debt-financed tax cut of ξ per capita for young members of generation t , combined with the credible announcement of a future (period $t+1$) increase in taxes of $\xi(1+r_{t,t+1})$ per capita on members of that same generation t will raise aggregate consumption by $N^H\xi$, the per capita tax cut times the number of borrowing-constrained members of generation t.

There will obviously be general equilibrium repercussions for equilibrium prices and quantities, but these are not our concern here. The main point is that the generational accounts are not a sufficient statistic for the government's treatment of individuals. The intertemporal equilibrium will be affected by changes in the *timing* of lifetime net payments to the government that leave individual generational accounts unchanged. The reason is that the generational accounts discount the future tax increase at the government's lending and borrowing rate $r_{t,t+1}$. The riskless interest factor, $1 + r_{t,t+1}$, however, is below the marginal rate of intertemporal substitution for the borrowing-constrained households. For generation t the relevant Euler equation is

$$\frac{c_t^2}{\alpha_1 c_t^1} = 1 + r_{t,t+1} + \frac{\lambda c_t^2}{\alpha_1}$$

where $\lambda \geq 0$ is the shadow price of the borrowing constraint 65 with $\lambda > 0$ if that constraint is binding.

In the asymmetric information literature, all contract can be enforced as long as the contingency specified in the contract can be observed by a third party. The timing of net tax payments over the life cycle of a household (holding constant their present value under discounting at the government's interest rate) has been shown not to matter in certain adverse selection models with collusive sharing of information by otherwise competitive lenders offering price-quantity contracts about the total amount of loans taken out from all lenders by a particular borrower (see e.g. [19], [14] and [15]. In each of these models, however, the future tax increase falling on an individual is assumed to be public information, shared equally between lender and borrower (and, one assumes, by the third party implicitly enforcing the contract). It is quite plausible to assume that, even if the aggregate tax burden is public information, the breakdown of the total among individuals is not public information. We do in fact in many countries have privacy laws about this. With wage income-related taxes such as the personal income tax and social security contributions, the assumption that future labor income is private information is tantamount to assuming that future tax payments are private information. With individual taxes and transfers unobservable, a tax cut of $100 for an individual borrower this period will not lead lenders to reduce loans to him by $100, even if it is known that future taxes for each borrower will go up by whatever tax cut they received times 1 plus the government's rate of interest.

It seems safe to start any discussion about the effect of the budget on saving from the presumption that some households are liquidity-constrained and that debt-financed tax cuts can relax these constraints. The quantitative importance of such liquidity constraints is of course still an open issue. I summarize this as the last proposition of the paper.

Proposition 5 *Liquidity constraints or borrowing constraints, due either to the inalienability of future after-tax labor income or to private information, may cause the timing of the net taxes paid to the government over the lifecycle of a household to matter, in addition to (or rather than) their present value, calculated using the government's discount rate.*

7 Conclusion.

There are three main conclusions. (1) The informativeness of generational accounts and their usefulness in budgetary policy evaluation or design lives or dies with the life-cycle model of consumption. (2) Even if the life-cycle of consumption is valid, there is no straightforward interpretation of the generational accounts unless there are no significant general equilibrium effects of budgetary policy on before-tax and before-transfer incomes, that is, unless incidence questions can be ignored. (3) Before any conclusion can be reached about the effect of the budget on intergenerational distribution and welfare, the intergenerational distribution of the benefits from public consumption has to be allowed for.

Despite these three cautionary reminders of potential pitfalls in the use and interpretation of generational accounts, I consider the construction of generational accounts to be a valuable enterprise that will benefit all researchers interested in saving behavior or in the effect of the budget on the intergenerational distribution of private resources. However, without (1) explicit consideration of the intergenerational distributional implications of the government consumption program, (2) convincing evidence that the life-cycle model adequately characterizes private consumption behavior and (3) a model of the general equilibrium repercussions of budgetary policy, the generational accounts don't speak clearly. Indeed they barely whisper, and what they appear to whisper could be untrue.

Notes

[1]By Ricardian gift motive I mean the following: (1) the utility of other generations posively affects my utility; (2) there is no strategic behavior amoung the generations.

[2]There may be incomplete market participation, because both the unborn and the dead will clearly have trouble participating in transactions today.

[3]For evaluating the usefulness of generational accounts in assessing the impact of the budget on generational welfare and national saving behavior, the complete markets assumption can be relaxed somewhat. It is quite acceptable for future before-tax labor income (or other future income that is not the return on some tradable financial claim) to be illiquid or hard to collateralize. The key assumption that must be satisfied for the generational accounts to be informative is that, *given* the *present value of lifetime (lump-sum) taxes net of transfers* (discounted using government discount factors), the *timing* of these taxes and transfers over each generation's life span is irrelevant In this case the present value (using the government discount rate) of the life-time transfers net of taxes received by any household is equivalent (in terms of liquidity and as regards value as collateral for consumpion loans) to a traded financial asset of equal value. See e.g. Hayashi [1987], Yotsuzuka [1987] and Kotlikoff [1989]. These papers all consider examples where future before-tax labor income in imperfectly collateralizable but future government transfer payments (discounted at the government's rate of interest) are equally liquid as a current holding of government debt of equal value.

[4]Any social welfare function that is *individualistic (*in the sense that it respects individual preference orderings over private and public consumption goods), will require that the evaluation of both the efficiency and the distributional aspects of budgetary policy ultimately be reduced to the consideration of how the policy affects the(contingent) sequences of individual-specific consumption of private and public goods. Government financing, including the generational accounts, matters only to the extent that it affects the sequences of public and private consumption that can be supported.

[5]Henceforth 'before-tax' will mean before tax, transfer,benefit and subsidy. The same applies, *mutatis mutandis*, to 'after-tax'..

[6]For expositional simplicity, depreciation is ignored.

[7]It ignores the impact of future taxes (not necessarily paid by the current holders of the assets) on the market value of the assets.

[8]The continuous time counterpart of equation 11 may be more familiar. Integration by parts shows that

$$\int_t^{\infty} i(s)M(s)e^{-\int_t^s i(z)dz}ds = M(t) + \int_t^{\infty} \dot{M}(s)e^{-\int_t^s i(z)dz}ds$$

or, equivalently, letting $r$denote the instantaneous riskless real rate of interest

$$\int_t^{\infty} i(s)\frac{M(s)}{P(s)}e^{-\int_t^s r(z)dz}ds = \frac{M(t)}{P(t)} + \int_t^{\infty} \frac{\dot{M}(s)}{P(s)}e^{-\int_t^s r(z)dz}ds$$

[9]The pecuniary rate of return on non-interest-bearing money will not in general be equal to the returns on the non-monetary assets. This is consistent with the absence of arbitrage opportunities because of the non-pecuniary returns from holding money.

[10]They use the generational accounts equation in forward-looking evalations of existing or alternative policies. With some hand-waving in the direction of certainty equivalence, they therefore require expected or ex-ante returns rather than ex-post or realized returns to be equalized.

[11]This utility function is extremely simple but suffices and is not very restrictive for the purpose of this paper. Money in the direct utility function could be replaced by some implicit transactions function or by a cash in advance (or in arrears) model. Separability within a period could be relaxed. Relaxation of time-additivity would be much messier.

[12]This is not necessarily a universal feature of the human condition. In France, indigent parents have a legal rigth to some measure of financial support from their children.

[13]This is not a self-evident assumption. While generation t chooses its consumption when old and the bequest it makes to generation t+1 after generation t-1 has chosen its bequest to generation t, the consumption when young of generation t is chosen before or at the same time that generation t-1 chooses its bequest to generation t. In principle, strategic interaction could occur.

[14]The equilibrium (after-tax) nominal rate of interest should be positive for short bonds to be held.

[15]The utility functional specifies the utility of bequests as depending on the utility of a representative descendant. The same utility of bequest function is consistent with a "the more the merrier" view of descendants' utility through a simple reinterpretation of the generational discount rate as being the product of the true generational discount rate and the number of descendants one has.

[16]In Kotlikoff [1989], the fiscal balance rule is only specified for a steady state with constant values for all real quantities and intertemporal prices, including a constant value of real government consumption. That paper does, however, refer to use being made of the fiscal balance rule in the numerical simulations of Auerbach and Kotlikoff [1987]. While I was unable to trace the rule in that monograph, the non-steady state version of the fiscal balance rule proposed in this paper seems the obvious generalization of Kotlikoff's steady state rule.

[17]The size of generation 0 and the level of productivity in period zero are set equal to 1.

[18]The same holds for total government consumption per capita and for total government consumption per unit of efficiency labor. Note that with purely non-rival public consumption goods, consumption per capita equals total consumption.

[19]and indeed by heterogenous agents within a generation.

[20]If occupation by the enemy is one of the consequences of being defeated

militarily, the young, who would be living longest under the foreigh yoke could be expected to be most in favor of a strong defense.

[21]The elasticity of substitution between capital is labor is given by $\sigma = \frac{[f(k)-kf'(k)]f'(k)}{-kf''(k)f(k)}$

[22]The difference between this example and the first is that the interest rate now equals the before-tax rental rate of capital.

[23]It is therefore very much in the spirit of the other legal-institutional assumption made in Section 2.4, that bequests could not be negative.

[24]Alternatively we could have made one group of households extremely patient and the other group extremely impatient, while keeping their endowments identical.

References

[1] Atkinson, Anthony B. and Joseph E. Stiglitz [1980], *Lectures on Public Economics*, McGraw-Hill Book Company, New York, NY.

[2] Auerbach, Alan J. and Laurence J. Kotlikoff [1987], *Dynamic Fiscal Policy*, New York: Cambridge University Press.

[3] Auerbach, Alan J. [1991], Jagadeesh Gokhale and Laurence J. Kotlikoff, "Generational Accounts, A Meaningful Alternative to Deficit Accounting", NBER Working Paper No. 3589, January, published in David Bradford ed. [1991], *Tax Policy and the Economy, vol. 5.* Cambridge: National Bureau of Economic Research and the MIT Press.

[4] Auerbach, Alan. J., Jagadeesh Gokhale and Laurence J. Kotlikoff [1992], Generational Accounting: A New Approach to understanding the effects of fiscal policy on saving., Scandinavian Journal of Economics 94(2), pp. 303-319.

[5] Auerbach, Alan J., Jagadeesh Gokhale and Laurence J. Kotlikoff [1994], "Generational Accounting: A Meaningful Way to Evaluate Fiscal Policy," Journal of Economic Perspectives, 8, pp. 73- 94.

[6] Buiter, Willem H. [1980], "'Crowding out' of private capital formation by government borrowing in the presence of intergenerational gifts and bequests", Greek Economic Review, 2, pp. 111-142, reprinted in Willem H. Buiter, *Macroeconomic Theory and Stabilization Policy*, University of Michigan Press, Ann Arbor, Michigan.

[7] Buiter, Willem H. [1983], "Measurement of the public sector deficit and its implications for policy evaluation and design", International Monetary Fund Staff Papers, 30, June, pp. 306-49.

[8] Buiter, Willem H. [1990], "Debt neutrality, redistribution and consumer heterogeneity: a survey and some extensions", in Willem H. Buiter *Principles of Budgetary and Financial Policy*, MIT Press, Cambridge, Massachusetts, pp. 183-222.

[9] Buiter, Willem H. and Kenneth M. Kletzer [1994], "Ponzi Finance, Government Solvency and the Redundancy or Usefulness of Public Debt", Revised Version, Mimeo, Yale University, April.

[10] Diamond, Peter [1970], "Incidence of an Interest Income Tax", Journal of Economic Theory, 2, pp. 211-24.

[11] Eaton, Jonathan and Mark Gersovitz [1981a], "Debt with potential repudiation: theoretical and empirical analysis", Review of Economic Studies, 48, 152, pp. 289-309.

[12] Eaton, Jonathan and Mark Gersovitz [1981b], *Poor-country borrowing in private financial markets and the repudiation issue*, Princeton, N.J. International Finance Section.

[13] Fisher, Eric O'N. and YoungSoo Woo [1994], A New Measure of the Korean Current Account, mimeo.

[14] Hayashi, Fumio [1987], Tests for liquidity constraints: a critical survey and some new observations. Chapter 13 in Truman F. Bewley editor, *Advances in Econometrics Fifth World Congress, Volume II.* Cambridge University Press, pp. 91-120.

[15] Kotlikoff, Laurence J. [1989], "From deficit delusion to the fiscal balance rule: looking for an economically meaningful way to assess fiscal policy", NBER Working Paper No, 2841, February

[16] Kotlikoff, Laurence, J. [1992] *Generational Accounting: Knowing Who Pays, and When, for What We Spend,* The Free Press, New York, NY.

[17] Musgrave, Richard A., K.E. Case and H. Leonard [1974], "The distribution of fiscal burdens and benefits", Public Finance Quarterly, 2, pp. 259-311.

[18] Wright, Gavin [1995], "The origins and economic significance of free labor in America", mimeo, Faculty of Economics and Politics, University of Cambridge.

[19] Yotsuzuka, Toshiki [1987], Ricardian equivalence in the presence of capital market imperfections. Journal of Monetary Economics *20* , pp. 411-436.