Keynesian Balance of Payments Models: Comment

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In a recent paper, Edward Kuska delivers a sweeping indictment of Keynesian balance of payments theory: "...almost all of the models of the Keynesian balance-of-payments literature suffer from internal contradictions and deficiencies which make them unsuitable for balance-of-payments theory" (p. 659). His critique of Keynesian balance of payments theory is expressed in four propositions (p. 664). Propositions 2 and 4 are incorrect. Proposition 3 is correct except in a world without capital mobility, which was the case considered by Kuska. Proposition 1 is correct (and familiar) and has been incorporated into Keynesian models of the balance of payments without changing any of their essential features or policy implications. Propositions 2 and 4 are based on a fundamental stock-flow confusion while Proposition 3 reflects a misspecification of budget constraints or balance sheet constraints in different countries.

Section I provides a brief general discussion of asset-market equilibrium and Walras' Law that brings out the fundamental error in Kuska's analysis. The results cited in this section are not new, but have not before been brought together in a concise and simple framework. Section II develops a two-country model akin to Kuska's that combines asset-market equilibrium with balance of payments disequilibrium and derives its most important short- and long-run properties. This model is specified to satisfy the "adding-up constraints" emphasized by William Brainard and James Tobin and Tobin, thus avoiding the pitfall referred to in Proposition 1. It should be noted that practically no one has violated the adding-up requirement since the publication of Brainard and Tobin's 1968 paper. One implication of the model is that international trade in financial assets prevents the monetary authority in one of the two countries from maintaining even short-term control over its own domestic money stock. This result is independent of the degree of substitutability between the bonds of the two countries. All that is required is that the private sector in each country can hold foreign bonds and that bonds are traded in perfect markets. The Keynesian models so summarily dismissed by Kuska are special cases of our consistent stock-flow model.[1]

We do not argue that the simple Keynesian models which Kuska attacks, those of James Meade, Robert Mundell, J. M. Fleming, Egon Sohmen, and S. C. Tsiang, for instance, are free of serious deficiencies: they focus almost exclusively on the short run; commodity and factor prices are fixed and capacity constraints ignored; the relationship between income-expenditure equilibrium and the flow of funds is often neglected. Nevertheless, these models are internally consistent. Furthermore, much recent Keynesian macroeconomic and balance of payments theory is aimed at correcting these deficiencies. William Branson (1976a, b), Peter Kenen, Stephen Turnovsky (1976), Buiter (1978), and Gary Smith and William Starnes, for example, explicitly incorporate long-run effects of financial asset flows and limited price flexibility into Keynesian open-economy models.[2]

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[1]In a longer version of this paper (1979) we illustrate this with a popular model of the balance of payments in a Keynesian economy which consistently combines balance of payments disequilibrium with asset-market equilibrium.

[2]The role of exchange rate expectations and the forward discount under a flexible exchange rate regime was never properly analysed in these models. It is here that the monetary or asset-market approach has made its most significant contributions, for example, Jacob Frenkel, Michael Mussa, Rudiger Dornbusch (1976), J. F. O. Bilson, and Eaton.

While Kuska directs his attack at "Keynesian" models of the balance of payments, his Propositions 2 and 4, if they were true, would apply equally to a number of authors adopting the "monetary approach" to the balance of payments. Jacob Frenkel and Carlos Rodriguez, for instance, in an article which Kuska cites among others as "largely free of [these] criticisms" (p. 659) do *in fact* develop a monetary model of the balance of payments which incorporates simultaneous money-market equilibrium and balance of payments disequilibrium. Rudiger Dornbusch (1975) provides another example of a model adopting the "monetary approach" which combines continuous asset-market equilibrium with a nonzero international payments balance. Our absolution of Keynesian balance of payments models thus applies equally to these (nonindicted) monetary models.

I. On Asset Market Equilibrium and Walras' Law

A number of recent papers on asset-market equilibrium in discrete and continuous time models have illuminated the distinction between two different notions of asset-market equilibrium: beginning-of-period and end-of-period equilibrium (Josef May, Sheetal Chand, Duncan Foley, Buiter (1979, 1980), Buiter and Geoffrey Woglom, Turnovsky (1977), Turnovsky and Edwin Burmeister). The distinction between the two is most easily demonstrated in discrete time.

Let $S^i(t)$ denote the stock of good i extant at the beginning of period t or, equivalently, the stock carried over from period $t-1$. The term $Q^i(t)$ denotes the planned production (issue or inflow) of good i and $C^i(t)$ the planned consumption (liquidation or outflow) of commodity i during period t. Let $p^i(t)$ denote the price of good i in period t. Finally, let $D^i(t_1, t_2)$ be the demand for good i, planned and expressed in the market at time t_1, to be held at time t_2.[3] Thus $D^i(t, t+1)$ denotes the demand for commodity i in period t to be carried over to the beginning of period $t+1$. We refer to this magnitude as *end-of-period* demand. Total market demand for good i during period t consists of demand for current consumption $C^i(t)$ plus end-of-period demand. Total market supply consists of initial stocks, $S^i(t)$ and period t production, $Q^i(t)$.

Thus,

$$S^i(t)+Q^i(t)=C^i(t)+D^i(t,t+1) \tag{1}$$

is a statement that supply equals demand for commodity i during period t.[4] When markets clear, $\Delta S^i(t) \equiv S^i(t+1)-S^i(t)=Q^i(t)-C^i(t)=D^i(t,t+1)-S^i(t)$. *Walras' Law* holds that the sum of the excess demand for all goods, real and financial, valued at $p^i(t)$, equals zero, that is, that

$$\sum_{i=1}^{n} p^i(t)\big[D^i(t,t+1)+C^i(t) -S^i(t)-Q^i(t)\big]\equiv 0 \tag{2}$$

where n denotes the total number of goods. If condition (1) obtains for $n-1$ goods, then it must obtain for all n. One market equilibrium condition can be suppressed.

Implicit in Kuska's Propositions 2 and 4 is a notion of money-market equilibrium equating end-of-period demand $D^i(t,t+1)$, to beginning-of-period supply $S^i(t)$, rather than end-of-period supply, that is,

$$D^i(t,t+1)=S^i(t) \tag{1'}$$

replaces (1). Clearly (1′) is correct only if $Q^i(t)=C^i(t)$. This will hold for all t only in stationary states when asset supplies are constant. A balance of payments disequilibrium implies that asset supplies are changing: the deficit country loses assets to the surplus country. Condition (1), the proper notion of end-of-period asset-market equilibrium, is perfectly consistent with $C^i(t)\neq Q^i(t)$. From the end-of-period equilibrium point of view, Kuska's error arises in his equations (21) and

[3] More generally, we could allow the time at which plans are made to differ from the time at which they are expressed in the market. Buiter (1980) discusses this distinction. See also footnotes 7 and 8.

[4] Equilibrium condition (1) applies equally to nondurables, in which case equilibrium implies $Q^i(t)=C^i(t)$, and to "old masters," where it implies $S^i(t)=D^i(t,t+1)$ (ignoring destruction).

(22) where he imposes an erroneous notion of Walras' Law based on the market equilibrium condition (1′). We return to this point in Section II.

The alternative notion of asset-market equilibrium is that of beginning-of-period equilibrium: initial stock supply equals demand for the stock at the same instant or

$$D^i(t,t)=S^i(t). \tag{3}$$

Beginning-of-period demands are related by the balance sheet constraint which states that any one of the asset-market equilibrium conditions can be suppressed,

$$\sum_{i=1}^{m} p^i(t)\left[D^i(t,t)-S^i(t)\right]\equiv 0, \tag{4}$$

which is independent of the flows in period t. The number of durable commodities is $m\leqslant n$. Goods are ordered such that the first m are durables. Market flow equilibrium for nondurables is given by

$$C^j(t)=Q^j(t);\, j=m+1,\dots,n. \tag{5}$$

Kuska's equations (21) and (22) are equally erroneous from the beginning-of-period viewpoint of equilibrium since they sum instantaneous stock excess demands and flow excess demands at current market prices. (Note that with durable capital and investment spending I, the output market equilibrium condition $C+I=Y$ is an end-of-period equilibrium condition. See Buiter (1979) and Turnovsky (1977) for a further discussion).[5]

In a continuous-time formulation of both beginning-of-period and end-of-period equilibrium, instantaneous rates of flow of production and consumption at any instant do not affect stocks at that instant. Denote $Q^i(t)=q^i(t)\Delta t$ and $C^i(t)=c^i(t)\Delta t$ where Δt is the length of the unit period. As Δt goes to zero, both (1) and (3) reduce to

$$D^i(t,t)=S^i(t);\, i=1,\dots,m. \tag{6}$$

Instantaneous asset-stock demands are related by the balance sheet constraint:

$$\sum_{i=1}^{m} p^i(t)\left[D^i(t,t)-S^i(t)\right]\equiv 0. \tag{7}$$

Again, any one of the *asset*-market equilibrium conditions can be omitted. Market equilibrium for nondurables, for which $D^i(t,t)\equiv S^i(t)\equiv 0$, is given by

$$c^j(t)=q^j(t);\, j=m+1,\dots,n. \tag{8}$$

When markets clear, $\dot{S}^i(t)=q^i(t)-c^i(t)$, $i=1,\dots,m$.

In the continuous-time formulation the instantaneous rates of flow in (8) and the stocks in (7) are not even dimensionally commensurate. They cannot be added together into a single Walras' Law for this reason alone.[6, 7]

[5]A consistent planning requirement or "psychological" (as opposed to market) flow budget constraint relates stocks and flows in the beginning-of-period formulation. Let $D^i(t_1,t_2,t_3)$ denote the demand for good i, planned at time t_1, to be expressed in the market at time t_2, to be held at time t_3; $\hat{p}^i(t,t+1)$ denotes the price of good i expected, in period t, to prevail during period $t+1$. Consistent planning requires

$$\sum_{j=1}^{m} \hat{p}^j(t,t+1)\left[D^j(t,t+1,t+1)-D^j(t,t,t)\right.$$
$$\left.+C^j(t)-Q^j(t)\right]+\sum_{i=m+1}^{n} p^i(t)\left[C^i(t)-Q^i(t)\right]\equiv 0$$

Consistent production and consumption in period t should allow the realization of planned asset purchases in period $t+1$, $D^j(t,t+1,t+1)$, at the asset prices expected to prevail in $t+1$. Note that this is not a constraint on current market excess demands. An interpretation of Kuska's error in terms of the beginning-of-period specification is that he replaces the psychological budget constraint by the invalid market budget constraint:

$$\sum_{j=1}^{m} p^j(t)\left[D^i(t,t,t)-S^i(t)\right]$$
$$+\sum_{i=m+1}^{n} p^i(t)\left[C^i(t)-Q^i(t)\right]\equiv 0$$

[6]Neo-Keynesian models with real capital include a flow equilibrium specification for investment rather than a capital stock equilibrium condition. Turnovsky (1977) justifies this approach by introducing adjustment costs.

[7]Consistency requires that

$$\sum_{j=1}^{m} p^j(t)\left[D_2^j(t,t,t)+D_3^j(t,t,t)\right]$$
$$=\sum_{i=1}^{n}\left[q^i(t)-c^i(t)\right]p^i(t)$$

The only three consistent sets of market equilibrium conditions and budget or balance sheet constraints are (1) and (2) in the discrete-time end-of-period model, (3), (4), and (5), in the discrete-time beginning-of-period model, and (6), (7), and (8) in the continuous-time version of either model. Kuska's equations (21) and (22) do not correspond to any of these consistent configurations. His Propositions 2 and 4 are consequently invalid. The next section illustrates the compatability of balance of payments disequilibrium with asset market equilibrium in a discrete-time end-of-period model.[8]

II. The Two-Economy *IS-LM* Model: Discrete-Time End-of-Period Specification

Consider two countries each of which produces a distinct output in perfectly elastic supply at a price fixed in terms of domestic currency. The two countries trade their outputs. As we consider only a fixed exchange rate regime, relative price changes and changes in the general price level are ignored.[9] The governments of both countries issue two forms of debt, money and bonds. Money bears no interest. M_t and M_t^* are the amounts of money issued at the beginning of period t by home and foreign governments, respectively. Each country's bond is a perpetuity paying one unit of currency in each period. Let A_t and A_t^* denote the number of domestic and foreign bonds respectively outstanding at the beginning of period t. The values of the stocks of bonds outstanding at the beginning of period t are thus A_t/r_t and A_t^*/r_t^*, respectively, while the values of net new bond issues during the period are $(A_{t+1}-A_t)/r_t$ and $(A_{t+1}^*-A_t^*)/r_t^*$, respectively. Interest payments on the outstanding debt during period t are given by A_t and A_t^*, respectively. For simplicity we ignore physical capital as an asset and investment as a flow demand for output.

First we state the budget constraints for the public (consolidated treasury and central bank) and private (consolidated corporate and household) sectors from which we derive the balance of payments or external sector accounts. We then impose market equilibrium conditions and behavioral relationships. Kuska's equations (9) through (20) constitute a special case of this model. We generalize it by including nonhuman wealth as an argument in money and consumption-demand functions, thus overcoming the problem raised by Kuska's valid Proposition 1. The model has the property that simultaneous money-market equilibrium and balance of payments disequilibrium are compatible, contrary to Kuska's Propositions 2 and 4. In addition, in the case of zero capital mobility, one asset-market equilibrium condition is redundant *in each country*, in contradiction with Kuska's Proposition 3.

A. *Sectoral Budget Constraints*

1. *The Public Sector*

Home government liabilities outstanding at the beginning of period t consist of bonds held by the domestic private sector A_t^d, and the foreign private sector A_t^f, plus money held by the domestic private sector in amount M_t^d, and the foreign government in amount M_t^f. The corresponding magnitudes for the foreign government are A_t^{*f}, A_t^{*d}, M_t^{*f}, and M_t^{*d}. In each case magnitudes are measured in terms of local currency. Government assets consist of reserves in the form of foreign money, held in amount $e_t M_t^{*d}$ by the home country government and $(1/e_t)M_t^f$ by the foreign government. Here e denotes the spot price of foreign currency in terms of domestic currency. The Appendix summarizes our variable definitions for this model. The budget constraint for the home-country

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in the continuous-time version of the end-of-period model and that

$$\sum_{j=1}^{m} p^j(t)\left[D_3^j(t,t,t)\right]=\sum_{i=1}^{n} p^i(t)\left[q^i(t)-c^i(t)\right]$$

in the continuous-time version of the beginning-of-period model. (We assume $\hat{p}^i(t,t)=p^i(t)$). The D_k^j denotes the partial derivative of the demand for asset j with respect to the time at which demand is expressed in the market ($k=2$) and with respect to the time of delivery ($k=3$).

[8]The continuous time case is not considered in detail. For a complete treatment, see our earlier paper.

[9]Buiter (1978) considers the consequences of allowing import prices and exchange rate changes to affect the overall price level and thus the real value of assets denominated in domestic currency in a Keynesian open-economy model.

government is given by:[10]

$$(9)\quad G_t + A_t^d + A_t^f - T_t \equiv \frac{1}{r_t}\Delta A_t^d + \frac{1}{r_t}\Delta A_t^f + \Delta M_t^d + \Delta M_t^f - e_t \Delta M_t^{*d}$$

Here Δ denotes the forward difference operator; $\Delta Z_t = Z_{t+1} - Z_t$. We assume that government spending and financing plans are always fulfilled so that (9) holds both *ex ante* and *ex post*. We assume, for simplicity, that the government does not buy foreign goods. Similarly, in the foreign country,[11]

$$(10)\quad G_t^* + A_t^{*d} + A_t^{*f} - T_t^* \equiv \frac{1}{r_t^*}\Delta A_t^{*d} + \frac{1}{r_t^*}\Delta A_t^{*f} + \Delta M_t^* d + \Delta M_t^{*f} - \frac{1}{e_t}\Delta M_t^f$$

The government must finance its current account deficit, public consumption plus debt service less tax revenue, by increasing its liabilities or spending its reserves. The official settlements surplus of the home country (the deficit of the foreign country) is $e_t \Delta M_t^{*d} - \Delta M_t^f$, measured in terms of the home currency.

2. *The Private Sector*

The private sector in the home country holds as assets only domestic money and foreign and domestic bonds. In period t, $Y_t + A_t^d + e_t A_t^{*d} - T_t$ is earned after taxes, where Y_t denotes total income. Home-country *ex ante* private absorption in period t consists of C_t^h of domestic output and $e_t C_t^{*d}$ of foreign output.

If we denote home-country end-of-period demand for domestic bonds as $H_{t,t+1}$, home-country demand for foreign bonds as $J_{t,t+1}$[12] and home-country demand for money as $L_{t,t+1}$, the *ex ante* private sector budget constraint is

$$(11)\quad Y_t - T_t + A_t^d + e_t A_t^{*d} - C_t^d - e_t C_t^{*d} \equiv H_{t,t+1} - \frac{1}{r_t}A_t^d + J_{t,t+1} - \frac{e_t}{r_t^*}A_t^{*d} + L_{t,t+1} - M_t^d$$

Assuming consumption plans are realized, which will be the case if commodity markets clear, the *ex post* budget constraint is[13]

$$(12)\quad Y_t - T_t + A_t^d + e_t A_t^{*d} - C_t^d - e_t C_t^{*d} \equiv \frac{1}{r_t}\Delta A_t^d + \frac{e_t}{r_t^*}\Delta A_t^{*d} + \Delta M_t^d$$

The corresponding *ex ante* (13) and *ex post* (14) budget constraints for the private sector of the foreign country are

$$(13)\quad Y_t^* - T_t^* + A_t^{*f} + \frac{1}{e_t}A_t^f - C_t^{*f} - \frac{1}{e_t}C_t^f \equiv H_{t,t+1}^* - \frac{1}{e_t r_t}A_t^f + J_{t,t+1}^* - \frac{1}{r_t^*}A_t^{*f} + L_{t,t+1}^* - M_t^{*f}$$

$$(14)\quad Y_t^* - T_t^* + A_t^{*f} + \frac{1}{e_t}A_t^f - C_t^{*f} - \frac{1}{e_t}C_t^f \equiv \frac{1}{r_t^*}\Delta A_t^{*f} + \frac{1}{e_t r_t}\Delta A_t^f + \Delta M_t^{*f}$$

[10] We note that Kuska misstates the government budget constraint in his equations (31) and (32). The *stock* demand for bonds is equated to the *increment* in the money supply. This error is carried over into the subsequent equations (33) and (34).

[11] In continuous time the *ex post* public sector budget constraint is

$$g + A^d + A^f - \tau \equiv \frac{1}{r}(\dot{A}^d + \dot{A}^f) + \dot{M}^d + \dot{M}^f - e\dot{M}^{*d}$$

where lowercase letters denote instantaneous flows and τ instantaneous tax receipts.

[12] Asset demands are *real* demands. Thus L, H, and J are demands for certain amounts of purchasing power in terms of the domestic price level while L^*, H^*, and J^* are demands for certain amounts of purchasing power in terms of the foreign price level.

[13] In continuous time the *ex post* private sector budget constraint is

$$c^d + ec^{*d} + \frac{1}{r}\dot{A}^d + \frac{e}{r^*}\dot{A}^{*d} + \dot{M}^d \equiv y - \tau + A^d + eA^{*d}$$

3. *The Balance of Payments*

Adding the public (9) and private *ex post* (12) budget constraints of the home country yields

$$(15)\quad \left[G_t+C_t^d+e_tC_t^{*d}+A_t^f-e_tA_t^{*d}-Y_t\right] + \left[\frac{e_t}{r_t^*}\Delta A_t^{*d}-\frac{1}{r_t}\Delta A_t^f\right] + \left[e_t\Delta M_t^{*d}-\Delta M_t^f\right]\equiv 0$$

The term in the first square brackets of (15), which we denote $-B_T$, corresponds to the current account deficit of the home country. The second term, denoted $-F$, is the capital account deficit or net outflow of funds. Finally, the third term, B, denotes the official settlements balance.[14] Thus $-B_T-F+B\equiv 0$. Similarly, for the foreign country

$$(16)\quad \left[G_t^*+C_t^{*f}+\frac{1}{e_t}C_t^f+A_t^{*d}-\frac{1}{e_t}A_t^f-Y_t^*\right] + \left[\frac{1}{e_tr_t}\Delta A_t^f-\frac{1}{r_t^*}\Delta A_t^{*d}\right]+\left[\frac{1}{e_t}\Delta M_t^f-\Delta M_t^{*d}\right] \equiv -B_T^*-F^*+B^*\equiv 0$$

Since $B=-eB^*$ and $F=-eF^*$, we have $B_T=-eB_T^*$. The *ex ante* balance of payments constraints can be found for the home country by adding (9) and (11), and for the foreign country by adding (10) and (13).

B. *Market Equilibrium*

In the most general version of the model, there are six goods, money, bonds, and output, each of which may originate from the home or foreign country. We now show that asset-market equilibrium is consistent with nonzero values of the current account and/or the official settlements balance.

The six market clearing conditions are[15, 16]

$$(17)\quad L_{t,t+1}=M_t^d+\Delta M_t^d$$

$$(18)\quad L_{t,t+1}^*=M_t^{*f}+\Delta M_t^{*f}$$

$$(19)\quad H_{t,t+1}+e_tH_{t,t+1}^* =(1/r_t)\left[A_t^d+\Delta A_t^d+A_t^f+\Delta A_t^f\right]$$

$$(20)\quad J_{t,t+1}^*+(1/e_t)J_{t,t+1} =(1/r_t^*)\left[A_t^{*f}+\Delta A_t^{*f}+A_t^{*d}+\Delta A_t^{*d}\right]$$

$$(21)\quad G_t+C_t^d+C_t^f=Y_t$$

$$(22)\quad G_t^*+C_t^{*d}+C_t^{*f}=Y_t^*$$

1. *Kuska's Proposition* 3

In general, only one equilibrium condition can be suppressed. Substitute the money and output market equilibrium conditions, (17) and (21), and the government budget con-

[14] In continuous time the current, capital, and official settlements balances for the home country are

$$b_T\equiv y+eA^{*d}-A^f-g-c^d-ec^{*d};$$

$$f\equiv\frac{1}{r}\dot{A}^f-\frac{e}{r^*}\dot{A}^{*d};\ b\equiv e\dot{M}^{*d}-\dot{M}^f$$

Note that $b_T+f-b\equiv 0$.

[15] We can rephrase (21) and (22) as *IS* equilibrium conditions. Adding and subtracting $(e_tC_t^{*d}+A_t^f-e_tA_t^{*d})$ from (21) we obtain

$$G_t+\left(C_t^d+e_tC_t^{*d}\right)+A_t^f-e_tA_t^{*d}+\left(C_t^f-e_tC_t^{*d}\right) =Y+A_t^f-e_tA_t^{*d}$$

Let $C_t=C_t^d+e_tC_t^{*d}$ be total home-country consumption, $X_t=C_t^f+e_tA_t^{*d}$ be home-country exports, including services, and $Z_t=e_tC_t^{*d}+A_t^f$ be home-country imports, including services. Thus (21) is equivalent to $G_t+C_t+X_t-Z_t=Y_t+e_tA_t^{*d}-A^f$. A similar derivation, from (22), obtains for the foreign country.

[16] In continuous time, the six market equilibrium conditions are

$$L=M^d;\ L^*=M^{*f};\ H+eH^*=\frac{1}{r}(A^d+A^f);$$

$$J^*+\frac{1}{e}J=\frac{1}{r^*}(A^{*f}+A^{*d});$$

$$g+c^d+c^f=y;\ g^*+c^{*f}+c^{*d}=y^*$$

straint (9) into the household budget constraint (12) and use the fact that when commodity markets clear, the *ex post* balance of payments identity can be written as

$$C_t^f - e_t C_t^{*d} + e_t A_t^{*d} + \frac{1}{r_t}\Delta A_t^f$$
$$-\frac{e}{r^*}\Delta A_t^{*d} + \Delta M_t^f - e\Delta M_t^{*d} \equiv 0.$$

This yields[17]

$$(23)\qquad \left(H_{t,t+1} - \frac{1}{r_t}\left(A_t^d + \Delta A_t^d\right)\right) + \left(J_{t,t+1} - \frac{e_t}{r_t^*}\left(A_t^{*d} + \Delta A_t^{*d}\right)\right) = 0$$

In the same way we find for the foreign country:

$$(24)\qquad \left(H_{t,t+1}^* - \frac{1}{e_t r_t}\left(A_t^f + \Delta A_t^f\right)\right) + \left(J_{t,t+1}^* - \frac{1}{r_t^*}\left(A_t^{*f} + \Delta A_t^{*f}\right)\right) = 0$$

Commodity and money markets are in equilibrium, but an excess demand may exist for one type of bond with an offsetting excess supply for the other. Adding together (23) and e_t times (24) yields the sum of (19) and e_t times (20). If money markets and output markets clear and (19) holds, then so does (20). *One* market, in general, may be suppressed. If Kuska's Proposition 3 had been directed at models in which bonds are traded internationally, it would have been correct. When bonds are *not* traded, however, which is the case Kuska considers, the second term in (23) and the first term in (24) become zero. These equations reduce to the equilibrium conditions in the bond markets, equations (19) and (20). When bonds are not traded, then, bond-market equilibrium *in each country* is implied by money-market equilibrium *in that country* and equilibrium in the two commodity markets. Both (19) and (20) must be suppressed, contrary to Kuska's Proposition 3.[18]

2. *Kuska's Propositions* 2 *and* 4

Equilibrium conditions (17), (18), (19), and (20) equate end-of-period asset demands to end-of-period supplies. Beginning-of-period supplies, inherited from the previous period, are exogenous while current period flows are endogenous or policy determined. Substituting (17) through (22) into the balance of payments conditions (15) and (16) reveals that asset-market equilibrium does not imply either that $B_T = 0$ or $B = 0$. We show this explicitly in Section *D* below.

C. *Asset Demand Functions and Kuska's Proposition* 1

We complete our specification of an *IS-LM* Keynesian model by introducing asset and commodity demand functions. The resulting two-country model corresponds in essential respects to Kuska's equations (9)–(20). One modification, however, meets the criticism of the treatment of wealth or asset stocks in asset demand functions raised in Kuska's Proposition 1.

Brainard and Tobin have provided ample examples of the often bizarre implications for the suppressed excess demand function implied by the specification of nonsuppressed equations and Walras' Law. An appropriate response to this criticism is to include nonhuman wealth as an argument in private sector demand functions. We define home-country and foreign-country private wealth, respectively, as W_t and W_t^* where[19]

$$(25)\qquad W_t \equiv M_t^d + \frac{1}{r_t}A_t^d + \frac{e_t}{r_t^*}A_t^{*d}$$

$$(26)\qquad W_t^* \equiv M_t^{*f} + \frac{1}{r_t^*}A_t^{*f} + \frac{1}{r_t e_t}A_t^f$$

[17] In continuous time the *ex ante* private sector balance sheet constraints for the two countries are

$$L + H + J \equiv M^d + \frac{1}{r}A^d + \frac{e}{r^*}A^{*d};$$
$$L^* + H^* + J^* \equiv M^{*f} + \frac{1}{r^*}A^{*f} + \frac{1}{er}A^f$$

[18] The assignment of the bond-market equilibrium conditions as the residual ones is arbitrary. We follow the approach adopted by Kuska.

[19] The issue as to whether or not domestically held government bonds constitute net wealth need not detain us here.

We also define home-country and foreign disposable income, $\tilde{Y}_t$ and $\tilde{Y}_t^*$, as

$$(27) \qquad \tilde{Y}_t \equiv Y_t + A_t^d + e_t A_t^{*d} - T_t$$

$$(28) \qquad \tilde{Y}_t^* \equiv Y_t^* + A_t^{*f} + \frac{1}{e_t} A_t^f - T_t^*$$

We specify end-of-period asset demands and consumption demands as follows:

$$(29) \qquad L_{t,t+1} = L(r_t, r_t^*, \tilde{Y}_t, W_t);$$

$$L_r \leqslant 0, \quad L_{r^*} \leqslant 0, \quad L_{\tilde{Y}} \geqslant 0, \quad 1 \geqslant L_W \geqslant 0$$

$$(30) \qquad H_{t,t+1} = H(r_t, r_t^*, \tilde{Y}_t, W_t);$$

$$H_r \geqslant 0, \quad H_{r^*} \leqslant 0, \quad 1 \geqslant H_W \geqslant 0$$

$$(31) \qquad C_t^d = C^d(r_t, r_t^*, e_t, \tilde{Y}_t, W_t);$$

$$C_r^d \leqslant 0, \quad C_{r^*}^d \leqslant 0, \quad C_e^d \geqslant 0$$

$$0 \leqslant C_{\tilde{Y}}^d \leqslant 1, \quad C_W^d \geqslant 0$$

$$(32) \qquad C_t^{*d} = C^{*d}(r_t, r_t^*, e_t, \tilde{Y}_t, W_t);$$

$$C_r^{*d} \leqslant 0, \quad C_{r^*}^{*d} \leqslant 0; \quad \frac{\partial}{\partial e}(eC^{*d}) \leqslant 0$$

$$0 \leqslant C_{\tilde{Y}}^{*d} \leqslant 1, \quad C_W^{*d} \geqslant 0$$

$$(33) \qquad L_{t,t+1}^* = L^*(r_t, r_t^*, \tilde{Y}_t^*, W_t^*),$$

$$L_r^* \leqslant 0, \quad L_{r^*}^* \leqslant 0, \quad L_{\tilde{y}}^* \geqslant 0, \quad 1 \geqslant L_{W^*}^* \geqslant 0$$

$$(34) \qquad H_{t,t+1}^* = H^*(r_t, r_t^*, \tilde{Y}_t^*, W_t^*),$$

$$H_r^* \geqslant 0, \quad H_{r^*}^* \leqslant 0, \quad 1 \geqslant H_{W^*}^* \geqslant 0$$

$$(35) \qquad C^{*f} = C^{*f}(r_t, r_t^*, e_t, \tilde{Y}_t^*, W_t^*),$$

$$C_r^{*d} \leqslant 0, \quad C_{r^*}^{*d} \leqslant 0, \quad C_e^{*d} \leqslant 0$$

$$0 \leqslant C_{\tilde{Y}^*}^{*d} \leqslant 1, \quad C_{W^*}^{*d} \geqslant 0$$

$$(36) \qquad C_t^f = C^f(r_t, r_t^*, e_t, \tilde{Y}_t^*, W_t^*),$$

$$C_r^f \leqslant 0; \quad C_{r^*}^f \leqslant 0; \quad \frac{\partial}{\partial e}\left(\frac{1}{e} C^f\right) \geqslant 0$$

$$0 \leqslant C_{\tilde{Y}^*}^f \leqslant 1, \quad C_{W^*}^f \geqslant 0$$

From the private sector budget constraints

$$(37) \quad L_{t,t+1} + H_{t,t+1} + J_{t,t+1} + C_t^d + e_t C_t^{*d} \equiv W_t + \tilde{Y}_t$$

$$(38) \quad L_{t,t+1}^* + H_{t,t+1}^* + J_{t,t+1}^* + C_t^{*f} + \frac{1}{e_t} C_t^f \equiv W_t^* + \tilde{Y}_t^*$$

equations (29) through (32) and (33) through (36) define demand for bond functions[20]

$$(39) \quad J(r_t, r_t^*, Y_t^d, W_t);$$

$$J_r \leqslant 0, J_{r^*} \geqslant 0; J_W \geqslant 0$$

$$(40) \quad J^*(r_t, r_t^*, Y_t^{*d}, W_t^*);$$

$$J_r^* \leqslant 0, J_{r^*}^* \geqslant 0, J_{W^*}^* \geqslant 0$$

These demand functions have the properties that: 1) assets are gross substitutes; 2) the demand for all assets and consumption increase in wealth; 3) the demand for money and consumption increase in income; and 4) an increase in the exchange rate shifts consumption away from imports toward domestic output. Since W_t appears as an argument in all domestic demand functions, it is no longer true that "... *any* exogenous increase in the quantity of bonds will be willingly held..." (Kuska, p. 660). An exogenous increase in bond supply, raising wealth, will affect demands for all other assets and commodities. After substituting the demand functions (29)–(36) into the six equilibrium conditions (17) through (22) and using Walras' Law to omit one market-clearing condition, we have a system of five independent equations. The endogenous variables are r_t, r_t^*, Y_t, Y_t^*, and either ΔM_t^d or ΔM_t^{*f}. As long as bonds are traded internationally, one government cannot control its money supply even in the short run, regardless of the degree of substitutability between foreign

[20]The restrictions we impose on the signs of the partial derivatives of J and J^* are not the only possible ones. They are consistent with the restrictions imposed in (29)–(36) and the budget constraints.

and domestic bonds.[21] If, however, bonds are *not* traded at all, only four of (29) through (36) are independent and both ΔM_t and ΔM_t^* can be controlled by the policy authorities; subject to their budget constraints, both governments can choose their money stocks independently.

D. *A Model Without Capital Mobility*

Kuska's equations (9)–(20) assume no international trade occurs in bonds. Since this assumption simplifies the analysis and permits us to illustrate the compatibility of asset-market equilibrium with a nonzero balance of payments in a particularly simple way, we also adopt it here. To incorporate this assumption set $J_{t,t+1} \equiv H_{t,t+1}^* \equiv A_t^{*d} \equiv A_t^f \equiv \Delta A_t^f \equiv \Delta A_t^{*d} \equiv 0$. Foreign interest rates no longer appear as arguments in domestic behavioral relationships, and vice versa. For simplicity we set

$$T_t = A_t^d \tag{41}$$

$$T_t^* = A_t^{*f} \tag{42}$$

so that $\tilde{Y}_t = Y_t$ and $\tilde{Y}_t^* = Y_t^*$. Kuska also sets $G_t = G_t^* = 0$. We will not yet incorporate this assumption.

As a result of these assumptions, then, we have four *independent* market equilibrium conditions.

$$L(r_t, Y_t, W_t) = M_t^d + \Delta M_t^d \tag{43}$$

$$L^*(r_t^*, Y_t^*, W_t^*) = M_t^{*f} + \Delta M_t^{*f} \tag{44}$$

$$G_t + C^d(r_t, e_t, Y_t, W_t) + C^f(r_t^*, e_t, Y_t^*, W_t^*) = Y_t \tag{45}$$

$$G_t^* + C^{*f}(r_t^*, e_t, Y_t^*, W_t^*) + C^{*d}(r_t, e_t, Y_t, W_t) = Y_t^* \tag{46}$$

The public and private sector budget constraints are[22]

$$G_t \equiv \frac{1}{r_t}\Delta A_t^d + \Delta M_t^d + \Delta M_t^f - e_t \Delta M_t^{*d} \tag{47}$$

$$G_t^* \equiv \frac{1}{r_t^*}\Delta A_t^{*f} + \Delta M_t^{*f} + \Delta M_t^{*d} - (1/e_t)\Delta M_t^f \tag{48}$$

$$C_t^d + e_t C_t^{*d} + \frac{1}{r_t}\Delta A_t^d + \Delta M_t^d \equiv Y_t \tag{49}$$

$$C_t^{*f} + \frac{1}{e_t}C_t^f + \frac{1}{r_t^*}\Delta A_t^{*f} + \Delta M_t^{*f} \equiv Y_t^* \tag{50}$$

The private wealth definitions are

$$W_t \equiv M_t^d + \frac{1}{r_t}A_t^d \tag{51}$$

$$W_t^* \equiv M_t^{*f} + \frac{1}{r_t^*}A_t^{*f} \tag{52}$$

Note that the private sector budget constraints (49) and (50) are implied by equations (43)–(48).

After substituting for W_t and W_t^*, equations (43) through (46) determine equilibrium values of r_t, r_t^*, Y_t and Y_t^*. Alternatively, if the government in the home country pegs r_t, ΔM_t is endogenous and r_t exogenous. Similarly, if the foreign government pegs r_t^*, ΔM_t^* is endogenous.

1. *Kuska's Propositions 2 and 4 Again*

The balance of payments B, which is now equivalent to the current account balance B_T, is from (16)

$$B_T = B = Y_t - G_t - C_t^d - e_t C_t^{*d} \tag{53}$$

[21] If we generalize this to n countries, $n-1$ countries are unable to control their domestic money supplies. Note also that the asset-market equilibrium conditions are only completely specified when we know the financing policies of the government whose money supply is not endogenously determined. If this is the home country, we need to know $(1/r)(\Delta A^d + \Delta A^f)$ and $(1/r)\Delta M^d$ (assuming G and T to be given).

[22] The asset-market equilibrium conditions (43) and (44) are only fully and completely specified when the public sectors have decided upon their mixes of domestic borrowing and domestic credit creation: $(1/r)\Delta A^d$ and ΔM^d for the home country government sector, $(1/r)^*\Delta A^{*f}$ and ΔM^{*f} for the foreign country government sector.

Substituting (21) into (53) gives

$$B_T = B = C_t^f - e_t C_t^{*d} \tag{54}$$

while (16) implies[23]

$$B_T = B = e_t \Delta M_t^{*d} - \Delta M_t^f \tag{55}$$

The first expression corresponds to the "absorption approach" view of the balance of payments, the second to the"elasticities approach" and the third to the "monetary approach." The three are of course equivalent, as implied by the sectoral budget constraints. Using the monetary approach equation, expression (55), the home-country government budget constraint, (47), implies that[24]

$$B \equiv \frac{1}{r_t} \Delta A_t^d + \Delta M_t^d - G_t \tag{56}$$

which, if asset markets are in end-of-period equilibrium, becomes

(57)

$$B = \left(H_{t,t+1} - \frac{1}{r_t} A_t^d \right) + \left(L_{t,t+1} - M_t^d \right) - G_t$$

Asset-market equilibrium does not imply that $B=0$. It simply implies that the sum of the planned and actual changes in the money stock and bond stock equal the sum of the balance of payments surplus and government deficit. If $G_t=0$ as Kuska implicitly assumes, then the govenment can fully sterilize the deficit by setting $\Delta A_t^d / r_t = B$, while zero sterilization implies $\Delta A_t^d / r_t = 0$. In the first case not only is the beginning-of-period money stock exogenous, but so is the end-of-period stock. A deficit country can, of course, continue to sterilize only so long as $A_t^d > 0$ or until it either runs out of international reserves or finds the surplus country unwilling to accumulate its currency in the form of reserves. Even if a sterilization policy is pursued, so that the money supply remains constant, an imbalance of payments implies that the private sector in the deficit country is losing assets, thus lowering its wealth and therefore its demand for imports. Similarly the surplus country is gaining assets, increasing its demand for imports. Nevertheless, end-of-period asset-market equilibrium is maintained each period. If the model is stable the transfer of wealth will cause the two countries eventually to attain balance of payments equilibrium. This will be a *stationary equilibrium* in which asset supplies are constant.

Kuska's error is apparent in our equation (56). His equations (21) and (22) erroneously define asset-market equilibrium as the equality of beginning-of-period supplies and end-of-period demands. When these are equal, B is indeed constrained at 0 (as long as the government balances its budget). The requirement that all stocks remain constant does indeed imply that $B=0$. This is a property of long-run stationary equilibrium and not of end-of-period asset-market equilibrium.

III. Conclusion

Standard textbook treatments of Keynesian balance of payments theory (see, for example, David J. Ott, Attiat F. Ott and Jang H. Yoo, and Branson, 1979) are nested within the general framework presented in Section II. We have shown elsewhere (1979) that the model developed in Section II can be described in terms of the "*IS-LM-BP*" analysis used, for instance, by Branson (1979, ch. 15). While balance of payments theorists should always pay careful attention to balance sheet constraints, budget constraints and Walras' Law, the literature which Kuska attacks is free of conceptual error on these grounds.

Appendix

G	domestic government spending on domestic output; $G = g\Delta t$
T	domestic taxes net of transfers [excluding interest on public debt]; $T = \tau\Delta t$
G^*	foreign government spending on foreign output; $G^* = g^*\Delta t$

[23]In continuous time, the trade balance which in this case equals the overall balance of payments is given by $b_T \equiv b \equiv y + eA^{*d} - A^f - g - c^d - ec^{*d} = c^f - ec^{*d} + eA^{*d} - A^f = e\dot{M}^{*d} - \dot{M}^f$.

[24]In continuous time this becomes $b \equiv (1/r)\dot{A}^d + \dot{M}^d - g$.

T^* foreign taxes net of transfers [excluding interest on public debt]; $T^* = \tau^* \Delta t$
C^d Domestic consumption demand for domestic output; $C^d = c^d \Delta t$
C^{*d} domestic consumption demand for foreign output; $C^{*d} = c^{*d} \Delta t$
C^f foreign consumption demand for domestic output; $C^f = c^f \Delta t$
C^{*f} foreign consumption demand for foreign output; $C^{*f} = c^{*f} \Delta t$
L private domestic demand for domestic money
L^* private foreign demand for foreign money
H private domestic demand for domestic government bonds
H^* private foreign demand for domestic government bonds
J private domestic demand for foreign government bonds
J^* private foreign demand for foreign government bonds
Y domestic output; $Y = y \Delta t$
Y^* foreign output; $Y = y^* \Delta t$
$\tilde{Y}$ domestic personal disposable income; $\tilde{Y} = \tilde{y} \Delta t$
$\tilde{Y}^*$ foreign personal disposable income; $\tilde{Y}^* = \tilde{y}^* \Delta t$
B_T domestic current account surplus; $B_T = b_T \Delta t$
F domestic private capital account surplus; $F = f \Delta t$
B domestic official settlements account surplus; $B = b \Delta t$
B_T^* foreign current account surplus; $B_T^* = b_T^* \Delta t$
F^* foreign capital account surplus; $F^* = f^* \Delta t$
B^* foreign official settlements account surplus; $B^* = b^* \Delta t$
W private domestic nonhuman wealth
W^* private foreign nonhuman wealth
A^d domestic private holdings of domestic government debt
A^f foreign private holdings of domestic government debt
M^d domestic private holdings of domestic money
M^f foreign official holdings of domestic money
A^{*f} foreign private holdings of foreign government debt
A^{*d} domestic private holdings of foreign government debt
M^{*f} foreign private holdings of foreign money
M^{*d} domestic official holdings of foreign money
r domestic interest rate
r^* foreign interest rate
e foreign exchange rate [spot]
Δ forward difference operator
$\dot{x}$ $\equiv dx/dt$

Time subscripts of stocks refer to the beginning of the period.

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