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FISCAL POLICY INTERDEPENDENCE AND EFFICIENCY

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ABSTRACT

This paper uses a two-country overlapping generations model to study the international transmission of fiscal policy among open interdependent economies under free international capital mobility. With only lump-sum taxes and transfers, international transmission involves only pecuniary externalities: barring dynamic inefficiency, only distributional issues (intergenerational and international) are involved. With age-specific taxes and transfers, the ability to run deficits and issue debt does not enhance the choice set of the governments.

Source-based taxes on the rentals from capital and residence-based taxes on all property income are also studied.

Willem H. Buiter P.O. Box 1972 Yale Station Department of Economics Yale University 28 Hillhouse Avenue New Haven, CT 06520-1972 (203)432-3580 Kenneth M. Kletzer P.O. Box 1972 Yale Station Department of Economics Yale University 28 Hillhouse Avenue New Haven, CT 06520-1972 (203)432-3608 [1987], when we move away from competitive equilibria in which all the assumptions for Pareto optimality are satisfied, market prices may do more than equate supply and demand and distribute income:

In economies with incomplete contingent markets, prices span the subspace in which consumption plans can be chosen. In economies with asymmetric information, prices transmit information. When agents affect prices, they affect the welfare of the other agents by altering their feasible consumption sets or their information structures. (p. 264)

In such economies the distinction between pecuniary and technological externalities vanishes because changes in prices do more than create or destroy rents. It is possible that the arguments in favor of the coordinated international management of what are prima facie international pecuniary fiscal externalities are (implicitly) based on such a non-Walrasian world view. It is however surely desirable to bring out explicitly either the reason(s) for the breakdown of the first fundamental theorem of welfare economics or the distributional criteria that support the cooperative fiscal policy prescription.

If demand-deficient Keynesian equilibria are likely to result from contractionary fiscal policy actions, a prima facie case for policy intervention, including international coordination of policies, exists. In a Keynesian unemployment equilibrium the value of output foregone exceeds the value of the extra leisure "enjoyed" by the unemployed. This holds even in a closed economy. In addition, in an open economy, part of any demand contraction (originating from the public or private sectors) will spill over to the rest of the world through the deflating country's demand for imports and supply of exports. This creates a nonpecuniary externality as the prices of goods and labor do not measure their marginal social opportunity costs.

In the rest of this paper the focus is not on Keynesian or other non-Walrasian equilibrium characterizations of goods markets and labor markets but on the distributional and efficiency arguments for coordination when many (or all) of the conditions required for Walrasian equilibria to be efficient are satisfied. Since stabilization policy is not our concern, our use of a nonmonetary model represents a considerable gain in simplicity with a relatively small cost in terms of loss of generality.

The efficient use of policy instruments such as exhaustive public spending (with or without international externalities) will in general require international policy coordination (see e.g. Kehoe [1986a, b]). The same holds for the efficient use of distortionary taxes and transfers (assuming no lump-sum taxes and transfers are available). The efficient use of lump-sum taxes and transfers will not in general require international policy coordination.

In a sequel to this paper (Buiter and Kletzer [1990a]) it is shown that in the absence of preexisting distortions or distortionary policy instruments and with unrestricted access to lump-sum taxes and transfers between generations within each country, policy cooperation is required in order to achieve a global equilibrium that is not just Pareto efficient with respect to individual household preferences but also represents a Pareto optimum with respect to the utilitarian social welfare functions of two national (and nationalistic) planners. Cooperation is also required to achieve a restricted optimum with respect to a utilitarian global social welfare function reflecting a global planner's international and intergenerational distribution preferences. This optimum is restricted because international lump-sum transfers are not permitted. International transfers (or side payments) are in general required to achieve the unrestricted global command optimum.

To put the key issues as concisely and precisely as possible we consider a two-country overlapping generations (OLG) model. The OLG model without operative gift and bequest motives has heterogeneity between the young and the old. This is essential if the proposition that fiscal coordination is welfare enhancing is to have a chance when only deficit financing and lump-sum taxes or transfers are considered. Representative agent models have "first-order" debt neutrality: holding constant the paths of exhaustive public spending and of distortionary taxes, transfers, tariffs and subsidies (including monetary financing), intertemporal redistributions of lump-sum taxes and transfers (and any associated variations in public sector deficits and debt) that are consistent with government solvency do not alter the equilibrium of the economy.

We proceed as follows. This paper focuses on the positive economics of fiscal policy in open interdependent economies and on the Pareto optimality (with respect to the individual private utility functions) of the equilibria generated by a range of fiscal policy actions. The analysis is restricted to the case of a nonmonetary economy. The sequel (Buiter and Kletzer [1990a]) studies Nash and cooperative policy behavior in this economy.

In Section 2 of the paper, we introduce a two-country Diamond model (Diamond [1965, 1970]). It is a competitive world economy whose private consumption sectors consist of the two-period Allais-Samuelson OLG model and whose production technology is given by a neoclassical production function. There are no distortions or technological externalities. Note that the lack of trading across generations that characterizes OLG models¹ is not in itself a distortion and a sufficient reason for inefficiency. Absent other distortions, any competitive equilibrium in an OLG model with a finite terminal date (which may be beyond the lifetimes of all generations currently

alive) is a Pareto optimum. Even if the economy goes on forever, dynamic inefficiency in only a possibility, not a necessary implication of absence of unrestricted trading across all generations that have been, are and ever will be. It is shown in Buiter and Kletzer [1990a] that a dynamically inefficient equilibrium cannot occur if national governments have utilitarian (national) objective functions and have access either to unrestricted age-specific lump sum taxes and transfers, or to age-independent lump sum taxes and transfers and unbalanced budget deficits and surpluses. Even if the two (large) governments play Nash and pursue strictly nationalistic objectives, any competitive equilibrium with perfect international capital will be dynamically efficient. The logic is simple: no government will adopt policies that keep its own interest rate below its growth rate forever. If the interest rate is above the growth rate in one country, perfect international capital mobility (and the assumption of equal growth rates (at any rate in the long run)) ensures that it holds for the whole world economy.

In Section 2 the only taxes are lump-sum taxes (transfers) paid by (to) the young or the old. Both the case of age- and generation-specific and age- and generation-independent lump-sum taxes are considered. The consideration of arbitrary age-specific taxes and transfers made the Yaari-Blanchard-Weil OLG model a less attractive vehicle as easy aggregation is lost.²

We consider the effects of policies that increase one nation's public debt on domestic and foreign interest rates, and capital stocks. Efficiency and (international and intergenerational) distribution aspects are reviewed. The consequences of the existence of long-dated debt (with a variable market price) are discussed in Section 3. In Section 4 distortionary taxes (both source-based and residence-based) and exhaustive public spending are analyzed.

2. GOVERNMENT BORROVING: THE CASE OF LUMP-SUM TAXES

We consider the two-country, one-good version of Diamond's [1965] two-period OLG model used in Buiter [1981] to which we add government borrowing, lump-sum taxes and transfers and, in later sections, distortionary taxes. Home country variables and functions will be unstarred, and foreign country variables and functions will be starred.

Individuals of the same generation within a country are identical. Successive generations have the same utility functions. People live for two periods, work in the first period of their lives and retire in the second. There is no intergenerational gift or bequest motive. Labor supply is inelastic and scaled to unity for each young worker. The home country's competitive representative individual's optimization problem is given in equations (1) and (2).

(1)
$$\max_{\substack{c_t^1, c_t^2}} u(c_t^1, c_t^2)$$

subject to

(2)
$$w_t - \tau_t^1 - c_t^1 \ge (c_t^2 + \tau_t^2)(1 + \tau_{t+1})^{-1}$$
 $c_t^1, c_t^2 \ge 0$.

The utility function u is twice continuously differentiable, strictly quasiconcave, and increasing in c^1 and c^2 . We assume that

The utility of lifetime consumption is maximized subject to the lifetime budget constraint (2). The government is assumed not to pursue obviously infeasible policies, so

(3)
$$w_t \geq \tau_t^1 + \tau_t^2 (1 + r_{t+1})^{-1}$$
.

Provided w is positive and r is bounded--which will be assured by the labor market assumptions and the constraints put on the production technology below--positive solutions will be obtained for c^1 and c^2 .

The interior first-order conditions for the individual are

(4)
$$\frac{u_{1}(c_{t}^{1}, c_{t}^{2})}{u_{2}(c_{t}^{1}, c_{t}^{2})} = 1 + r_{t+1}.$$

Equations (2) (holding with equality) and (4) can be solved for c^1 and c^2 .

(5a)
$$c_t^1 = c^1(w_t - \tau_t^1, \tau_t^2, 1 + r_{t+1})$$

(5b)
$$c_t^2 = c^2(w_t - \tau_t^1, \tau_t^2, 1 + r_{t+1})$$

The partial derivatives of these two consumption functions are summarized in equations (6a-f).

(6a)
$$c_w^1 \equiv \frac{\partial c_t^1}{\partial (w_t - \tau_t^1)} = -(u_{12} - (1 + r_{t+1})u_{22})(1 + r_{t+1})\Delta^{-1}$$

(6b)
$$c_{\tau^2}^1 \equiv \frac{\partial c_t^1}{\partial \tau_t^2} = -\left[\frac{1}{1 + r_{t-1}}\right] c_w^1$$

(6c)
$$c_w^2 = \frac{\partial c_t^2}{\partial (w_t - \tau_t^1)} = (1 + r_{t+1})(1 - c_w^1)$$

$$(6d) c_{\tau^2}^2 = \frac{\partial c_{\tau^2}^2}{\partial \tau_t^2} = -(1 - c_w^1)$$

(6e)
$$c_r^1 = \frac{\partial c_t^1}{\partial (1+r_{t+1})} = \left[u_2 - (c_t^2 + \tau_t^2)(1+r_{t+1})^{-1}(u_{12} - (1+r_{t+1})u_{22}) \right] \Delta^{-1}$$

$$(6f) c_r^2 = \frac{\partial c_t^2}{\partial (1 + r_{t+1})} = -(1 + r_{t+1}) c_r^1 + \frac{(c_t^2 + \tau_t^2)}{1 + r_{t+1}}$$

where $\Delta \equiv u_{11} - (1 + r_{t+1}) [2u_{12} - (1 + r_{t+1})u_{22}] < 0$ by strict quasiconcavity.

If consumption in both periods is a normal good, then $1>c_w^1$, $c_w^2(1+r)^{-1}>0$. We assume this to be the case in what follows.

Output I is produced by a twice continuously differentiable production function with constant returns to scale capital I and labor L.

$$\frac{y_t}{L_t} = y_t = f(\frac{I_t}{L_t}) = f(k_t) , f' > 0; f'' < 0 ; f(0) = 0 ;$$

$$\lim_{k\to 0} f'(k) = \lim_{k\to \infty} \frac{1}{f'(k)} = + \infty \; ; \; k \ge 0 \; .$$

The labor market and capital rental market are competitive and clear, so

(7)
$$w_t = f(k_t) - k_t f'(k_t) = w(k_t)$$
,

(8)
$$r_t = f'(k_t) .$$

The Inada conditions imposed on the production function assume that, provided the initial capital stock is positive, the capital stock will always be positive.

The government imposes lump-sum taxes (transfers when negative) on the young and/or the old, and satisfies its budget identity by borrowing or lending. In this section all debt consists of one-period real bonds. The home country government budget identity is $B_{t+1} \equiv (1+r_t)B_t - \tau_t^2L_t - \tau_{t-1}^2L_{t-1}$. In intensive form with $b = \frac{B}{L}$ and $L_{t+1} = L_t(1+n)$ this can be rewritten as

(9)
$$b_{t+1}(1+n) = (1+r_t)b_t - \tau_t^1 - \tau_{t-1}^2(1+n)^{-1}.$$

The corresponding equations for the foreign country are, with self-explanatory notation:

(10)
$$w_{t}^{*} - \tau_{t}^{*1} - c_{t}^{*1} = (c_{t}^{*2} + \tau_{t}^{*2})(1 + \tau_{t+1}^{*})^{-1}$$

(11)
$$\frac{u_1^*(c_t^{*1}, c_t^{*2})}{u_2^*(c_t^{*1}, c_t^{*2})} = 1 + r_{t+1}^*$$

(12a)
$$c_t^{*1} = c^{*1}(w_t^* - \tau_t^{*1}, \tau_t^{*2}, 1 + r_{t+1}^*)$$

(12b)
$$c_{t}^{*2} = c_{t}^{*2} (w_{t}^{*} - \tau_{t}^{*1}, \tau_{t}^{*2}, 1 + \tau_{t+1}^{*})$$

(13)
$$w_t^* = f^*(k_t^*) - k_t^* f^{*'}(k_t^*) = w^*(k_t^*)$$

$$r_t^* = f^*/(k_t^*)$$

(15)
$$b_{t+1}^{*} (1+n^{*}) = (1+r_{t}^{*})b_{t}^{*} - r_{t}^{*1} - r_{t-1}^{*2}(1+n^{*})^{-1}.$$

We assume perfect international capital mobility. Since the same commodity is produced at home and abroad (though not necessarily with the same technology), this implies

(16)
$$r = r^*.$$

Because nothing essential hinges on it, we assume that the population growth rates and the population sizes of the two countries are the same i.e. $n=n^*$, and $L_t=L_t^*$.

The global capital market equilibrium condition is given by:

$$B_{t+1} + B_{t+1}^* + I_{t+1} + I_{t+1}^* = (w_t - \tau_t^1 - c_t^1) L_t + (w_t^* - \tau_t^{*1} - c_t^{*1}) L_t^*$$

or in intensive form

$$(17) \ (b_{t+1} + b_{t+1}^* + k_{t+1} + k_{t+1}^*) (1 + n) = w_t - \tau_t^1 - c_t^1 + w_t^* - \tau_t^{*1} - c_t^{*1}.$$

Given the assumptions we made about tastes and technology, a competitive equilibrium exists in this economy for positive initial capital stocks k_0 and

 k_{θ}^{*} and bounded initial net external assets $h_{\theta} = (c_{-1}^{2} + \tau_{-1}^{2})(1 + r_{\theta})^{-1}(1 + n)^{-1} - b_{\theta} - k_{\theta} .$

Competitive equilibria of OLG models can be dynamically inefficient (e.g. the model can have the property that the interest rate is forever below the growth rate). We shall focus on equilibria that are not dynamically inefficient, partly because the empirical evidence seems to suggest this (see e.g. Abel et al. [1989]), and partly because many of the conventional laws of economics are suspended when there is dynamic inefficiency and it is hard to conduct any sensible analysis at all. If the interest rate exceeds the growth rate, it makes sense to impose the government's solvency constraint given below.

$$B_{t} = \sum_{i=0}^{\infty} (\tau_{t+i}^{1} L_{t+i} + \tau_{t+i-1}^{2} L_{t+i-1}) \Delta_{t+i}^{4}$$

$$\Delta_{t+i} \equiv \prod_{j=0}^{i} \left[\frac{1}{1+r_{t+j}} \right] \text{ if } i \geq 0$$

$$= 1 \text{ if } i = -1$$

In intensive form this becomes

(18)
$$b_{t} = \sum_{i=0}^{\infty} \left[\frac{\tau_{t+i}^{1}}{t+n} + \left[\frac{1}{t+n} \right]^{2} \tau_{t+i-1}^{2} \right] \delta_{t+i} ,$$

(19)
$$\delta_{t+i} = \prod_{j=0}^{i} \left[\frac{1+n}{1+r_{t+j}} \right] \text{ if } i \geq 0.$$

$$= 1 \text{ if } i = -1$$

Similarly, for the rest of the world's governments we have

(20)
$$b_t^* = \sum_{i=0}^{\infty} \left[\frac{\tau_{t+i}^*}{t+n} + \left[\frac{1}{t+n} \right]^2 \tau_{t+i-1}^* \right] \delta_{t+i}.$$

In addition to the direct distributional effects of fiscal policy-the changes in lifetime resources associated with (possibly age- and generation-specific) changes in lump-sum taxes and transfers at given relative prices--these tax-transfer policy changes will also in general change relative prices. In our one-commodity model the only price is the intertemporal relative price, the interest discount factor $(1+r_t)^{-1}$. Redistribution through changes in endogenous relative prices is therefore limited to redistribution between net borrowers and net lenders.

Typically, unless second-period transfers are very large ($-\tau^2$ is very large), the young will be net savers or net lenders and the old net dissavers. The young in period t will therefore benefit from a high value of r_{t+1} (as well as from a low value of r_t which would reflect a high value of the capital stock k_t and therefore a high wage w_t).

Internationally a high saving, low investment country will lend abroad. Its young will therefore benefit from a higher return on their net foreign investment (a high value of r_{t+1} for those young in period t). The higher rate of interest will, if there are net foreign assets, be at the expense of the next generation abroad rather than the next generation at home. In Buiter

[1981] these issues are considered for two countries with identical production technologies but different rates of pure time preference.

If the model were extended to include a two-commodity ("our goods" and "their goods") structure, changes in the terms of trade (a static relative price) would provide another "endogenous" mechanism for redistribution of income and welfare. Net exporters of the home country good would benefit from an increase in the relative price of home goods. Atemporal and intertemporal relative prices can be intimately related. Uncovered real interest parity requires that the real interest rate in terms of home goods should equal the real interest rate in terms of foreign goods plus the expected percentage rate of change in the relative price of foreign goods.

While for practical policy purposes the endogeneity of the terms of trade is likely to be important, the methodological message of this paper can be conveyed almost equally well in a one-good setting. Occam's razor therefore leads us to excise the static terms of trade.

We now state the first proposition about the competitive equilibrium characterized by equations (2, 4, 7, 8, 10, 11, 13, 14, 17, 18, 19 and 20).

Proposition 1

Given initial values k_0 , k_0^* , b_0 and b_0^* , any equilibrium for k_t , k_t^* , c_t^1 , c_t^2 , c_t^{*1} and c_t^{*2} for all $t \geq 1$ with arbitrary paths of debts $(b_t$ and b_t^*), and of lump-sum taxes and transfers $(\tau_t^1$, τ_t^2 , τ_t^{*1} and τ_t^{*2}) can be replicated without debts and deficits merely by using lump-sum taxes and transfers.

Proof

Consider paths b_t , τ_t^1 , τ_t^2 , b_t^* , τ_t^{*1} and τ_t^{*2} for all t that support equilibrium paths k_t , c_t^1 , c_t^2 , k_t^* , c_t^{*1} , c_t^{*2} for all $t \geq 1$ for given k_0 , k_0^* , b_0 and b_0^* .

We show that for any other set of debt paths $\overset{=}{b}_t$, $\overset{=}{b}_t$, $t \geq 1$, there exists associated paths for lump-sum taxes and transfers $\overset{=}{\tau_t}$, $\overset{=}{\tau_t}$, $\overset{=}{\tau_t}$, and $\overset{=}{\tau_t}$ that support the same equilibrium paths $\overset{=}{k}_t$, $\overset{=}{c}_t^1$, $\overset{=}{c}_t^2$, $\overset{=}{k}_t^*$, $\overset{=}{c}_t^{*1}$ and $\overset{=}{c}_t^{*2}$. Let

$$(21) \stackrel{=}{b}_{t+1} - \stackrel{=}{b}_{t+1} + \stackrel{=}{b}_{t+1} - \stackrel{=}{b}_{t+1} = \begin{bmatrix} \frac{1}{l+n} \end{bmatrix} \begin{bmatrix} \tau_1^l & -\frac{1}{l+n} \end{bmatrix} + \begin{bmatrix} \frac{1}{l+n} \end{bmatrix} \begin{bmatrix} \tau_1^* & -\frac{1}{t} \\ \tau_t^l & \tau_t^l \end{bmatrix} \text{ for all } t$$

and

$$(22a) - \begin{bmatrix} = 1 & -\tau \\ \tau_t^1 & -\tau_t^1 \end{bmatrix} = \frac{1}{1+\tau_{t+1}} \begin{bmatrix} = 2 & -\tau_t^2 \\ \tau_t^2 & -\tau_t^2 \end{bmatrix}$$
 for all t

(22b)
$$-\begin{bmatrix} -*_{t} & -*_{t} \\ \tau_{t} & -\tau_{t}^{*_{1}} \end{bmatrix} = \frac{1}{1+\tau_{t+1}} \begin{bmatrix} -*_{t} & -*_{t} \\ \tau_{t} & -\tau_{t}^{*_{2}} \end{bmatrix}$$
 for all t

Equation (21) ensures that the global capital market equilibrium condition (17) will be satisfied for the same values of c_t^1 , c_t^2 , k_t and k_t^* (and therefore also the same values of \mathbf{w}_t and \mathbf{w}_t^*). Equation (22a) ensures that the home country private sector budget constraint will be satisfied for the same values of c_t^1 , c_t^2 , \mathbf{w}_t and r_{t+1} . Equation (22b) ensures the same for c_t^{*1} , c_t^{*2} , \mathbf{w}_t^* and r_{t+1} .

Substituting (22a) into the government solvency constraint (18) we get

$$\begin{array}{l} = \\ \stackrel{-}{b}_t - \stackrel{-}{b}_t = \\ \sum_{i=0}^{\infty} \left[\begin{bmatrix} = 1 \\ \tau_{t+i} - \stackrel{-}{\tau_{t+i}} \end{bmatrix} \begin{bmatrix} \frac{1}{1+n} \end{bmatrix} - \frac{(1+r_{t+i})}{1+n} \begin{bmatrix} = 1 \\ \tau_{t+i-1} - \frac{1}{\tau_{t+i-1}} \end{bmatrix} \begin{bmatrix} \frac{1}{1+n} \end{bmatrix} \right] \delta_{t+i}$$

i.e.

In the same manner, after substituting (22b) into the foreign government solvency constraint (20), we get

(23b)
$$b_{t}^{=*} - b_{t} = \begin{bmatrix} -*_{1} & -*_{1} \\ \tau_{t-1} - \tau_{t-1} \end{bmatrix} \frac{1}{1+n} .$$

The sum of these last two expressions is equation (21).

The remaining equilibrium conditions (4, 7, 8, 11, 13 and 14) are also satisfied. Using (22a, b) it is easily seen that with the new fiscal policy the initial consumption levels \overline{c} and \overline{c}^* are not only feasible but also optimal. This is because each generation's wealth is unchanged by the new policy. Equilibrium consumption demands and real interest rates therefore remain unchanged. To get Proposition 1 we choose $b_t = b_t = 0$ for all $t \geq 1.0$

Thus any equilibrium with government debt and deficits (or surpluses) can be replicated by an economy in which the government budgets are balanced

period-by-period (and the stocks of debt are zero) by an appropriate choice of age- and generation-specific lump-sum taxes and transfers.

Proposition 1 is a generalization of the well-known proposition that an equilibrium with positive public debt financed by taxes on the young is equivalent to a balanced budget, pay-as-you-go (or unfunded) social security retirement scheme in which lump-sum taxes on the young are paid out as lump-sum transfers to the old. Proposition 2 is an open-economy generalization of a point made by Calvo and Obstfeld [1988a, b] providing the converse of Proposition 1.

Proposition 2

Given initial values k_0 and k_0^* , any equilibrium for k_t , k_t^* , c_t^1 , c_t^2 , c_t^{*1} and c_t^{*2} for all $t \ge 1$ with zero public debt (and with public sector deficits balanced country-by- country), and age-dependent lump-sum taxes and transfers can be replicated with age-independent lump-sum taxes and transfers, and time-varying public debt and deficits.

Proof

Let variables with single overbars represent the balanced-budget case with age-dependent taxes and transfers and variables with double overbars the age-independent tax/transfer case with unbalanced taxes. Note that $b_t = b_t^* = 0$, $\overline{\tau}_t^1 = -(1+n)^{-1}\overline{\tau}_{t-1}^2$, $\overline{\tau}_t^{*1} = -(1+n)^{-1}\overline{\tau}_{t-1}^{*2}$, $\overline{\tau}_{t-1}^{*2} = \overline{\tau}_t^1$ and $\overline{\tau}_{t-1}^{*2} = \overline{\tau}_t^1$ for all $t \ge 1$. From the two government budget identities (9 and 15) and equation (17), it follows that

$$(24) \qquad \stackrel{=}{b}_{t+1}^{=} + \stackrel{=}{b}_{t+1}^{=} = - (1 + \tau_t) \begin{bmatrix} = & = * \\ b_t + b_t^* \end{bmatrix} + \begin{bmatrix} \frac{2}{t} + \frac{n}{n} \end{bmatrix} \begin{bmatrix} \tau_t^1 + \overline{\tau}_t^{*1} \end{bmatrix} .$$

Equation (24) has to hold for all $\overset{=}{b}_t$, $\overset{=}{b}_t^*$, $\overset{*}{\tau}_t^I$ and $\overset{*}{\tau}_t^{I}$ for all t including $\overset{=}{b}_t = \overset{=}{b}_{t+1} = \tau_t^I = \theta$. This can only be true if (24) holds country-by-country i.e.

and

Note that $\overset{=}{b}$ and $\overset{=}{b}$ are per capita debt stocks and that r_t is the real interest rate, not the real interest rate net of the rate of population growth. Per capita world public debt is likely to zigzag from a positive value in one period to a negative value in the next. If $\overline{\tau}_t^1 + \overline{\tau}_t^{*1}$ is constant over time, the sawtooth pattern of the public debt (explosive if r > 0) signaled in Calvo and Obstfeld [1988b] is immediately apparent.

For the private budget sets to remain unchanged at unchanged interest rates and wage rates it must be true (from (22a, 22b) and the characterization of the two regimes) that:

(26a)
$$\frac{1}{1+\tau_{t+1}} \bar{\tau}_{t+1}^{1} = -\bar{\tau}_{t}^{1} + \bar{\tau}_{t}^{1} - \frac{1+n}{1+\tau_{t+1}} \bar{\tau}_{t+1}^{1}$$

and

(26b)
$$\frac{1}{1+r_{t+1}} \bar{\tau}_{t+1}^{*1} = -\bar{\tau}_{t}^{*1} + \bar{\tau}_{t}^{*1} - \frac{1+n}{1+r_{t+1}} \bar{\tau}_{t+1}^{*1}.$$

Consider the home country. At a given rate of interest the unfunded balanced budget social security scheme increases the present discounted value of lifetime resources of the generation born in period t by $-\overline{\tau}_t^l + \frac{1+n}{l+r_{t+1}} \, \overline{\tau}_{t+1}^l \,.$ Without loss of generality, think of the change in the present value of lifetime resources as positive. To achieve the same increase in lifetime resources of the t^{th} generation with the alternative scheme under which the tax on or transfer to both generations alive during any given period must be the same, requires that equation (26a) should hold. If $\overline{\tau}_t^l$ was positive, that is generation t paid a tax when young, then $\overline{\tau}_{t+1}^l$ must be negative. The transfer received by old and young alike in period t+1 must be equal in present discounted value to the tax paid by generation t when young plus $-\overline{\tau}_t^l + \frac{1+n}{l+r_{t+1}} \, \overline{\tau}_{t+1}^l$. The homogeneous part of the tax equation changes sign each period and grows at a proportional rate t+r in absolute value.

From the home country government budget identity we have

(27)
$$(1+n)\overset{=}{b}_{t+1} = (1+r_t)\overset{=}{b}_t - \frac{2+n}{l+n} = \overset{=}{\tau}_t^1.$$

Adding (27) and its foreign counterpart, and using equations (26a, b and 17), it is easily seen how the sawtooth pattern of taxes is transmitted to a sawtooth pattern for the world stock of public debt.

One way of approaching the solvency issue is to solve equation (24) (or the individual governments' equations (25a, b)) forward in time. Provided we impose the terminal condition given in (29), this yields equation (28).

(29)
$$\lim_{s \to \infty} \prod_{j=0}^{s} \left[-\frac{1}{1+r_{t+j}} \right] \left[b_{t+1+s} + b_{t+1+s} \right] = 0.$$

It is, however, hard to interpret (28) and (29) in terms of the usual "no-Ponzi game" condition requiring that the discounted value of the terminal debt be nonpositive in the limit. Per capita debt in (29) is discounted using real interest rates, not real interest rates net of real growth rates, and the discount factor changes sign each period. A more direct approach is the following. Consider the home country government. Its conventional solvency constraint is obtained by solving the single period budget identity forward in time as in (30) below.

$$(30) b_0 = \sum_{i=0}^{T-1} \left[\frac{\tau_i^1}{l+n} + \left[\frac{1}{l+n} \right]^2 \tau_{i-1}^2 \right] \delta_i + \delta_{T-1} b_T$$

The conventional solvency constraint is $\lim_{T\to \infty} \delta_{T-1} b_T \leq \theta$. This condition will not in general be satisfied under the age-independent tax-transfer scheme. While the per capita debt stock under the age-independent tax-transfer scheme $\stackrel{=}{b}$ will zigzag explosively from period-to-period even if τ^1 is constant (see equation (24)), per capita debt over a two-period interval will tend to be well-behaved whenever the net intergenerational transfer under the balanced budget unfunded social security retirement scheme is well-behaved. From the home country government budget identity under the age-independent tax-transfer scheme in (27) it follows that

Using this expression and (26a) it follows that

$$(31) \begin{array}{l} = \\ b_{t+2} = \frac{(1+r_{t+1})(1+r_t)}{(1+n)(1+n)} \begin{array}{l} = \\ b_t + \frac{(2+n)}{(1+n)^2} \end{array} \left[(1+n)^{-1} \tau_{t+1}^1 - (1+r_{t+1})^{-1} \tau_t^1 \right] .$$

 $(1+n)\bar{\tau}_{t+1}^1$ - $(1+r_{t+1})\bar{\tau}_t^1$ is the value (in period t+1) of the net lifetime resources transferred to a member of generation t by the balanced budget social security retirement scheme. Solving (31) forward in time we get

$$(32) \stackrel{=}{b}_{t} = -\frac{(2+n)}{(1+n)^{2}} \sum_{i=0}^{T} \delta_{t+1+2i} \left[(1+n)^{\frac{1}{\tau}}_{t+1+2i} - (1+r_{t+1+2i})^{\frac{1}{\tau}}_{t+2i} \right] + \delta_{t+2T+1} \stackrel{=}{b}_{t+2(T+1)}.$$

The natural solvency condition for this model is that for each t we have

(33)
$$\lim_{T\to\infty} \delta_{t+2T+1} \stackrel{=}{b}_{t+2(T+1)} \leq 0.$$

Our proposed solvency criterion is therefore the following. Consider the sequence $\{\delta_{T-1}b_T^{}\}_{T=0}^{\infty}$. Even if this is not a convergent sequence, we will call the home country government solvent if the sequence possesses a convergent infinite subsequence $\{\delta_{T_j-1}b_{T_j}^{}\}_{T_j=0}^{\infty}$ such that the limit, as j goes to infinity of $\delta_{T_j-1}b_{T_j}$, is nonpositive. There clearly exist sequences of balanced-budget unfunded social security retirement schemes $\{\tau_t^{}l_t^{}\}_{t=0}^{\infty}$ for which this solvency constraint is satisfied.

We conclude that all equilibria supported by unfunded lump-sum, balanced-budget social security schemes that satisfy (33) can be replicated with age-independent lump-sum taxes and transfers, and endogenous public debt and deficits⁵.

Proposition 1 states that public debt and deficits are redundant policy instruments as long as the policy authority has age-specific lump-sum taxes and transfers. Proposition 2 emphasizes that an authority without access to age-specific lump-sum taxes can use public debt and deficits as perfect substitutes for the missing age-specific taxes and transfers.

To simplify notation and analysis somewhat we specialize in what follows the private utility functions to the time-additive form given below.

$$u_t = u(c_t^1, c_t^2) = v(c_t^1) + \beta v(c_t^2), \qquad v' > 0; v'' < 0;$$

$$\lim_{c \to 0} v'(c^i) = \lim_{c \to \infty} \frac{1}{v'(c^i)} = + \infty ,$$
 $i = 1, 2$

and with similar Inada conditions

$$u_{t}^{*} = u_{t}^{*}(c_{t}^{*1}, c_{t}^{*2}) = v_{t}^{*}(c_{t}^{*1}) + \beta_{t}^{*}v_{t}^{*}(c_{t}^{*2}) .$$

 $(1 - \beta)\beta^{-1}$ and $(1 - \beta^*)\beta^{*-1}$ are the domestic and foreign private rates of time preference; $\theta < \beta$, β^* .

This yields the home country consumption functions

$$c_{t}^{1} = c^{1}(w_{t} - \tau_{t}^{1}, \tau_{t}^{2}, 1 + r_{t+1}, \beta)$$

$$c_t^2 = c^2(w_t - \tau_t^1, \tau_t^2, 1 + r_{t+1}, \beta)$$

with

$$\begin{split} c_{w}^{I} &= \frac{\left(1 + r_{t+1}\right)^{2}}{\left(1 + \delta\right)} \; v''(c_{t}^{2}) \tilde{\Delta}^{-1} \\ c_{\tau}^{I} &= -\frac{1}{1 + r_{t+1}} \; c_{w}^{I} \\ c_{w}^{2} &= \left(1 + r_{t+1}\right) \; \left(1 - c_{w}^{I}\right) \\ c_{\tau}^{2} &= -\left(1 - c_{w}^{I}\right) \\ c_{\tau}^{I} &= \left[v'(c_{t}^{2}) + \frac{\left(c_{t}^{2} + r_{t}^{2}\right)}{1 + \delta} \; v''(c_{t}^{2})\right] \tilde{\Delta}^{-1} \\ c_{\tau}^{2} &= -\left(1 + r_{t+1}\right) \; c_{\tau}^{I} + \frac{c_{t}^{2} + r_{t+1}^{2}}{1 + r_{t+1}} \\ \tilde{\Delta} &= v''(c_{t}^{I}) + \frac{\left(1 + r_{t+1}\right)^{2}}{1 + \delta} \; v''(c_{t}^{2}) < 0 \; . \end{split}$$

Exactly analogous expressions can be found for the foreign country.

The following proposition shows that national age-dependent lump-sum tax/transfers which need not be coordinated can be chosen to support Pareto optimal equilibria in our economy.

Proposition 3

A Pareto optimal plan can be attained as a competitive equilibrium allocation under free capital mobility and unfunded national social security schemes. Coordination of the latter is unnecessary.

Proof

This proposition can be argued by noting the necessary conditions for the global Pareto problem with the Lagrangean given below. λ_t and λ_t^* are strictly positive weights with $\sum_{i=-1}^\infty \lambda_i^* < +\infty$ and $\sum_{i=-1}^\infty \lambda_i^* < +\infty$, and ψ_t is the Lagrange multiplier on the global resource constraint in period t.

$$\begin{split} \sum_{t=0}^{\infty} \lambda_{t} (v(c_{t}^{1}) + \beta v(c_{t}^{2})) + \sum_{t=0}^{\infty} \lambda_{t}^{*} (v^{*}(c_{t}^{*1}) + \beta^{*}v^{*}(c_{t}^{*2})) \\ + \lambda_{-1} \beta v(c_{-1}^{2}) + \lambda_{-1}^{*} \beta^{*}v^{*}(c_{-1}^{*2}) \\ + \sum_{t=0}^{\infty} \psi_{t} \left[f(k_{t}) + f(k_{t}^{*}) + k_{t} + k_{t}^{*} - (1 + n)k_{t+1} \\ - (1 + n)k_{t+1}^{*} - (c_{t}^{1} + c_{t}^{*1}) - \frac{(c_{t-1}^{2} + c_{t-1}^{*2})}{1 + n} \right] \end{split}$$

The first-order conditions are

$$\lambda_{t}v'(c_{t}^{1}) = \lambda_{t}^{*}v^{*}'(c_{t}^{*1}) = \psi_{t}$$

$$\lambda_{t}\beta v'(c_{t}^{2}) = \lambda_{t}^{*}\beta^{*}v^{*}'(c_{t}^{*2}) = \frac{\psi_{t+1}}{t+1}$$

$$\psi_{t+1}(1 + f'(k_{t+1})) = \psi_t(1+n)$$

$$\psi_{t+1}(1 + f^*(k_{t+1}^*)) = \psi_t(1+n)$$
.

These conditions include the necessary conditions for a competitive equilibrium

(34a)
$$v'(c_t^1) - \beta(1 + r_{t+1}) v'(c_t^2) = 0$$
,

(34b)
$$v''(c_t^{*1}) - \beta'(1 + r_{t+1}) v''(c_t^{*2}) = 0$$
 and

(34c)
$$f'(k_{t+1}) = f^{*'}(k_{t+1}) = r_{t+1}$$
,

along with restrictions ensuring dynamic efficiency on $\{\psi_t\}$, $\psi_t \geq 0$ for all t, and $\lim_{t \to \infty} \psi_t \geq 0$ and $\lim_{t \to \infty} \psi_t k_t = \lim_{t \to \infty} \psi_t k_t^* = 0$. Note that for this to hold in steady state, it is sufficient that $\lim_{t \to \infty} \lambda_t / \lambda_{t-1} = \lim_{t \to \infty} \lambda_t^* / \lambda_{t-1}^* < 1$ which is satisfied if the Pareto weights λ_t and λ_t^* are exponentially declining.

As Calvo and Obstfeld [1988] point out, the household budget constraints given in equations (2) and (10) along with equations (34a, b, c) imply that only the net discounted subsidy/tax to the household is needed to influence its behavior. With separate national balanced-budget, age-dependent transfer schemes, each government has one instrument τ_t^2 and τ_t^{*2} for one target k_{t+1} and k_{t+1}^* (or equivalently home country and foreign country saving by the young) respectively. With free capital mobility equation (34c) holds, and further it ensures that independent fiscal policies using only lump-sum transfers suffice.

Note that for arbitrary lump-sum taxes a competitive equilibrium under free capital mobility need not satisfy the conditions for dynamic efficiency, but there exist national lump-sum tax policies that will ensure dynamic efficiency.

The short-run effect of domestic or foreign government policies leading to larger public debts can be inferred from equations (35a, b).

$$(35a) \qquad k_{t+1} + k_{t+1}^* = \left[\frac{1}{l+n}\right] \left[w(k_t) - \tau_t^l - c^l \left[w(k_t) - \tau_t^l, \tau_t^2, 1 + f'(k_{t+1})\right] \right]$$

$$+ w^*(k_t^*) - \tau_t^{*l} - c^{*l} \left[w(k_t^*) - \tau_t^{*l}, \tau_t^{*2}, 1 + f^{*'}(k_{t+1}^*)\right]$$

$$- \left[1 + f'(k_t)\right] b_t + \tau_{t-1}^2 (1+n)^{-l} - \left[1 + f^{*'}(k_t^*)\right] b_t^* + \tau_{t-1}^{*2} (1+n)^{-l}$$

(35b)
$$f'(k_{t+1}) = f''(k_{t+1})$$

Solving from (35b) for \boldsymbol{k}_{t+1}^{*} as a function of \boldsymbol{k}_{t+1} we get

(36)
$$k_{t+1}^* = \xi(k_{t+1})$$
; $\xi' = \frac{f''}{f''} > 0$.

Substituting (36) into (35a) we obtain a first-order (nonlinear) difference equation in k. A linear approximation at a stationary equilibrium k_0 , b_0 , b_0^* , τ_0^1 , τ_0^{*1} , τ_0^{*2} and τ_0^{*2} yields

Local stability requires that

$$(38) \left| \left[\frac{1}{1+n} \right] f'' \left[(c_w^1 - 1)k + (c_w^{*1} - 1)k^* - (b+b^*) \right] \mathfrak{N}^{-1} \right| < 1.$$

Note that normality of consumption when young $(0 < c_w^1, c_w^{*1} < 1)$ and a nonnegative effect of a higher interest rate on saving by the young $(c_r^1, c_r^{*1} \leq 0)$ do not suffice to ensure local stability. A nonnegative interest rate responsiveness of saving by the young is sufficient to ensure that Ω is positive. This permits us to obtain the short-run effects of changes in initial conditions and in policy on the capital stocks in the two countries, and thus on the rate of interest.

Provided 0 > 0, we have

Note that k_{t+1}^* increases, and r_{t+1} decreases whenever k_{t+1} increases.

Higher taxes in either country in period t on the old or young reduce the interest rate, and boost next period's domestic and foreign capital stocks. The anticipation by the young in period t of higher taxes next period also reduces their consumption, boosts the next period's capital stocks and lowers the interest rate. Higher initial stocks of domestic or foreign public debt lower the capital stocks and raise the interest rate.

The unanticipated introduction in period t of a one unit per capita tax increase on the young with the proceeds transferred to the old in the same country boosts aggregate consumption and lowers the world capital stock, since $\frac{c_w-1}{\Pi(I+n)} < 0$ (the pleasantly surprised t-1 generation has a shorter remaining life span than the unpleasantly surprised t generation).

If following the unanticipated introduction of this scheme in period t the young expect the next generation to provide for them in the same manner,

the negative effect on the capital stock in period t+1 (equal to $\frac{1}{\prod (I+n)} \left[\frac{c_w^I(r-n)}{I+r} - 1 \right]$) is of course reinforced.

Direct international redistribution using lump-sum transfers is discussed in Buiter and Kletzer [1990a]. It should be clear however that the positive economics of direct lump-sum international redistribution are straightforward. Consider for concreteness redistribution from the young in the foreign country to the young in the home country. At a given world rate of interest this will raise (lower) next period's global capital stock if the marginal propensity to save in the home country exceeds that in the foreign country (if $c_w^1 < c_w^{*1}$). If, say, the global capital stock increases, the associated reduction in the world rate of interest will cause further (indirect) international redistribution from net lenders to net borrowers. Therefore in addition to the direct redistributional effect of the lump-sum international transfer, the change it causes in the intertemporal relative price will further redistribute income among the young and the old, within and between the two countries. In the case of a reduction in the rate of interest in period t+1 this redistribution will typically be from those old in period t+1 to the young.

In steady state, the two-country equilibrium is given by

$$(b + b^{*} + k + k^{*})(1 + n) = w(k) - \tau^{1} - c^{1}[w(k) - \tau^{1}, \tau^{2}, (1 + f'(k))]$$

$$+ w^{*}(k^{*}) - \tau^{*1} - c^{*1}[w^{*}(k^{*}) - \tau^{*1}, \tau^{*2}, (1 + f^{*}(k^{*}))]$$

$$(n - f'(k))b = -\tau^{1} - \frac{\tau^{2}}{1 + n}$$

$$(n - f^{*}(k^{*}))b^{*} = -\tau^{*1} - \frac{\tau^{*2}}{1 + n}$$

with

$$k^* = \xi(k)$$

and

$$\tau = f'(k) = f^{*\prime}(k^*) .$$

Note that one of b, τ^1 and τ^2 and one of b^* , τ^{*1} and τ^{*2} is implied by the others. Without loss of generality we treat τ^1 , τ^2 , τ^{*1} and τ^{*2} as exogenous policy instruments, and b and b^* as endogenous.

The effect of changes in τ^1 , τ^{*1} , τ^2 and τ^{*2} on k (with b and b^* endogenous) is given by equation (40) if $n \neq \tau$.

$$(40) dk = h^{-1} \left[\frac{1}{1+n} \left(c_w^1 - 1 \right) + \frac{1}{n-\tau} \right] d\tau^{\frac{1}{2}} + h^{-1} \left[\frac{1}{1+n} \left(c_w^{\frac{1}{2}} - 1 \right) + \frac{1}{n-\tau} \right] d\tau^{\frac{1}{2}} + \frac{h^{-1}}{1+n} \left[\frac{1}{1+\tau} c_w^{\frac{1}{2}} + \frac{1}{n-\tau} \right] d\tau^{\frac{1}{2}}$$

where

$$\dot{A} = 1 + \frac{f^{\prime\prime}}{f^{*\prime\prime}} + \left[\frac{1}{1+n}\right] f^{\prime\prime} \left[k(1-c_w^1) + k^*(1-c_w^{*1}) + c_\tau^1 + c_\tau^1 + (1+n)\frac{(b+b^*)}{n-f^\prime}\right] \ .$$

If the model is locally stable (see equation (38)), then $\Lambda > 0$.

An increase in the scale of an unfunded social security retirement scheme (e.g. an increase in τ^1 and a reduction in τ^2 by (1 + n) times the increase in

 τ_I) will lower the steady-state capital stocks at home and abroad, and raise the world rate of interest⁷ if the economy is not dynamically inefficient.

From equations (37 and 38) we can determine the precise nature of the pecuniary externalities that the government of a country can impose on the other country in the short run. Equation (40) provides the same information for the long run.

3. LONG-DATED GOVERNMENT DEBT

We now analyze how our model changes when long-dated public debt and the associated possibility of capital losses and capital gains are added to the model. To illustrate, we add to the one-period bonds in the model consols or perpetuities i.e. financial claims paying a fixed coupon (scaled to one unit of output) each period from now till kingdom come. Let p_t be the "ex dividend" or ex-coupon price of a home country government consol and p_t^* the price of a foreign government consol. B_t^L denotes the stock of home government consols at the beginning of period t and d and d the stock of foreign government consols. d and d denote the domestic and foreign government stocks of one-period debt respectively.

Since capital markets are efficient and there is no sovereign risk, we have $p_t = p_t^*$, and

$$\frac{1}{p_t} + \frac{p_{t+1} - p_t}{p_t} = r_{t+1} = r_{t+1}^* = f'(k_{t+1}) = f^{*'}(k_{t+1}).$$

Ex ante all assets earn the same expected rate of return. Domestic and foreign private investors are indifferent between holding domestic or foreign, short or long government debt.

The home country government budget identity becomes

$$p_{t}(B_{t+1}^{L} - B_{t}^{L}) + B_{t+1}^{s} - B_{t}^{s} \equiv B_{t}^{L} + \tau_{t}B_{t}^{s} - \tau_{t}^{1}L_{t} - \tau_{t-1}^{2}L_{t-1}.$$

Ruling out speculative asset bubbles, 9 the period t price of the consol is

(41)
$$p_t = \sum_{i=1}^{\infty} \prod_{j=1}^{i} \frac{1}{1 + r_{t+j}}.$$

Does the existence of long-dated debt provide the home country authorities with additional means to tax or expropriate domestic or foreign residents that is does it provide additional instruments of policy that were unavailable when there was only short debt and age- and generation-specific lump-sum taxes?

Since the old own all financial wealth, they are the only ones that will be affected in any given period by any "news" causing asset prices that period to be different from what they were expected to be.

Consider the budget constraints of the old in period t:

$$\begin{aligned} (42a) & L_{t-1} \Big[c_{t-1}^2 + \tau_{t-1}^2 \Big] \leq (1 + \tau_t) \Big[a_t^3 \Big[B_t^3 + B_t^{*3} \Big] + \tilde{B}_t^3 \Big] \\ & + (1 + p_t) \Big[a_t^L \Big[B_t^L + B_t^{*L} \Big] + \tilde{B}_t^L \Big] \\ & + (1 + f'(k_t)) a_t^k I_{t}^+ (1 + f^{*'}(k_t^*)) a_t^{k^*} I_t^* \end{aligned}$$

$$(42b) \qquad L_{t-1} \left[c_{t-1}^* + \tau_{t-1}^{*2} \right] \leq (1 + r_t) \left[(1 - a_t^s) \left[B_t^s + B_t^{*s} \right] - \tilde{B}_t^s \right]$$

$$+ (1 + p_t) \left[(1 - a_t^L) \left[B_t^L + B_t^{*L} \right] - \tilde{B}_t^L \right]$$

$$+ (1 + f'(k_t)) (1 - a_t^k) \mathbf{I}_{t}^+ (1 + f^{*'}(k_t^*)) (1 - a_t^{k*}) \mathbf{I}_t^* .$$

 a_t^L is the share of total "outside" long bonds (issued by the home and foreign governments) held in period t by the old in the home country; a_t^S is the share of the total stock of home and foreign short government bonds held by the home country old, a_t^k the share of the home country capital stock and a^{k*} the share of the foreign capital stock held by the home country old. \tilde{B}_t^L and \tilde{B}_t^S are the amounts of private long debt and private short debt respectively owned by the home country old and owed by the foreign old (the "inside" debt).

The short rate r_t is a bygone by the time period t arrives because $r_t = f'(k_t)$, and k_t is predetermined in period t. The price of long debt p_t however is nonpredetermined. From equation (41) it is a function of all expected future one-period rates of return, and through them of current expectations of all future taxes and transfers imposed by the domestic and foreign governments.

Note that in a model with only single-period debt the government of a country (say the home country) can, through unanticipated variations in taxes on (transfers to) its own old, reproduce any effect on the resources available to the old in that country that can be achieved through unexpected capital gains or losses on long debt. Let \boldsymbol{E}_{t-1} be the expectation operator conditional on information available at t-1. Variables with a single overbar refer to a situation without long-dated bonds and variables with a double

overbar to a situation with long-dated bonds. By choosing an unexpected change in $\tilde{\tau}_{t-1}^2$ of the magnitude given in (43), the ability to achieve unexpected variations in the value of long-dated debt can be seen to be a redundant instrument for a government interested only in influencing the opportunity set of its own old citizens.

$$(43) L_{t-1} \left[\tilde{\tau}_{t-1}^{2} - B_{t-1} \tilde{\tau}_{t-1}^{2} \right] = L_{t-1} \left[\tilde{\tau}_{t-1}^{2} - B_{t-1} \tilde{\tau}_{t-1}^{2} \right]$$

$$+ \left[a_{t}^{L} \left[B_{t}^{L} + B_{t}^{*L} \right] + \tilde{B}_{t}^{L} \right] \left[p_{t}^{-} B_{t-1} p_{t} \right]$$

Since in the current setup the government of a country is assumed not to have any formal powers to tax citizens of the other country, 10 the ability to influence the price of long-dated debt through its current and/or anticipated future policy actions does give the government a direct handle to influence the resource constraint faced by the old in the other country. With only one-period debt this channel of influence is absent. By influencing the resource constraint faced by the foreign old (with nationalistic governments this would typically involve imposing an unexpected capital loss on the foreign old that is an unexpected fall in p_t when $(1 - a_t^L) \left[B_t^L + B_t^{*L} \right] - \tilde{B}_t^L > 0 \right)$ the amount of resources available to the young in both countries will be altered (increased in the example just given). Any undesired effects on the own old of the unexpected change in p_t can be neutralized by a matching unexpected change in r_{t+1}^2 .

To interpret this finding the following points should be recognized.

First, only through unexpected capital gains or losses on long bonds can a government obtain an additional instrument to influence the resources available to the current old in the rest of the world. If history is just the

unfolding of a preordained policy scenario without any surprises, the dynamically efficient equilibria in an economy with only short debt are equivalent to those in an economy with long debt. In this economy without exogenous uncertainty, any time-consistent equilibrium with rational expectations and more generally any rational expectations equilibrium with policy instrument values determined by credible policy rules will not be characterized by long debt price surprises or tax surprises. 11

Second, even if there is no public sector long debt outstanding, private agents could have large gross positions in long maturity assets $(\tilde{B}_t^L > 0)$ in equations (42a, b)). Without uncertainty and risk aversion, equalization of expected rates of return does not suffice to determine individual private agent holdings of individual assets. For example an equilibrium with zero stocks of public debt in both countries and with no net international asset position for either country is consistent with, say, the home country private sector owning long claims on the foreign private sector and owing short liabilities to same $(p_t \tilde{B}_t^L = -\tilde{B}_t^3 > 0)$. To determine the consequences of policy actions (current or anticipated) that drive up the yield on long bonds, one must know the global distribution of long assets and liabilities, private and public. Just considering foreign holdings of long home country public debt does not in general provide enough information to determine the costs and benefits of policies aimed at (unexpectedly) changing long bond prices.

Finally, the introduction of long-dated debt does not affect the results concerning the Pareto optimality of competitive equilibria with perfect international capital mobility under different lump-sum tax and transfer schemes. If the government actions causing capital gains or losses are anticipated, asset prices will adjust to equate expected rates of return. If

they are unanticipated, the resulting price changes are nondistortionary capital levies or subsidies.

4. DISTORTIONARY TAXES AND EXHAUSTIVE PUBLIC SPENDING

In this section we consider briefly the positive economic consequences of two kinds of distortionary taxes and of public spending on goods and services.

4.1 Nondiscriminatory Source Based Taxation of Capital Income

Consider the imposition by the home country government of a proportional tax on the income from domestic capital. The tax is nondiscriminatory in that the same tax rate $\theta < 1$ is applied to the rental income from all physical capital located within the national boundaries regardless of whether it is owned by domestic or foreign residents. The foreign country similarly imposes a nondiscriminatory proportional tax rate $\theta^* < 1$ on the rental income from all physical capital located within its national boundaries. The only equations of our model that are affected by these changes are the arbitrage conditions relating the interest rate to the returns on capital and the two government budget identities. g_t and g_t^* denote respectively home country and foreign country exhaustive public spending in period t per member of generation t.

(44a)
$$r_t = (1 - \theta_t)f'(k_t)$$

(44b)
$$r_t = (1 - \theta_t^*) f^{*'}(k_t^*)$$

(45a)
$$g_t - \tau_t^1 - (1+n)^{-1} \tau_{t-1}^2 - \theta_t k_t f'(k_t) + \tau_t b_t = (1+n) b_{t+1} - b_t$$

(45b)
$$g_{t}^{*} - \tau_{t}^{*1} - (1+n)^{-1} \tau_{t-1}^{*2} - \theta_{t}^{*} k_{t}^{*} f^{*} (k_{t}^{*}) + r_{t} b_{t}^{*} = (1+n) b_{t+1}^{*} - b_{t}^{*}$$

In the presence of distortionary taxes and in the absence of lump-sum taxes and transfers, optimal budgetary policy will in general involve unbalanced government deficits even in models with "first-order" debt neutrality such as representative agent models. Given the need for nonzero public sector revenues, tax smoothing considerations may make it desirable to spread the unavoidable excess burden of distortionary taxes over time so as to minimize its total impact on a utilitarian social welfare function. However, since nondistortionary taxes and transfers are still available in our model, any desired smoothing of distortionary taxes can be achieved with continuously balanced national budgets by varying the time paths of lump-sum taxes and transfers.

With balanced national budgets (45a) and (45b) become

(46a)
$$g_t - \tau_t^1 - (1+n)^{-1} \tau_{t-1}^2 - \theta_t k_t f'(k_t) = 0$$
, and

(46b)
$$g_t^* - \tau_t^{*1} - (1 + n)^{-1} \tau_{t-1}^{*2} - \theta_t^* k_t^* f^{*\prime}(k_t^*) = 0$$
.

In what follows we shall take g_t , τ_t^1 and θ_t as exogenous with τ_{t-1}^2 adjusting endogenously to satisfy the home country balanced-budget condition (46a). In the same way g_t^* , τ_t^{*1} and θ_t^* will be taken as exogenous with τ_{t-1}^{*2} adjusting to satisfy (46b). Government exhaustive spending is assumed to be public consumption that enters into private utility functions in an additively separable manner. It will therefore only affect private behavior through the private budget constraints.

Possible reasons for the use of distortionary taxes despite the presence of lump-sum taxes are discussed at greater length in Buiter and Kletzer [1990a]. They include additional leverage over the world rate of interest and

the domestic stock of capital, and the scope offered for taxing foreign residents if foreigners own part of the domestic capital stock.

With nondiscriminatory source-based taxation of capital income, the aftertax rates of return on capital located in the home country and capital located in the foreign country will be equalized if there is perfect international financial capital mobility. The beforetax rates of return will differ unless the same proportional tax rate is levied in both countries $(\theta = \theta^*)$. Even if $\theta = \theta^*$, there is still (unless $\theta = \theta^* = 0$) a wedge between the world-wide private rate of return to saving τ and the social rate of return to capital formation $f' = f^{*'}$.

Equation (47) gives \boldsymbol{k}_{t}^{*} as a function of \boldsymbol{k}_{t} , $\boldsymbol{\theta}_{t}$ and $\boldsymbol{\theta}_{t}^{*}$.

$$(47) k_t^* = \xi(k_t, \theta_t, \theta_t^*)$$

with

$$\xi_k = (1 - \theta)(1 - \theta^*)^{-1} \frac{f''}{f^{*''}} > 0$$

$$\xi_{\theta} = -(1 - \theta^*)^{-1} \frac{f'}{f^{*''}} > 0$$

$$\xi_{\theta}^* = (1 - \theta^*)^{-1} \frac{f_{''}^*}{f_{'''}^*} < \theta .$$

From the global capital market equilibrium condition and the two consumption functions given below, the impact effects on the capital stocks and thus on the interest rate of changes in θ , θ^* , g and g^* can be determined.

$$\begin{aligned} &(1+n)\left[k_{t+1} + \xi(k_{t+1} \ , \ \theta_{t+1} \ , \ \theta_{t+1}^*)\right] = w(k_t) - \tau_t^1 - c_t^1 + w^*(k_t^*) - \tau_t^{*1} - c_t^{*1} \\ &c_t^1 = c^1[w(k_t) - \tau_t^1 \ , \ (1+n)(g_{t+1} - \tau_{t+1}^1 - \theta_{t+1}k_{t+1}f'(k_{t+1})) \ , \\ &1 + (1-\theta_{t+1})f'(k_{t+1})] \\ &c_t^{*1} = c^{*1}[w^*(k_t^*) - \tau_t^{*1}, \ (1+n)(g_{t+1}^* - \tau_{t+1}^{*1} - \theta_{t+1}^*k_{t+1}^*f''(k_{t+1}^*)) \ , \\ &1 + (1-\theta_{t+1}^*)f''(k_{t+1}^*)] \end{aligned}$$

This yields:

$$\begin{cases} (1+n) \left[1 + \xi_{k} - c_{\tau}^{1} g \theta(f' + k f'') - c_{\tau}^{*1} g \theta^{*}(f^{*'} + k^{*} f^{*'}) \xi_{k} \right] \\ + c_{\tau}^{1} (1-\theta) f'' + c_{\tau}^{*1} (1-\theta^{*}) f^{*'} \xi_{k} \right] dk_{t+1} \\ = -(1+n) c_{\tau}^{1} g dg_{t+1} - (1+n) c_{\tau}^{*1} g dg_{t+1}^{*} \\ + \left\{ -(1+n) \left[\xi_{\theta} - c_{\tau}^{1} g k f' - c_{\tau}^{*1} g \theta^{*}(f^{*'} + k^{*} f^{*'}) \xi_{\theta} \right] \right. \\ + c_{\tau}^{1} f' - c_{\tau}^{1} (1-\theta^{*}) f^{*'} \xi_{\theta} d\theta_{t+1} \\ + \left\{ -(1+n) \left[\xi_{\theta}^{*} - c_{\tau}^{*1} g k^{*} f^{*'} - c_{\tau}^{*1} g \theta^{*}(f^{*'} + k^{*} f^{*'}) \xi_{\theta}^{*} \right] \right. \\ + c_{\tau}^{*1} f^{*'} - c_{\tau}^{1} (1-\theta^{*}) f^{*'} \xi_{\theta}^{*} d\theta_{t+1}^{*} . \end{cases}$$

If we evaluate all these impact multipliers at $\theta=\theta^*=0$, the results are unambiguous. Consider first an increase in public spending, g_{t+1} . The expression in equation (48) represents the case where the increase in g_{t+1} is anticipated in period t. As regards its impact on private saving by the young in period t, this is equivalent to an anticipated increase in τ_t^2 . The result is an increase in saving and larger values of k_{t+1} and k_{t+1}^* . The world interest rate in period t+1 falls. If the increase in g_{t+1} is unexpected, there is of course no effect on k_{t+1} as saving by the young in period t will not have been influenced by it. An anticipated increase in g_{t+1}^* also raises the two capital stocks in period t+1 and reduces the world rate of interest.

A larger value of θ_{t+1} lowers the domestic capital-labor ratio in period t+1. The well-known result of Diamond [1970] that in OLG models an increase in the rate of capital income taxation may lower the capital-labor ratio to such an extent that the aftertax marginal product of capital and thus the interest rate actually rise can occur in the two-country model as well. Note that it can happen even if the home country capital income tax is the only distortion in the economy and if the (infinitesimal) change in θ is evaluated at $\theta=0$.

The effect of an increase in θ_{t+1} on k_{t+1}^* is ambiguous. Given k_{t+1} , a larger value of θ_{t+1} implies an increase in k_{t+1}^* . The reduction in k_{t+1} as a result of the increase in θ_{t+1} will have the effect of lowering k_{t+1}^* .

The total world capital stock k_{t+1} + k_{t+1}^* declines unambiguously as the result of an increase in θ_{t+1} (or in θ_{t+1}^*) if the home country and the foreign country are identical. At the initial capital stock, the higher tax rate on capital income means a lower aftertax return on physical capital and therefore a lower interest rate. This we assume will reduce saving by the young $(c_r^1 < \theta)$. In addition the higher tax rate on home country capital income

means that lump-sum taxes on the home country old will be reduced. Given our assumption of normal consumption in both periods, this too reduces saving by the young who anticipate the future tax cut. Another way of looking at it is that the increase in θ_{t+1} and the reduction in τ_t^2 have (because they represent balanced-budget operations) offsetting "income effects." That leaves just the substitution effect of the lower aftertax rate of return on capital associated with a higher value of θ_{t+1} which depresses saving.

Qualitatively the steady-state effects of permanent changes in g, g^* , θ and θ^* are the same as the impact effects, provided the model is locally stable.

The effects of nondiscriminatory source-based taxation of, say, domestic capital income on the relative stocks of domestic and foreign capital, and on wages in period t+1 and beyond are not surprising. Such taxes (or subsidies) are often introduced or manipulated precisely because of their expected effects on fixed capital formation. It is important to note however that in general equilibrium such capital taxes will also affect private saving, both because they affect the interest rate and because the change in these taxes may alter the lifetime budget constraints faced by different generations. Unless Diamond's OLG paradox occurs, higher capital taxation will reduce the interest rate. In our model with lump-sum taxation on the old assigned the role of passive residual adjusting to balance the public sector budget, higher capital taxes will at a given interest rate tend to reduce saving by the young. If alternative budgetary rules are specified (say, with τ_{t+1}^1 or g_{t+1} adjusting to balance the government budget), different results will obtain.

4.2 Residence-Based Taxation of All Asset Income

The polar opposite of nondiscriminatory source-based taxation of capital income is the application by a nation of an equal proportional tax rate to all nonwage income earned by its residents, regardless of whether the assets that are the source of the income are located at home or abroad. Let θ now denote the home country's tax rate on its residents' asset income and θ^* the foreign country's tax rate on the asset income of its residents. r is the riskless one-period interest rate on home country debt and r^* the riskless one-period interest rate on foreign debt. With residence-based taxation of all asset income and perfect international capital mobility (49a, b) hold.

(49a)
$$f'(k)(1-\theta) = f^{*'}(k^*)(1-\theta) = r(1-\theta) = r^*(1-\theta)$$

(49b)
$$f'(k)(1-\theta^*) = f^{*'}(k^*)(1-\theta^*) = r(1-\theta) = r^*(1-\theta^*)$$

Now it is the beforetax rates of return on investment that are equalized globally by perfect international capital mobility: $f'(k) = f^{*'}(k^*) = r = r^*$. While there is no distortion in the global allocation of physical capital between countries, there can be distortions both between home country and foreign private saving, and in global saving and investment.

First, home country private savers face the intertemporal relative price $(1+r(1-\theta))^{-1}$ while foreign private savers face the intertemporal relative price $(1+r(1-\theta))^{-1}$. Unless $\theta=\theta^*$, these intertemporal relative prices differ for domestic and foreign savers. Second, even if $\theta=\theta^*$ but with $\theta=\theta^*\neq 0$, the common home and foreign intertemporal relative price faced by savers differs from the common marginal rates of intertemporal transformation through capital formation $(1+f'(k))^{-1}=(1+f^{*'}(k^*))^{-1}$.

With residence-based taxation of asset income and balanced government budgets, equations (50a, b) hold. Exhaustive government spending is omitted.

(50a)
$$\tau_t^1 + (1+n)^{-1}\tau_{t-1}^2 + (1+n)^{-1}\theta_t\tau_t(w_{t-1} - c_{t-1}^1 - \tau_{t-1}^1) = 0$$

(50b)
$$\tau_{t}^{*1} + (1+n)^{-1}\tau_{t-1}^{*2} + (1+n)^{-1}\theta_{t}^{*}r_{t}(w_{t-1}^{*} - c_{t-1}^{*1} - \tau_{t-1}^{*1}) = \theta$$

Noting again that beforetax marginal products of capital are equalized at home and abroad, that $r(1 - \theta)$ governs the home country private saving decision and that $r(1 - \theta^*)$ governs the foreign private saving decision, we can determine the effect of anticipated changes in $\boldsymbol{\theta}_{t+1}$ and $\boldsymbol{\theta}_{t+1}^*$ on the interest rate \boldsymbol{r}_{t+1} and the world stock of capital \boldsymbol{k}_{t+1} + \boldsymbol{k}_{t+1}^* . We again assume that $\boldsymbol{\tau}_t^{\pmb{z}}$ and $\boldsymbol{\tau}_t^{\pmb{z}}$ adjust to maintain budget balance when $\boldsymbol{\theta}_{t+1}$ or $\boldsymbol{\theta}_{t+1}^{\pmb{z}}$ are varied. If $c_r^1 < \theta$, a higher asset income tax rate in the home country will reduce home country private saving at a given capital-labor ratio. In addition the higher asset income tax rate means a reduction (anticipated by the young generation born in period t) in τ_t^2 . With normality this too will tend to reduce saving by the home country young in period t. The result is a reduction in \boldsymbol{k}_{t+1} and in \boldsymbol{k}_{t+1}^* . The beforetax rate of interest r therefore rises. Again the Diamond OLG paradox can occur (even when the distortionary $tax \theta$ is the only distortion in the economy and the (infinitesimal) increase in θ is evaluated at $\theta = 0$): the home country aftertax rate of interest can increase when θ is raised. The foreign aftertax rate of interest of course rises. If the model is locally stable, the long-run effects of a permanent increase in θ are in the same direction as the impact effects.

5. CONCLUSION

The subject matter of this paper is the positive economics and Pareto efficiency of fiscal policy in open interdependent economies. The first three sections consider the positive economics and welfare economics of the pecuniary externalities associated with the use of national fiscal policy instruments in a two-country OLG model. Without public goods, technological externalities or other departures from the conventional competitive ideal type, interdependence and the transmission of policy shocks (or other disturbances) are mediated through competitive market clearing prices.

Disregarding the possibility of dynamic inefficiency--which can occur in the class of competitive OLG models considered in this paper--the equilibria generated by different lump-sum tax-transfer and borrowing policies in the two countries will all be Pareto efficient. Because of the assumption of perfect international capital mobility, national tax-transfer and debt policies will affect foreign as well as domestic interest rates and capital-labor ratios.

When government debt has a single-period maturity, the ability to run public sector deficits or surpluses does not enhance the range of equilibria that can be supported by government policy if governments have access to unrestricted age-dependent lump-sum taxes and transfers. Without age-specific lump-sum taxes and transfers, the ability to unbalance the public sector budgets allows governments to support the same equilibria that could be supported with age-specific lump-sum taxes and transfers. This restatement in a two-country setting of a result of Calvo and Obstfeld [1988] is true with longer maturity debt only if time-consistent or other credible policies alone are considered.

The central role of the world interest rate as a key determinant of the distribution of the gains from international lending and borrowing in our

model is shared by the role of the static terms of trade as a key determinant of the distribution of the gains from static trade in more general models (see Buiter and Kletzer [1990b]). The analysis also suggests that without a mechanism for achieving nondistortionary international redistribution, global social welfare could be enhanced by the use of such distortionary instruments as taxes on the income from capital, source or residence taxation of foreign factor income, the subsidization of foreign lending or borrowing etc. Pareto efficiency would then be sacrificed in the pursuit of social welfare.

In Section 4 of the paper source- and residence-based taxation of capital income and other asset income is studied together with a brief analysis of the consequences of variations in exhaustive public spending.

If there are distortions (global public goods, international production or consumption externalities, noncompetitive market structures etc.), the use of nonlump-sum tax, transfer and subsidy instruments need not be at the expense of Pareto efficiency. We explore some of these issues further in the sequel to this paper (Buiter and Kletzer [1990a]) which also contains an analysis of the welfare economics of cooperative and noncooperative fiscal policy design.

NOTES

 1 In OLG models with infinite lived agents, all agents alive at time t can trade with each other and (in due course) with all agents born in periods beyond t. The Yaari-Blanchard model with a zero death rate is an example of such a model. The key property of absence of debt neutrality is preserved as long as the birth rate is positive. What is required for our analysis is that the government be able to redistribute resources between heterogeneous agents, either directly (say through balanced budget tax-transfer schemes) or indirectly (say through taxes and borrowing). The OLG model provides a convenient (and plausible) way of introducing the required heterogeneity.

²Examples of the use of the two-period OLG model in two-country models are Buiter [1981], Buiter and Eaton [1983], Kehoe [1986a], Hamada [1986] and Sibert [1988]. The Yaari-Blanchard OLG model has been applied to two-country models in Frenkel and Razin [1987], Obstfeld [1989] and Buiter [1989].

 3 The conditions on z ensure that equation (2) will hold with equality.

⁴This is obtained by solving the government's single-period budget identity forward in time and imposing the terminal condition $\lim_{i\to\infty} t_{i+1} = 0.$

⁵It is clear that the optimal consumption path under the balanced budget and age-dependent taxes and transfers remains optimal under age-independent taxes and transfers and unbalanced budgets.

⁶If n = r changes in debt and taxes have no real effects.

$$\begin{array}{c|c} 7 \frac{\partial k}{\partial \tau^I} & c \\ d\tau^2 & = -(1+n)d\tau^I \end{array} = \frac{(r-n)}{(1+n)\Lambda} (c_w^I - 1) < 0 \text{ if } r > n .$$

⁸Note that any steady state with a positive price of bonds will have a positive real interest rate as in steady state $r = \frac{1}{P} > 0$. With a zero population growth rate, the substitution of consols for short bonds therefore would eliminate dynamically inefficient steady states. Since the coupon is positive (and equal to 1 in our example), the only possibly steady states have a constant price of consols. In many OLG models noninterest bearing outside fiat money is effectively a "zero coupon consol." It will only be held if the expected proportional rate of capital gains on money is no less than the rate of return on alternative assets. With such a zero coupon consol, all steady states are at the golden rule.

⁹i.e. assuming that
$$\lim_{s\to\infty} \prod_{j=1}^{s} \left[\frac{1}{1+\tau_{t+j}} \right] P_{t+s} = 0$$
.

¹⁰Nor can a government make positive transfers to the other country in our setup. This assumption is relaxed in Section 4 when the taxation of capital income permits a government to tax foreign residents if they own home country capital.

11 This credibility may be due to reputation effects, to credible punishment strategies or other trigger strategies in models of games with memory or to the presence of a compulsively honest government satisfying the George Washington maxim ("I cannot tell a lie").

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